Higher-Spin Asymptotic Symmetries, Charges and Soft Theorems

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Outline

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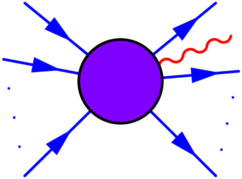
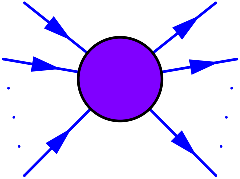
Weinberg's Soft Theorem

Let us consider a scattering process involving a soft particle with spin s and momentum q^μ. For $s = 1, 2, 3, \ldots$ and any D, to leading order in the soft momentum q^μ



$$= \left\{ \sum_{n \in \text{in/out}} g_n^{(s)} \frac{[\epsilon_n(q) \cdot p_n]^s}{q \cdot p_n} \right\}$$



Enforcing gauge invariance leads to:

- the conservation of electric charge for $s = 1$,
- the equivalence principle for $s = 2$,
- $\boxed{g_n^{(s)} = 0}$ for $s \geq 3$ by momentum conservation.

Soft Theorems as Ward Identities

It was pointed out [*Strominger et al.* 2013] that:

- Weinberg's soft *photon* theorem follows from invariance of QED under large gauge transformations;
- Weinberg's soft *graviton* theorem is a consequence of BMS symmetry for asymptotically flat spacetimes.

What about higher spins?

Can Weinberg's theorem be recast as the Ward identities of an underlying symmetry also for $s \geq 3$? Can we identify it?

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Recent Interest in Asymptotic Symmetries

Why asymptotic symmetries?

- New insights into **soft theorems** in particle physics [A. Strominger et al., 2013].
- Potentially observable effects: **gravitational memory** [A. Strominger and A. Zhiboedov, 2014] and **electromagnetic memory** [L. Bieri and D. Garfinkle, 2013].
- Insight (?) into the black hole **information paradox** [S. Hawking, M. Perry and A. Strominger, 2016] (but see also [M. Porrati et al., 2017]).

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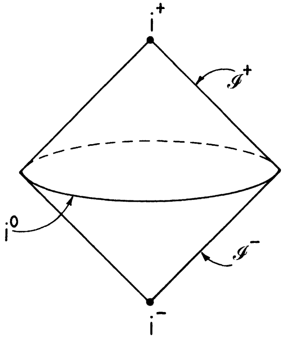
Asymptotic Flatness

Let us consider an **asymptotically flat** spacetime in $D = 4$.
Near $\mathscr{I}^+$ let $x^\mu = r, u, z, \bar{z}$ be retarded Bondi coordinates (denoted with Greek indices):



- a radial coordinate r
- retarded time u and
- angular coordinates $z, \bar{z}$

$$z = e^{i\phi} \cot \frac{\theta}{2}, \qquad \bar{z} = e^{-i\phi} \cot \frac{\theta}{2}.$$

$\gamma_{z\bar{z}}$ = metric on the Euclidean sphere
D_z = covariant derivative on the sphere.

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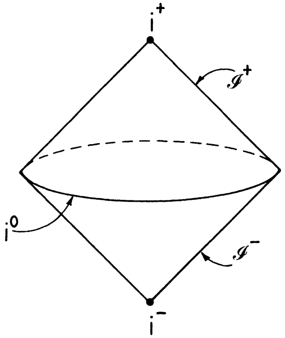

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Asymptotic Flatness

The Bondi gauge

Such a spacetime can be described by a fluctuation $h_{\mu\nu}$ with respect to the Minkowski metric $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu},$$

subject to the conditions (Bondi gauge): $h_{r\mu} = 0 = h_{z\bar{z}}$,

$$h_{uu} = \frac{2m_B}{r}\,, \qquad h_{uz} = -U_z\,, \qquad h_{zz} = rC_{zz} \quad (\& \ \ z \leftrightarrow \bar{z})\,,$$

where m_B, U_z and C_{zz} are functions of u, z and $\bar{z}$ only.

- These falloffs follow from the general analysis of the **nonlinear** theory,
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BMS Symmetry and Soft Graviton Theorem

The **linearized diffeomorphisms** which (locally) preserve the Bondi gauge are:

- supertranslations [*Bondi, van der Burg, Metzner, Sachs*] specified by an arbitrary angular function $T(z, \bar{z})$,
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$$D_z Y_z(z, \bar{z}) = 0\,, \qquad D_{\bar{z}} Y_{\bar{z}}(z, \bar{z}) = 0\,.$$

The associated vector fields generate the **BMS algebra.**

Using the Ward identities of **supertranslation symmetry**, we were able to retrieve Weinberg's soft graviton theorem without assuming, but rather deriving

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The "Bondi-like gauge"

A natural way to generalize the Bondi-gauge falloffs to spin three in $D = 4$ Minkowski space is ("Bondi-like gauge"): $\varphi_{r\mu\nu} = 0 = \varphi_{\alpha z\bar{z}}$,

$$\varphi_{uuu} = \frac{B}{r}\,, \quad \varphi_{uuz} = U_z\,, \quad \varphi_{uzz} = rC_{zz}\,, \quad \varphi_{zzz} = r^2 B_{zzz} \quad (\& \ \ z \leftrightarrow \bar{z})\,,$$

where B, U_z, C_{zz} and B_{zzz} are functions of u, z, $\bar{z}$ only.

- This is a set of gauge/falloff conditions **consistent** with the free equations of motion.
- The extension to spin s is $\varphi_{r\mu_2\ldots\mu_s} = 0 = \varphi_{z\bar{z}\mu_3\ldots\mu_s}$ and, letting d denote the number of "z" indices,

$$\varphi_{uu\ldots uzz\ldots z} = \mathcal{O}\left(r^{d-1}\right).$$

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Spin-s Supertranslations

We now look for **large gauge transformations** $\delta\varphi_{\mu_1\dots\mu_s} = \nabla_{(\mu_1}\epsilon_{\dots\mu_s)}$ that preserve the Bondi-like gauge.

- For any s, they are found to comprise an **infinite-dimensional** family of symmetries;
- Higher-spin supertranslations $\epsilon^{p}_{d,c}$ (where p=number of u's and c=number of $z\bar{z}$ pairs) are specified by a single arbitrary angular function $T(z,\bar{z})$ as

$$\epsilon^{p}_{d,0} = -\frac{r^d D_z^d T_p}{\prod_{k=1}^{d}(s-p-k)},$$
$$\epsilon^{p}_{d,c+1} = -\frac{1}{2}\gamma_{z\bar{z}} r^2 \left(\epsilon^{p}_{d,c} - 2\epsilon^{p+1}_{d,c}\right),$$
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Spin-three Supertranslations and Superrotations

For spin three, the complete set of large gauge symmetries preserving the Bondi-like gauge is parametrized by:

- an arbitrary angular function

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- conformal Killing 2-tensors on the celestial sphere

$$D_z K_{zz}(z,\bar{z}) = 0\,, \qquad D_{\bar{z}} K_{\bar{z}\bar{z}}(z,\bar{z}) = 0\,,$$

- and the solutions to

$$D_z^2 \rho_z(z,\bar{z}) = 0\,, \qquad D_{\bar{z}}^2 \rho_{\bar{z}}(z,\bar{z}) = 0\,.$$

In both cases, these equations have an infinite-dimensional space of solutions.

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Supertranslation Charge

- We can compute the supertranslation surface charge for any integer spin s:

$$Q^{+} = s\int_{\mathscr{I}^{+}_{-}} dz d\bar{z}\, \gamma_{z\bar{z}} B(-\infty, z, \bar{z}) T(z, \bar{z})$$

where $\varphi_{uu\ldots u} = B/r$.

- We assume that supertranslation symmetry is generated by the corresponding charge

$$[Q^{+}, \Phi] = \frac{s}{2} g^{(s)} T(z, \bar{z})(i\partial_u)^{s-1}\Phi \,.$$

- Considering S-matrix elements, we obtain the Ward identity

$$\langle \text{out}|Q^{+}S - SQ^{-}|\text{in}\rangle = \frac{s}{2}\sum_n g_n^{(s)} T(z, \bar{z}) E_n^{s-1} \langle \text{out}|S|\text{in}\rangle \,.$$

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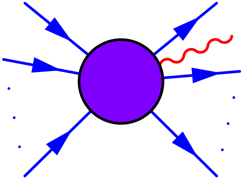
Weinberg's Factorization from Higher-Spin Supertranslations

Applying D_z^{s-1} to the so-obtained Ward identity allows us to retrieve Weiberg's theorem written in terms of the coordinates $z, \bar{z}$ on the celestial sphere.
To summarize:

$$\langle \text{out}|Q^+ S - SQ^-|\text{in}\rangle = \frac{s}{2}\sum_n g_n^{(s)} T(z,\bar{z}) E_n^{s-1} \langle \text{out}|S|\text{in}\rangle ,$$

$$\Downarrow$$



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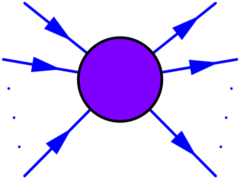

Infrared Issues and Higher Spins

Potential insight into the complicated infrared behavior of higher-spin fields:

- the corresponding **physical quanta** should decouple at low energy (Weinberg's theorem);
- **interactions** of massless higher-spin fields appear to be in conflict with (perturbative) locality on flat space (*at least*).

Two kinds of infrared regulators (apparently) allow to avoid such issues:

- a **cosmological constant** $\Lambda \neq 0$ (Vasiliev theory),
- the **string tension** $1/\alpha'$ (string theory).

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- The analysis presented so far admits a *straightforward generalization* to any spacetime dimension (both even and odd).
- Let us focus on the self-interacting spin-one case: pure Yang-Mills.
- This case also allows us to keep track of the relevant nonlinear effects (unlike for higher spins).

In radial gauge $\mathcal{A}_r = 0$, this analysis highlights two paticularly relevant terms in the asymptotic $1/r$ expansion:

- Radiation terms $\boxed{A_u r^{(2-D)/2}}$, giving rise to finite nonzero energy and color flux across null infinity,
- Coulombic term $\boxed{\tilde{A}_u r^{3-D}}$ contributing to the color charge integral at a given retarded time.

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$$\mathcal{P}(u) = -\int_{S_u} \gamma^{ij} \mathrm{tr}(\partial_u A_i \partial_u A_j) d\Omega_{D-2}\,,$$

where z^i and γ_{ij} are coordinates and the metric on the celestial sphere.

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- Higher-spin supertranslation symmetry can be held responsible for Weinberg's soft factorization theorem.
- The asymptotic analysis can be extended to any D and allows for consistent expressions of charge and energy flux.

Outlook

- Is there a **non-Abelian algebra** underlying higher-spin asymptotic symmetries?
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