Novel phase structures in $\Box^k$ scalar theories

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New Frontiers In Theoretical Physics,
XXXVI Convegno Nazionale di Fisica Teorica
Cortona, May 25, 2018

Based on:

M.S, G.P.Vacca

[arXiv:1711.08685 [hep-th]], Eur.Phys.J.C

[arXiv:1708.09795 [hep-th]], Phys.Rev.D

Introduction

Conformal invariant QFTs appear in different areas of physics

- *Critical phenomena*
  - *Ferromagnet-paramagnet*
  - *Liquid-vapour*

    ⋮
- *Beyond the Standard Model*
  - *Conformal Technicolor*
  - *Dynamical electroweak symmetry breaking*

    ⋮

Introduction

(Analytic) approaches to conformal invariant QFT

- *Renormalization group* $\begin{cases} \textit{Exact RG} \\ \textit{Epsilon expansion} \end{cases}$

  [Wilson 1971]

  [Wilson and Fisher 1972] . . .

- *Conformal field theory* [Rychkov, Tan '15]

  [Basu, Krishnan '15]

  [Nii '16]

  [Hasegawa, Nakayama '16]

  [Gopakumar, Kaviraj, Sen, Sinha '16]

  [Gliozzi, Guerrieri, Petkou, Wen '16, '17]

  [Alday '16]

  [Codello, M.S, Vacca, Zanusso '17] . . .

Introduction

Main Goal:

Use ϵ expansion to study critical properties of scalar theories with higher-derivative kinetic terms:

$$\mathcal{L} = \frac{1}{2}\phi(-\Box)^k\phi + \text{interactions}, \qquad k \in \mathbb{N}$$

- *Concentrate mainly on renormalization group method*
- *But also discuss CFT method using Schwinger-Dyson equations*

Introduction

Already applied to single-scalar theories with standard kinetic terms:

- Renormalization group

  Codello, M.S, Vacca, Zanusso [arXiv:1706.06887 [hep-th]], Phys.Rev.D

  Codello, M.S, Vacca, Zanusso [arXiv:1705.05558 [hep-th]], Eur.Phys.J.C

- Conformal symmetry and Schwinger-Dyson equations

  Codello, M.S, Vacca, Zanusso [arXiv:1703.04830 [hep-th]], JHEP

Multi-field scalar theories with permutation symmetry:

Codello, M.S, Vacca, Zanusso *To appear*

Introduction

Motivation

- *Theory of elasticity* Riva-Cardy, [Nakayama '16]
- *Physics of certain polymers* [Schwahn, et.al '99]
- *de Sitter holography* [Brust, Hinterbichler '16]

  ⋮

- *Theoretical motivations . . .*

Introduction

Previous studies

- *[Bonanno, Zappala '15]*
  Lifshitz critical behaviour
- *[Osborn, Stergiou '16]*
  C_T *(coefficient of the e.m. two-point function) in Free CFT*
- *[Brust, Hinterbichler '16]*
  $U(N)$ *Free CFT*
- *[Gliozzi, Guerrieri, Petkou, Wen '16, '17]*
  Analytic structure of conformal blocks for box^k theories
- *[Gracey, Simms '17]*
  Quartic $O(N)$ *models at large* N

$\vdots$

$\Box^k$ scalar theories: setup and general considerations

Consider critical theories that are marginal deformations of Higher-derivative Free CFTs

$$\mathcal{L}_{HFT} = \tfrac{1}{2}\phi(-\Box)^k\phi, \qquad k \in \mathbb{N}$$

Fix critical dimension by requiring that ϕ^{2n} be a marginal operator

$$d_c = \frac{2nk}{n-1}$$

$\Box^k$ scalar theories: setup and general considerations

$$\phi^{2n}, \cdots, \phi\Box^k\phi$$

- $k, n-1$ *relatively prime:*

  Either no other marginal operators or can be consistently ignored ($\mathbb{Z}_2$-odd)

- $k, n-1$ *have common divisor:*

  There are $\mathbb{Z}_2$-even (and possibly $\mathbb{Z}_2$-odd) marginal operators

$\Box^k$ scalar theories: setup and general considerations

We distinguish two types of models

- $k, n-1$ *relatively prime: First type*
- $k, n-1$ *have common divisor: Second type*

Theories of the first type

$\Box^k$ scalar theories of first type

- *One can consistently consider pure potential deformations*

$$\mathcal{L} = \tfrac{1}{2}\phi\,(-\Box)^k\phi + V(\phi)$$

- *Propagator satisfies* $(-\Box_x)^k G_{xy} = \delta^d_{xy}$ *and is given by*

$$G_{xy} = \frac{1}{(4\pi)^k\Gamma(k)}\,\frac{\Gamma(\delta)}{\pi^\delta}\,\frac{1}{|x-y|^{2\delta}}, \qquad \delta \equiv [\phi] = \tfrac{d}{2} - k$$

Functional perturbative RG

[Jack, Osborn '83], . . .
[O'Dwyer, Osborn '07]
[Codello, M.S, Vacca, Zanusso '17]
[Osborn, Stergiou '17]

- *Gives more insight into the structure of flow*
- *Allows to calculate many critical quantities in one shot*

Functional perturbative RG

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + g_1\phi + g_2\phi^2 + g_3\phi^3 + g_4\phi^4 \qquad d = 4$$

Dimensionful beta functions (dim.reg. and $\overline{MS}$)

$$\begin{aligned}
\beta_1 &= 12\, g_2 g_3 - 108\, g_3^3 - 288\, g_2 g_3 g_4 + 48\, g_1 g_4^2 \\
\beta_2 &= 24\, g_4 g_2 + 18\, g_3^2 - 1080\, g_3^2 g_4 - 480\, g_2 g_4^2 \\
\beta_3 &= 72\, g_4 g_3 - 3312\, g_3 g_4^2 \\
\beta_4 &= 72\, g_4^2 - 3264\, g_4^3
\end{aligned}$$

Functional perturbative RG

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi) \qquad d = 4$$

$$\beta_V = a(V^{(2)})^2 + bV^{(2)}(V^{(3)})^2$$

Functional perturbative RG

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + V(\phi) \qquad d = 4$$

$$\beta_V = \tfrac{1}{2}(V^{(2)})^2 + \tfrac{1}{2}V^{(2)}(V^{(3)})^2$$

Functional perturbative RG

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi) \qquad d = 4$$

$$\beta_V = \frac{1}{2}(V^{(2)})^2 + \frac{1}{2}V^{(2)}(V^{(3)})^2$$

$1 - \text{loop}$

Functional perturbative RG

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + V(\phi) \qquad d = 4$$

$$\beta_V = \frac{1}{2}(V^{(2)})^2 + \frac{1}{2}V^{(2)}(V^{(3)})^2$$

$$2 - \text{loops}$$

$\Box^k$ scalar theories of first type

$$\beta_v = -dv + \frac{d-2k+\eta}{2}\varphi\, v' + \frac{1}{n!}\, v^{(n)\,2}$$
$$-\Gamma(n\delta_c)\,\frac{1}{3}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!}\; v^{(r+s)}v^{(s+t)}v^{(t+r)}$$
$$-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\, v^{(n)}v^{(n+s)}v^{(n+t)}$$

Quantities expressed in units of RG scale μ

$$v(\varphi) = \mu^{-d}\, V(\mu^{\delta}\,\varphi) \qquad d = d_c - \epsilon$$

$\Box^k$ scalar theories of first type

$$\begin{aligned}\beta_v = &- dv + \frac{d-2k+\eta}{2}\varphi\, v' + \frac{1}{n!}\, v^{(n)\,2} \\ &-\Gamma(n\delta_c)\,\frac{1}{3}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!}\; v^{(r+s)}v^{(s+t)}v^{(t+r)} \\ &-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\, v^{(n)}v^{(n+s)}v^{(n+t)}\end{aligned}$$

$$v \to \frac{[(4\pi)^k\Gamma(k)]^n\Gamma(n\delta_c)}{\Gamma(\delta_c)^n}\, v \qquad \delta_c = \frac{k}{n-1}$$

$\Box^k$ scalar theories of first type

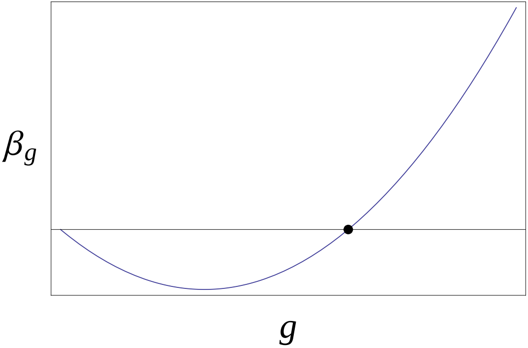
$$\beta_v = -dv + \frac{d-2k+\eta}{2}\varphi\, v' + \frac{1}{n!}\, v^{(n)\,2}$$
$$-\Gamma(n\delta_c)\,\frac{1}{3}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!}\; v^{(r+s)}v^{(s+t)}v^{(t+r)}$$
$$-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\, v^{(n)}v^{(n+s)}v^{(n+t)}$$

$$K^{n,k}_{rst} \equiv \frac{\Gamma\left((n-r)\delta_c\right)\Gamma\left((n-s)\delta_c\right)\Gamma\left((n-t)\delta_c\right)}{\Gamma\left(r\delta_c\right)\Gamma\left(s\delta_c\right)\Gamma\left(t\delta_c\right)}$$

$$J^{n,k}_{st} \equiv \psi(n\delta_c) - \psi(s\delta_c) - \psi(t\delta_c) + \psi(1)$$

$\Box^k$ scalar theories of first type

$$\beta_g = -(n-1)\epsilon g + \frac{(2n)!^3}{n!^3} g^2 + \mathcal{O}(g^3), \qquad v(\varphi) = g\varphi^{2n}$$



$\Box^k$ scalar theories of first type

Example $k = 2,\, n = 2$

$$\beta_v = -\,dv + \frac{d-4+\eta}{2}\varphi\, v' + \frac{1}{2}\,(v^{(2)})^2 + \frac{1}{12}\; v^{(2)}(v^{(3)})^2$$

$\Box^k$ scalar theories of first type

Example $k = 3,\, n = 2$

$$\beta_v = -\,dv + \frac{d-6+\eta}{2}\varphi\, v' + \frac{1}{2}\,(v^{(2)})^2 + \frac{43}{120}\; v^{(2)}(v^{(3)})^2$$

$\Box^k$ scalar theories of first type

Example $k=3$, $n=3$

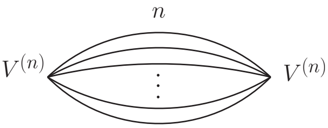
$$\beta_v = -dv + \frac{d-6+\eta}{2}\varphi\, v' + \frac{1}{6}\,(v^{(3)})^2 - \frac{35\pi^2}{8192}\,(v^{(4)})^3 + \frac{31}{1260}\,v^{(3)}v^{(4)}v^{(5)} - \frac{7}{144}v^{(2)}(v^{(5)})^2$$

$\Box^k$ scalar theories of first type

$$\beta_v = -dv + \frac{d-2k+\eta}{2}\varphi\, v' + \boxed{\frac{1}{n!}\, v^{(n)\,2}} \quad (n-1)-\text{loops}$$

$$-\Gamma(n\delta_c)\,\frac{1}{3}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!}\, v^{(r+s)}v^{(s+t)}v^{(t+r)}$$

$$-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\, v^{(n)}v^{(n+s)}v^{(n+t)}$$

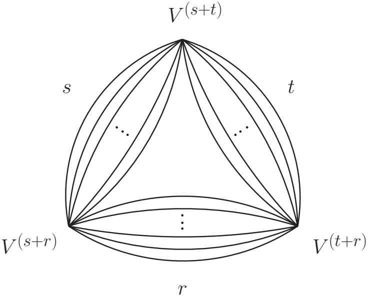


$\Box^k$ scalar theories of first type

$$\beta_v = -dv + \frac{d-2k+\eta}{2}\varphi\, v' + \frac{1}{n!}\, v^{(n)\,2}$$

$$\boxed{-\Gamma(n\delta_c)\,\frac{1}{3}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!}\; v^{(r+s)}v^{(s+t)}v^{(t+r)}} \quad 2(n-1)-\text{loops}$$

$$-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\; v^{(n)}v^{(n+s)}v^{(n+t)}$$



$\Box^k$ scalar theories of first type

$$\beta_v = -\,dv + \frac{d-2k+\eta}{2}\varphi\, v' + \frac{1}{n!}\, v^{(n)\,2}$$

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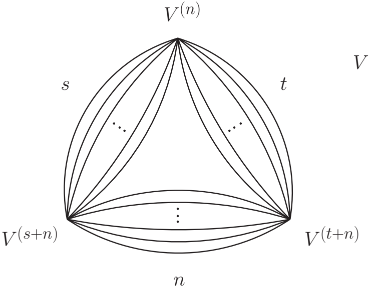
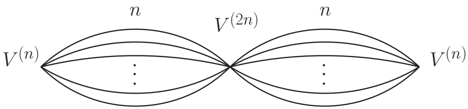
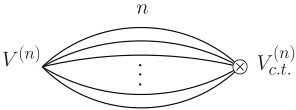
$$\boxed{-\frac{1}{n!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\; v^{(n)}v^{(n+s)}v^{(n+t)}} \qquad 2(n-1)-\text{loops}$$

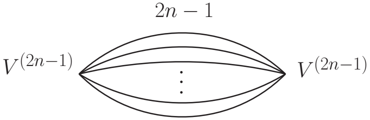






$\Box^k$ scalar theories of first type

Running of wavefunction renormalization induced by potential



$$\eta = (-1)^{k+1} \frac{n(\delta_c)_k}{k(\delta_c+k)_k} \, 4(2n)! \, g^2 \qquad g \equiv \frac{v^{(2n)}(0)}{(2n)!}$$

$$= (-1)^{k+1} \frac{n(\delta_n)_k}{k(\delta_n+k)_k} \frac{4(n-1)^2 n!^6}{(2n)!^3} \epsilon^2, \qquad (a)_b \equiv \frac{\Gamma(a+b)}{\Gamma(a)}$$

Critical quantities from RG

Critical quantities are encoded in the expansion coefficients around the scale invariant point $\beta^i(g_*) = 0$

$$\beta^k_{g_*+\delta g} = M^k{}_i\,\delta g^i + N^k{}_{ij}\,\delta g^i \delta g^j + O(\delta g^3)$$

Express in basis with definite scaling $\mathcal{S}^a{}_i\,\delta g^i \equiv \lambda^a$

$$\mathcal{S}^a{}_i\,M^i{}_j\,(\mathcal{S}^{-1})^j{}_b = -\theta_a\,\delta^a{}_b \qquad \mathcal{S}^a{}_i\,N^i{}_{jk}\,(\mathcal{S}^{-1})^j{}_b\,(\mathcal{S}^{-1})^k{}_c = \tilde{C}^a{}_{bc}$$

$$\beta^a_\lambda = -\theta_a\,\lambda^a + \tilde{C}^a{}_{bc}\,\lambda^b\lambda^c + O(\lambda^3)$$

J. Cardy *"Scaling and Renormalization in Statistical Physics"*

Codello, M.S, Vacca, Zanusso [arXiv:1705.05558 [hep-th]], Eur.Phys.J.C

$\Box^k$ scalar theories of first type

Coupling anomalous dimensions

$$\begin{aligned}\tilde{\gamma}_i \;=\; & \frac{\eta}{2} i + 2n\,\eta\,\delta_i^{2n} + \frac{(n-1)i!}{(i-n)!}\,\frac{2n!}{(2n)!}\left[\epsilon - \frac{n}{n-1}\,\eta\right] \\ + \; & \frac{(n-1)^2 i! n!^6}{(2n)!^2}\Gamma(n\delta_c)\sum_{r,s,t}\frac{K^{n,k}_{rst}}{(r!s!t!)^2}\left[\frac{2n!}{3(i-n)!} - \frac{r!}{(i-2n+r)!}\right]\epsilon^2 \\ + \; & \frac{(n-1)^2 i! n!^4}{(2n)!^2}\sum_{s,t}\frac{J^{n,k}_{st}}{s!^2 t!^2}\left[\frac{n!}{(i-n)!} - \frac{2s!}{(i-2n+s)!}\right]\epsilon^2\end{aligned}$$

$$(r+s+t=2n,\; s+t=n)$$

Shadow relation

$$\tilde{\gamma}_1 + \tilde{\gamma}_{2n-1} = (n-1)\epsilon \quad \Leftrightarrow \quad \theta_1 + \theta_{2n-1} = d$$

$\Box^k$ scalar theories of first type

Dimensionless OPE coefficients $l = i + j - 2n$

$$
\begin{aligned}
\tilde{C}^{l}_{\ ij} \;=\;& \frac{1}{n!}\,\frac{i!}{(i-n)!}\,\frac{j!}{(j-n)!} + 2(2n)!\,\left(i\delta_j^{2n} + j\delta_i^{2n} + 2n\,\delta_i^{2n}\delta_j^{2n}\right)\epsilon \\
& -\; \Gamma(n\delta_c)\,\frac{(n-1)n!^3}{(2n)!}\sum_{r,s,t}\frac{K^{n,k}_{rst}}{r!s!t!^2}\,\frac{j!}{(j-s-t)!}\,\frac{i!}{(i+s-2n)!}\,\epsilon \\
& -\; \frac{(n-1)n!^2}{(2n)!}\sum_{s,t}\frac{J^{n,k}_{st}}{s!t!}\,\left[\frac{1}{n!}\,\frac{j!}{(j-n-s)!}\,\frac{i!}{(i-n-t)!}\right. \\
& +\; \left.\frac{1}{s!}\,\frac{i!}{(i-n)!}\,\frac{j!}{(j-n-s)!} + \frac{1}{s!}\,\frac{j!}{(j-n)!}\,\frac{i!}{(i-n-s)!}\right]\epsilon
\end{aligned}
$$

Consistent with CFT literature

Gliozzi, Guerrieri, Petkou, Wen [arXiv:1702.03938 [hep-th]], JHEP
Codello, M.S, Vacca, Zanusso [arXiv:1703.04830 [hep-th]], JHEP

$\Box^k$ scalar theories of first type

CFT approach using Schwinger-Dyson equation

$$\mathcal{L} = \tfrac{1}{2}\phi(-\Box)^k\phi + g\phi^{2n}$$

- *Conformal invariance*

$$\langle O_a(x) O_b(y)\rangle = \frac{c_a\,\delta_{ab}}{|x-y|^{2\Delta_a}} \qquad \Delta_a = a\delta + \gamma_a$$

$$\langle O_a(x) O_b(y) O_c(z)\rangle = \frac{C_{abc}}{|x-y|^{\Delta_a+\Delta_b-\Delta_c}\,|y-z|^{\Delta_b+\Delta_c-\Delta_a}\,|z-x|^{\Delta_c+\Delta_a-\Delta_b}}$$

- *Schwinger-Dyson equation*

$$\left\langle \frac{\delta S}{\delta\phi}(x)\, O_1(y)\, O_2(z)\dots \right\rangle = 0$$

$\Box^k$ scalar theories of first type

CFT approach using Schwinger-Dyson equation

$$\mathcal{L} = \tfrac{1}{2}\phi(-\Box)^k\phi + g\phi^{2n}$$

- *Conformal invariance*

$$\langle O_a(x) O_b(y)\rangle = \frac{c_a\,\delta_{ab}}{|x-y|^{2\Delta_a}} \qquad \Delta_a = a\delta + \gamma_a$$

$$\langle O_a(x) O_b(y) O_c(z)\rangle = \frac{C_{abc}}{|x-y|^{\Delta_a+\Delta_b-\Delta_c}\,|y-z|^{\Delta_b+\Delta_c-\Delta_a}\,|z-x|^{\Delta_c+\Delta_a-\Delta_b}}$$

- *Schwinger-Dyson equation (leading order)*

$$\left\langle \Box_x^k\phi(x)\; O_1(y)\, O_2(z)\ldots \right\rangle = 2ng(-1)^{k-1}\left\langle \phi(x)^{2n-1}\; O_1(y)\, O_2(z)\ldots\right\rangle$$

$\Box^k$ scalar theories of first type

Anomalous dimension $\eta = 2\gamma$

$$\Box_x^k \Box_y^k \langle \phi_x \phi_y \rangle = \langle \Box_x^k \phi_x \, \Box_y^k \phi_y \rangle$$

$$\Box_x^k \Box_y^k \frac{c}{|x-y|^{2\Delta_1}} = (2ng)^2 \left\langle \phi(x)^{2n-1} \phi(y)^{2n-1} \right\rangle$$

$$4^{2k} \gamma (-1)^{k-1} k!(k-1)! \prod_{i=2}^{2k} (\delta_c + i - 1) = (2n)^2 g^2 c_{\text{free}}^{2n-2}$$

$$\gamma = (-1)^{k+1} \frac{n(\delta_c)_k}{k(\delta_c + k)_k} \, 2(2n)! \, g^2$$

$$c_{\text{free}} = \frac{1}{(4\pi)^k \Gamma(k)} \frac{\Gamma(\delta_c)}{\pi^{\delta_c}} \qquad g \to \frac{[(4\pi)^k \Gamma(k)]^n \Gamma(n\delta_c)}{\Gamma(\delta_c)^n} \, g$$

$\Box^k$ scalar theories of first type

Critical exponents γ_i

$$\Box_x^k \langle \phi_x \phi_y^i \phi_z^{i+1} \rangle = \langle \Box_x^k \phi_x \phi_y^i \phi_z^{i+1} \rangle$$

OPE coefficients $C^1{}_{2k,2l+1}$

$$\Box_x^k \langle \phi_x \phi_y^{2k} \phi_z^{2l+1} \rangle = \langle \Box_x^k \phi_x \phi_y^{2k} \phi_z^{2l+1} \rangle \quad k - l \neq 0, 1$$

$$\vdots$$

Theories of the second type

$\Box^2$ scalar theories of second type

$$\mathcal{L} = \tfrac{1}{2}\phi\,\Box^2\phi + \tfrac{1}{2}Z(\phi)(\partial\phi)^2 + V(\phi)$$

$$n = 2m+1, \qquad m = 1, 2, \cdots$$

$$d_c = 4 + \frac{2}{m}, \qquad \delta_c = \frac{1}{m}$$

There are two marginal operators in this case:

$$\phi^{2(2m+1)}, \qquad \phi^{2m}(\partial\phi)^2$$

$\Box^2$ scalar theories of second type

$$m = 1$$

1	ϕ	ϕ^2	ϕ^3	ϕ^4	ϕ^5	ϕ^6	ϕ^7	$\cdots$
				$(\partial\phi)^2$	$\phi(\partial\phi)^2$	$\phi^2(\partial\phi)^2$	$\phi^3(\partial\phi)^2$	$\cdots$
						$(\partial^2\phi)^2$	$\phi(\partial^2\phi)^2$	$\cdots$

$\Box^2$ scalar theories of second type

$$m = 1$$

1	ϕ	ϕ^2	ϕ^3	ϕ^4	ϕ^5	ϕ^6	ϕ^7	$\cdots$
				$(\partial\phi)^2$	$\phi(\partial\phi)^2$	$\phi^2(\partial\phi)^2$	$\phi^3(\partial\phi)^2$	$\cdots$
						$(\partial^2\phi)^2$	$\phi(\partial^2\phi)^2$	$\cdots$

$\Box^2$ scalar theories of second type

$$v(\varphi) = \mu^{-d}\, V(\mu^{\delta}\, \varphi), \qquad z(\varphi) = \mu^{-2}\, Z(\mu^{\delta}\, \varphi)$$

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

$$\beta_z = -2z + \frac{d-4}{2}\varphi z' + 2\frac{v^{(2m+1)} z^{(2m+1)}}{(2m+1)!} + \frac{z^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{3m+2}{2(2m+1)}\, \frac{(z^{(m)})^2}{(m+1)!} - \frac{2(m+1)}{(2m+1)}\, \frac{(v^{(3m+2)})^2}{(3m+1)!}$$

$$v \to (m+1)\, \frac{(4\pi)^{2(2m+1)}}{m^2 \Gamma^{2m}(\delta_c)}\, v, \qquad z \to \frac{(4\pi)^{2m+1}}{m \Gamma^m(\delta_c)}\, z$$

$\Box^2$ scalar theories of second type

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

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$m - \text{loop}$

$\Box^2$ scalar theories of second type

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

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$2m - \text{loop}$

$\Box^2$ scalar theories of second type

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

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$2m - \text{loop}$

$\Box^2$ scalar theories of second type

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

$$\beta_z = -2z + \frac{d-4}{2}\varphi z' + 2\frac{v^{(2m+1)} z^{(2m+1)}}{(2m+1)!} + \frac{z^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{3m+2}{2(2m+1)} \frac{(z^{(m)})^2}{(m+1)!} - \frac{2(m+1)}{(2m+1)} \frac{(v^{(3m+2)})^2}{(3m+1)!}$$

$m - \text{loop}$

$\Box^2$ scalar theories of second type

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{(v^{(2m+1)})^2}{(2m+1)!}$$

$$\beta_z = -2z + \frac{d-4}{2}\varphi z' + 2\frac{v^{(2m+1)} z^{(2m+1)}}{(2m+1)!} + \frac{z^{(m+1)} z^{(m-1)}}{(m+1)!} + \frac{3m+2}{2(2m+1)}\frac{(z^{(m)})^2}{(m+1)!} - \frac{2(m+1)}{(2m+1)}\frac{(v^{(3m+2)})^2}{(3m+1)!}$$

$3m - \text{loop}$

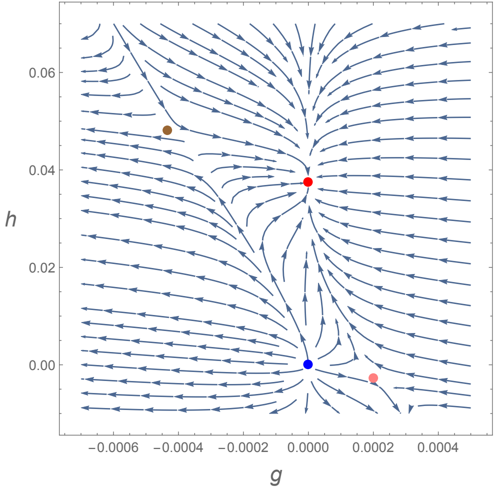
$\Box^2$ scalar theories of second type

The $n=3$ case:

$$\beta_v = -dv + \frac{d-4}{2}\varphi v' + \frac{v^{(2)} z}{2} + \frac{(v^{(3)})^2}{6}$$

$$\beta_z = -2z + \frac{d-4}{2}\varphi z' + \frac{v^{(3)} z^{(3)}}{3} + \frac{z^{(2)} z}{2} + \frac{5(z^{(1)})^2}{12} - \frac{(v^{(5)})^2}{18}$$

$\Box^2$ scalar theories of second type



$$\mathcal{L} = \tfrac{1}{2}\phi\,\Box^2\phi + \tfrac{1}{2}h\,\phi^2(\partial\phi)^2 + g\phi^6 \qquad (n = 3,\ \epsilon = 0.1)$$

$\Box^2$ scalar theories of second type

Let us consider the IR fixed point

$$h = \frac{2m(m+1)(2m+1)}{2+7m(m+1)}\,\frac{(m+1)!m!^2}{(2m)!^2}\,\epsilon, \quad g = 0$$

Field anomalous dimension

$$\eta = \frac{\Gamma(\delta_c)}{\Gamma(2+\delta_c)}\,\frac{2(m+1)^2(2m+1)^2}{(3m+1)(2+7m(m+1))^2}\,\frac{(m+1)!^2 m!^4}{(2m)!^3}\,\epsilon^2$$

ϕ^i-coupling anomalous dimension (all i)

$$\tilde{\gamma}_i = \frac{i!}{(i-m-1)!}\,\frac{2m(2m+1)}{2+7m(m+1)}\,\frac{m!}{(2m)!}\,\epsilon$$

$\phi^i(\partial\phi)^2$-coupling anomalous dimension (all i)

$$\tilde{\omega}_i = i!\left[\frac{3m+2}{2m+1}\,\frac{m+1}{(i-m)!} + \frac{1}{(i-m-1)!} + \frac{m(m+1)}{(i-m+1)!}\right]\frac{2m(2m+1)}{2+7m(m+1)}\,\frac{m!}{(2m)!}\,\epsilon$$

$\Box^2$ scalar theories of second type

Conformal invariance and Schwinger-Dyson equations

$$\mathcal{L} = \tfrac{1}{2}\phi\,\Box^2\phi + \tfrac{1}{2}h\,\phi^{2m}(\partial\phi)^2 + g\phi^{2(2m+1)}$$

$$\Box^2\phi + 2(2m+1)g\phi^{4m+1} - mh\phi^{2m-1}(\partial\phi)^2 - h\phi^{2m}\Box\phi = 0$$

$\Box^2$ scalar theories of second type

Anomalous dimension $\eta = 2\gamma$:

$$\Box_x^2 \Box_y^2 \langle \phi_x \phi_y \rangle = \langle \Box_x^2 \phi_x \Box_y^2 \phi_y \rangle$$

$$\Box_x^2 \Box_y^2 \langle \phi_x \phi_y \rangle \stackrel{\mathrm{LO}}{=} -2^9 \gamma \, c \, |x-y|^{-\frac{2}{m}-8} \prod_{i=0}^{3} (i + 1/m)$$

$$\begin{aligned} \langle \Box_x^2 \phi_x \Box_y^2 \phi_y \rangle \stackrel{\mathrm{LO}}{=} & \; 4(2m+1)^2 (4m+1)! g^2 c^{4m+1} |x-y|^{-\frac{2}{m}-8} \\ & - \; 8(2m+1)! m^{-2} h^2 c^{2m+1} |x-y|^{-\frac{2}{m}-8} \end{aligned}$$

$\square^2$ scalar theories of second type

Critical exponents γ_i:

$$\square_x^2 \left\langle \phi_x \, \phi_y^i \, \phi_z^{i+1} \right\rangle = \left\langle \square_x^2 \phi_x \, \phi_y^i \, \phi_z^{i+1} \right\rangle$$

$$\gamma_{i+1} - \gamma_i = \binom{i}{m} \frac{(2m)!}{(m+1)!} h + 4 \binom{i}{2m} \frac{(4m+1)!}{(2m)!} g$$

Can be solved with the boundary condition $\gamma_1 = \mathcal{O}(\epsilon^2)$,
and reproduces RG result

$\Box^2$ scalar theories of second type

Cubic corrections in $d_c = 6$ ($m = 1$)

$$\begin{aligned}
\beta_v =& -d\,v + \frac{d-4+\eta}{2}\varphi\,v^{(1)} + \frac{1}{6}\,v^{(3)\,2} + \frac{1}{2}z\,v^{(2)} \\
& -\frac{1}{12}v^{(4)\,3} - \frac{1}{12}v^{(3)}v^{(4)}v^{(5)} + \frac{35}{1296}v^{(2)}v^{(5)\,2} \\
& +\frac{1}{12}z^{(2)}v^{(2)}v^{(4)} - \frac{1}{16}z\,v^{(4)\,2} - \frac{1}{16}z^{(2)}\,v^{(3)\,2} - \frac{5}{16}z^{(1)}\,v^{(3)}v^{(4)}
\end{aligned}$$

$\square^2$ scalar theories of second type

Cubic corrections in $d_c = 6$ ($m = 1$)

$$\beta_v = -d\,v + \frac{d-4+\eta}{2}\varphi\,v^{(1)} + \frac{1}{6}\,v^{(3)\,2} + \frac{1}{2}z\,v^{(2)}$$
$$-\frac{1}{12}v^{(4)\,3} - \frac{1}{12}v^{(3)}v^{(4)}v^{(5)} + \frac{35}{1296}v^{(2)}v^{(5)\,2}$$
$$+\frac{1}{12}z^{(2)}v^{(2)}v^{(4)} - \frac{1}{16}z\,v^{(4)\,2} - \frac{1}{16}z^{(2)}\,v^{(3)\,2} - \frac{5}{16}z^{(1)}\,v^{(3)}v^{(4)}$$

$2 - \mathrm{loop}$

$\square^2$ scalar theories of second type

Cubic corrections in $d_c = 6$ ($m = 1$)

$$\beta_v = -d\,v + \frac{d-4+\eta}{2}\varphi\,v^{(1)} + \frac{1}{6}\,v^{(3)\,2} + \frac{1}{2}z\,v^{(2)}$$
$$-\frac{1}{12}v^{(4)\,3} - \frac{1}{12}v^{(3)}v^{(4)}v^{(5)} + \frac{35}{1296}v^{(2)}v^{(5)\,2}$$
$$+\frac{1}{12}z^{(2)}v^{(2)}v^{(4)} - \frac{1}{16}z\,v^{(4)\,2} - \frac{1}{16}z^{(2)}\,v^{(3)\,2} - \frac{5}{16}z^{(1)}\,v^{(3)}v^{(4)}$$

$4 - \mathrm{loop}$

$\Box^2$ scalar theories of second type

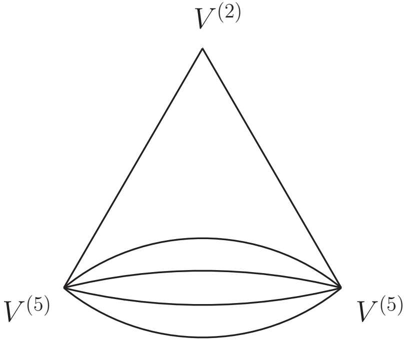
Cubic corrections in $d_c = 6$ ($m = 1$)

$$\beta_v = -d\,v + \frac{d-4+\eta}{2}\varphi\,v^{(1)} + \frac{1}{6}\,v^{(3)\,2} + \frac{1}{2}z\,v^{(2)} - \frac{1}{12}v^{(4)\,3} - \frac{1}{12}v^{(3)}v^{(4)}v^{(5)} + \frac{35}{1296}v^{(2)}v^{(5)\,2} + \frac{1}{12}z^{(2)}v^{(2)}v^{(4)} - \frac{1}{16}z\,v^{(4)\,2} - \frac{1}{16}z^{(2)}\,v^{(3)\,2} - \frac{5}{16}z^{(1)}\,v^{(3)}v^{(4)}$$

$3 - \text{loop}$

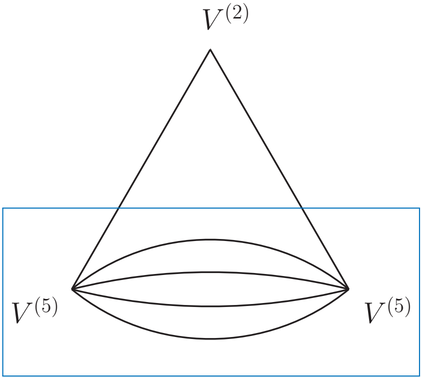
$\Box^2$ scalar theories of second type

- *Computing Z counter-terms is inevitable*



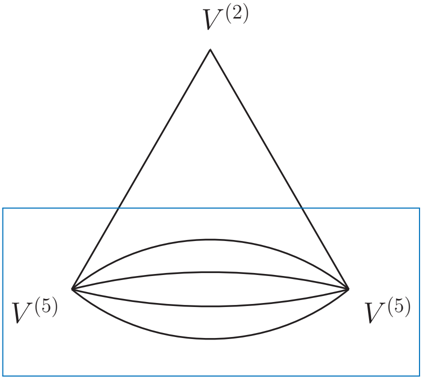
$\Box^2$ scalar theories of second type

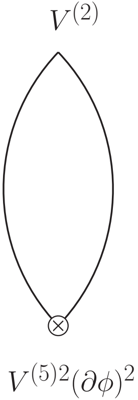
◦ *Computing Z counter-terms is inevitable*



$\Box^2$ scalar theories of second type

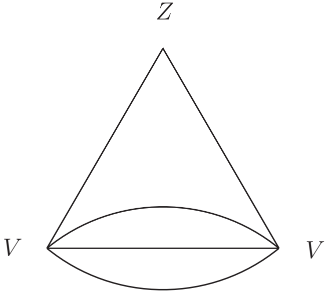
- *Computing Z counter-terms is inevitable*





$\Box^2$ scalar theories of second type

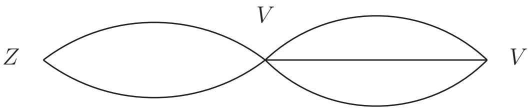
- *Non-local div appear in separate Feynman diagrams*



$$-\frac{1}{(4\pi)^9}\frac{1}{24}\left[\frac{1}{3\epsilon^2}+\left(\frac{7}{4}-\frac{\gamma}{2}\right)\frac{1}{\epsilon}\right]Z\,V^{(4)\,2}+\frac{1}{(4\pi)^9}\frac{1}{48\epsilon}\frac{1}{\Box}\left(Z^{(2)}\,(\partial\phi)^2\right)V^{(4)\,2}$$

$\Box^2$ scalar theories of second type

◦ *Non-local div appear in separate Feynman diagrams*

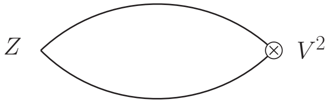


$$-\frac{1}{(4\pi)^9}\frac{1}{24}\left[\frac{1}{\epsilon^2}+\left(\frac{15}{4}-\frac{3\gamma}{2}\right)\frac{1}{\epsilon}\right] Z\, V^{(3)} V^{(5)}$$

$$+\frac{1}{(4\pi)^9}\frac{1}{48\epsilon}\,\frac{1}{\Box}\left(Z^{(2)}\,(\partial\phi)^2\right) V^{(3)} V^{(5)}$$

$\Box^2$ scalar theories of second type

- *Non-local div appear in separate Feynman diagrams*

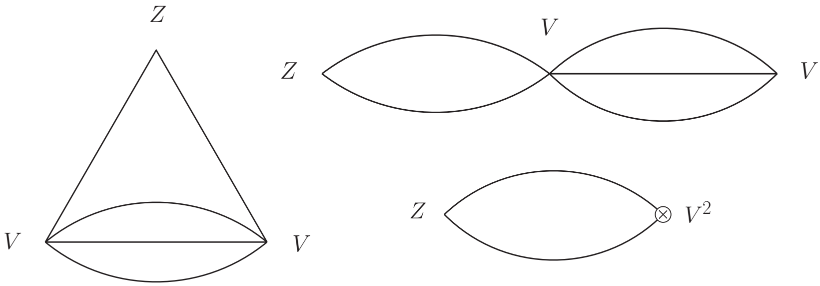


$$\frac{1}{(4\pi)^9}\frac{1}{48}\left[\frac{1}{\epsilon^2}+\left(\frac{3}{2}-\frac{\gamma}{2}\right)\frac{1}{\epsilon}\right]Z\left[V^{(3)\,2}\right]^{(2)}$$

$$-\ \frac{1}{(4\pi)^9}\frac{1}{96\epsilon}Z^{(2)}\,(\partial\phi)^2\,\frac{1}{\Box}\left[V^{(3)\,2}\right]^{(2)}$$

$\Box^2$ scalar theories of second type

◦ *Non-local div appear in separate Feynman diagrams*



$$\frac{1}{(4\pi)^9}\frac{1}{36}\left[\frac{1}{\epsilon^2}-\frac{3}{8\epsilon}\right] Z\, V^{(4)\,2} - \frac{1}{(4\pi)^9}\frac{9-4\gamma}{96\epsilon}\, Z\, V^{(3)}\, V^{(5)}$$

$\Box^2$ scalar theories of second type

OPE coefficients for IR fixed point ($g = 0, h \neq 0$)

$$\begin{aligned}\tilde{C}^l{}_{ij} = &\ \frac{1}{6}\frac{i!}{(i-3)!}\frac{j!}{(j-3)!}\\ -&\ \frac{1}{16}\frac{i!}{(i-4)!}\frac{j!}{(j-4)!}\,h - \frac{1}{8}\frac{i!}{(i-3)!}\frac{j!}{(j-3)!}\,h\\ +&\ \frac{1}{12}\frac{i!}{(i-2)!}\frac{j!}{(j-4)!}\,h - \frac{5}{16}\frac{i!}{(i-3)!}\frac{j!}{(j-4)!}\,h + (i \leftrightarrow j)\end{aligned}$$

$$\phi^i \times \phi^j \sim C^l_{ij}\,\phi^l, \qquad l = i + j - 6$$

Conclusions and future directions

- *First type theories behave, in many aspects, like standard theories*

- *Second type theories have several new features:*

  - *The phase diagram is more involved. There are no pure potential scale invariant deformations*

  - *Derivative interactions are unavoidable even at leading order in a derivative expansion*

  - *Nonlocal ultraviolet divergences appear in Feynman diagrams that sum up to local terms in amplitudes*

Conclusions and future directions

- *Including higher order corrections*
- *Extension to critical models with odd interactions*
- *Global symmetries, e.g.* $O(N)$ *models*

$$\vdots$$

- *Reproduce and extend results (for theories of the second type) using Conformal Bootstrap idea*

Thank You