Strongly deformed N=4 SYM in the double scaling limit as an integrable CFT

In collaborations with V. Kazakov, F. Levkovich-Maslyuk, E. Olivucci

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CONVEGNO NAZIONALE DI FISICA TEORICA

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[N.Beiser, R.Roiban '15]

The most general theory which admits an AdS_5 dual description in terms of a string σ-model

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classical level ⟶ conformal and integrable
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$$\mathcal{L} = N_c \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^\mu\phi_i^\dagger D_\mu\phi^i + i\bar{\psi}_A^{\dot\alpha}D_{\dot\alpha}^\alpha\psi_\alpha^A\right] + \mathcal{L}_{\mathrm{int}}$$

where $i = 1,2,3\,,\ A = 1,2,3,4\,,\ D_{\dot\alpha}^\alpha = D_\mu(\sigma^\mu)_{\dot\alpha}^\alpha$ and

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Deformation parameters $q_j = e^{-\frac{i}{2}\gamma_j}\quad j = 1,2,3$, related to the Cartan subalgebras $u(1)^3 \subset su(4) \cong so(6)$, break supersymmetry!!

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$$\beta_{\alpha_{jj}^2} = \frac{g^4}{\pi^2} \sin^2 \gamma_j^+ \sin^2 \gamma_j^- + \frac{\alpha_{jj}^4}{4\pi^2}$$

At weak coupling the beta function has two fixed points (they should persist for any g, N_c)

$$\alpha_{jj}^2 = \pm 2 i g^2 \sin \gamma_j^+ \sin \gamma_j^- + \mathcal{O}(g^4)$$

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The double scaling limit

DS limit = Weak coupling + Large imaginary twists $\longrightarrow$ $g \to 0 \quad q_i \to \infty$

$\xi_i := 4\pi q_i g$ fixed

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In the DS limit the gauge field decouples [O.Gurdogan and V.Kazakov '16]

$$\mathcal{L}_{\phi\psi} = N_c \mathrm{Tr}\left(-\frac{1}{2}\partial^\mu \phi_j^\dagger \partial_\mu \phi^j + i\bar{\psi}_j^{\dot{\alpha}}(\sigma^\mu)_{\dot{\alpha}}^{\alpha}\partial_\mu \psi_\alpha^j\right) + \mathcal{L}_{\mathrm{int}}$$

and only a certain class of Yukawa and 4-scalar interactions survive

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We can build a family of other theories playing with the couplings

- $\xi_1 \to 0$: 3-scalar, 2-fermion theory
- $\xi_1, \xi_2 \to 0$: 2-scalar theory
- $\xi_1 = \xi_2 = \xi_3 = \xi$: β-deformed theory

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All these theories are chiral = their action is not invariant w.r.t. hermitian conjugation.

Missing terms with opposite chirality can be retrieved in the opposite DS limit → $g \to 0 \quad q_i \to 0$, $\xi_i := 4\pi q_i/g$ fixed

Fishnet Theories & Integrability

Consider the two point function $\langle \mathrm{Tr}\phi_i^L(0)\mathrm{Tr}\phi_i^{\dagger L}(x)\rangle$ in the 2-scalar theory

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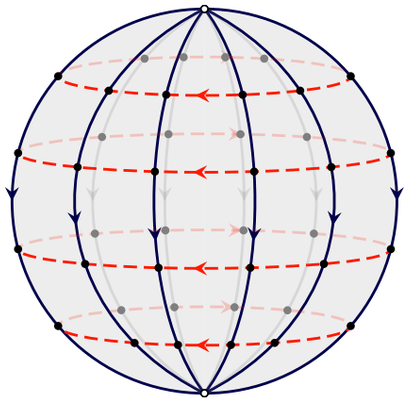

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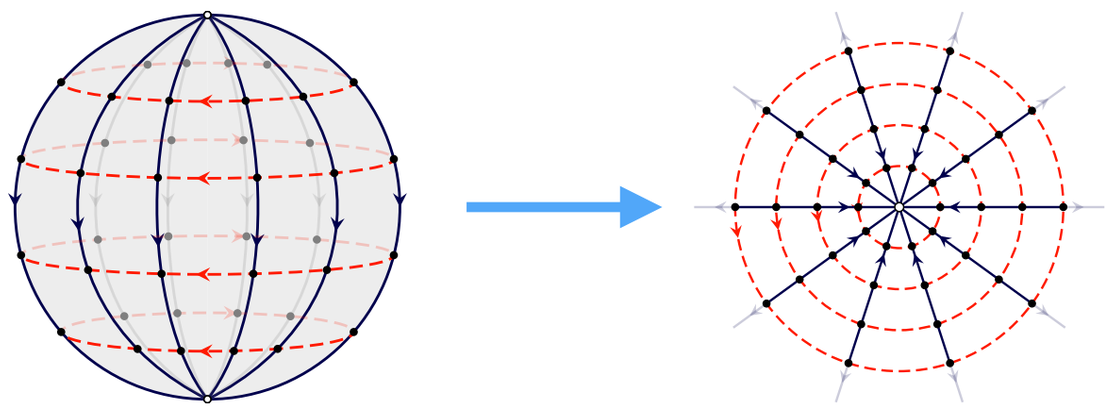

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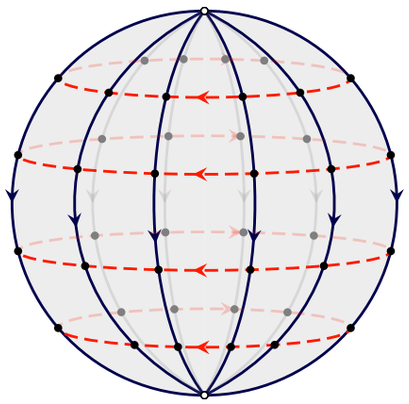

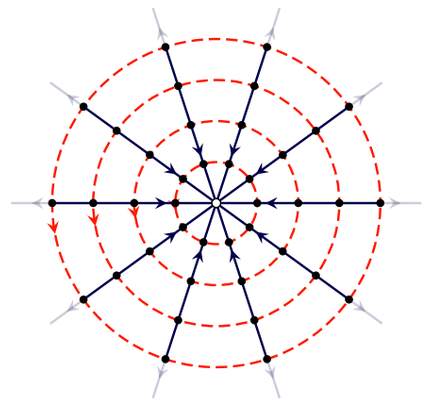

Hamiltonian evolution in the radial direction $(x_{L+1} = x_1)$

$$\mathcal{H}_L = \prod_{l=1}^{L} \frac{1}{(x_{l+1} - x_l)^2} \prod_{l=1}^{L} \Delta_{x_l}^{-1}$$

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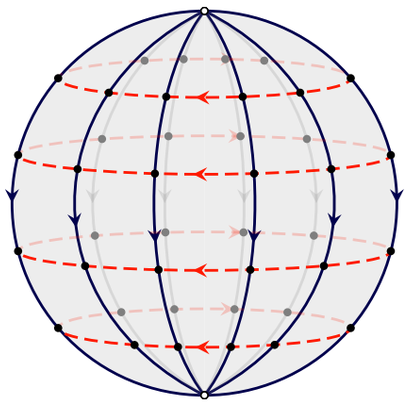

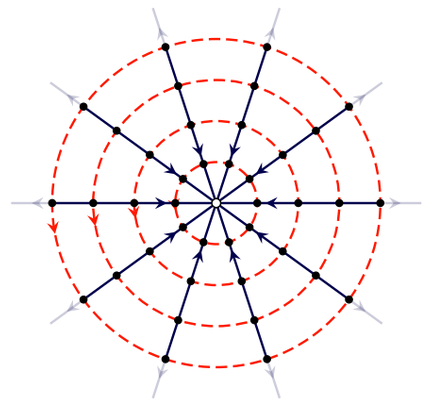

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Identified as a transfer matrix of a non-compact Heisenberg spin-chain

[N. Gromov, V. Kazakov, G.Korchemsky, S.Negro, G.Sizov '17]

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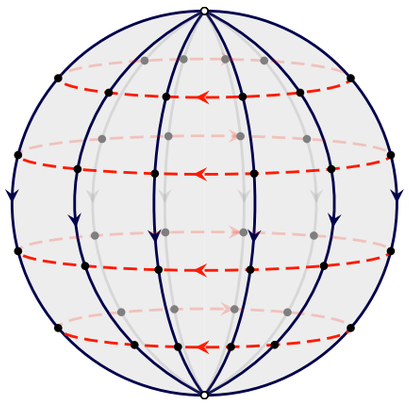

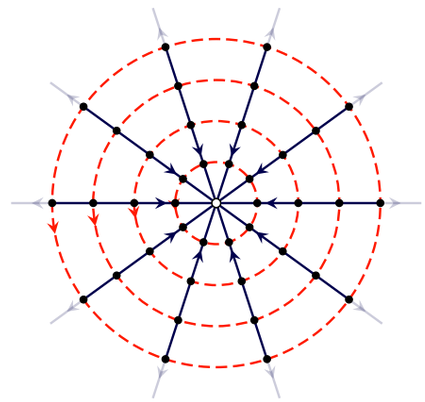

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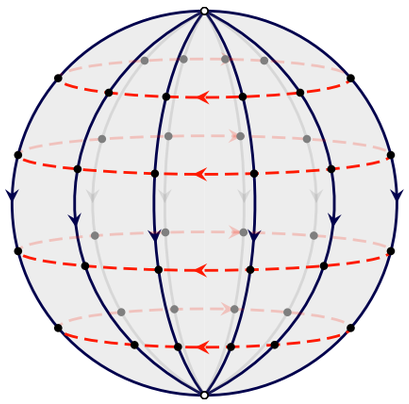

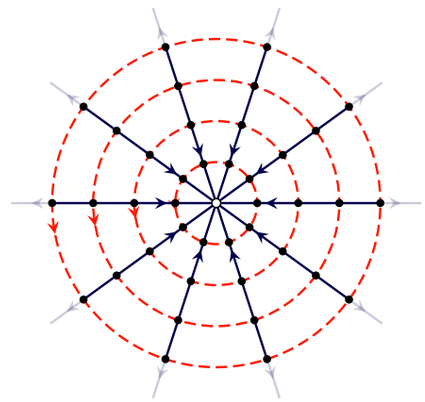

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- Can I reproduce the same result from the 2-scalar field theory?
- Is it possible to compute the spectrum of the most general DS theory in both ways?

Fermionic fishnet!!

Spectrum of 2-scalar theory for L=2

Consider the 4-point function of the only length-two non-protected operator at spin S=0

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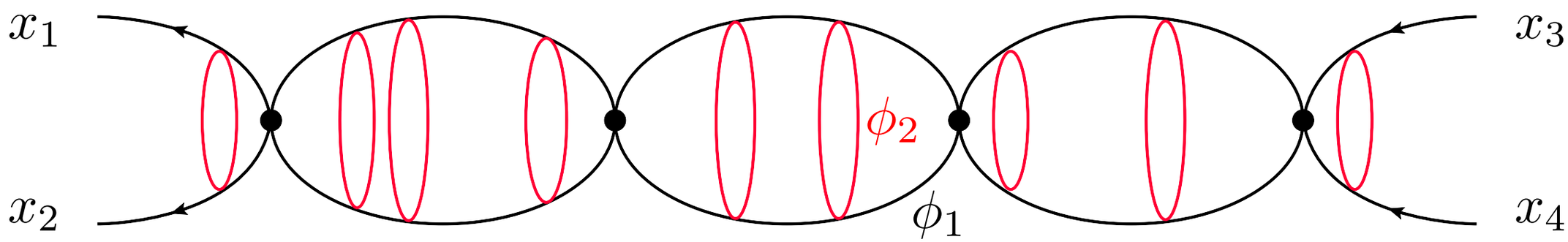

Spectrum of 2-scalar theory for L=2

Consider the 4-point function of the only length-two non-protected operator at spin S=0

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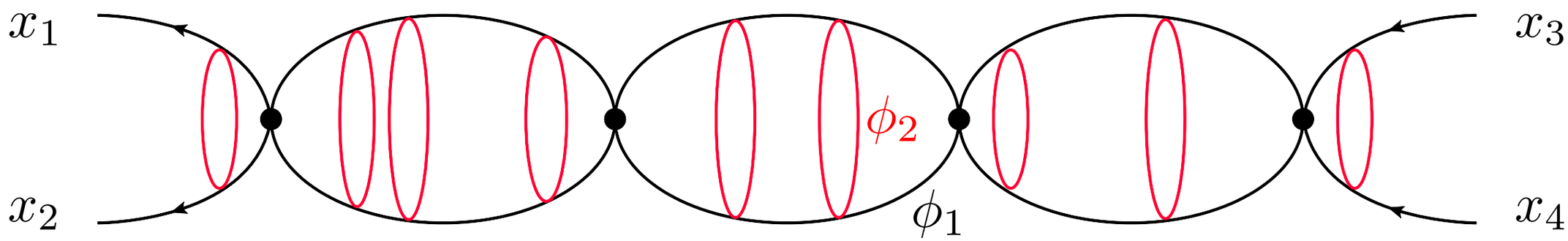

$$G = \sum_{k=0}^{\infty}(\alpha^2\mathcal{V} + \xi^4\mathcal{H})^k \frac{1}{x_{13}^2 x_{24}^2} = \int \frac{d^4x_{1'}d^4x_{2'}}{x_{1'3}^2 x_{2'4}^2}\langle x_1, x_2|\frac{1}{1-\alpha^2\mathcal{V}-\xi^4\mathcal{H}}|x_{1'}, x_{2'}\rangle$$

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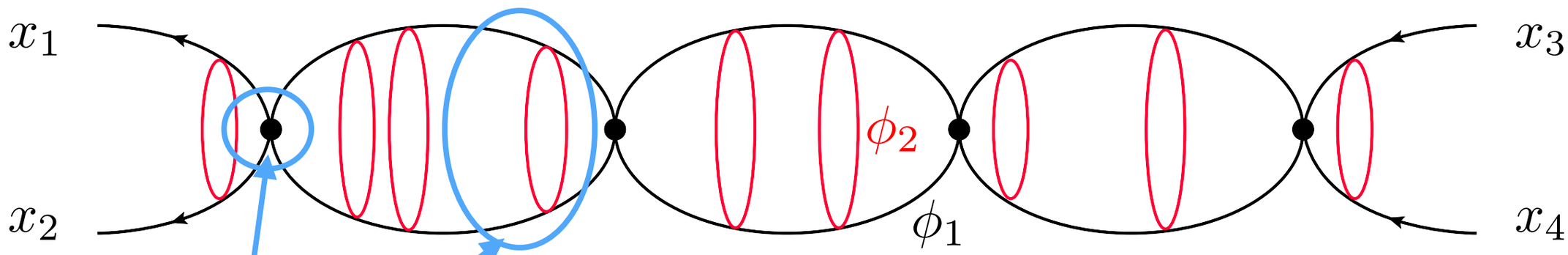

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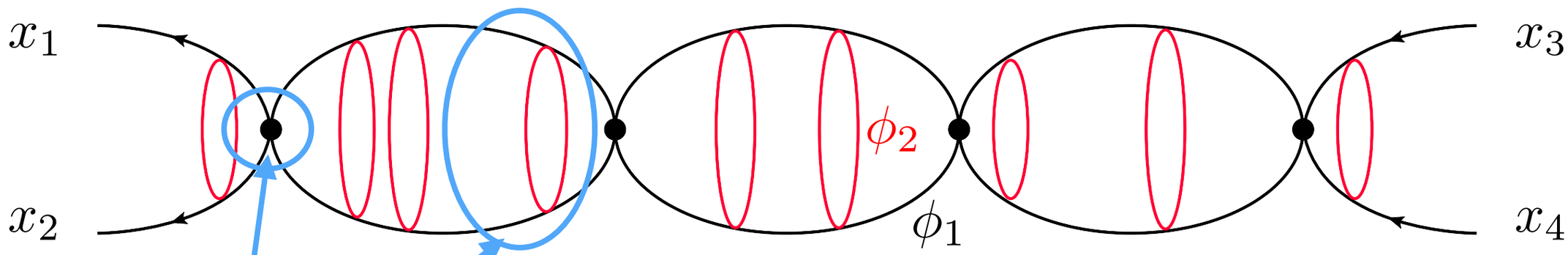

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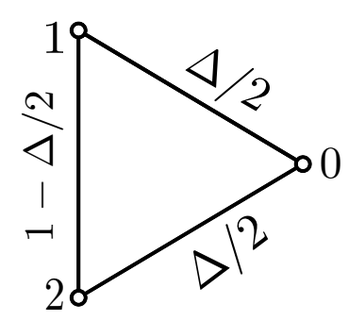
The integral kernels $\mathcal{V}$ and $\mathcal{H}$ commute with the (2,0,0)x(2,0,0) spin-chain (generators of the conformal group). This property fixes their eigenstates

$$\Phi_\Delta(x_1, x_2, x_0) = \frac{1}{x_{12}^2}\left(\frac{x_{12}^2}{x_{10}^2 x_{20}^2}\right)^{\Delta/2} =$$



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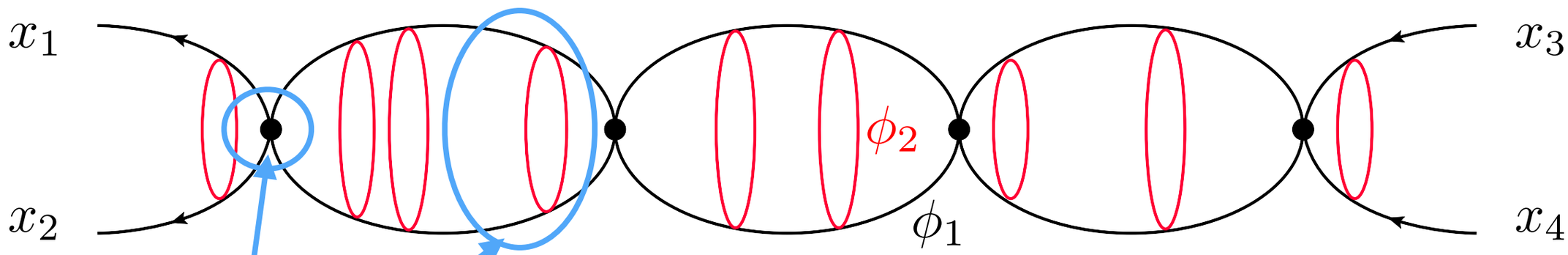

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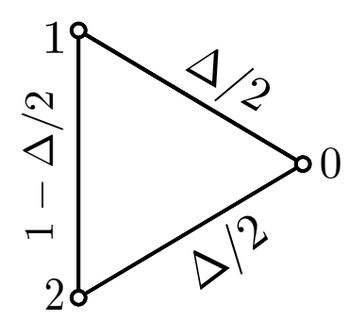
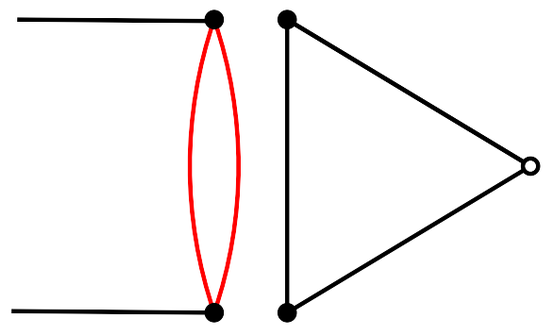
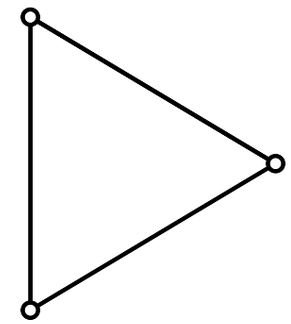


Expanding G on this basis of states, it will depend only on h the eigenvalue of $\mathcal{H}$.

Taking the residue at $h^{-1} = \xi^4$, we can compute the OPE coefficients!

Spectrum of 2-scalar theory for L=2

To compute the scaling dimension we have to solve $h^{-1} = \xi^4$computing the eigenvalue

$\mathcal{H}\Phi_\Delta =$  $= h$  $= h\Phi_\Delta$

where black dots are integrated. Solving it with the “star-triangle” relations we have

$$(\Delta - 4)(\Delta - 2)^2\Delta = 16\xi^4$$

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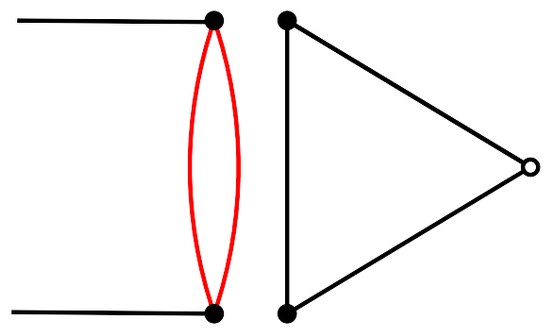
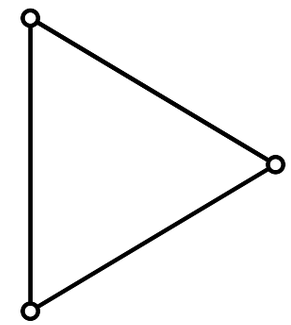
$\mathcal{H}\Phi_\Delta =$  $= h$  $= h\Phi_\Delta$

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Solution of this equation together with OPE coefficients define exact conformal data of operators appearing in the OPE of $\mathrm{Tr}[\phi_1(x_1)\phi_1(x_2)]$

- twist-2 operators $\Delta = 2 - 2i\xi^2 + i\xi^6 - \frac{7}{4}i\xi^{10} + \mathcal{O}(\xi^{14})$
- twist-4 operators $\Delta = 4 + \xi^4 - \frac{5}{4}\xi^8 + \frac{21}{8}\xi^{12} + \mathcal{O}(\xi^{16})$
- 2 shadow operators $\Delta \to 4 - \Delta$

[D.Grabner, N.Gromov, V.Kazakov, G.Korchemsky '17]

Spectrum of the DS theories for L=2

Consider the same 4-point function $G = \langle \mathrm{Tr}[\phi_1(x_1)\phi_1(x_2)]\mathrm{Tr}[\phi_1^\dagger(x_3)\phi_1^\dagger(x_4)]\rangle$

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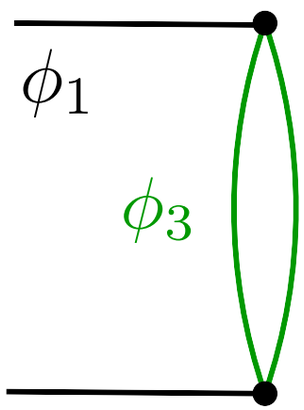

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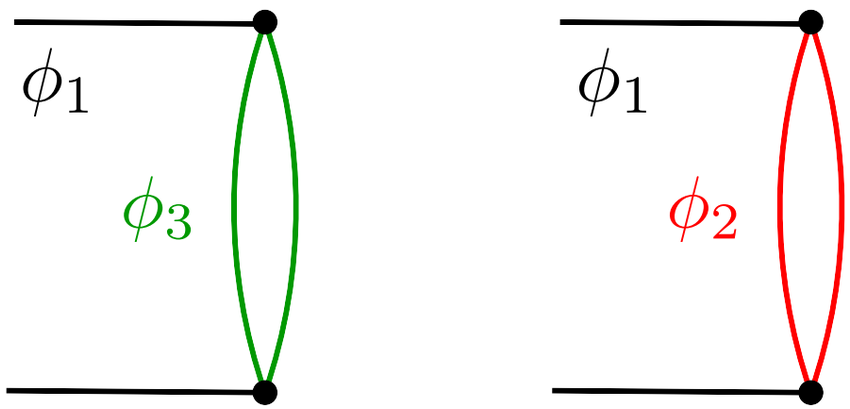

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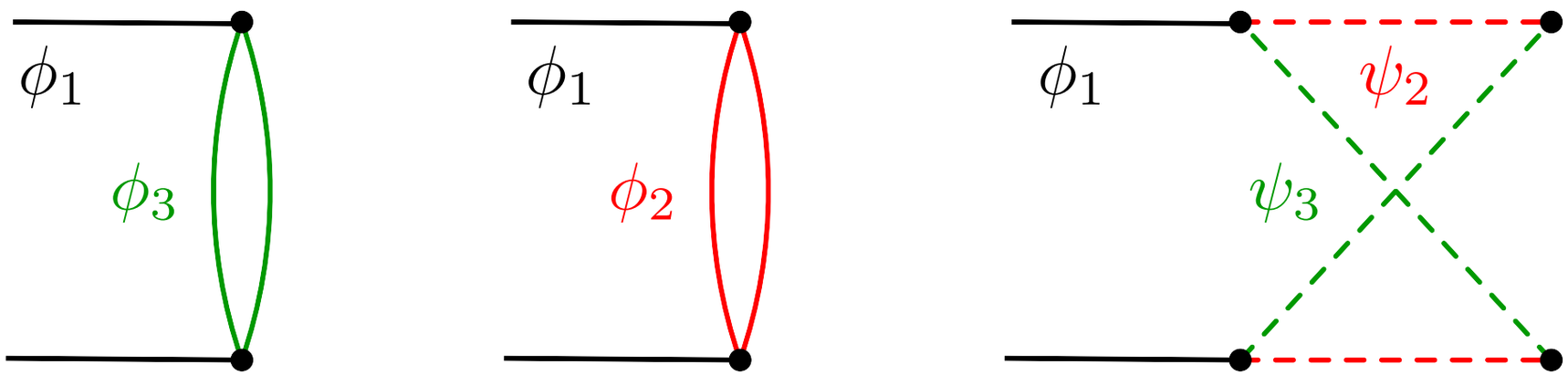

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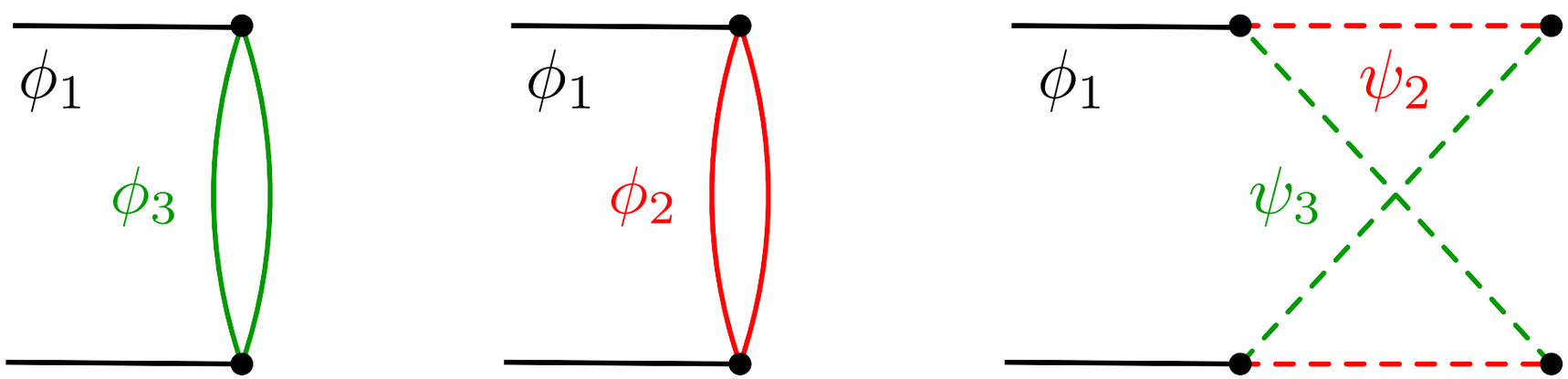

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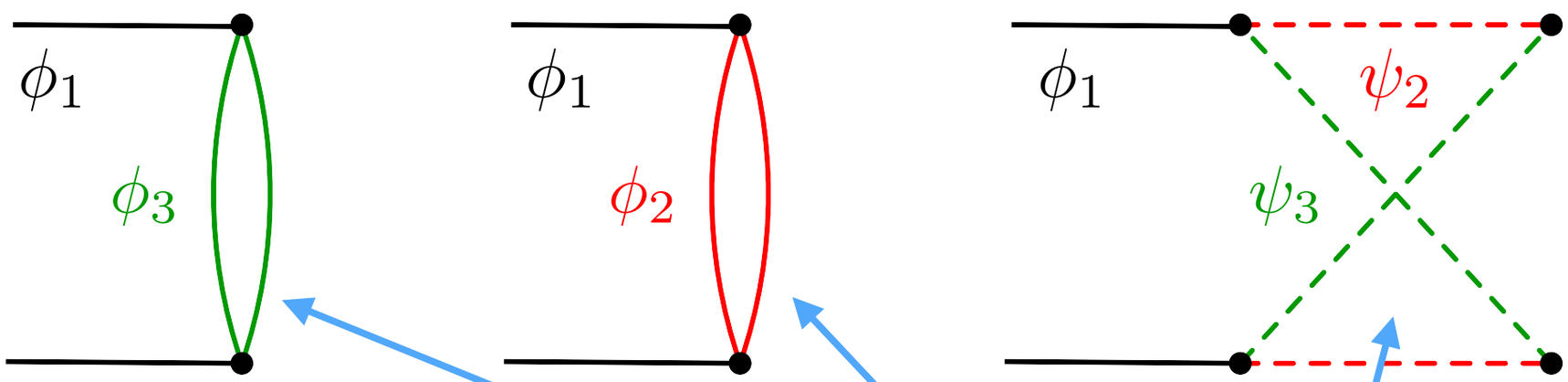

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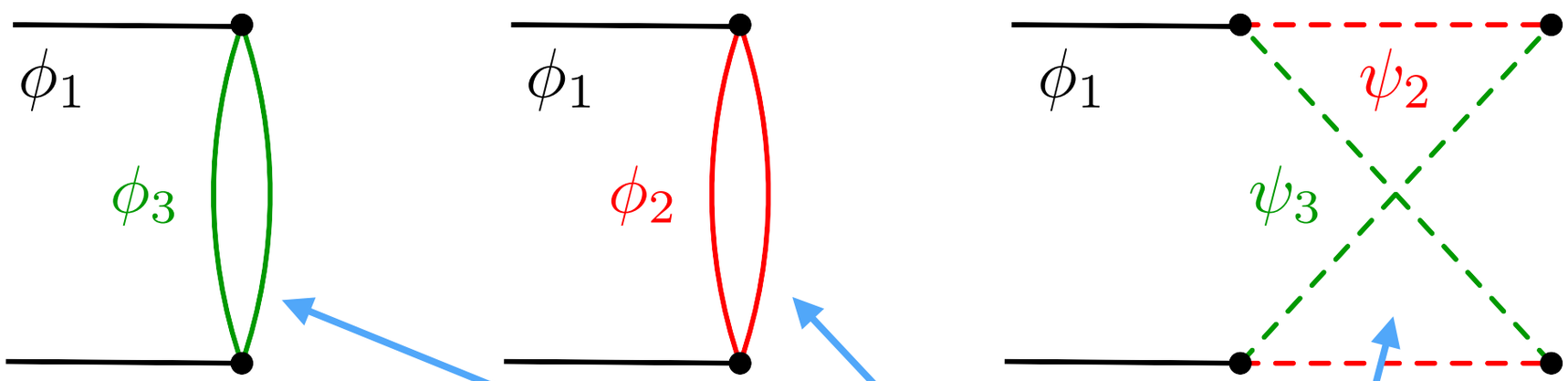

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Then the spectral equation reads $h^{-1} - \xi_2^2\xi_3^2 h^{-1} h_f = \xi_2^4 + \xi_3^4$ where

$$h_f = -2\left[h + \Gamma(\tfrac{\Delta}{2}-1)\Gamma(1-\tfrac{\Delta}{2})\left(\frac{{}_3F_2(1,2,\frac{\Delta}{2};\frac{\Delta}{2}+1,\frac{\Delta}{2}+1|1)}{\frac{\Delta}{2}\,\Gamma(\frac{\Delta}{2}+1)\Gamma(2-\frac{\Delta}{2})} + \pi\cot\pi(4-\tfrac{\Delta}{2})\right)\right]$$

Spectrum of the DS theories for L=2

Let's focus only on the scaling dimension of the twist-2 operator $\mathrm{Tr}[\phi_1(x_1)\phi_1(x_2)]$

$$\Delta_1 = 2 - i\sqrt{\xi_-^2}\left[2 - \left(\xi_-^2 - 6\xi_{23}^2\zeta_3\right) + \frac{1}{4}\left(7\xi_-^4 - 12\xi_{23}^2\xi_-^2(3\zeta_3 + 5\zeta_5) + 108\xi_{23}^4\zeta_3^2\right)...\right]$$

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Spectrum of the DS sub-theories? $(i, j, k = 1, 2, 3)$

- $\xi_i \to 0$ $\longrightarrow$ $\Delta_i\,,\ \Delta_{j\neq i} = \Delta^{\text{2-scalar}}$
- $\xi_i, \xi_{j\neq i} \to 0$ $\longrightarrow$ $\Delta_{k\neq i,j} = 2\,,\ \Delta_i = \Delta_j = \Delta^{\text{2-scalar}}$
- $\xi_1 = \xi_2 = \xi_3 = \xi$ $\longrightarrow$ $\Delta_i = 2$ in agreement with [J.Fokken, C.Sieg, M.Wilhelm '14]

Spectrum of the DS theories for L=2

The general DS theory has a second non-protected operator, then we want to study also

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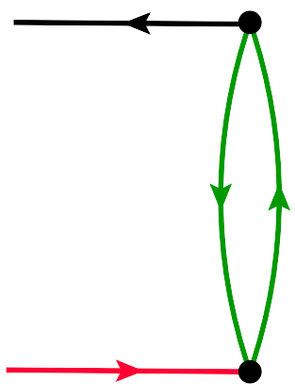

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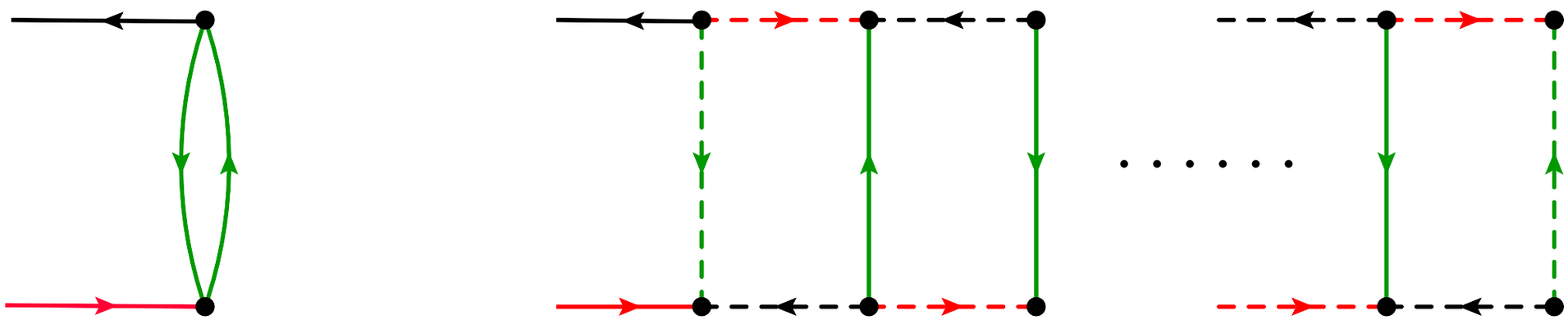

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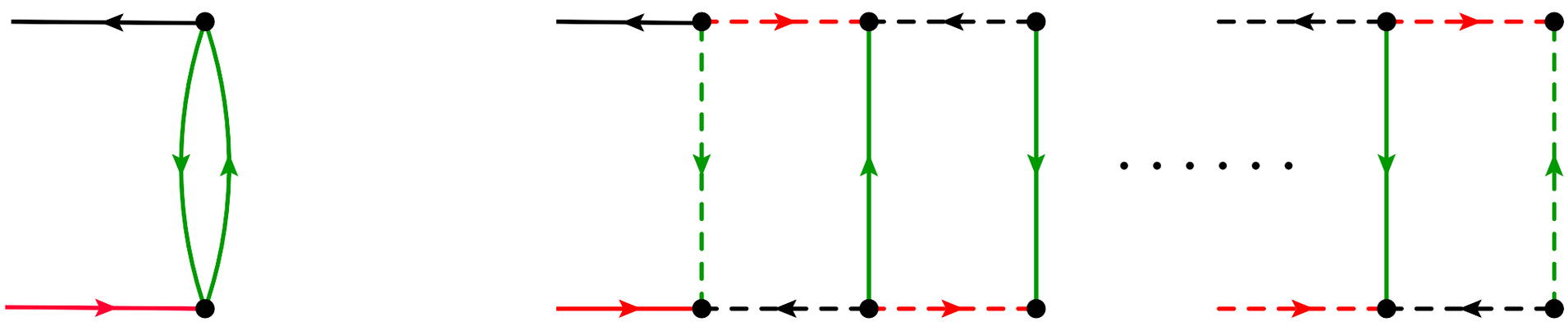

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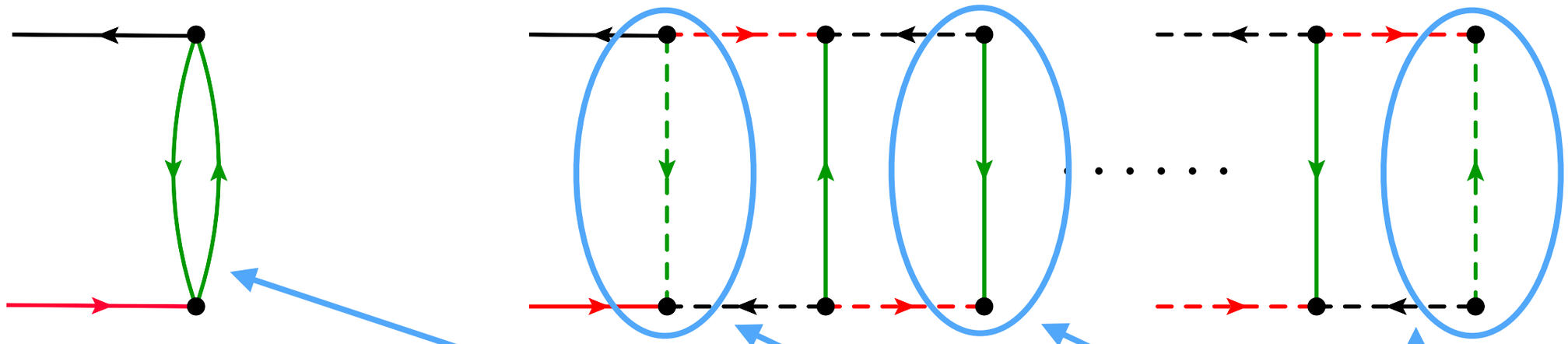

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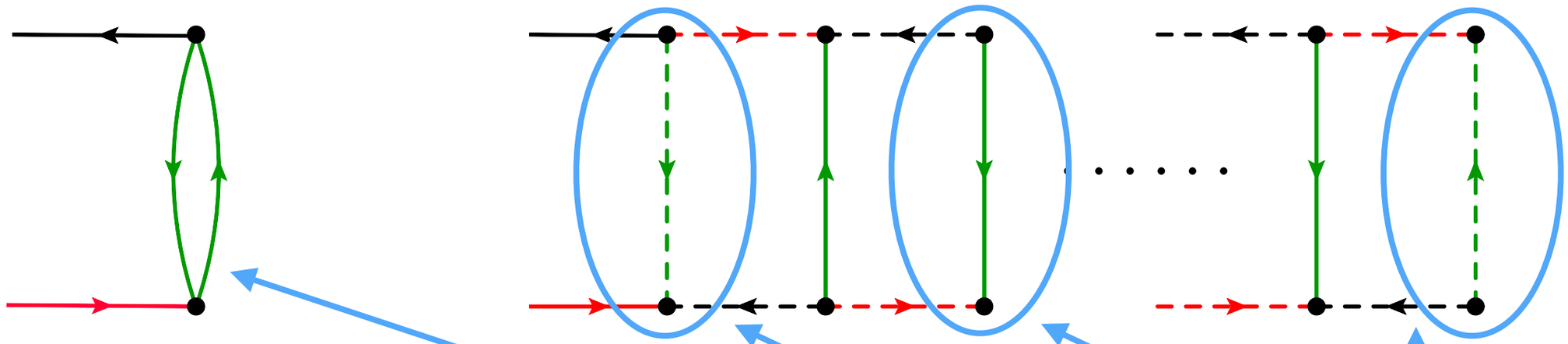

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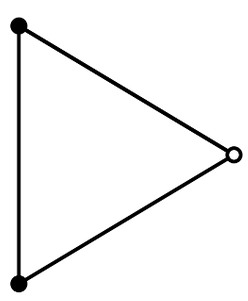
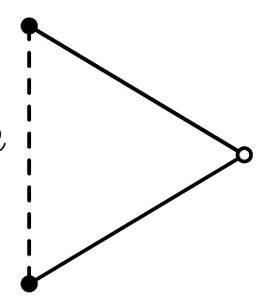
The combination of all the fermionic Hamiltonian commutes with the bosonic spin-chain

$$\underbrace{\hat{\mathcal{H}}\mathcal{H}_b^n\tilde{\mathcal{H}}}_{\mathcal{H}_f}$$



$$= \tilde{h}\hat{\mathcal{H}}\mathcal{H}_b^n$$



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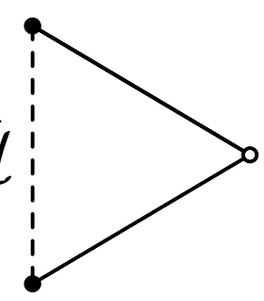
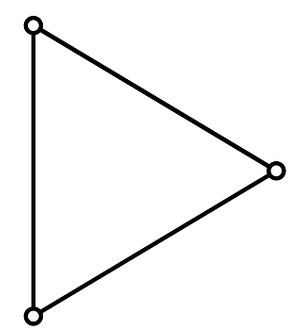


$$= \underbrace{\hat{h}h_b^n\tilde{h}}_{h_f}$$



Spectrum of the DS theories for L=2

The spectral equation reads $h^{-1} + \frac{\xi_1 \xi_2 \xi_3^2}{h_b^{-1} - \xi_1 \xi_2} = \xi_1^2 \xi_2^2$

Let's focus only on the scaling dimension of the twist-2 operator $\mathrm{Tr}[\phi_1(x_1)\phi_2^\dagger(x_2)]$

$$\Delta_{12} = 2 - 2i\sqrt{\xi_{12}\left(\xi_{12} + \xi_3^2\right)} - \frac{i\xi_{12}^2\xi_3^2}{\sqrt{\xi_{12}\left(\xi_{12} + \xi_3^2\right)}} + \frac{i}{4}\left(\frac{\xi_{12}}{\xi_{12} + \xi_3^2}\right)^{3/2}\left(4\xi_{12}^3 + 12\xi_{12}^2\xi_3^2 + 17\xi_{12}\xi_3^4 + 8\xi_3^6\right) + \ldots$$

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The spectrum of the other two operators can be written in terms of Δ_{12}

$\Delta_{23} = \Delta_{12}(\xi_{12} \to \xi_{23}, \xi_3 \to \xi_1)$ and $\Delta_{31} = \Delta_{12}(\xi_{12} \to \xi_{31}, \xi_3 \to \xi_2)$

Spectrum of the DS theories for L=2

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Spectrum of the DS sub-theories? $(i, j, k, l = 1, 2, 3)$

- $\xi_i \to 0 \quad \longrightarrow \quad \Delta_{jk} = \begin{cases} \Delta^{\text{2-scalar}}(\xi^2 \to \xi_{jk}) & j \neq k \neq i \\ 2 & j = i \text{ or } k = i \end{cases}$
- $\xi_i, \xi_{j\neq i} \to 0 \quad \longrightarrow \quad \Delta_{kl} = 2$
- $\xi_1 = \xi_2 = \xi_3 = \xi \quad \longrightarrow \quad \Delta_{ij} = 2 - 2i\sqrt{2}\xi^2 - \dfrac{i\xi^4}{\sqrt{2}} + \dfrac{41i\xi^6}{8\sqrt{2}} + \ldots$

Final Remarks

We have computed the scaling dimension of operator for L=2 for the family of DS theories

The next steps are [Work in progress…]

- Check of the spectrum with the QSC
- Check with direct computations of the diagrams
- Computation of the β- function and critical points at higher order

Biggest future goal: solve QSC (numerics) for the general deformed theory

[Work in progress…]

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THANK YOU