Modified Unruh effect from GUP

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* F. Scardigli, M. Blasone, G. L. and R. Casadio, arXiv:1804.05282.

Outline

- Motivations
- Historical excursus on the **G**eneralized **U**ncertainty **P**rinciple (GUP)
- Revising the Unruh effect in the context of GUP
  - Heuristic (semiclassical) approach
  - Field Theoretical approach
- Conclusions and outlook

Motivations

- Testing (at a theoretical level) the effects of GUP in a context different from string theory or other theories of quantum gravity

- Investigating the relation between the deviation from thermality of Unruh radiation induced by GUP and those found in different contexts (e.g., Casimir-Polder forces*, flavor mixing and oscillations†)

* J. Marino, A. Noto and R. Passante, Phys. Rev. Lett. **113** (2014)
† M. Blasone, G. Lambiase and G. L., Phys. Rev. D **96** (2017)

Generalized Uncertainty Principles (GUPs)

- Various theories of quantum gravity predict the existence of a **minimum length scale**, which leads to the modification of the standard Heisenberg uncertainty principle (HUP)

$$\delta x \geq \frac{\hbar}{2\,\delta p}$$

- Research on GUP has several decades of history*

- Last 30 years: **string theory** suggest that, in gedanken experiments on high energy string scatterings involving Gravity, the "effective" uncertainty relation should read†

$$\delta x \geq \frac{\hbar}{2\,\delta p} + 2\,\beta\, l_p^2\, \frac{\delta p}{\hbar}$$

* C.N. Yang, 1947 - H.Snyder, 1947 - C.A. Mead, 1964 - F. Karolyhazy, 1966
† G. Veneziano, 1987 - D. J. Gross, 1987

- A similar relation is obtained via gedanken experiments on the Hawking radiation coming from **large black holes**[*]...

  ...and on the formation of **micro black holes**[†]

$$\delta x \geq \frac{\hbar}{2\,\delta p} + 2\,G\,\delta p$$

- Kempf, Mann, Vagenas, Brau, etc. translate GUP into a **deformed commutator**[‡] and develop a deformed Quantum Mechanics

$$[\hat{x}, \hat{p}] = i\hbar \left[1 + \beta \left(\frac{\hat{p}}{m_{\mathrm{p}}} \right)^2 \right]$$

[*] M. Maggiore, Phys. Lett. B **319** (1993) 83

[†] F. Scardigli, Phys. Lett. B **452** (1999) 39

[‡] A. Kempf, R. B. Mann, 1995 - Brau, 1999 - Vagenas, 2008

The essence of the Unruh effect

"... *the behavior of particle detectors under acceleration is investigated where it is shown that an accelerated detector even in flat spacetime will detect particles in the vacuum...*

... This result is exactly what one would expect of a detector immersed in a thermal bath of scalar photons of temperature $T_{\mathrm{U}} = a/2\pi \, (= \hbar \, a/2\pi \, k_{\mathrm{B}})$"*

* W. G. Unruh, Phys. Rev. D **14** (1976) 870

Unruh effect from HUP: heuristic (semiclassical) derivation

Consider a particle of mass m at rest in an uniformly accelerated frame. The **kinetic energy** it acquires while the frame moves a distance δx is

$$E_k = m\, a\, \delta x$$

Suppose this energy is barely enough to create N pairs of the same kind of particles from the quantum vacuum (i.e. $E_k \simeq 2\, N\, m$), so that

$$\delta x \simeq 2\,\frac{N}{a} \quad \underset{HUP}{\Longrightarrow} \quad \delta E \simeq \frac{\hbar}{2\,\delta x} \simeq \frac{\hbar\, a}{4\, N}$$

If we interpret δE as a classical **thermal agitation** of the particles (i.e. $\frac{3}{2}\, k_{\mathrm{B}}\, T \simeq \delta E$), then we have

$$T = \frac{\hbar\, a}{6\, N\, k_{\mathrm{B}}} = T_{\mathrm{U}}, \quad \text{for } N = \pi/3 \simeq 1$$

Modified Unruh effect from GUP: heuristic derivation

GUP

$$\delta x \simeq \frac{\hbar}{2\,\delta E} + 2\,\beta\,\ell_{\rm p}^2\,\frac{\delta E}{\hbar} = \ell_{\rm p}\left(\frac{m_{\rm p}}{\delta E} + \beta\,\frac{\delta E}{m_{\rm p}}\right)$$

$$\delta x \simeq 2\,\frac{N}{a}\,,\quad \delta E \simeq \frac{1}{2}\,k_{\rm B}\,T \quad \Longrightarrow \quad \frac{2\,\pi}{a} \simeq \frac{\hbar}{k_{\rm B}\,T} + \beta\,\ell_{\rm p}^2\,\frac{k_{\rm B}\,T}{\hbar}$$

For $\beta\,k_{\rm B}\,T/m_{\rm p} \sim \beta\,m/m_{\rm p} \ll 1$

Modified Unruh temperature

$$T \simeq T_{\rm U}\left(1 + \frac{\beta}{4}\,\frac{\ell_{\rm p}^2\,a^2}{\pi^2}\right) \simeq T_{\rm U}\left[1 + \frac{\beta}{4}\left(\frac{k_{\rm B}\,T_{\rm U}}{m_{\rm p}}\right)^2\right]$$

Unruh effect from canonical QFT

Quantization of the scalar field in Minkowski space

- **Plane-wave** expansion

$$\phi(\mathbf{x}) = \int dk \left[a_k U_k(\mathbf{x}) + a_k^\dagger U_k^*(\mathbf{x})\right], \quad [a_k, a_{k'}^\dagger] = \delta(k-k'),$$

$$a_k\,|0\rangle_{\mathrm{M}} = 0 \;\; \forall k, \qquad U_k(\mathbf{x}) = (4\,\pi\,\omega_k)^{-\frac{1}{2}}\, e^{i(k\,x-\omega_k\,t)}$$

- **Boost-mode** expansion

$$\phi(\mathbf{x}) = \int_0^{+\infty} d\Omega \sum_{\sigma=\pm} \left[d_\Omega^{(\sigma)}\widetilde{U}_\Omega^{(\sigma)}(\mathbf{x}) + d_\Omega^{(\sigma)\dagger}\widetilde{U}_\Omega^{(\sigma)*}(\mathbf{x})\right]$$

$$\widetilde{U}_\Omega^{(\sigma)}(\mathbf{x}) = \int dk\, p_\Omega^{(\sigma)*}(k)\, U_k(\mathbf{x}), \quad p_\Omega^{(\sigma)}(k) = \frac{1}{\sqrt{2\,\pi\,\omega_k}}\left(\frac{\omega_k+k}{\omega_k-k}\right)^{i\,\sigma\,\Omega/2}$$

- Boost-mode operators

$$d_\Omega^{(\sigma)} = \int dk \, p_\Omega^{(\sigma)}(k) \, a_k$$

- **Canonical** commutators

$$\left[a_k, a^\dagger_{k'}\right] = \delta(k-k') \Longleftrightarrow \left[d_\Omega^{(\sigma)}\,,\, d_{\Omega'}^{(\sigma')\,\dagger}\right] = \delta_{\sigma\sigma'}\,\delta(\Omega-\Omega')$$

- **Invariance** of quantum vacuum

$$d_\kappa^{(\sigma)}|0\rangle_{\mathrm{M}} = 0$$

Rindler space

- **Rindler coordinates**

$$t = \xi \sinh \eta, \quad x = \xi \cosh \eta$$

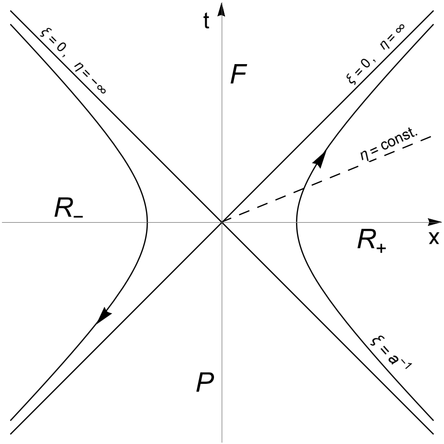
- Rindler vs Minkowski metrics

$$ds^2_{\mathrm{M}} = (dt)^2 - (dx)^2 \rightarrow$$

$$ds^2_{\mathrm{R}} = \xi^2 d\eta^2 - d\xi^2$$

- Worldline of a Rindler observer

$$\eta = a\tau, \quad \xi = \mathrm{const} \equiv a^{-1}$$



Quantization of the scalar field in Rindler space*

$$\phi(\mathbf{x}) = \int_0^{+\infty} d\Omega \sum_{\sigma=\pm} \left[b_\Omega^{(\sigma)}\, u_\Omega^{(\sigma)}(\mathbf{x}) + b_\Omega^{(\sigma)\dagger}\, u_\Omega^{(\sigma)*}(\mathbf{x}) \right], \quad \Omega = \omega/a$$

$$\left[b_\kappa^{(\sigma)}\,,\, b_{\kappa'}^{(\sigma')\,\dagger} \right] = \delta_{\sigma\sigma'}\,\delta\,(\Omega - \Omega'), \quad b_\Omega^{(\sigma)}|0\rangle_{\mathrm{R}} = 0$$

$$u_\Omega^{(\sigma)} = \frac{\theta(\sigma\,\xi)}{2\,\Omega\,\sqrt{2\,\pi}}\, h_\kappa^{(\sigma)}(\xi)\, e^{-i\,\sigma\,\Omega\,\eta}$$

* S. Takagi, Prog. Theor. Phys. Suppl. **88** (1986) 1

Minkowski (boost-mode) quantization: Rindler quantization:

$$\phi = \int d\Omega \sum_{\sigma=\pm} \left[d_\Omega^{(\sigma)} \, \widetilde{U}_\kappa^{(\sigma)} + d_\Omega^{(\sigma)\,\dagger} \, \widetilde{U}_\kappa^{(\sigma)*} \right], \qquad \phi = \int d\Omega \sum_{\sigma=\pm} \left[b_\Omega^{(\sigma)} \, u_\Omega^{(\sigma)} + b_\Omega^{(\sigma)\,\dagger} \, u_\Omega^{(\sigma)*} \right]$$

$$\searrow \qquad \swarrow$$

Bogoliubov transformation

$$b_\Omega^{(\sigma)} = \frac{e^{\pi\Omega}}{\sqrt{e^{2\pi\Omega}-1}} \, d_\Omega^{(\sigma)} + \frac{1}{\sqrt{e^{2\pi\Omega}-1}} \, d_\Omega^{(-\sigma)\,\dagger}$$

- Minkowski vacuum is a **thermal state** for the Rindler observer

$$_{\mathrm{M}}\langle 0 | b_\Omega^{(\sigma)\,\dagger} \, b_{\Omega'}^{(\sigma')} | 0 \rangle_{\mathrm{M}} = \left(e^{a\Omega/T_U} - 1 \right)^{-1} \delta_{\sigma\sigma'} \, \delta(\Omega - \Omega')$$

- Unruh temperature

$$T_{\mathrm{U}} = a/2\pi \, (= \hbar \, a/2\,\pi \, k_{\mathrm{B}})$$

Modified Unruh effect from GUP: QFT derivation

Goal

Can we deform the algebra of the field operators to mimic the modified Unruh temperature?

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Quantum one-dimensional harmonic oscillator

$$A = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega\hat{x} + i\hat{p}), \qquad A^\dagger = \frac{1}{\sqrt{2m\hbar\omega}}(m\omega\hat{x} - i\hat{p})$$

$$[A, A^\dagger] = \frac{1}{i\hbar}[\hat{x}, \hat{p}]$$

$$[\hat{x}, \hat{p}] = i\hbar\left[1 + \beta\left(\frac{\hat{p}}{m_{\mathrm{p}}}\right)^2\right] \Longrightarrow [A, A^\dagger] = \frac{1}{1-\alpha}\left[1 - \alpha\left(A^\dagger A^\dagger + A A - 2A^\dagger A\right)\right]$$

$$\alpha = \beta\frac{m\hbar\omega}{2m_{\mathrm{p}}^2}$$

- Scalar field in the plane-wave representation

$$[A_k, A_{k'}^\dagger] = \frac{1}{1-\tilde{\alpha}}\left[1 - \tilde{\alpha}\left(A_k^\dagger\, A_{k'}^\dagger + A_k\, A_{k'} - 2\, A_k^\dagger\, A_{k'}\right)\right]\delta(k-k')$$

$$\tilde{\alpha} = \beta\,\frac{\hbar^2\omega_k^2}{2\,m_{\mathrm{p}}^2} = 2\,\beta\,\ell_{\mathrm{p}}^2\,\omega_k^2$$

- Scalar field in the boost-mode representation (for $\beta p^2/m_{\mathrm{p}}^2 \ll 1$)

$$\left[D_\Omega^{(\sigma)},\, D_{\Omega'}^{(\sigma')\dagger}\right] = \frac{1}{1-\gamma}\left[1 - \gamma\,\left(D_\Omega^{(\sigma)\dagger}\, D_{\Omega'}^{(-\sigma')\dagger} + D_\Omega^{(\sigma)}\, D_{\Omega'}^{(-\sigma')}\right.\right.$$

$$\left.\left. - D_\Omega^{(\sigma)\dagger}\, D_{\Omega'}^{(\sigma')} - D_\Omega^{(-\sigma)\dagger}\, D_{\Omega'}^{(-\sigma')}\right)\right]\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega')$$

$$\gamma = \beta\,\frac{\hbar^2 a^2\,\Omega^2}{2\,m_{\mathrm{p}}^2} = 2\,\beta\,\ell_{\mathrm{p}}^2\,a^2\,\Omega^2$$

About the *conjectured* commutator...

- The deforming parameter $\tilde{\alpha}$ has been redefined *ad hoc* to suit the boost-mode representation (i.e., $\omega_k \to a\,\Omega$)
- It has been constructed so that the D-operators in the wedges R_+ and R_- are still commuting with each other
- It has been symmetrized with respect to σ and $-\sigma$, so that

$$\left[D_{\Omega}^{(\sigma)}, D_{\Omega'}^{(\sigma')\dagger}\right] = \left[D_{\Omega}^{(-\sigma)}, D_{\Omega'}^{(-\sigma')\dagger}\right]$$

Remark

The deformation of the D-algebra leads to an analogous modification of the commutator between the Rindler B-operators

GUP effects on the Unruh distribution

$$\langle 0_{\mathrm{M}}|\, B_{\Omega}^{(\sigma)\dagger}\, B_{\Omega'}^{(\sigma')}\, |0_{\mathrm{M}}\rangle \;=\; \frac{\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega')}{\left(e^{2\pi\Omega}-1\right)(1-\gamma)} \;\underset{\gamma\ll 1}{\simeq}\; \frac{\delta_{\sigma\sigma'}\,\delta(\Omega-\Omega')}{e^{2\pi\Omega-\gamma}-1}$$

This can be interpreted as a B-E thermal distribution with a **shifted Unruh temperature** T such that

$$2\pi\Omega-\gamma \;=\; \frac{\hbar\, a\,\Omega}{k_{\mathrm{B}}\,T_{\mathrm{U}}}-\gamma \;\equiv\; \frac{\hbar\, a\,\Omega}{k_{\mathrm{B}}\,T} \;\Longrightarrow$$

$$T=\frac{T_{\mathrm{U}}}{1-\beta\,\pi\,\Omega\,k_{\mathrm{B}}^{2}\,T_{\mathrm{U}}^{2}/m_{\mathrm{p}}^{2}} \simeq T_{\mathrm{U}}\left(1+\beta\,\pi\,\Omega\left(\frac{k_{\mathrm{B}}\,T_{\mathrm{U}}}{m_{\mathrm{p}}}\right)^{2}\right)=T_{\mathrm{U}}\left(1+\beta\,\pi\,\Omega\,\frac{\ell_{\mathrm{p}}^{2}\,a^{2}}{\pi^{2}}\right)$$

Remark

T contains an explicit dependence on the Rindler frequency $\Omega=\omega/a$

Thermodynamical argument

Small deformations of HUP $\Longrightarrow$ The modified Unruh radiation is (approximately) a thermal black body radiation $\Longrightarrow$ The majority of Unruh quanta are emitted around a frequency ω such that $\hbar\omega = k_{\mathrm{B}} T_{\mathrm{U}}$

Modified Unruh temperature

$$T \simeq T_{\mathrm{U}} \left(1 + \frac{\beta}{2} \frac{\ell_{\mathrm{p}}^2 a^2}{\pi^2}\right) = T_{\mathrm{U}} \left(1 + \frac{\beta}{2} \left(\frac{k_{\mathrm{B}} T_{\mathrm{U}}}{m_{\mathrm{p}}}\right)^2\right)$$

...to be compared with the heuristic result

$$T \simeq T_{\mathrm{U}} \left(1 + \frac{\beta}{4} \frac{\ell_{\mathrm{p}}^2 a^2}{\pi^2}\right) \simeq T_{\mathrm{U}} \left[1 + \frac{\beta}{4} \left(\frac{k_{\mathrm{B}}\, T_{\mathrm{U}}}{m_{\mathrm{p}}}\right)^2\right]$$

Conclusions and Outlook

- Deviation from thermality of Unruh radiation in the context of GUP
- Small deformations of HUP $\Longrightarrow$ The resulting Unruh distribution exhibits a thermal spectrum with a **modified temperature**

$$T \simeq T_{\mathrm{U}}\Big(1 + \beta\,\mathcal{O}(a^2)\Big)$$

- Agreement between the heuristic and field theoretical approaches

- How does the GUP-induced deviation from thermality match with the non-thermal behaviors of Unruh radiation found in other contexts (Casimir effect, flavor mixing)?

- What happens beyond the approximation of equal deformed algebras for the *A*- and *D*- operators? (Deformation of algebra for field operators)

- Extension of GUP to the Hawking effect (for which the heuristic derivation of the modified temperature can be performed in a way similar to the Unruh radiation*)

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* F. Scardigli, Nuovo Cim. B **110** (1995) 1029


THANK
YOU
GRACIAS
ARIGATO
SHUKURIA
DANKSCHEEN
TASHAKKUR ATU
YAQHANYELAY
SUKSAMA
EKHMET
MEHRBANI
GRAZIE
MAAKE
KOMAPSUMNIDA
GOZAIMASHITA
EFCHARISTO
JUSPAXAR
PALDIES
BOLZİN
MERCI
BİYAN
SHUKRIA
TINGKI

This quantum mechanical approach gives:

- upper bounds on β of non-gravitational origin

$$\text{Landau levels: } \beta \le 10^{46}, \qquad \text{Lamb shift: } \beta \le 10^{20}$$

$$\text{Hydrogen levels: } \beta \le 10^{34}, \quad \text{Nano mech. oscillators: } \beta \le 10^{18}$$

As yet*, it is known that $\beta = \frac{82\pi}{5}$

- **non linear** representations of the fundamental variables $X = f(x)$, $P = g(p)$ in the deformed commutator

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* F. Scardigli, G. Lambiase and E. Vagenas, Phys. Lett. B **767** (2017) 242