$\lambda\varphi^4$ at *NNNNNNNNLO*

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based on `1612.04376`, `1702.04148`, `1805.05882`,
with Marco Serone and Giovanni Villadoro

New Frontiers in Theoretical Physics

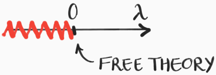
Perturbation theory

$$S[\varphi] = \int d^d x \left[\tfrac{1}{2}(\partial\varphi)^2 + V(\varphi)\right]$$

$$F = \int \mathcal{D}\varphi \; \varphi(x_1)\ldots\varphi(x_M)\, e^{-\frac{S[\varphi]}{\lambda}}$$

Perturbative expansion

$$F \approx \sum_n c_n \lambda^n \longrightarrow \infty$$



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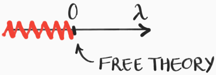
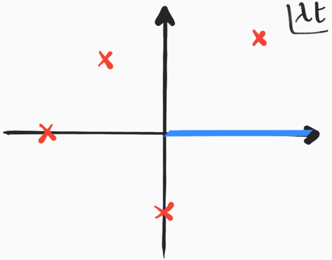


Borel resummation

$$F_B \equiv \int_0^\infty dt\, e^{-t} \overbrace{\left[\sum_n \frac{c_n}{n!}(\lambda t)^n\right]}^{\text{convergent}}$$



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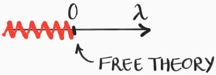

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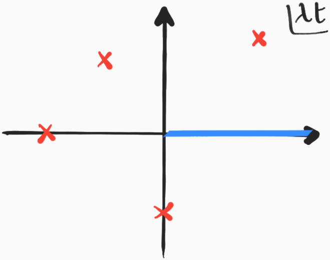

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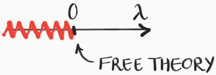

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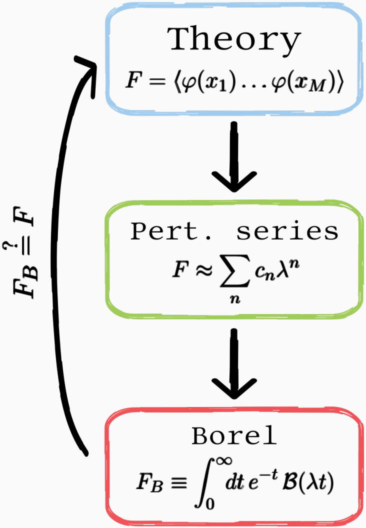


Criterion for Borel summability

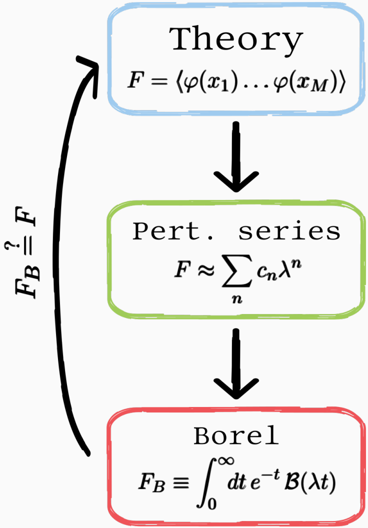
$$S[\varphi] = \int d^dx \left[\frac{1}{2}(\partial\varphi)^2 + V(\varphi)\right]$$

super-renormalizable $V(\varphi)$

- $d = 2$ generic polynomial
- $d = 3$ polynomial of degree 4

Criterion
If $\exists$ unique *real* solution to $S'[\varphi] = 0$ with $S[\varphi_0] < \infty$

- F_B well defined
- $F_B = F$



Derrick's theorem

$$S[\varphi] = \int d^d x \left[\frac{1}{2}(\partial\varphi)^2 + V(\varphi) \right]$$

Unique *real* solution to e.o.m.

- $d \geq 3$ always
- $d = 2$ if no continuous degenerate vacua

Proof (Derrick's theorem)

$$S[\varphi_0(x)] = A(\varphi_0) + B(\varphi_0) \qquad A \geq 0\,, B \geq 0$$

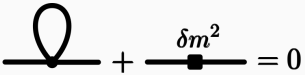
$$\varphi_\lambda(x) \equiv \varphi_0(\lambda x)$$

$$(2-d)A(\varphi_0) - dB(\varphi_0) = 0$$

$\lambda\varphi^4$ theory in $d = 2$

$$V(\varphi) = \frac{1}{2}\varphi^2 + \lambda\varphi^4$$

- Renormalized by normal ordering

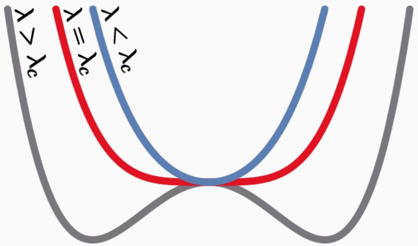


- Phase transition λ_c: spontaneous breaking of $\varphi \to -\varphi$
- Critical theory: 2d Ising Model
- Studied with other methods: `Hamiltonian truncation`, ϵ `expansion`, …

Phase transition vs Borel resummation

Phase transition
non-analytic

Borel resummation
analytic

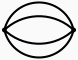
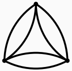
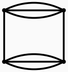
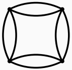
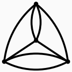


Spontaneous symmetry breaking $\varphi \to -\varphi$

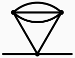
- add explicit breaking $S_\epsilon[\varphi] = S[\varphi] + \epsilon \int d^2x\, \varphi(x)$
- $\epsilon \to 0$ after $V \to \infty$

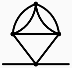
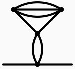
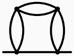
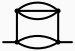
Borel resummation with fixed $\epsilon = 0$

$$\lim_{|x|\to\infty} \langle \varphi(x)\varphi(0)\rangle_{\text{conn.}} \neq 0 \qquad \text{when } \lambda > \lambda_c$$

$\Lambda = -$  $\lambda^2 +$  $\lambda^3 - \Big[$  $+$  $+$  $\Big]\, \lambda^4$

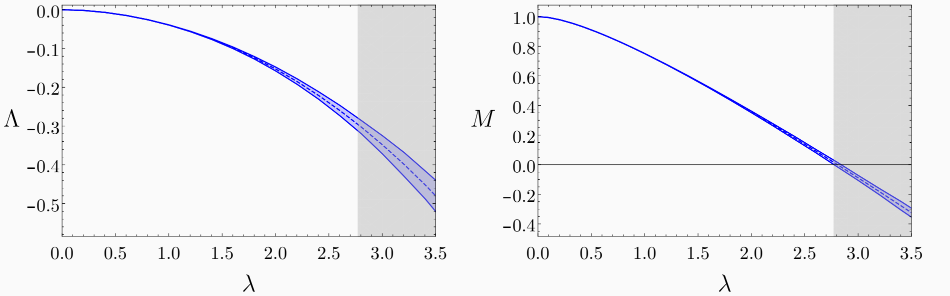
$+\,(\texttt{6 diags})\,\lambda^5 - (\texttt{19 diags})\,\lambda^6 + (\texttt{50 diags})\,\lambda^7 - (\texttt{204 diags})\,\lambda^8$

$\Gamma_2 = -$  $\lambda^2 + \Big[$  $+$  $\Big]\, \lambda^3$

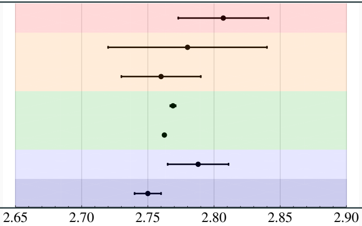
$- \Big[$  $+$  $+$  $+$  $+$  $+$  $\Big]\, \lambda^4$

$+\,(\texttt{19 diags})\,\lambda^5 - (\texttt{75 diags})\,\lambda^6 + (\texttt{317 diags})\,\lambda^7 - (\texttt{1622 diags})\,\lambda^8$

Resummation



Critical point



λ_c	Method
2.807(34)	Borel RPT [This work]
2.78(6)	HT [2015]
2.76(3)	HT [2017]
2.769(2)	MPS I [2013]
2.7625(8)	MPS II [2013]
2.788(15)(8)	LMC [2015]]
2.75(1)	LMC+Borel RPT [2015]

ν	η	Method
1	$\frac{1}{4}$	Exact
0.92(30)	0.08(20)	$\mathrm{PT}_{4\mathrm{pt}}$, 4loops [1978]
0.97(8)	0.13(7)	$\mathrm{PT}_{4\mathrm{pt}}$, 4loops [1980]
0.966(?)	0.146(?)	$\mathrm{PT}_{4\mathrm{pt}}$, 5loops [2000]
0.952(14)	0.237(27)	ϵ-expansion, 6loops [2017]
0.96(6)	0.244(28)	$\mathrm{PT}_{2\mathrm{pt}}$, 8($\nu$),6 ($\eta$)loops [This work]

Conclusions

- Borel summability in 2*d* and 3*d*
- Interpretation of beyond λ_c
- Resummation at strong coupling

Outlook

- Broken phase
- 3d theories
- Theories with gauge symmetries

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Thank You