Volume and complexity for Warped AdS black holes

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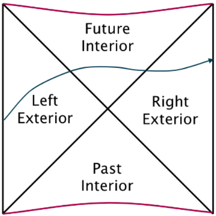
Outline

1. Complexity=volume conjecture
2. Black holes in Warped AdS_3
3. Computation of the volume
4. Conclusions and perspectives

ER=EPR

Consider the Kruskal extension of the AdS black hole.
The dual interpretation is the existence of a thermofield double state

$$|\Psi_{TFD}\rangle \propto \sum_n e^{-E_n\beta/2 - iE_n(t_L+t_R)} |E_n\rangle_R |E_n\rangle_L .$$



Correlators between the two CFTs are non-zero due to entanglement:

$$\langle\Psi_{TFD}|\mathcal{O}_1\mathcal{O}_2|\Psi_{TFD}\rangle \neq 0 \tag{1}$$

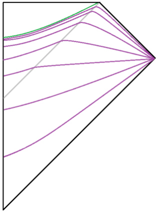
Boundaries are disconnected; the only way to communicate is through the interior regions ⇒ the existence of the Einstein-Rosen bridge allows spacelike correlations (ER=EPR) [Maldacena, Susskind, 2013].

Evolution of Einstein-Rosen bridge

The Einstein-Rosen bridge grows with time far after the black hole reaches thermal equilibrium.
In order to follow the history of the interior region, we foliate spacetime with global spacelike slices [Susskind, 2014]:

- Geodesically complete causal curves must intersect these slices once
- Slices must stay away from curvature singularities
- The entire region outside the horizon must be foliated by these slices

Given the set of spacelike slices anchored on a spatial sphere with infinite radius, we choose the one with maximum volume.
Varying t, we foliate the spacetime with maximal slices.



What represents in the dual theory the growth of the Einstein-Rosen bridge?

Computational complexity

Consider a space of states and the concepts of simple state and simple operation.

Example: a system composed of K classical bits

- Simple state: $(00000000\ldots)$
- Generic state: $(0010111001\ldots)$
- Simple operation: flip a single bit $(0 \leftrightarrow 1)$

Computational complexity is the least number of simple operations needed to obtain a generic final state starting from a simple one.

Classical physical quantities:

- Maximum entropy $S = K \log 2$
- Thermalization time $t_{\rm therm} \sim K^p$
- Maximum complexity $C = K/2$
- Time to get maximally complex $t_{\rm compl} \sim K^p$

Quantum complexity

Quantum mechanically, we assume the existence of an Hilbert space.
Example: a system of K qubits

- Simple state $|0\rangle = |00000\ldots\rangle$
- Generic state $|\psi\rangle = \sum_{i=1}^{2^K} \alpha_i |i\rangle$
- Simple operation: act on 2 qubits

Complexity is the minimum number of simple unitary operators required to transform a simple state into a generic one.
Quantum physical quantities:

- Maximum entropy $S = K \log 2$
- Thermalization time $t_{\mathrm{therm}} \sim K^p$
- Maximum complexity $C = e^K$
- Time to get maximally complex $t_{\mathrm{compl}} \sim e^K$

Complexity=Volume conjecture

Conjecture (Susskind, 2014)

The complexity of the boundary state is proportional to the spatial volume V of a maximal slice sitting behind the horizon:

$$C \sim \frac{\mathrm{Max}(V)}{Gl} \tag{2}$$

Requirements about complexity from the gravity side:

- Complexity is extensive and proportional to the degrees of freedom of the system [Stanford, Susskind, 2014]:

$$\frac{dC}{dt} \sim TS \tag{3}$$

- Extremal black holes are ground states and therefore static ⇒ they have vanishing complexity

We investigate Complexity=Volume conjecture in spacetimes with an holographic dual

Black holes in Warped AdS_3

Warped AdS_3 is a non-trivial modification of AdS_3 which breaks the isometry group from $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ to $SL(2,\mathbb{R})_L \times U(1)_R$.
Black holes in Warped AdS_3 [Anninos, Padi, Song, Strominger, 2008]:

$$\frac{ds^2}{l^2} = dt^2 + \frac{dr^2}{(\nu^2+3)(r-r_+)(r-r_-)} + \left(2\nu r - \sqrt{r_+ r_-(\nu^2+3)}\right) dt d\theta + \frac{r}{4}\left[3(\nu^2-1)r + (\nu^2+3)(r_+ + r_-) - 4\nu\sqrt{r_+ r_-(\nu^2+3)}\right] d\theta^2 \,. \tag{4}$$

- If $\nu = 1$ we recover the BTZ black hole in AdS spacetime
- If $\nu^2 < 1$ the solution admits closed timelike curves [Banados, Barnich, Compère, Gomberoff, 2005]
- In Einstein gravity ν is related to central charges via [Anninos, 2009]

$$c_L = c_R = \frac{12 l \nu^2}{G(\nu^2+3)^{3/2}} \,. \tag{5}$$

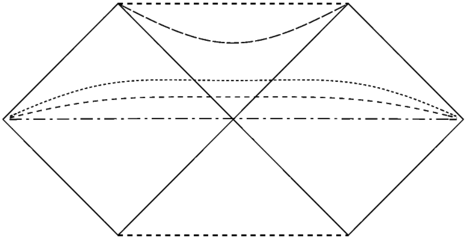
Extremal volume

Time translation symmetry in Schwarzschild coordinates corresponds to invariance under the time evolution in the boundary WCFT with Hamiltonian $H = H_L - H_R$:

$$t_L \to t_L + \Delta t \,, \qquad t_R \to t_R - \Delta t \,. \tag{6}$$

It is not restrictive to consider for the extremal volume the symmetric configuration

$$t_L = t_R \tag{7}$$



Computation of the volume (non-rotating case)

We follow the strategy of [Carmi, Chapman, Marrochio, Myers, Sugishita, 2017]. We put $r_- = 0, r_+ = r_0$.
The volume functional is chosen along the angular direction giving

$$V = 2 \cdot 2\pi \int_{\lambda_{\min}}^{\lambda_{\max}} d\lambda \, l^2 \sqrt{\frac{\dot{u}^2 r}{4} \left[3(\nu^2 - 1)r + (\nu^2 + 3)r_0\right] - \left(\dot{u} r \nu - \frac{\dot{r}}{2}\right)^2}$$
$$= 4\pi \int d\lambda \, \mathcal{L}(r, \dot{r}, \dot{u}) \, ,$$

where u is an ingoing null coordinate.
Normalization condition:

$$\frac{\dot{u}^2 r}{4} \left[3(\nu^2 - 1)r + (\nu^2 + 3)r_0\right] - \left(\dot{u} r \nu - \frac{\dot{r}}{2}\right)^2 = 1 \, . \qquad (8)$$

Conserved quantity:

$$E = \frac{1}{l^2} \frac{\partial \mathcal{L}}{\partial \dot{u}} = \frac{\nu^2 + 3}{4} \dot{u} r (r_0 - r) + \frac{\nu r \dot{r}}{2} \, . \qquad (9)$$

Solving for $\{\dot{r}, \dot{u}\}$ we express the volume as

$$\frac{V}{4\pi l^2} = \int \frac{dr}{\dot{r}} = \frac{1}{2}\int_{r_{\min}}^{r_{\max}} \sqrt{\frac{r\left(3\left(\nu^2-1\right)r + \left(\nu^2+3\right)r_0\right)}{4E^2 + \left(\nu^2+3\right)r\left(r-r_0\right)}}\, dr\,, \tag{10}$$

where $r_{\min}$ is the turning point of the Einstein-Rosen bridge given by

$$\dot{r} = 0 \;\Rightarrow\; r_{\min}^2 - r_0 r_{\min} + \frac{4E^2}{(3+\nu^2)} = 0\,. \tag{11}$$

The difference of u coordinates is:

$$u(r_{\max}) - u(r_{\min}) = \int_{r_{\min}}^{r_{\max}} dr \frac{\dot{u}}{\dot{r}} =$$

$$= \int_{r_{\min}}^{r_{\max}} dr \left[\frac{2}{(\nu^2+3)(r_0-r)}\left(\frac{E}{r}\sqrt{\frac{r\left(3\left(\nu^2-1\right)r + \left(\nu^2+3\right)r_0\right)}{4E^2 + \left(\nu^2+3\right)r\left(r-r_0\right)}} - \nu\right)\right], \tag{12}$$

where

$$\lim_{r_{\max}\to\infty} u(r_{\max}) - u(r_{\min}) = t_R + r^*(r_{\max}) - r^*(r_{\min})\,. \tag{13}$$

The volume can be written as

$$\frac{V}{4\pi l^2} = E(u(r_{\rm max}) - u(r_{\rm min})) + \int_{r_{\rm min}}^{r_{\rm max}} dr \left\{ \frac{2\nu E}{(\nu^2+3)(r_0-r)} - \frac{\sqrt{r\left[4E^2 - r(r_0-r)(\nu^2+3)\right]\left[(\nu^2+3)r_0 + 3r(\nu^2-1)\right]}}{2(\nu^2+3)r(r_0-r)} \right\} . \tag{14}$$

Differentiating with respect to time we get

$$\frac{1}{2l}\frac{dV}{dt_R} = \frac{dV}{d\tau} = 2\pi l E \, . \tag{15}$$

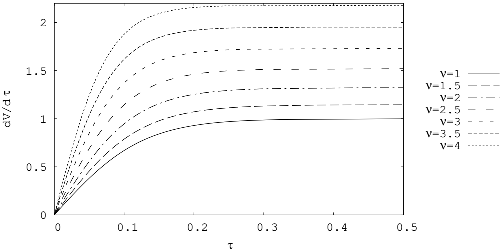


Figure 1: Time dependence of $dV/d\tau$ in units of πl, for $r_0 = 1$ and various values of the warping parameter ν

Rotating case

Conserved quantity and radius of the turning point:

$$E = \frac{1}{l^2}\frac{\partial \mathcal{L}}{\partial \dot{u}}\,, \qquad r_{\min} = \frac{r_+ + r_-}{2}\left(1 \pm \sqrt{1 - \frac{16E^2}{(\nu^2+3)(r_+ + r_-)^2}}\right) . \quad (16)$$

The result is still

$$\frac{dV}{d\tau} = 2\pi l E\,. \quad (17)$$

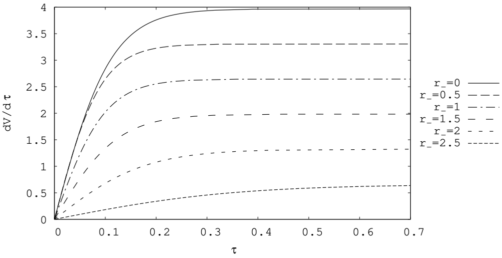


Figure 2: Time dependence of $dV/d\tau$ in units of πl, for $r_+ = 3, \nu = 2$ and various values of the inner radius r_-

Late time complexity

In the late time limit, the volume is invariant under translations in t and rotations in $\theta \Rightarrow$ the maximal slice sits at constant $r = \hat{r}$ [Susskind, 2014]. Extremizing the volume, the only possible constant-r slice sits at

$$\hat{r} = \frac{r_+ + r_-}{2} \Rightarrow \lim_{\tau \to \infty} \frac{dV}{d\tau} = \frac{\pi l}{2}(r_+ - r_-)\sqrt{3+\nu^2} \,. \tag{18}$$

Consistency checks:

- It vanishes in the extremal case
- It is proportional to the product

$$TS = \frac{(r_+ - r_-)(3+\nu^2)}{16G} \,. \tag{19}$$

- It satisfies a bound involving the conserved charges of the black hole [Cai, Ruan, Wang, Yang, Peng, 2016]

$$\frac{dV}{d\tau} \lesssim [(M - \Omega J)_+ - (M - \Omega J)_-] = TS \,. \tag{20}$$

Holographic dictionary for complexity

In AdS_D case the standard dictionary is

$$\lim_{\tau\to\infty}\frac{dV}{d\tau}=\frac{8\pi Gl}{D-1}TS\,,\qquad C=(D-1)\frac{V}{Gl}\,. \tag{21}$$

In the warped AdS_3 case we obtain

$$\lim_{\tau\to\infty}\frac{dV}{d\tau}=4\pi Gl\,\eta\,TS\,,\qquad \eta=\frac{2}{\sqrt{3+\nu^2}}\,. \tag{22}$$

Possible interpretations:

- Complexity approaches at late times ηTS with $\eta\le 1 \Rightarrow$ warping would make the complexity rate decrease
- The holographic dictionary is

$$C=\frac{2}{Gl\eta}V\,, \tag{23}$$

and the rate always saturates at TS

Conclusions and perspectives

Conclusions:

- Complexity rate is a monotonically increasing function of time
- It is proportional to TS at late times
- It satisfies a bound involving the conserved charges of the black hole

Future developments:

- Study of generalizations of Complexity=Volume conjecture for Warped black holes seen as solutions of NMG, TMG ... [Bueno, Min, Speranza, Visser, 2016],[Alishahiha et al., 2017]
- Study of complexity in the boundary WCFT [Caputa, Kundu, Miyaji, Takayanagi, Watanabe, 2017]
- Study of the Complexity=Action conjecture for Warped black holes [Brown, Roberts, Susskind, Swingle, Zhao, 2016]

Thank you for the attention!