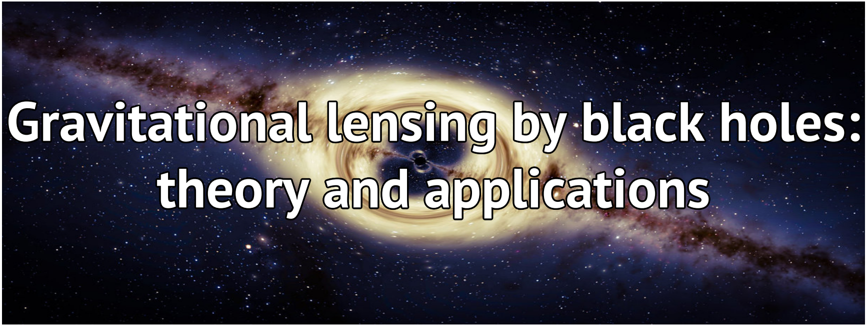

Gravitational lensing by black holes: theory and applications

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in collaboration with Valerio Bozza

Università degli Studi di Salerno, INFN Sezione di Napoli

New Frontiers in Theoretical Physics, Cortona, 25/05/2018

Outline

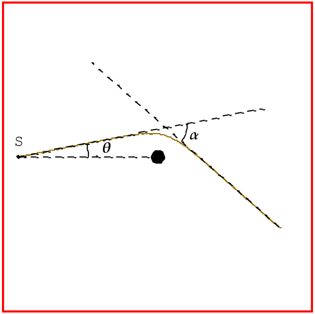
1. Gravitational lensing by black holes

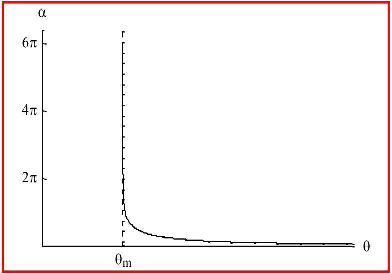
2. Strong deflection limit

3. Higher order images in astrophysical observables

4. Conclusions

Gravitational lensing by black holes





- The deflection angle α is related to the impact angle θ and diverges as θ tends to θ_{m}.

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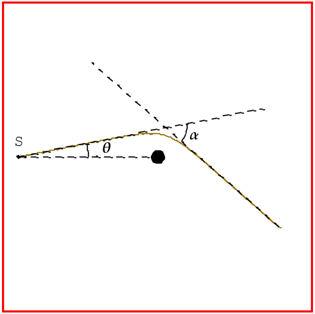

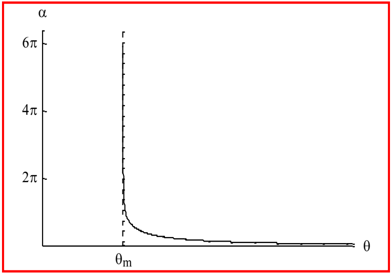

- The deflection angle α is related to the impact angle θ and diverges as θ tends to θ_{m}.
- No gravitational lensing images can be found within θ_{m} (Shadow of the black hole).

Darwin's formula and SDL

- Schwarzschild deflection angle can be exactly expressed as an elliptic integral [1].

[1] C. Darwin, Proc. R. Soc. Lond. A 249 180 (1959)

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- For large values of the impact angle, we recover the weak deflection limit calculated by Einstein

$$\alpha(\theta) \to \frac{4MG}{D_{OL}\theta}.$$

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- For small values of the impact angle, Darwin proposed the approximate form

$$\alpha(\theta) = -a \log\left(\frac{\theta}{\theta_m} - 1\right) + b + O(\theta - \theta_m),$$

with

$$D_{OL}\theta_m = 3\sqrt{3}/M, \quad a = 1, \quad b = \log(216(7 - 4\sqrt{3})) - \pi.$$

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Darwin's formula and SDL

- It can be proved that the divergence in the deflection angle for $\theta \to \theta_{\mathrm{m}}$ is always logarithmic for any kind of black hole metrics[2].

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- Therefore, one can write a universal formula for gravitational lensing by spherically symmetric black holes in the Strong Deflection Limit:

$$\alpha(\theta) = -a \log\left(\frac{\theta}{\theta_{\mathrm{m}}} - 1\right) + b + O(\theta - \theta_{\mathrm{m}}),$$

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- The coefficients $\theta_{\rm m}, a, b$ are functions of the black hole metric.
- On the observer plane $\theta_{\rm m}$ is the angular radius of the shadow of the black hole.

[2] V. Bozza, Phys. Rev. D 66 (2002) 103001

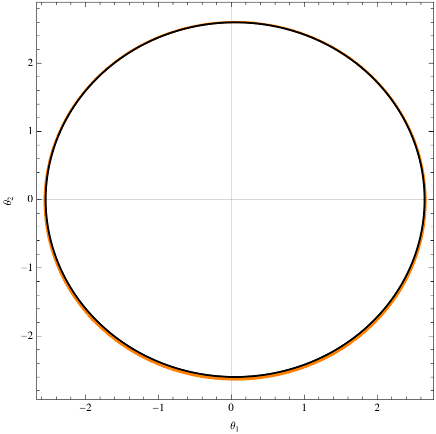
Images formation

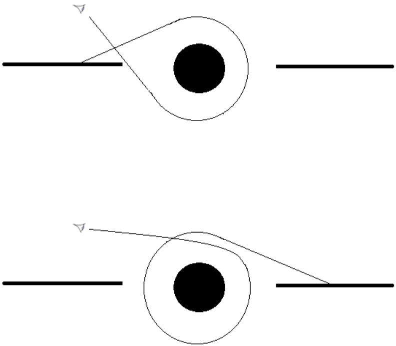
- The strong deflected photons will form an infinite number of images, named *higher-order images*.
- These images are very faint[3].
- Their positions are given by:

$$\theta = \theta_{\mathrm{m}} \left(1 + e^{\frac{b-\gamma}{a}}\right).$$

[3] V. Bozza, Phys. Rev. D 66 (2002) 103001

Images formation





Black hole lensing: where?

- By integrating geodesics in General Relativity we have found fascinating phenomena created by photons winding closely around black holes.

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- One of the effects due to the bending of light is the astrometric shift of S-stars around the black hole[4].
- Characterisation of the Fe Kα line emitted by an accretion disk surrounding a central black hole[5][6].

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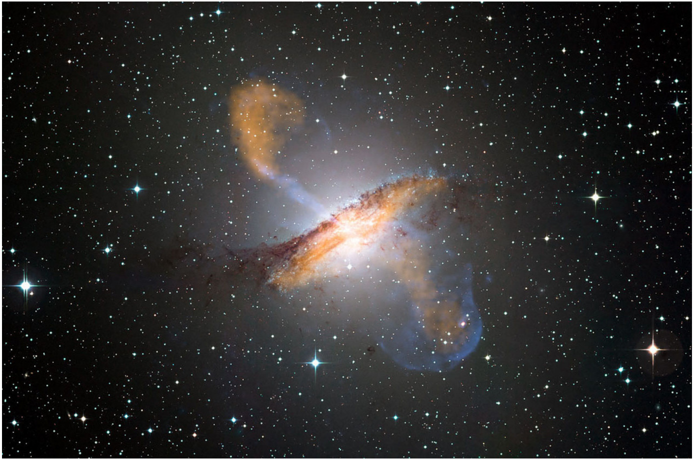
Iron Kα line emitted by an accretion disk



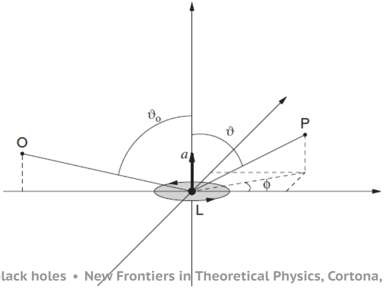
Credits: NASA and ESO

Iron Kα line emitted by an accretion disk

On the observer plane (θ_1, θ_2) ***the specific flux*** is given by:

$$F_o(\mu_o, E_o) = \int\int g^3 r_e^{-3}(1 + 2.06\mu_e)\,\mathrm{d}\theta_1\mathrm{d}\theta_2.$$

E_o is the observed energy, μ_e is the cosine of the emission angle of a single photon, g is the frequency shift factor, r_e is the distance between the source and the centre of the black hole and μ_o is the cosine of the inclination angle of the accretion disk with respect to the line of sight.



Higher order images in Fe Kα line spectra

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- We consider ***strongly deflected photons***, i.e., photons that perform one or more revolutions around the black hole before reaching the observer.
- Their contribution produces distortions of iron line profile.

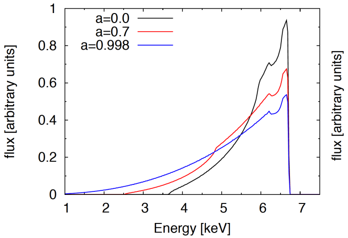


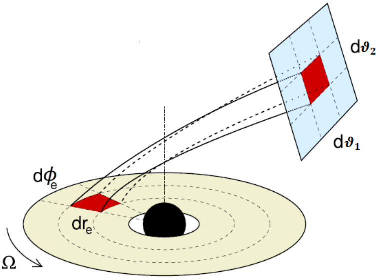
Figure: Comparison of iron line models produced with the KYRLINE model. (Svoboda J., arXiv:1007.5196v2, 2010)

Our approach

We have to solve the geodesics equation in the SDL[7] in order to link the observer plane to the source plane

Strategy

$$(r_e, \phi_e) \rightarrow \textit{Gravitational lensing} \rightarrow (\theta_1, \theta_2)$$



[7] Bozza V. and Scarpetta G., Physical Review D 76, 0830080 (2007).

Geodesics equations in Kerr

The geodesics equations are[8]

$$\pm \int \frac{1}{\sqrt{R}}\,\mathrm{d}r = \pm \int \frac{1}{\sqrt{\Theta}}\,\mathrm{d}\vartheta,$$

$$\phi_f - \phi_i = a \int \frac{r^2 + a^2 - aJ}{\Delta\sqrt{R}}\mathrm{d}r - a \int \frac{r}{\sqrt{R}}\mathrm{d}r + J \int \frac{\csc^2\vartheta}{\sqrt{\Theta}}\mathrm{d}\vartheta,$$

with

$$\Theta = Q + a^2\cos^2\vartheta - J^2\cot^2\vartheta, \qquad R = r^4 + (a^2 - J^2 - Q)r^2 + ((J-a)^2 + Q)r - a^2 Q.$$

Where a is the ***spin*** of the black hole and J and Q are respectively the projection of the ***specific angular momentum*** of the photon along the spin axis and the ***Carter integral***.

[8] Chandrasekar S., The Mathematical Theory of Black Holes, Oxford University Press (2009).

Geodesics equations in Kerr

- J and Q can be related to the angular coordinates in the sky (θ_1, θ_2) from which the photon reaches the observer[9]

$$\theta_1 = -\frac{J}{r_o\sqrt{1-\mu_o^2}},$$

$$\theta_2 = \pm r_o^{-1}\sqrt{Q + \mu_o^2\left(a^2 - \frac{J^2}{1-\mu_o^2}\right)}.$$

r_o is the distance of the observer from the black hole.

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- The photon trajectories around the black holes are characterized by the existence of an ***unstable circular orbit***.

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Geodesics equations in Kerr

- We introduce a parameter $\xi \in [-1, 1]$ along the shadow border and a parameter $\epsilon \in [-1, +\infty)$ quantifying the radial distance from the shadow.

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Transformation rules

$$r_e(\epsilon, \xi) = r_0(\epsilon, \xi) + a\,\delta r_e(\epsilon, \xi) + o(a^2)$$
$$\phi_e(\epsilon, \xi) = \phi_0(\xi) + a\,\delta\phi_e(\epsilon, \xi) + o(a^2)$$

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Results

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- Thanks to a numerical code, Fe Kα lines are processed for different inclinations of accretions disk with respect to the line of sight, from $\mu_o = 0.01$ (edge-on) to $\mu_o = 0.99$ (face-on).

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- The shapes are the same for a generic high order image.
- Each order is suppressed by a factor $e^{-\pi} = 0.043$ with respect to the previous one.

Results

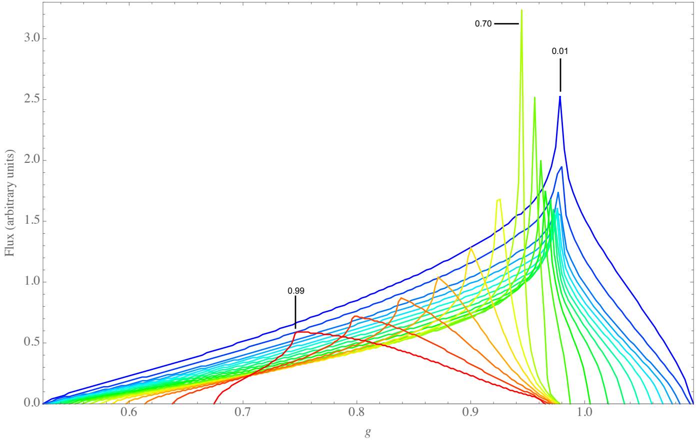

Figure: Contribution of the second order images of the accretion disk to a relativistic emission line for observer inclinations μ_o for a Schwarzschild black hole.

Results

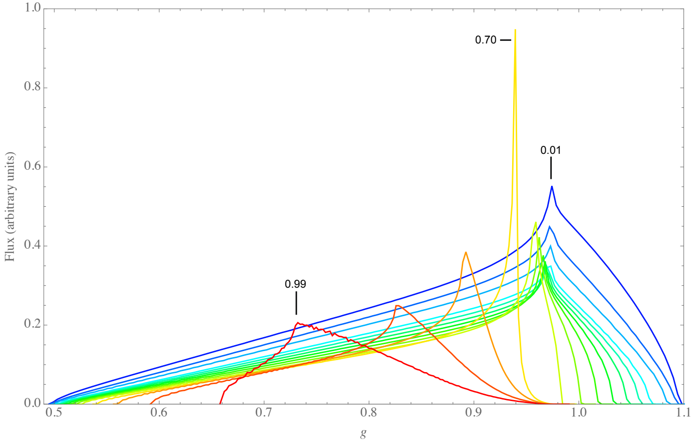

Figure: Contribution of second order images of the accretion disk to a relativistic emission line for observer inclinations μ_o for a Kerr black hole with $a = 0.03$.

Position of the peak

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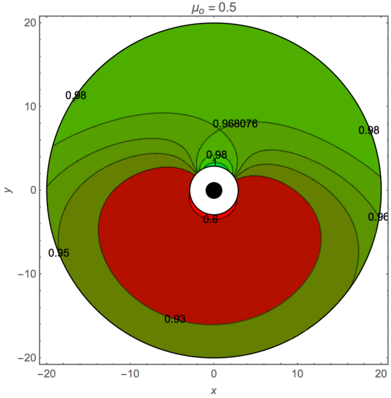
- It corresponds to a saddle point of the frequency shift factor g



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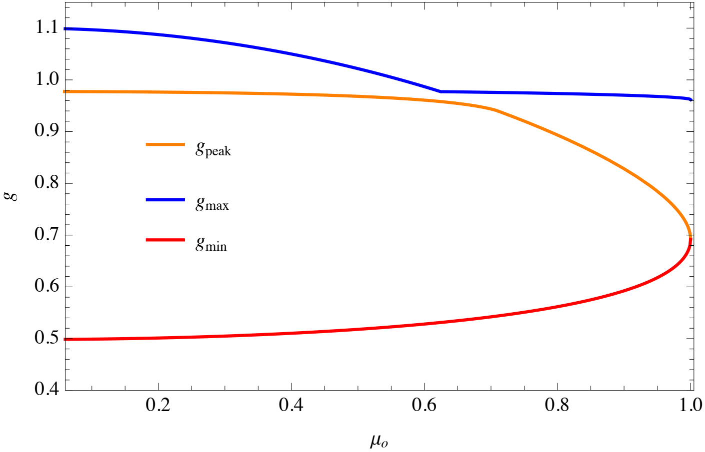

Figure: Peak frequency and boundaries in terms of μ_o.

Conclusions and perspectives

Conclusions

- Black holes are able to form an infinite number of images of a given source.
- The strong deflection limit formalism can be applied to several interesting observational contexts.
- Line profiles emitted by an accretion disk have a very simple shape.
- Interesting analytical results are produced.

Perspectives

- To figure out the phenomenology of the Fe Kα emission lines for a Kerr black hole deeper and deeper.
- Modify the nature of the source with more advanced scenarios.