Role of neutrino mixing in accelerated proton decay

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* M. Blasone, G. Lambiase, G. G. Luciano, L. P. , Phys. Rev. D **97**, 105008 (2018).

Outline

- Motivations
- Preliminary tools
  - Fulling-Davies-Unruh effect
  - Neutrino mixing
- Historical excursus on inverse β decay
- Inverse β decay in the context of neutrino mixing
- Conclusions and outlook

Motivations

- Testing the consistency of QFT in curved background by comparing the decay rate of an accelerated proton in the inertial and comoving frame: a ‘theoretical check" of the **Unruh effect**[*].

- Clarifying some conceptual problems in the context of the inverse β decay with mixed neutrinos[†].

- Investigating the issue of mass or flavor neutrino states as fundamental objects in QFT.[‡]

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[*] G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. D **59**, 094004 (1999).
[†] D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016).
[‡] M. Blasone and G. Vitiello, Annals Phys. **244**, 283 (1995).

Unruh effect

- **Rindler coordinates**

$$x^0 = \xi \sinh\eta, \quad x^3 = \xi \cosh\eta$$

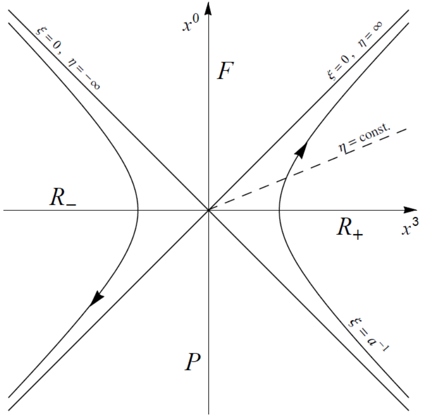
- Rindler vs Minkowski

$$ds^2_{\mathcal{M}} = (dx^0)^2 - (dx^3)^2 - (d\vec{x})^2 \rightarrow$$

$$ds^2_{\mathcal{R}} = \xi^2 d\eta^2 - d\xi^2 - (d\vec{x})^2$$

- Worldline of a Rindler observer

$$\eta = a\,\tau, \quad \xi = \mathrm{const} \equiv a^{-1}, \quad \vec{x} = \mathrm{const}$$



Fulling-Davies-Unruh effect

The Rindler observer perceives Minkowski vacuum as a **thermal bath**

$$\langle 0_{\mathcal{M}}|\widehat{N}(\omega)|0_{\mathcal{M}}\rangle \equiv n(\omega) = \frac{1}{e^{a\,\omega/T_{FDU}} + 1}$$

where

$$T_{FDU} = \frac{a}{2\pi}$$

is the **Fulling-Davies-Unruh** temperature.

Neutrino mixing

Pontecorvo mixing transformations (two flavor model)...

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$

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$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$

...lead to the quantum mechanical oscillation probability

$$P_{e\to\mu} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right).$$

Decay of accelerated particles

The decay properties of particles are less fundamental than commonly thought*.

* R. Muller, Phys. Rev. D **56**, 953 (1997).

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Inverse β decay

$$p \rightarrow n + e^{+} + \nu_e$$

...the proton decay is not kinematically forbidden!

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Remark

The lifetime of particles is not an absolute concept.

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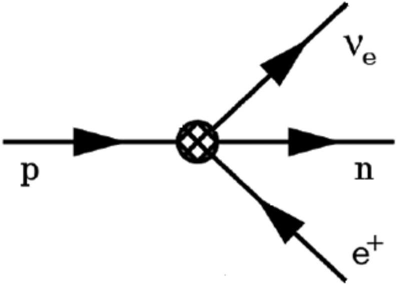
Inverse β decay (inertial frame)

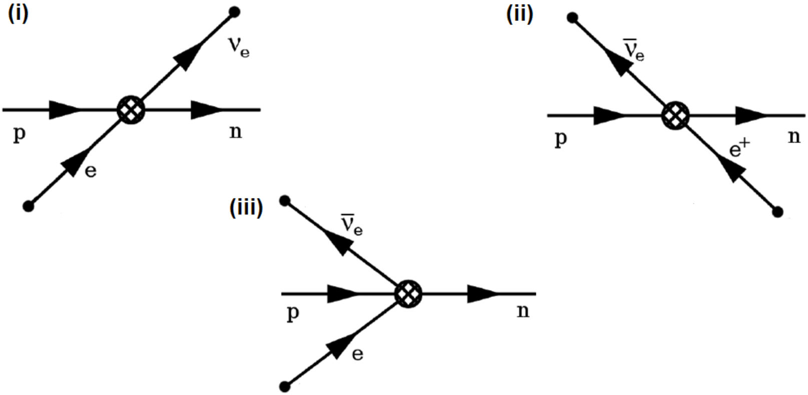
$$p \to n + e^{+} + \nu_e$$



Inverse β decay (comoving frame)

$$(i)\ p + e \to n + \nu_e \qquad (ii)\ p + \bar{\nu}_e \to n + e^+$$

$$(iii)\ p + e + \bar{\nu}_e \to n$$



Setting the stage

In 2D with **massless** neutrino ($a \ll M_{Z^0}, M_{W^\pm} \approx 10^{36} cm/s^2$)* :

$$\widehat{j}^\mu = \widehat{q}(\tau) u^\mu \delta\left(u - a^{-1}\right), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau} \widehat{q}_0 e^{-i\widehat{H}\tau}$$

$$\widehat{H}\,|n\rangle = m_n\,|n\rangle\,, \quad \widehat{H}\,|p\rangle = m_p\,|p\rangle\,, \quad G_F = |\langle p|\,\hat{q}_0\,|n\rangle|$$

In this regime a Fermi current-current interaction can be considered

$$\widehat{S}_I = \int d^2x \sqrt{-g}\,\widehat{j}_\mu \left(\widehat{\overline{\Psi}}_\nu \gamma^\mu \widehat{\Psi}_e + \widehat{\overline{\Psi}}_e \gamma^\mu \widehat{\Psi}_\nu\right).$$

*D. A. T. Vanzella and G. E. A. Matsas, Phys. Rev. Lett. **87** 151301 (2001).

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm}\int_{-\infty}^{+\infty} dk \left[\widehat{b}_{k\sigma}\,\psi_{k\sigma}^{(+\omega)} + \widehat{d}_{k\sigma}^{\dagger}\,\psi_{-k-\sigma}^{(-\omega)}\right], \qquad \omega = \sqrt{m^2+\mathbf{k}^2}$$

$$\psi_{k+}^{(\pm\omega)} = \frac{e^{i\left(\mp\omega x^0 + kx^3\right)}}{\sqrt{2\pi}} \begin{pmatrix} \pm\sqrt{(\omega\pm m)/2\omega} \\ 0 \\ k/\sqrt{2\omega\,(\omega\pm m)} \\ 0 \end{pmatrix}$$

$$\psi_{k-}^{(\pm\omega)} = \frac{e^{i\left(\mp\omega x^0 + kx^3\right)}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \pm\sqrt{(\omega\pm m)/2\omega} \\ 0 \\ -k/\sqrt{2\omega\,(\omega\pm m)} \end{pmatrix}$$

Inertial frame calculation

The tree-level transition amplitude...

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I \, |0\rangle \otimes |p\rangle$$

Inertial frame calculation

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... and the related differential transition rate...

$$\frac{d^2\mathcal{P}^{p\to n}_{in}}{dk_e dk_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}|^2\,, \qquad \frac{\mathcal{P}^{p\to n}}{T} = \Gamma^{p\to n}$$

Inertial frame calculation

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... give the inertial decay rate

$$\Gamma^{p\to n}_{in} = \frac{4G_F^2 a}{\pi^2 e^{\pi\Delta m/a}} \int_0^\infty d\tilde{k}_e \int_0^\infty d\tilde{k}_\nu K_{2i\Delta m/a}\left[2\left(\tilde{\omega}_e + \tilde{\omega}_\nu\right)\right].$$

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \left[\widehat{b}_{\omega\sigma}\, \psi_{\omega\sigma} + \widehat{d}^{\dagger}_{\omega\sigma}\, \psi_{-\omega-\sigma}\right]$$

$$\psi_{\omega+} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \\ -K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \end{pmatrix} e^{-i\omega\eta/a}$$

$$\psi_{\omega-} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \\ K_{i\omega/a+1/2}(m\xi) - iK_{i\omega/a-1/2}(m\xi) \end{pmatrix} e^{-i\omega\eta/a}$$

Comoving frame calculation

The transition amplitude for each process...

$$\mathcal{A}^{p\to n}_{(\mathcal{I})} \;=\; \langle n| \otimes \langle\, emit\, |\widehat{S}_I|\, abs\, \rangle \otimes |p\rangle\,, \quad \mathcal{I} = i, ii, iii$$

Comoving frame calculation

The transition amplitude for each process...

$$\mathcal{A}^{p\to n}_{(\mathcal{I})} = \langle n| \otimes \langle\, emit\, |\widehat{S}_I|\, abs\,\rangle \otimes |p\rangle\,, \quad \mathcal{I} = i, ii, iii$$

... and the respective differential transition rates...

$$\frac{d^2\mathcal{P}^{p\to n}_{\mathcal{I}}}{d\omega_e d\omega_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} \left|\mathcal{A}^{p\to n}_{\mathcal{I}}\right|^2 n_F^{(abs)}(\omega_{e(\nu)})[1 - n_F^{(emit)}(\omega_{\nu(e)})],$$

$$n_F(\omega) = \frac{1}{1 + e^{2\pi\omega/a}}$$

Comoving frame calculation

... give the total decay rate

$$\Gamma_{com}^{p\to n} = \Gamma_{(i)}^{p\to n} + \Gamma_{(ii)}^{p\to n} + \Gamma_{(iii)}^{p\to n}$$

$$= \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a) K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\left(\omega-\Delta m\right)/a\right]}.$$

Comoving frame calculation

... give the total decay rate

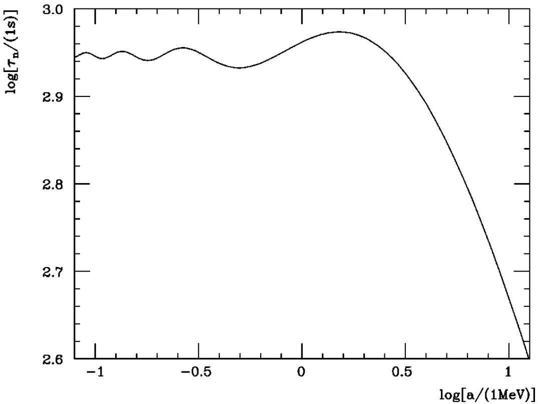
$$\Gamma^{p\to n}_{com} = \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}$$

$$= \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a)K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\left(\omega-\Delta m\right)/a\right]}.$$

Result

At tree level

$$\Gamma^{p\to n}_{in} = \Gamma^{p\to n}_{com}$$



Remarks

The equality of the two decay rates confirms:

- the necessity of Unruh effect in QFT
- the General Covariance of QFT in curved background

Generalizing to 4D with **massive neutrino***...

$$\widehat{j}^{\mu} = \widehat{q}(\tau) u^{\mu} \delta\left(u - a^{-1}\right) \delta(x^1) \delta(x^2), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau} \widehat{q}_0 e^{-i\widehat{H}\tau}$$

$$\widehat{H} \left| n \right\rangle = m_n \left| n \right\rangle, \quad \widehat{H} \left| p \right\rangle = m_p \left| p \right\rangle, \quad G_F = \left| \left\langle p \right| \hat{q}_0 \left| n \right\rangle \right|$$

$$\widehat{S}_I = \int d^4x \sqrt{-g}\, \widehat{j}_{\mu} \left(\widehat{\overline{\Psi}}_{\nu} \gamma^{\mu} \widehat{\Psi}_e + \widehat{\overline{\Psi}}_e \gamma^{\mu} \widehat{\Psi}_{\nu} \right)$$

* H. Suzuki and K. Yamada, Phys. Rev. D **67** (2003).

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int d^3k \left[\widehat{b}_{\mathbf{k}\sigma} \psi^{(+\omega)}_{\mathbf{k}\sigma} + \widehat{d}^{\dagger}_{\mathbf{k}\sigma} \psi^{(-\omega)}_{-\mathbf{k}-\sigma} \right]$$

$$\psi^{(\pm\omega)}_{\mathbf{k}+}(x^0, \mathbf{x}) = \frac{e^{i(\mp\omega x^0 + \mathbf{k}\cdot\mathbf{x})}}{2^2 \pi^{\frac{3}{2}}} \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} m \pm \omega \\ 0 \\ k^3 \\ k^1 + ik^2 \end{pmatrix}$$

$$\psi^{(\pm\omega)}_{\mathbf{k}-}(x^0, \mathbf{x}) = \frac{e^{i(\mp\omega x^0 + \mathbf{k}\cdot\mathbf{x})}}{2^2 \pi^{\frac{3}{2}}} \frac{1}{\sqrt{\omega(\omega \pm m)}} \begin{pmatrix} 0 \\ m \pm \omega \\ k^1 - ik^2 \\ -k^3 \end{pmatrix}$$

Inertial frame calculation

Using the integral representation of the Bessel function

$$K_\mu(z) \;=\; \frac{1}{2}\int_{C_1}\frac{ds}{2\pi i}\Gamma(-s)\Gamma(-s-\mu)\left(\frac{z}{2}\right)^{2s+\mu},$$

together with the expansion formula...

$$(A+B)^z=\int_C\frac{dt}{2\pi i}\,\frac{\Gamma(-t)\,\Gamma(t-z)}{\Gamma(-z)}\,A^{t+z}\,B^t$$

Inertial frame calculation

...the decay rate in the inertial frame becomes

$$\Gamma_{in}^{p\to n} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{(\frac{m_e}{a})^2\, (\frac{m_\nu}{a})^2}{\Gamma(-s-t+3)\,\Gamma\,(-s-t+7/2)}$$

$$\times \Big[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+\,\mathrm{Re}\Big\{\Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4)\Big\}$$

$$\times\, \Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2))\Big],$$

where $C_{s(t)}$ is the path picking up all poles of gamma functions in $s(t)$ complex plane.

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \int d^2k \left[\widehat{b}_{\mathbf{w}\sigma}\psi^{(+\omega)}_{\mathbf{w}\sigma} + \hat{d}^{\dagger}_{\mathbf{w}\sigma}\psi^{(-\omega)}_{\mathbf{w}-\sigma}\right], \qquad \mathbf{w} \equiv (\omega, k^x, k^y)$$

$$\psi^{(\omega)}_{\mathbf{w}+} = N\,\frac{e^{i(-\omega\eta/a+k_x x+k_y y)}}{(2\pi)^{\frac{3}{2}}} \begin{pmatrix} i l K_{i\omega/a-1/2}(\xi l) + m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \\ i l K_{i\omega/a-1/2}(\xi l) - m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \end{pmatrix}$$

with $l = \sqrt{m^2 + (k^x)^2 + (k^y)^2}$.

Comoving frame calculation

Summing up the contributions of the three processes and using

$$x^\sigma K_\nu K_\mu = \frac{\sqrt{\pi}}{2} G^{40}_{24}\left(x^2 \middle| \begin{matrix} \frac{1}{2}\sigma,\ \frac{1}{2}\sigma\ +\ \frac{1}{2} \\ \frac{1}{2}(\nu+\mu+\sigma), \frac{1}{2}(\nu-\mu+\sigma), \frac{1}{2}(-\nu+\mu+\sigma), \frac{1}{2}(-\nu-\mu+\sigma)\end{matrix}\right),$$

the total decay rate in the comoving frame becomes

$$\Gamma^{p\to n}_{com} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{\left(\frac{m_e}{a}\right)^2 \left(\frac{m_\nu}{a}\right)^2}{\Gamma(-s-t+3)\,\Gamma\,(-s-t+7/2)}$$

$$\times \Big[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+\,\mathrm{Re}\Big\{\Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4)\Big\}$$

$$\times\, \Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2))\Big].$$

Proton decay and neutrino mixing: a theoretical paradox?

Recently, it has been argued that **neutrino mixing** can spoil the agreement between the two results*.

The *leitmotiv* is the violation of the **Kubo-Martin-Schwinger** (KMS) definition of thermal state for the accelerated neutrino vacuum in the context of mixing.

It is claimed that the contradiction must be solved **experimentally**.

* D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016).

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Remark

An experiment cannot be used as a tool for checking the internal consistency of theory against a theoretical paradox.

The question must be settled at a theoretical level, in conformity with the General Covariance of QFT in curved background.

An attempt to solve the ambiguity has been proposed*, but...

Inverse β decay with neutrino mixing

$$p \to n + \bar{\ell}_\alpha + \nu_i, \qquad \ell = \{e, \tau, \mu\}, \quad i = \{1, 2, 3\}$$

...there are several problems related to the use of definite mass neutrinos!

It is possible to prove that their choice leads to ambiguities†.

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*G. Cozzella, S. A. Fulling, A. G. S. Landulfo, G. E. A. Matsas and D. A. T. Vanzella, arXiv:1803.06400, to appear in Phys. Rev. D.

†S. M. Bilenky, F. von Feilitzsch and W. Potzel, J. Phys. G **38**, 115002 (2011).

Inertial frame calculation

Applying Pontecorvo transformations on both neutrino fields and states, the transition amplitude becomes*

$$\mathcal{A}_{in}^{p\to n} = G_F \Big[\cos^2\theta\, \mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_1}, \omega_e) + \sin^2\theta\, \mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_2}, \omega_e) \Big],$$

$$\mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_i}, \omega_e) = \int_{-\infty}^{+\infty} d\tau\, e^{i\Delta m \tau}\, u_\mu \left[\bar{\psi}_{\sigma_\nu}^{(+\omega_{\nu_i})}\, \gamma^\mu\, \psi_{-\sigma_e}^{(-\omega_e)} \right], \qquad i = 1, 2$$

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* M. Blasone, G. Lambiase, G. G. Luciano, L. P., Phys. Rev. D **97**, 105008 (2018).

Consequently, the total decay rate is

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} \,+\, \sin^4\theta\,\Gamma_2^{p\to n} \,+\, \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_j^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_j},\omega_e)\right|^2, \quad j=1,2,$$

$$\Gamma_{12}^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \Big[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e)\,\mathcal{I}^*_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e)+\mathrm{c.c.}\Big]$$

Comoving frame calculation

Assuming neutrino asymptotic states to be **mass eigenstates**, calculations in the comoving frame give for the process (*i*)

$$\mathcal{A}^{p\to n}_{(i)} = \frac{G_F}{a}\Big[\cos\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\Big],$$

$$\mathcal{J}_{\sigma_\nu\sigma_e} = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\eta}\, u_\mu\left[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\right]$$

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$$\mathcal{J}_{\sigma_\nu\sigma_e} = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\eta}\, u_\mu\left[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\right]$$

Similar calculations for the other two processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} \;&\equiv\; \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}\\ &=\; \cos^2\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\,\widetilde{\Gamma}^{p\to n}_2.\end{aligned}$$

Comoving frame calculation

$$\Gamma^{p\to n}_{com} = \cos^2\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\,\widetilde{\Gamma}^{p\to n}_2,$$

$$\begin{aligned}\widetilde{\Gamma}^{p\to n}_j = & \frac{2\,G_F^2}{a^2\,\pi^7\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega\,\int d^2k_\nu\,l_{\nu_j}\left|K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_j}}{a}\right)\right|^2\\ & \times\int d^2k_e\,l_e\left|K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 + m_{\nu_j}m_e\\ & \times\mathrm{Re}\left\{\int d^2k_\nu K^2_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_j}}{a}\right)\int d^2k_e K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right\}\end{aligned}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^2\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^2\theta\,\widetilde{\Gamma}_2^{p\to n},$$

Although:

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^2\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^2\theta\,\widetilde{\Gamma}_2^{p\to n},$$

Although:

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

- there is no counterpart of $\Gamma_{12}^{p\to n}$ in $\Gamma_{com}^{p\to n}$
- Pontecorvo matrix elements appear with different powers.

Remark

Neutrino asymptotic states must be inevitably **flavor eigenstates**

Violating the KMS condition?

Assuming asymptotic neutrinos to be **flavor eigenstates** *would violate the KMS definition of a thermal state of a quantum system by adding coherent, off-diagonal correlations in the density matrix. Consequently, the accelerated neutrino vacuum state would not be thermal, contradicting the essential characteristic of the Unruh effect*[*].

[*] D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016).

Non-thermal Unruh effect for mixed neutrinos

Two Bogoliubov transformations involved*:

$$\phi_{\mathcal{R}} \xrightarrow{\text{thermal Bogol. } (a)} \phi_{\mathcal{M}} \Rightarrow \text{condensate in } |0_{\mathcal{M}}\rangle$$

$$\phi_1, \phi_2 \xrightarrow{\text{mixing Bogol. } (\theta)} \phi_e, \phi_\mu \Rightarrow \text{condensate in } |0_{e,\mu}\rangle$$

*M. Blasone, G. Lambiase and G. G. Luciano, Phys. Rev. D **96** 025023 (2017).

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$$\phi_1, \phi_2 \xrightarrow{\text{mixing Bogol. } (\theta)} \phi_e, \phi_\mu \Rightarrow \text{condensate in } |0_{e,\mu}\rangle$$

How do they combine when flavor mixing for an accelerated observer is considered?

*M. Blasone, G. Lambiase and G. G. Luciano, Phys. Rev. D **96** 025023 (2017).

Non-thermal Unruh effect for mixed fields

Condensation density of **Rindler mixed neutrinos** in $|0\rangle_{\mathcal{M}}$:

$$\langle 0_{\mathcal{M}} | \widehat{N}(\theta,\omega) | 0_{\mathcal{M}} \rangle = \underbrace{\frac{1}{e^{a\,\omega/T_{FDU}}+1}}_{Unruh\ \ thermal\ \ spectrum} + \underbrace{\sin^2\theta \left\{ \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)\right\}}_{non-thermal\ \ mixing\ \ corrections} .$$

Remark

Non-thermal corrections only appear at orders higher than $\mathcal{O}\left(\frac{\delta m}{m}\right)$

Comoving frame calculation with flavor eigenstates

Taking neutrino asymptotic states to be **flavor eigenstates**, calculations in the comoving frame give for the process (*i*)

$$\mathcal{A}_{(i)}^{p\to n} \;=\; \frac{G_F}{a}\left[\cos^2\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\;+\;\sin^2\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\right],$$

$$\mathcal{J}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\eta}\, u_\mu\left[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\right]$$

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Analogous procedures for the other processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} &\equiv \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)} \\ &= \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12}\end{aligned}$$

$$\Gamma^{p\to n}_{com} = \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12},$$

$$\begin{aligned}
\widetilde{\Gamma}^{p\to n}_{12} &= \frac{2\,G_F^2}{a^2\,\pi^7\,\sqrt{l_{\nu_1} l_{\nu_2}}\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega \left\{ \int d^2k_e\, l_e \left| K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 \right. \\
&\times \int d^2k_\nu \left(\kappa_\nu^2 + m_{\nu_1} m_{\nu_2} + l_{\nu_1} l_{\nu_2}\right) \\
&\times \mathrm{Re}\left\{ K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_1}}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\} \\
&+ m_e \int d^2k_e \int d^2k_\nu \left(l_{\nu_1} m_{\nu_2} + l_{\nu_2} m_{\nu_1}\right) \\
&\times \mathrm{Re}\left\{ K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_1}}{a}\right) \right. \\
&\left.\left. \times\, K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\right\}, \quad \kappa_\nu \equiv (k_\nu^x, k_\nu^y)
\end{aligned}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

...what about the "off-diagonal" terms?

$$\Gamma_{12}^{p\to n} \stackrel{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Small neutrino mass approximation

Evaluating these terms is non-trivial.

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However, for $\frac{\delta m}{m_{\nu_1}} \equiv \frac{m_{\nu_2} - m_{\nu_1}}{m_{\nu_1}} \ll 1$,

$$\Gamma_{12}^{p\to n} = 2\Gamma_{1}^{p\to n} + \frac{\delta m}{m_{\nu_1}}\Gamma_{\delta m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

$$\widetilde{\Gamma}_{12}^{p\to n} = 2\widetilde{\Gamma}_{1}^{p\to n} + \frac{\delta m}{m_{\nu_1}}\widetilde{\Gamma}_{\delta m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

Taking a suitable limit

The calculation of Γ_{δ_m} and $\widetilde{\Gamma}_{\delta_m}$ for $m_{\nu_1} \neq 0$ is still an hard task.

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Taking a suitable limit

The calculation of Γ_{δ_m} and $\widetilde{\Gamma}_{\delta_m}$ for $m_{\nu_1} \neq 0$ is still an hard task.

Significant simplifications arise from the limit

$$m_{\nu_1} \to 0.$$

Remark

The limit has a purely mathematical meaning.

Result...

$$\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = \frac{\widetilde{\Gamma}_{\delta_m}}{m_{\nu_1}}$$

... and its full expression

$$\begin{aligned}\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = & \lim_{\varepsilon\to 0}\frac{G_F^2\, m_e\, a^3}{\pi^3\, e^{\pi\Delta m/a}}\int_{C_s}\frac{ds}{2\pi i}\int_{C_t}\frac{dt}{2\pi i}\left(\frac{\varepsilon}{a}\right)^{2s+2}\left(\frac{m_e}{a}\right)^{2t+2} \\ & \times\frac{\Gamma(-2s)\Gamma(-2t)\Gamma(-t-1)\Gamma(-s-1)}{\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-2s-2t)} \\ & \times\left[\Gamma\left(-s-t+1+i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1-i\frac{\Delta m}{a}\right)\right. \\ & \left.+\Gamma\left(-s-t+1-i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1+i\frac{\Delta m}{a}\right)\right]\end{aligned}$$

Conclusions

- Unruh radiation gets non-trivially modified in the context of flavor mixing.
- Neutrino asymptotic states must be inevitably flavor eigenstates for the General Covariance of QFT to be preserved.
- The agreement between the two decay rates is restored in the first-order approximation.

Outlook

What happens beyond the first-order approximation?

$$\Gamma_{in}^{p\rightarrow n} = \Gamma_{com}^{p\rightarrow n}$$

$$\downarrow$$

The paradox would be solved at a theoretical level

$$\Gamma_{in}^{p\rightarrow n} \neq \Gamma_{com}^{p\rightarrow n}$$

$$\downarrow$$

- neutrino mixing is at odds with General Covariance
- Unruh effect with neutrino mixing should be revised
- Pontecorvo transformations are not consistent with QFT*

*M. Blasone and G. Vitiello Annals Phys. **244** 283 (1995).

THANKS FOR YOUR ATTENTION
AND
PLEASE ASK BUT NOT TOO MUCH