A Cosmological Signature of the Standard Model Higgs Vacuum Instability: Primordial Black Holes as Dark Matter

with José Ramón Espinosa and Antonio Riotto. arXiv:1710.11196

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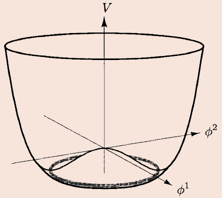
Higgs potential beyond the tree level

Tree level potential

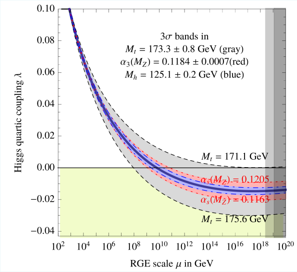
$$V(h) = \frac{1}{4}\lambda\, h^4 - \frac{1}{2}\mu^2 h^2$$

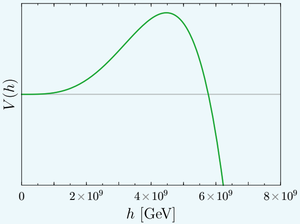


RG improved potential

$$V(h) = \frac{1}{4}\lambda(h)\, h^4$$

[1307.3536 Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia]



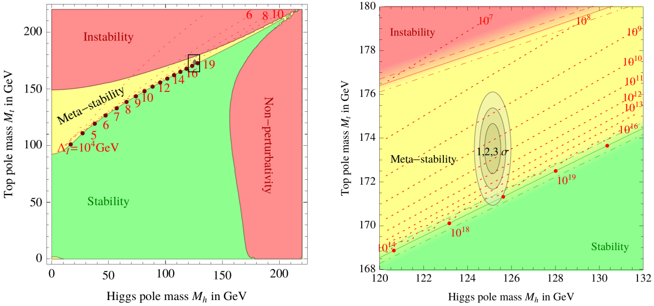


Implications of living in a false vacuum

['79 Cabibbo, Maiani, Parisi, Petronzio; Hung; '89 Sher; '94 Altarelli, Isidori; '96 Casas, Espinosa, Quirós; '07 Espinosa, Giudice, Riotto; ...]

Tunnelling today

Negligible probability, today we are safe. We live in a metastable Universe.



[1307.3536 Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia]

Implications of living in a false vacuum

['07 Espinosa, Giudice, Riotto; '15 Espinosa, Giudice, Morgante, Riotto, Senatore, Strumia, Tetradis; '16 East, Kearney, Shakya, Yoo, Zurek; '16 Salvio, Strumia, Tetradis, Urbano; ...]

During inflation

- Power spectrum of a scalar field in de Sitter is $\left(\frac{H}{2\pi}\right)^2 \Longrightarrow$ quantum jumps of the background value of the Higgs field of order $\sim \pm\frac{H}{2\pi}$, on a time scale H^{-1}.
- Depending on the value of H, these fluctuations could lead the Higgs beyond the barrier, and make it roll towards the true vacuum.
- This vacuum has large negative energy $\Longrightarrow$ AdS bubble, which can expand at the speed of light. $\Longrightarrow$ It didn't happen in our past lightcone.

Implications of living in a false vacuum

['07 Espinosa, Giudice, Riotto; '15 Espinosa, Giudice, Morgante, Riotto, Senatore, Strumia, Tetradis; ...]

During reheating

- Higgs interacts with thermal bath of SM particles. Overall effect: stabilisation of the potential through a thermal contribution V_T,

$$V_0(h) + V_T(h) = \frac{1}{4}\lambda(h)h^4 + \frac{1}{2}m_T^2 h^2\,, \qquad m_T^2 = 0.12\,T^2 \exp\left(-\frac{h}{2\pi T}\right)\,.$$

- If T_{RH} is high enough and h is not too far, thermal corrections can "rescue" the Higgs, bringing it back around 0.

Possible observational evidences?

Are there possible observational signatures of the Higgs instability?

- Tunnelling today: not here, until this morning.

> If this time is on the order of 10^9 yr, we have occasion for anxiety.

> This would be the appropriate case to study if we were currently living in a false vacuum whose apocalyptic decay is yet to occur.

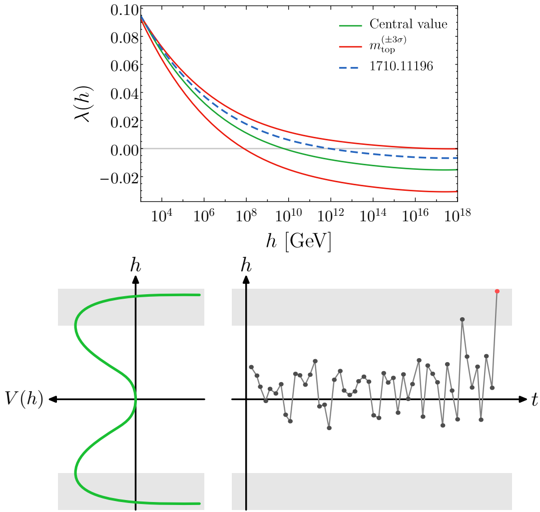
[Coleman]

- What if the Higgs probed the unstable region at the end of inflation, and was rescued back in time by thermal corrections at reheating?

Higgs instability and quantum fluctuations

- We assume that the Higgs potential turns negative at some scale.
- During inflation, the background field $h_c(t)$ has random fluctuations

$$\Delta_q h_c \sim \pm \frac{H}{2\pi} \,.$$

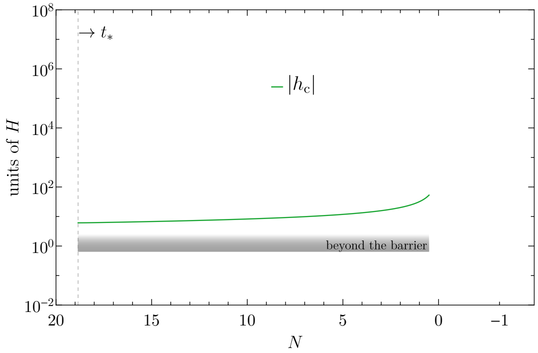


Higgs instability and quantum fluctuations

- If the classical evolution $\Delta h_c \sim \dot{h}_c \, \Delta t$ prevails over the quantum corrections, h_c begins to slow roll down the negative potential.

$$\overbrace{\frac{V'(h_c)}{3H^2}}^{\text{classical}} \gtrsim \overbrace{\frac{H}{2\pi}}^{\text{quantum}}$$

- From this starting point t_* we follow the classical evolution of the field h_c.



Fluctuations of the Higgs field

- Define the fluctuations of the Higgs, $h(t,\vec{x}) = h_c(t) + \delta h(t,\vec{x})$.
- E.o.m.[1]:

$$\delta\ddot{h}_k + 3H\delta\dot{h}_k + \frac{k^2}{a^2}\,\delta h_k + V''(h_c)\,\delta h_k = 0$$

Sub-Hubble

Oscillation term dominates.

Super-Hubble

After the Hubble crossing $k \sim aH$, which happens at t_k, dominates. $V''(h_c) < 0 \Rightarrow$ source term which drives δh_k.

[1] Feedback of the metric is negligible.

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Super-Hubble ($V''(h_{\rm c})\,\delta h_k$)
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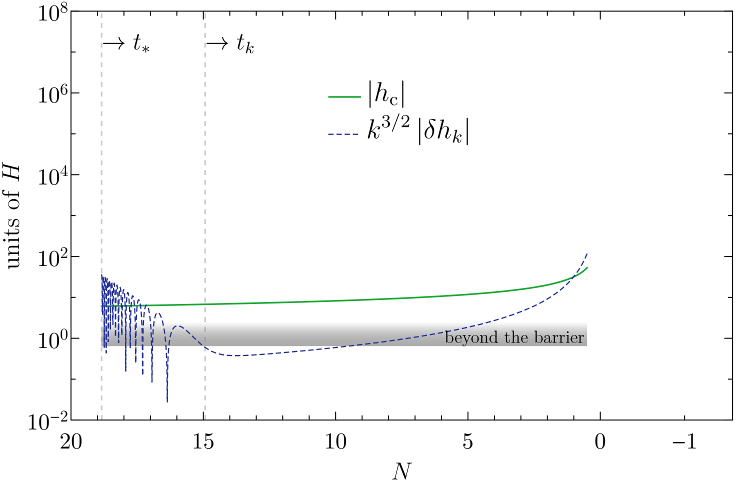
Fluctuations δh_k grow when they exit the Hubble radius



Comoving curvature perturbation ζ

- Perturbations are quantified by the gauge-invariant *comoving curvature perturbation* ζ, which is conserved on super-horizon scales.
- In flat gauge,

$$\zeta = H\frac{\delta\rho_{\text{tot}}}{\dot{\rho}} = \underbrace{\frac{\rho_{\text{st}}}{\rho_{\text{tot}}}\zeta_{\text{st}}}_{\text{standard inflation contr.}} + \underbrace{H\frac{\delta\rho_h}{\dot{\rho}_{\text{tot}}}}_{\text{contr. from } h \text{ at small } k}$$

- The largest contribution to ζ, in the last e-folds of inflation, comes from the Higgs.

End of inflation and reheating

- At the end of inflation, if the reheating temperature is high enough and Higgs has not gone too far down the potential, thermal corrections can rescue h and bring it back around 0.
- Positive contribution to $V(h)$:

$$\ddot{h}_c + 3H\dot{h}_c + m_T^2 \, h_c = 0$$

- Both h_c and δh_k oscillate around 0.
- Decay to radiation: in a very short time (damping rate, evaluated at 2 loops, $\sim 10^{-3}T$) the Higgs field decays to radiation.
- We end up with large fluctuations of *radiation* on small scales.
- If they're large enough, when they re-cross the Hubble radius can generate PBH.

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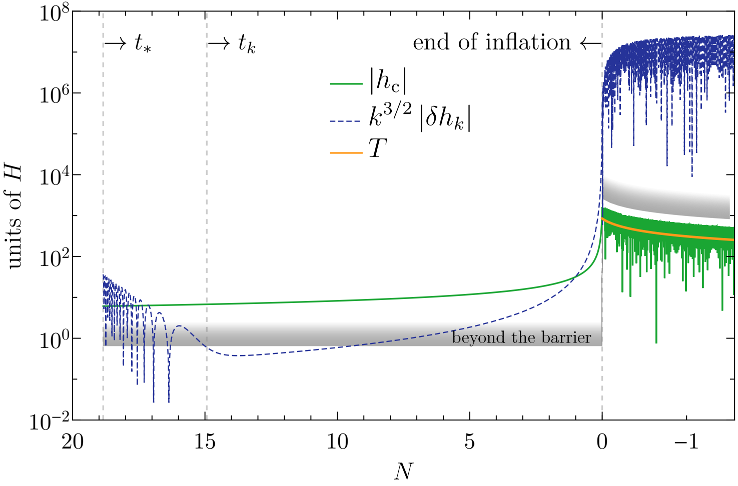
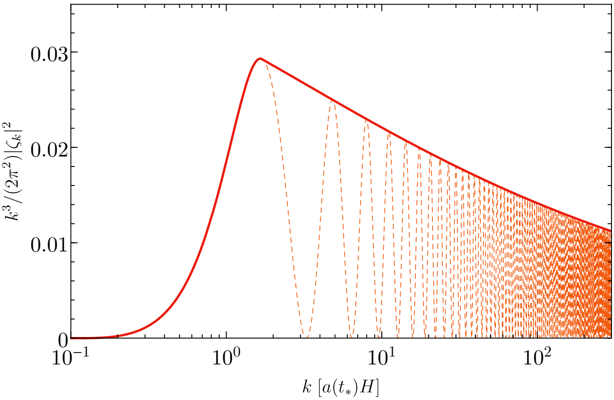
Evolution of Higgs background and fluctuations



Power spectrum of fluctuations generated by the Higgs

- Power spectrum $\mathcal{P}_\zeta = \dfrac{k^3}{2\pi^2}\,|\zeta_k|^2$ on small scales $k \gtrsim k_*$: much larger than the usual $\mathcal{O}(10^{-9})$ of CMB scales.



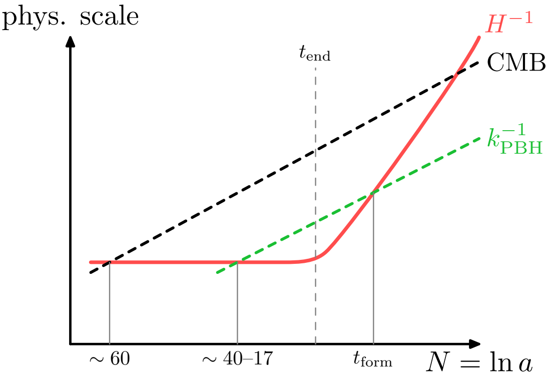
Primordial Black Holes

['67 Zeldovich, Novikov; '71 Hawking; '75 Chapline; '96 Garcia-Bellido, Linde, Wands; '16 Carr, Kühnel, Sandstad (and refs therein); '18 Carr, Silk; '18 Sasaki, Suyama, Tanaka, Yokoyama (and refs therein); ...]

- Black Holes formed early during radiation domination, and not from the final stages of a stellar evolution.
- They are a candidate for DM: they behave as collisionless cold DM.
- Similar phenomenology to MACHOs, with some distinguishing features:
  - any possible mass range, a priori;
  - light $M_{\text{PBH}} \lesssim 10^{-18} M_{\odot}$: evaporation;
  - heavy $M_{\text{PBH}} \gtrsim 10^{4} M_{\odot}$: CMB distortion.

Generation of PBH

- Large overfluctuations, when they re-enter the Hubble radius, can collapse into a PBH.



Generation of PBH: mass at formation

- Mass of the PBH: mass contained in a sphere of radius $\sim H^{-3}$.

$$M_{\rm PBH} = \underbrace{\gamma}_{\sim 0.2} \frac{4\pi}{3}\,\rho\,H^{-3} \approx M_\odot\, e^{2(N-36)}\,.$$

- Collapse happens when crossing a threshold $\Delta_{\rm c}$ of the density contrast

$$\Delta(t,\vec{x}) = \frac{4}{9}\left(\frac{1}{aH}\right)^2 \nabla^2\zeta(\vec{x})\,,$$

  physically corresponding to spatial curvature of the metric.
- Numerical calculations show $\Delta_{\rm c} \approx 0.45$.

Abundance of PBH

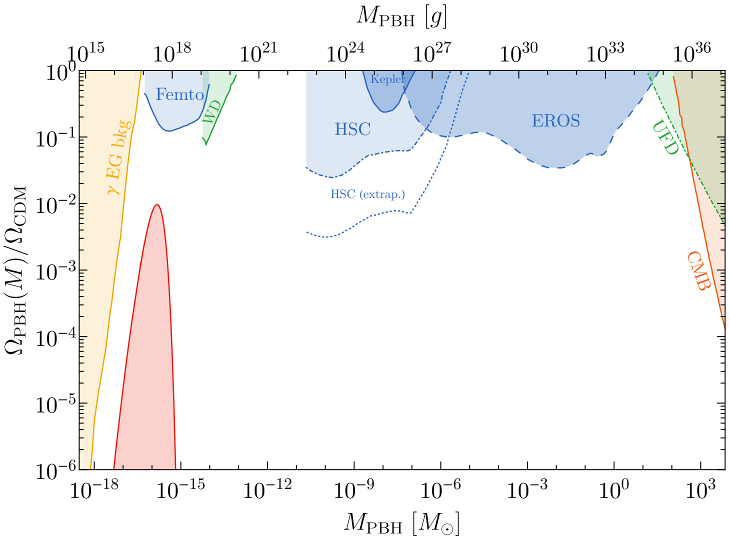
- Given the power spectrum, compute the smoothed density contrast $\sigma_\Delta(M)$.
- Formation rate: probability of exceeding threshold Δ_c.
  In Gaussian approximation

$$\beta(M) = \int_{\Delta_c}^{\infty} \frac{d\Delta}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\Delta^2/2\sigma_\Delta{}^2}.$$

- After formation, $\rho_{\rm PBH} \sim a^{-3}$ and behave as collisionless CDM. After equality, they scale as the rest of matter.

$$f(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm CDM}} = \frac{\beta(M)}{1.6 \cdot 10^{-16}} \left(\frac{\gamma}{0.2}\right)^{3/2} \left(\frac{g_*(T_f)}{106.75}\right)^{-1/4} \left(\frac{M}{10^{-15} M_\odot}\right)^{-1/2}.$$

Mass spectrum of PBHs and current constraints



Dependence of the mass function on parameters of our model

- Position of the peak: depends on t_*, the total number of e-folds of evolution.
  For the central running of λ, this would give too light PBH: need a slower evolution, i.e. less negative λ.
- Height of the peak: depends how much the fluctuations grew before the end of inflation.
- About fine-tuning: the formation rate depends exponentially on $\sigma_\triangle$ in any PBH model.
- In SM + inflation, DM is provided by this mechanism.
  One can invoke anthropic explanations for this fine-tuning: without DM there would be no Large Scale Structures today.

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Conclusions

- The metastability of the Higgs potential is an important byproduct of the SM.
  Understanding its implications could help us to shed light on BSM Physics thanks to the study of the Early Universe.
- How could we observe indirectly its presence?
- If the Higgs probed the unstable region at the end of inflation and was rescued back by reheating, PBHs could be an outcome of this mechanism.
- Today we have the possibility to probe extensively PBHs as a possible explanation for Dark Matter.
- Possible improvements:
  1. Evolution of the mass function under the effects of *accretion* and *merging*;
  2. Refine the calculation of the formation rate by including non-Gaussianity.

Thanks for your attention!

1. BACKUP SLIDES

Higgs instability and quantum fluctuations

- If the classical evolution Δh_c prevails over the quantum corrections, h_c begins to slow roll down the negative potential.

$$\text{e.o.m.:} \qquad \ddot{h}_c + 3H\dot{h}_c + V'(h_c) = 0$$

$$\text{slow roll:} \qquad \ddot{h}_c \ll 3H\dot{h}_c$$

$$\Delta h_c : \qquad \Delta h_c \sim \dot{h}_c \, \Delta t \sim \frac{V'(h_c)}{3H} H^{-1}$$

$$\overbrace{\frac{V'(h_c)}{3H^2}}^{\text{classical}} \gtrsim \overbrace{\frac{H}{2\pi}}^{\text{quantum}}$$

- With $V(h) = \frac{1}{4}\lambda(h)h^4$, we have

$$h_c \gtrsim \left(\frac{3}{2\pi\lambda}\right)^{1/3} H \, .$$

- From this starting point t_* we follow the classical evolution of the field h_c.

δh_k grows as $\dot{h}_c$

- We can find a relation between δh_k and h_c:

$$\begin{aligned} &\delta h_k \text{ super-hor.:} && \delta\ddot{h}_k + 3H\delta\dot{h}_k + V''(h_c)\delta h_k = 0 \\ &\text{e.o.m. for } h_c && \ddot{h}_c + 3H\dot{h}_c + V'(h_c) = 0 \\ &\to \mathrm{d/dt} && \left(\dot{h}_c\right)^{\cdot\cdot} + 3H\left(\dot{h}_c\right)^{\cdot} + V''(h_c)\,\dot{h}_c = 0 \end{aligned}$$

- δh_k at super-horizon scales follows the same evolution of $\dot{h}_c$, they are proportional:

$$\delta h_k(t) = C(k)\,\dot{h}_c(t)$$

- The constant $C(k)$ is found by matching with the standard sub-hor. solution at t_k:

$$\delta h_k \overset{k \ll aH}{=} \frac{1}{a\sqrt{2k}} \exp\left(i\frac{k}{aH}\right) \overset{t_k}{\simeq} \frac{H}{\sqrt{2k^3}} \quad \Longrightarrow \quad C(k) = \frac{H}{\dot{h}_c(t_k)\sqrt{2k^3}}\,.$$

- Fluctuations of the Higgs that leave the Hubble radius towards the end of inflation grow a lot.

Comoving curvature perturbation ζ

- Perturbations are quantified by the gauge-invariant *comoving curvature perturbation* ζ, which is conserved on super-horizon scales.
- In flat gauge,

$$\zeta = H\frac{\delta\rho_{\text{tot}}}{\dot{\rho}} = \underbrace{\frac{\rho_{\text{st}}}{\rho_{\text{tot}}}\zeta_{\text{st}}}_{\text{standard inflation contr.}} + \underbrace{H\frac{\delta\rho_h}{\dot{\rho}_{\text{tot}}}}_{\text{contr. from } h \text{ at small } k}$$

$$\delta\rho_h = \rho(h_c + \delta h) - \rho(h_c) = \dot{h}_c\,\dot{\delta h} + V'(h_c)\,\delta h$$

- On super-horizon scales, using the e.o.m. we find

$$\delta\rho_{h\,k} = (-3H)\,C(k)\left(\dot{h}_c\right)^2, \qquad \dot{\rho_h} = (-3H)\left(\dot{h}_c\right)^2,$$

$$\zeta_k(k \gtrsim k_*) = \frac{\rho_h}{\rho_{\text{tot}}}\Big(H\,C(k)\Big).$$

- The largest contribution to ζ, in the last e-folds of inflation, comes from the Higgs.

Abundance of PBH

- Formation rate: probability of exceeding threshold Δ_c.
  In Gaussian approximation[2],

$$\beta(M) = \int_{\Delta_c}^{\infty} \frac{d\Delta}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\Delta^2/2\sigma_\Delta^2} \overset{\sigma_\Delta \ll \Delta_c}{\approx} \frac{\sigma_\Delta}{\sqrt{2\pi}\,\Delta_c} \exp\left(-\frac{1}{2}\frac{\Delta_c^2}{\sigma_\Delta^2}\right).$$

- $\sigma_\Delta(M)$ is the smoothed density contrast

$$\sigma_\Delta^2\Big(M(k)\Big) = \int_0^\infty d\ln q\, \underbrace{W^2(q,k)}_{\text{Gaussian}} \underbrace{\frac{16}{81}\left(\frac{q}{k}\right)^4 \mathcal{P}_\zeta(k)}_{\mathcal{P}_\Delta(k)}.$$

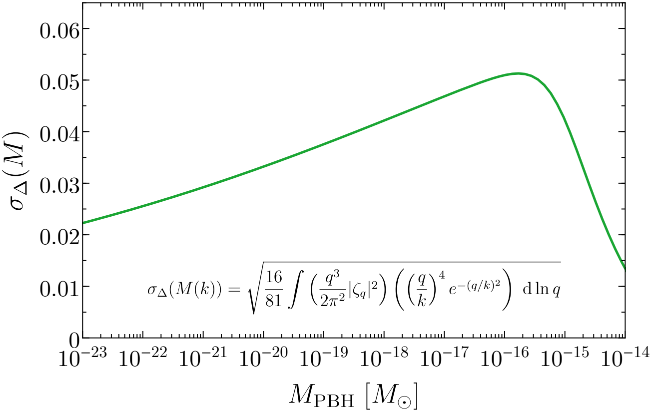
- After formation, $\rho_{\rm PBH} \sim a^{-3}$ and behave as collisionless CDM. After equality, they scale as the rest of matter.

$$f(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm CDM}} = \frac{\beta(M)}{1.6\cdot 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*(T_f)}{106.75}\right)^{-1/4}\left(\frac{M}{10^{-15}M_\odot}\right)^{-1/2}.$$

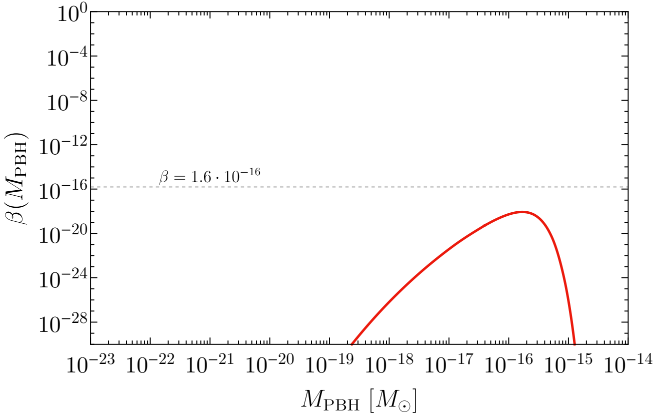
[2] Non-Gaussianities play an important role, given that we look at the tail of the distribution ['18 Franciolini, Kehagias, Matarrese, Riotto].

Smoothed variance of density contrast as a function of $M_{\rm PBH}$

- For PBHs of mass $\sim 10^{-15} M_\odot$, the right abundance is achieved for $\sigma_\Delta \sim 0.05$.

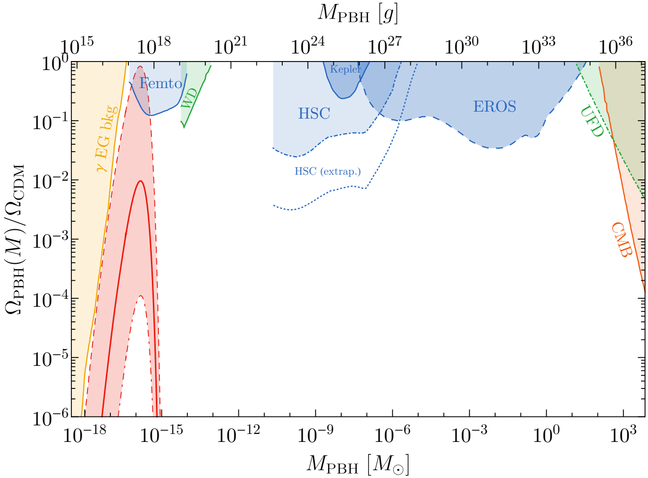


Formation rate $\beta(M)$

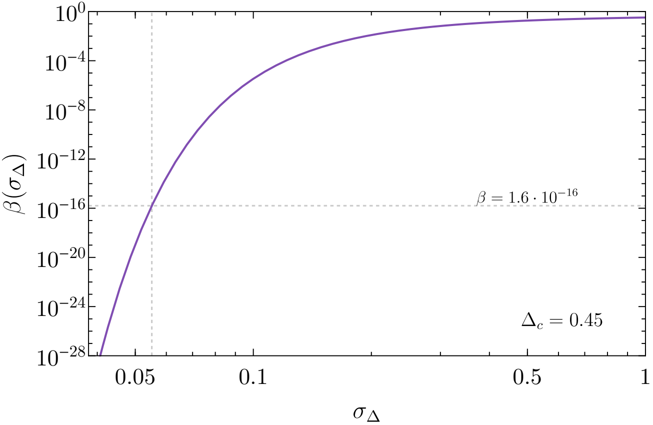


Non-Gaussianity: account for skewness $S_3 \equiv \langle \Delta^3 \rangle / \sigma_\Delta^4$

$$\beta(M) = \frac{1}{\sqrt{2\pi}\,\nu} \exp\left[-\frac{1}{2}\nu^2 \left(1 - S_3 \frac{\sigma_\Delta}{3} \left(\nu - 2 - \tfrac{1}{\nu^2} \right) \right) \right] , \qquad \nu \equiv \frac{\Delta_c}{\sigma_\Delta} .$$



Formation rate as a function of σ_Δ



Fine tuning of Ω_{PBH}