Localisation and Non-perturbative dynamics in supersymmetric theories

M. Billo, M.L. Frau, A. Lerda, F.Fucito, JFM, Y. Stanev and C. Wen

J. Aguilera, D. Correa, F.Fucito, V. Giraldo, JFM, L.Pando-Zayas

F.Fucito, JFM, R. Poghossian

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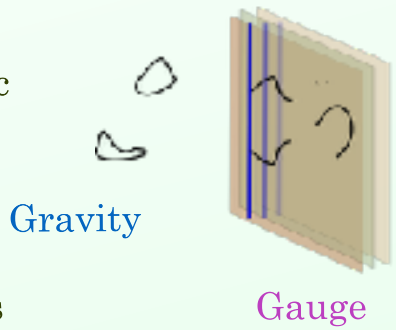

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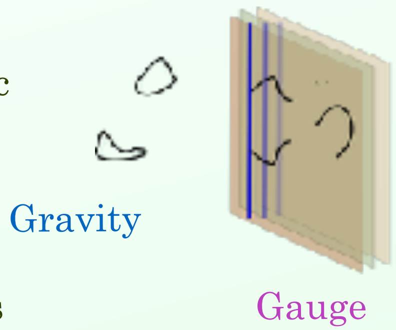

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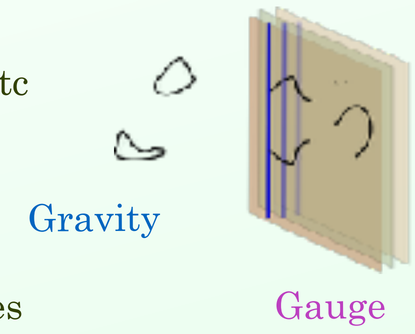

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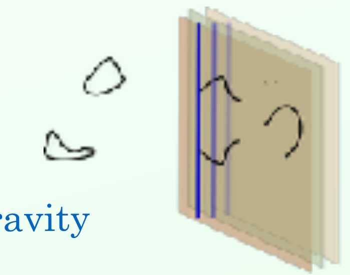

Gravity

Gauge

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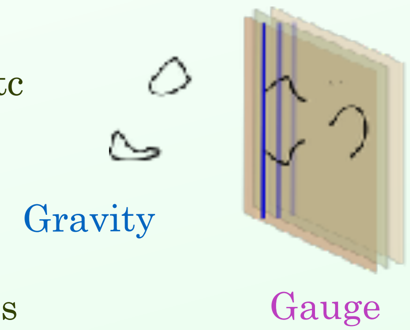

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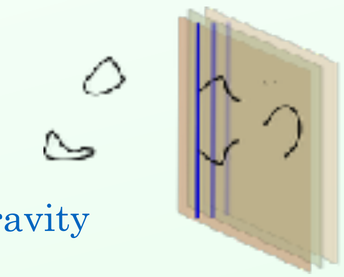

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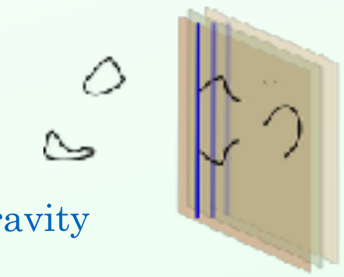

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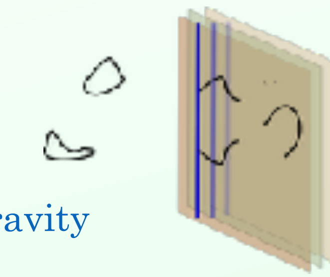

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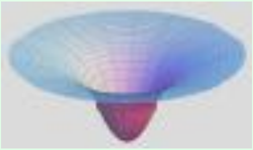

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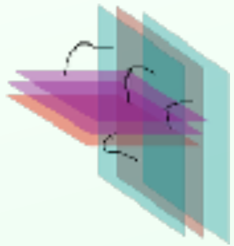

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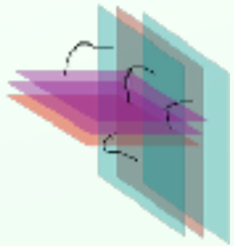

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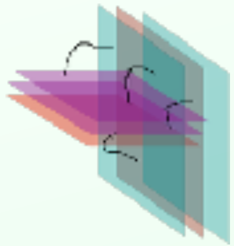

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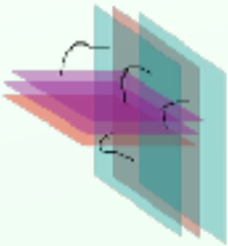

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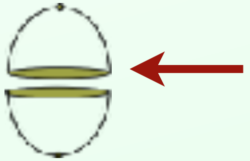

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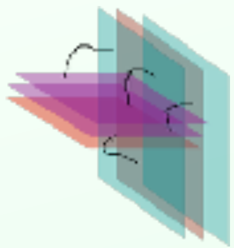

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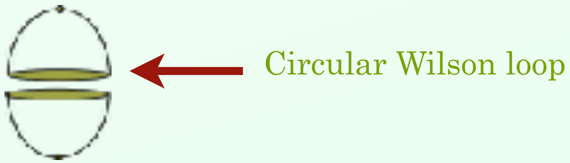

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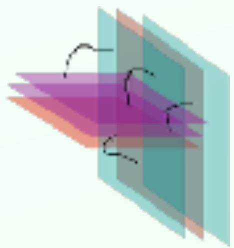

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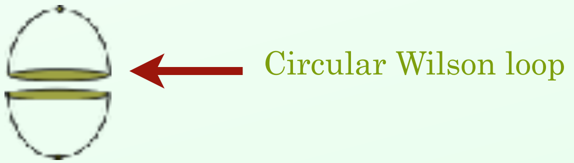

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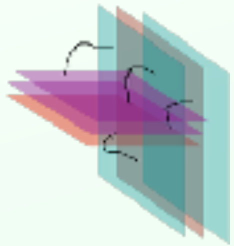

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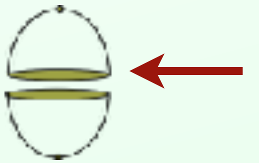

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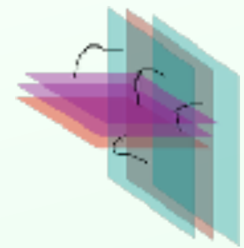

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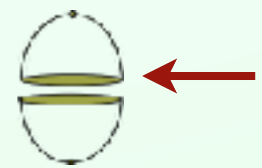

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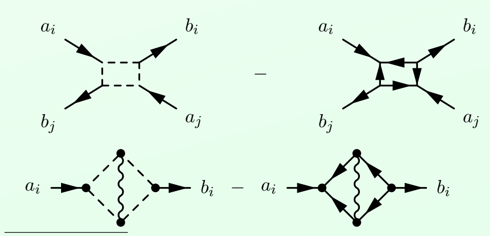

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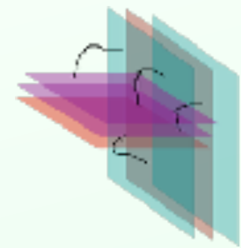

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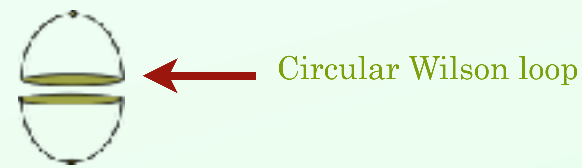

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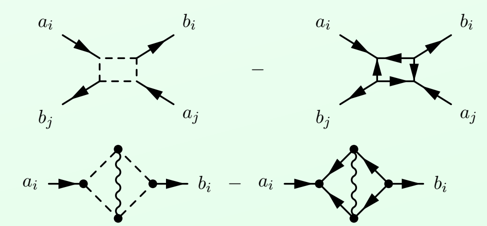

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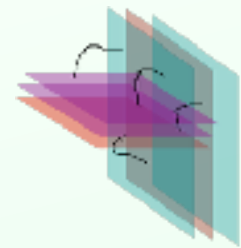

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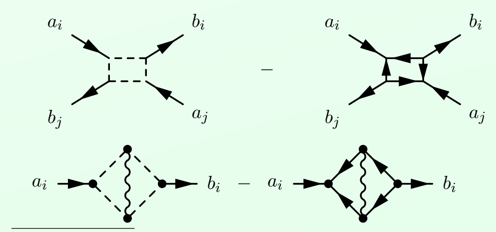

- **S-duality: N=2* prepotentials**
  Billo, Frau, Fucito, Lerda, JFM

$$S_{\text{eff}} = \int d^4x d^4\theta \mathcal{F}(\Phi)$$

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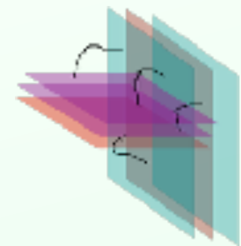

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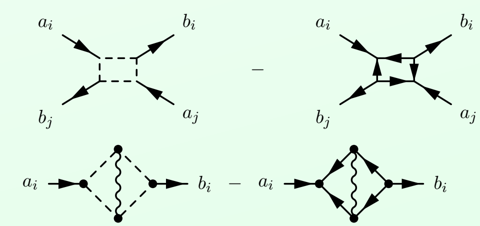

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Self-duality of SU(N) & SO(2r+1)/Sp(2r) duality

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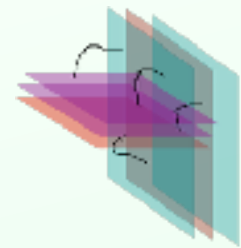

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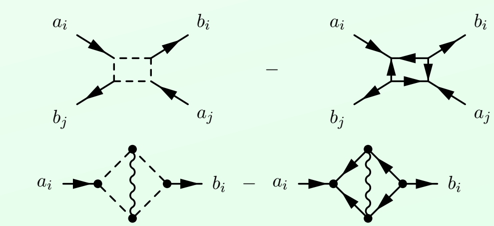

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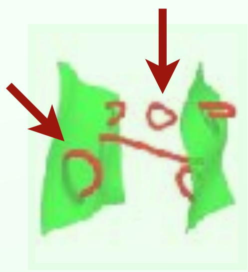
  - Closed strings: Gravity theories



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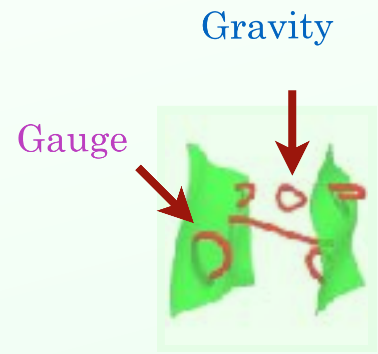

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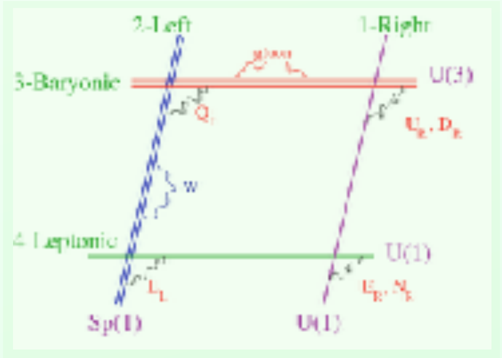
  - Matter fields: Brane intersections



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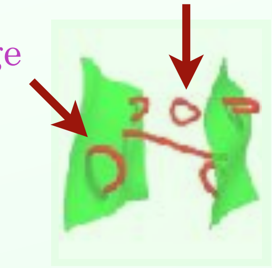

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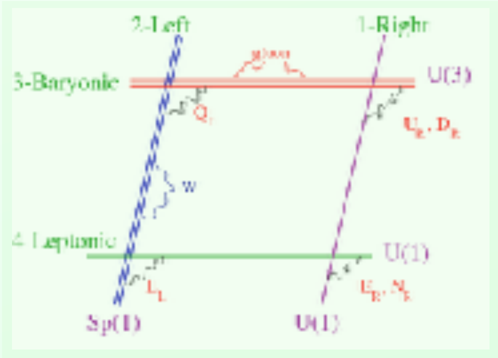

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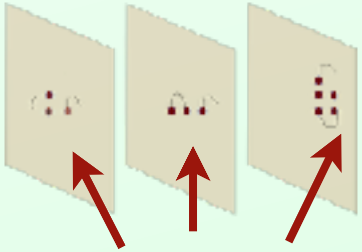

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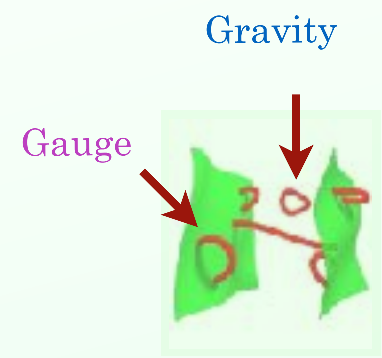

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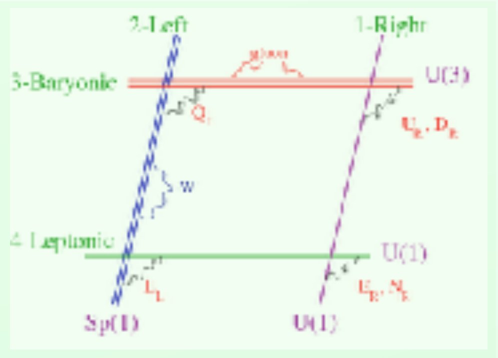

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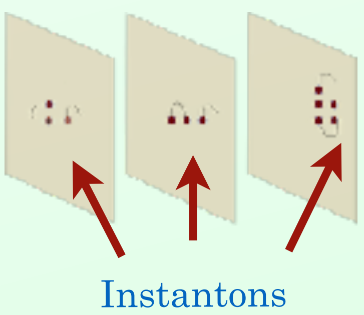
⇒

Non-perturbative effects from Matrix integrals !



Localization

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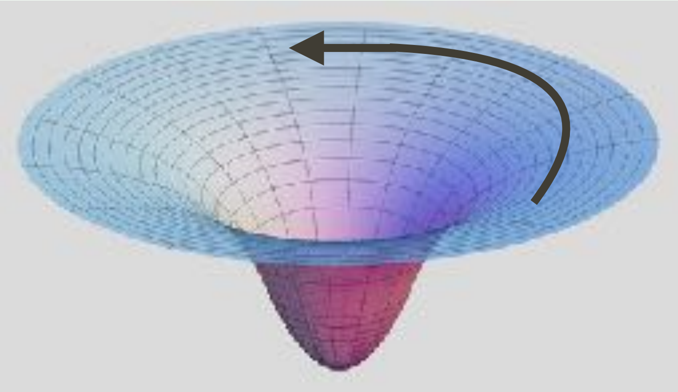
- Localization formula:

$$Q_\xi \equiv d + i_\xi \qquad i_\xi dx^i \equiv \delta_\xi x^i \qquad Q_\xi^2 = \delta_\xi$$



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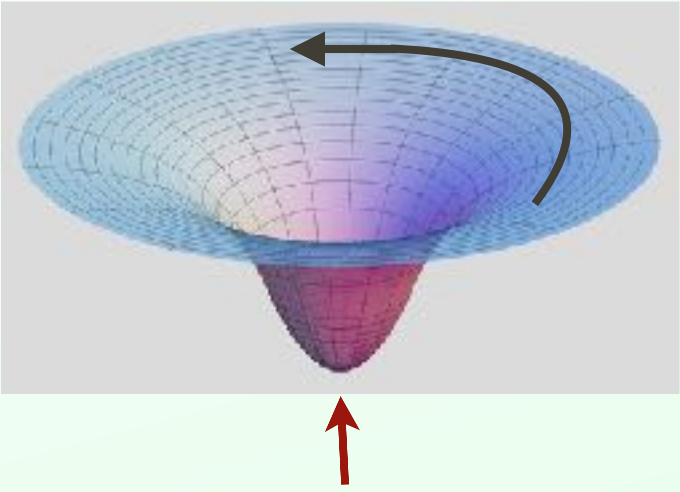

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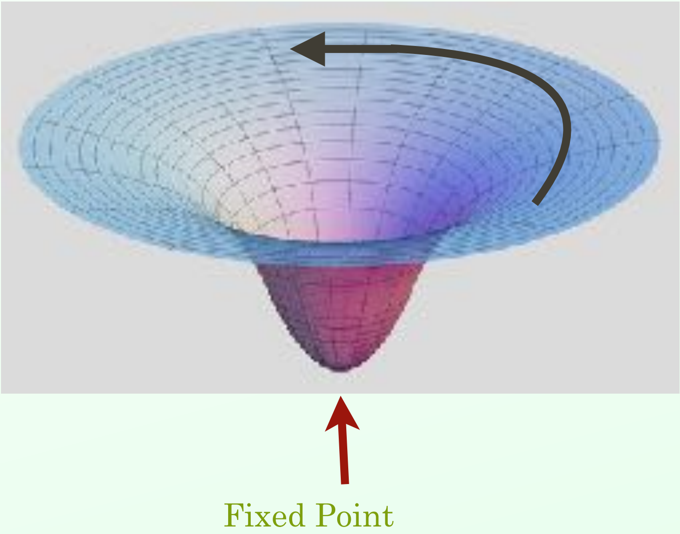

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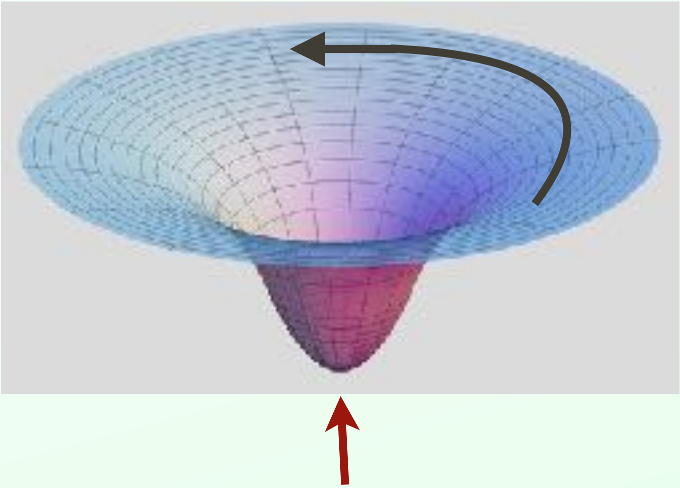

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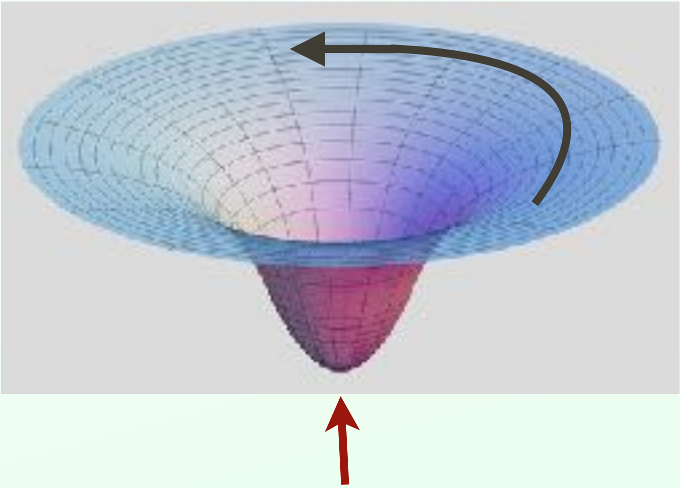

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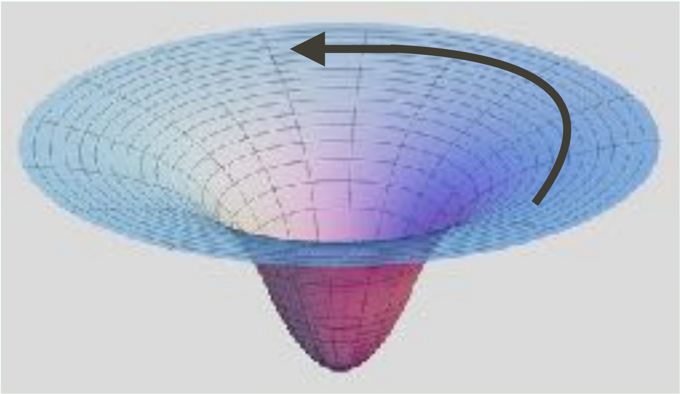

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- **Supersymmetric theories:** Q_ξ Supersymmetry charge and $\alpha = e^{-S}$

Gauge partition functions

Gauge partition functions

- Path integral

$$Z_{\mathbb{R}^4}(a,q) = \int \mathcal{D}\Phi\, e^{-S_{YM}} = \sum_{k=0}^{\infty} q^k \int d\mathfrak{M}_k\, e^{-S_{\rm inst,k}}$$

instanton number

ADHM instanton moduli:
positions , sizes, orientations

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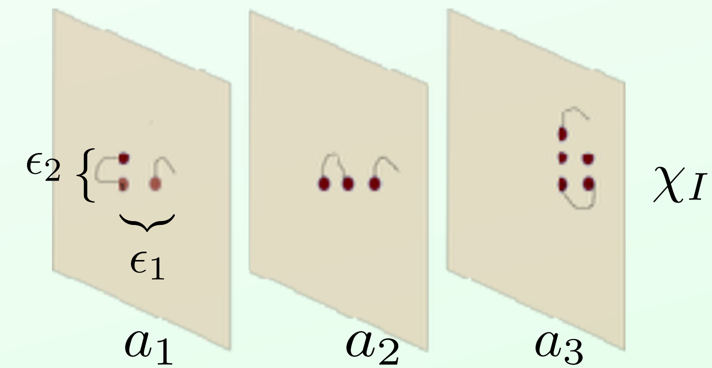

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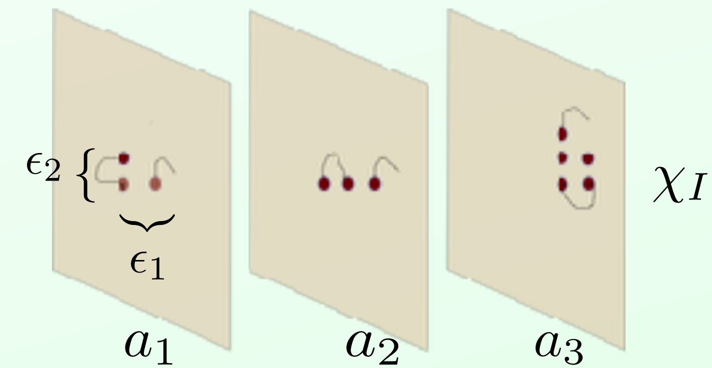

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Nekrasov

$$\mathcal{F} = -\lim_{\epsilon_l \to 0} \epsilon_1\epsilon_2 \ln Z(\epsilon_1,\epsilon_2;q)$$

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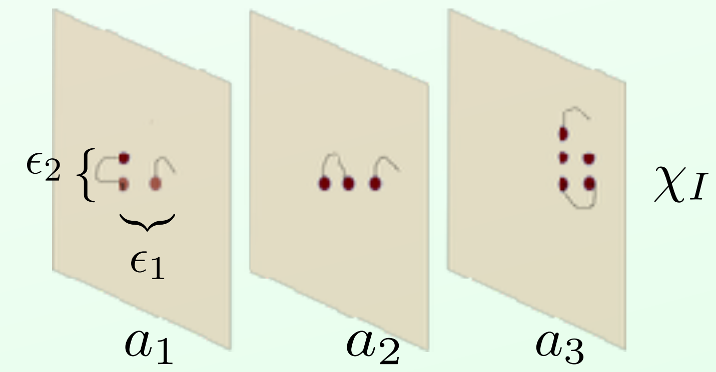

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- **N=4 theory:**

$$Z_{\text{inst}} = 1 \qquad |Z_{\text{oneloop}}|^2 = \Delta(a) = \prod_{u<v} a_{uv}^2$$

No-instantons !

N=2 gauge theories

N=2 gauge theories

- The prepotential

$$S_{\text{eff}} = \int d^4x d^4\theta \mathcal{F}(\Phi)$$

Prepotential

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Gauge instantons

$$q = e^{2\pi i \tau}$$

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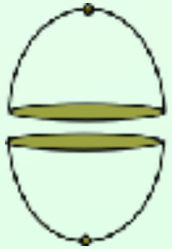
Prepotential

Instantons

$\epsilon_1 = \epsilon_2 =$



Anti-instantons

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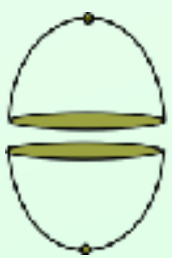

Anti-instantons

- **SUSY observables:** Prepotentials, Wilson loops, two-point correlators

Wilson loops

Wilson loops

- Supersymmetric Wilson loop:

$$\mathcal{C} = i \int_0^L (A_m \, \dot{x}^m + |\dot{x}| \, \varphi_1) ds$$

$$z_\ell(s) = r_\ell \, e^{i\epsilon_\ell s}$$

Circular Wilson loops

Squashed parameter

$$\frac{\epsilon_1}{\epsilon_2} = \frac{n_1}{n_2}$$

$$|\dot{x}| = 1$$

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$$\langle \mathrm{tr}\, e^{\mathcal{C}} \rangle_{S^4} = \frac{1}{Z} \int_\gamma d^N a \, \mathrm{tr}\, e^{\frac{2\pi i n_1 \, a}{\epsilon_1}} \; |Z_{\mathrm{one-loop}}(\, a) Z_{\mathrm{tree+inst}}(a, \vec{\tau}) \,|^2$$

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Wilson loop

N=4 theory

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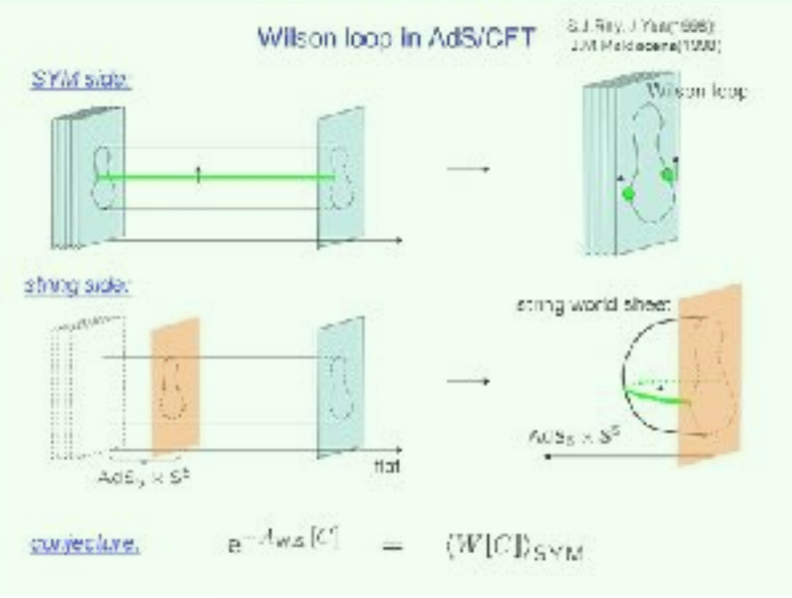
- Fundamental representation



N=4 theory

- **Fundamental representation**

$$W = \frac{2\, I_1\left(\sqrt{\lambda_{\rm YM}}\right)}{\sqrt{\lambda_{\rm YM}}} = 1 + \frac{\lambda_{\rm YM}}{8} + \frac{\lambda_{\rm YM}^2}{192} + \ldots$$

loops

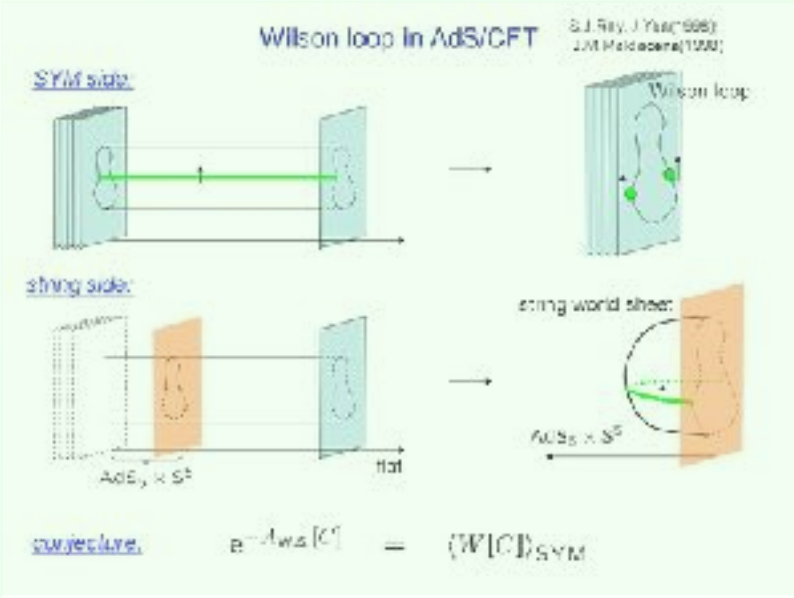
Weak coupling

$$= \frac{e^{\sqrt{\lambda_{\rm YM}}}}{4\sqrt{2\pi}\,\lambda_{\rm YM}^{\frac{5}{4}}}\left(-3 + 8\sqrt{\lambda_{\rm YM}} + \ldots\right)$$

holography

Strong coupling

$$\lambda_{YM} = g_{\rm YM}^2 N$$



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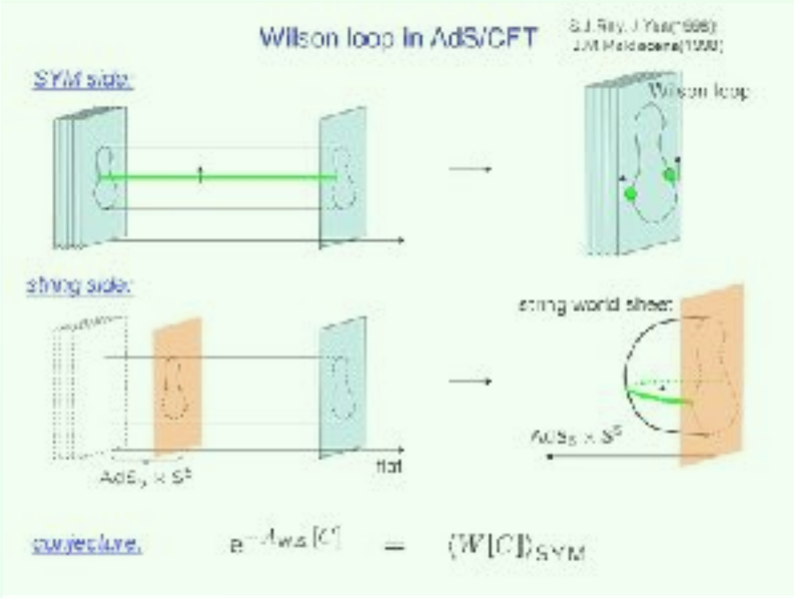
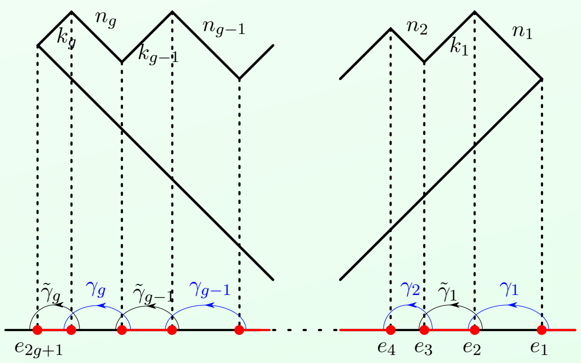


- Large representations



Dual Geometry

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + f_4^2 ds^2_{S^4} + d\Sigma^2\,.$$

given in terms of two polynomials

$$H_{2g+2}(z)^2 = \prod_{i=1}^{2g+2}(z - e_i) \qquad P_{g+1}(z)$$

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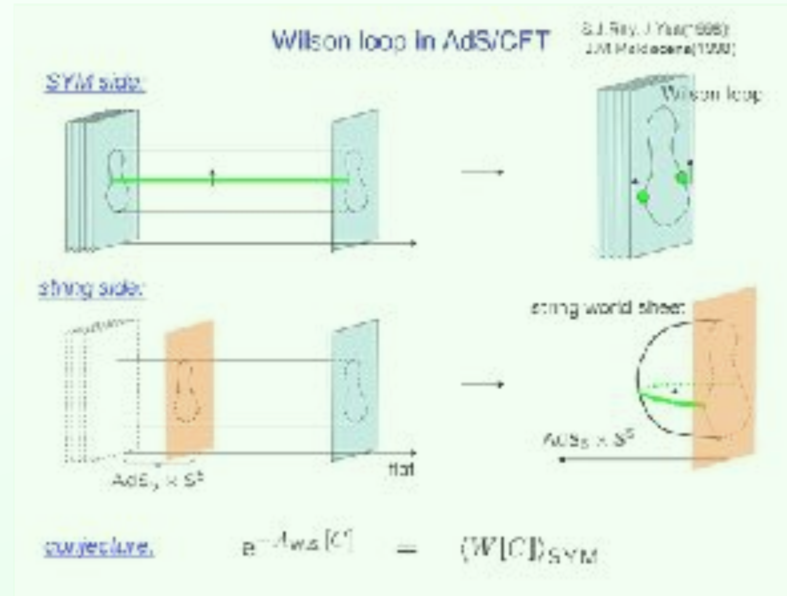

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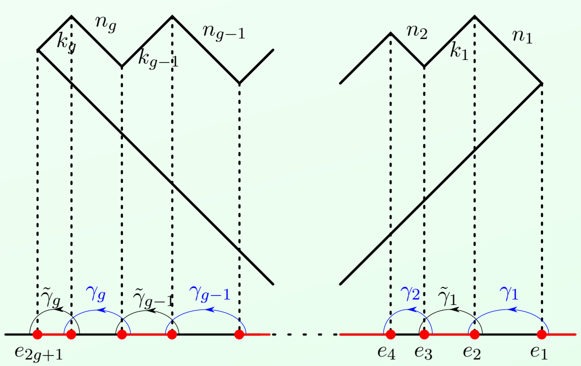

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- Minimal action: Fundamental string

$$S = \frac{1}{2\pi\alpha'}\int d^2\sigma\sqrt{\det(G_{MN}\partial_\alpha X^M \partial_\beta X^N)} + \frac{1}{2\pi\alpha'}\int B$$

- Fundamental string action

$$S=\frac{1}{2\pi\alpha'}\int d\phi\, d\rho\, \sinh\rho\, e^{\frac{\Phi(z)}{2}} f_1(z)^2\sqrt{1+\frac{4\,\sigma(z)^2}{f_1(z)^2}|z'|^2}+\frac{1}{2\pi\alpha'}\int d\phi\, d\rho\, \sinh\rho\, b_1(z)\,,$$

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- Equations of motion: $z(\rho) = \text{const}$ $\quad\quad$ $\partial_z \left(e^{\frac{\Phi}{2}} f_1^2 \right) = \partial_z b_1 = 0$

- **Fundamental string action**

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$\Longrightarrow$ large N and large λ

Agree with Sugra

$$\frac{\langle W_{\mathbf{R}}\, W_{\text{fund}}\rangle}{\langle W_{\mathbf{R}}\rangle} = \int_{-\infty}^{\infty} dx\, \rho(x)\, e^x \approx \frac{2}{\pi\lambda}\sum_{i=1}^{g+1}\int_{c_i-\mu_i}^{c_i+\mu_i} dx\, \sqrt{\mu_i^2-(x-c_i)^2}\, e^x \approx \sum_{i=1}^{g+1} e^{\sqrt{\frac{\lambda n_i}{N}} + \frac{\lambda}{4N}\left(K_i - \sum_j^g \frac{K_j n_j}{N}\right)}\,.$$

!

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$$\langle W \rangle = \frac{1}{Z_{S^4}} \int_\gamma da \, |Z_{\rm tree}(a,\tau) Z_{\rm one-loop}(a) Z_{\rm inst}(a,q)|^2 \, {\rm Tr}_{\rm fund} \, e^{\frac{2\pi i a}{\epsilon_1}}$$

Infinite sum over Young-tableaux

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$$Z_1(q) = {}_2F_1\left(\begin{smallmatrix}A_1,A_2\\ B\end{smallmatrix}\Big|q\right) \quad , \quad Z_2(q) = q^{1-B_2}\,{}_2F_1\left(\begin{smallmatrix}1-B+A_1,1-B+A_2\\ 2-B\end{smallmatrix}\Big|q\right)$$

$$A_u = \frac{m_1-\bar m_u+\epsilon_1-\epsilon_2}{\epsilon_1}-1 \quad , \quad B_u = \frac{m_1-m_u}{\epsilon_1}+1$$

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Exact formula !

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- **t'Hooft loop** $q \approx 1$ strong coupling

$$\langle W \rangle = \frac{\sum_{i,j}^N c_{ij} \tilde{Z}_j \tilde{\tilde{Z}}_j}{\sum_{j=1}^N r_j \, |Z_j|^2}$$

t'Hooft loop operators

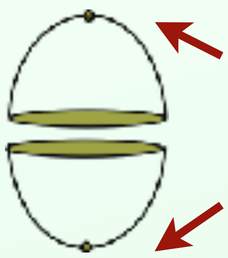
Two point correlators

Two point correlators

- **Localization formula** .

Baggio, Niarchos, Papadodimas
Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu,

$$\left\langle \mathcal{O}^i_{(n)}(\varphi(x_1))\, \bar{\mathcal{O}}^j_{(n)}(\bar{\varphi}(x_2)) \right\rangle_{\rm QFT} = \frac{1}{(4\pi^2 x_{12}^2)^n} \left\langle \mathcal{O}^i_{(n)}(a)\, \bar{\mathcal{O}}^j_{(n)}(a) \right\rangle$$



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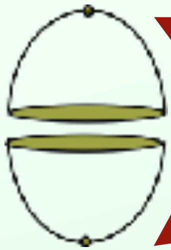

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loop & instantons

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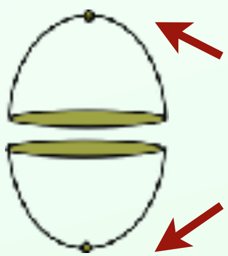

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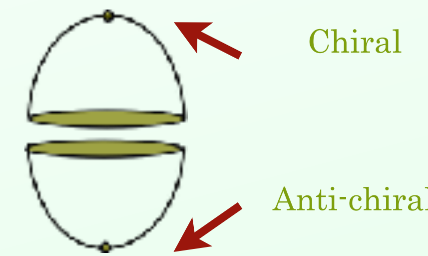

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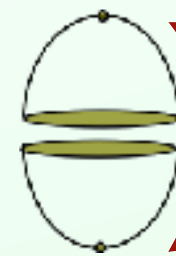

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Tree Adj Fund

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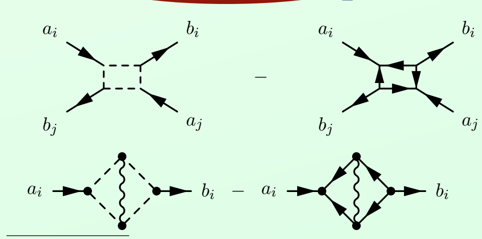
Nf=2N 2-loop



Non-conformal case

- **Divergences:**

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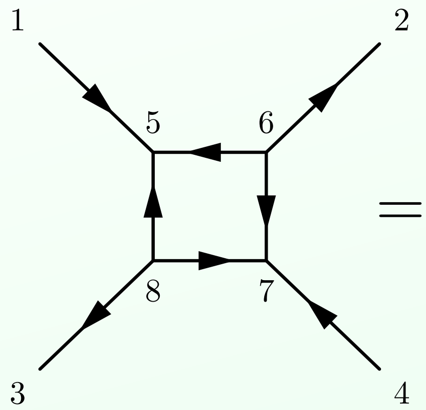
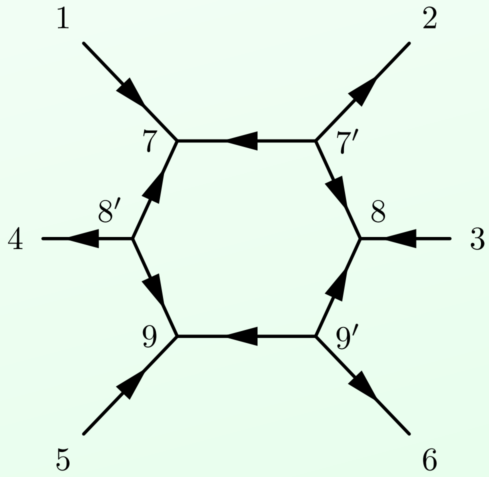
$$c = \frac{3-2N^2}{N(N^2+1)} \qquad c_1 = -\frac{3(2N^2-5)}{N(N^2+7)}\,, \qquad c_2 = \frac{(7\,N^4 - 39\,N^2 + 30)}{N^2(N^2+3)(N^2+7)}$$

Effective vertices

- Non-trivial contributions



$$= -\frac{6\,\zeta(3)}{2}\left(\frac{g^2}{8\pi^2}\right)^2\left[N_f\,\mathrm{tr}_{\mathrm{fund}}(\varphi\bar{\varphi})^2 - \mathrm{tr}_{\mathrm{adj}}(\varphi\bar{\varphi})^2\right]$$



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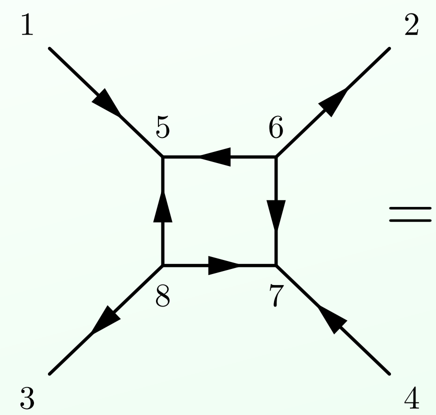

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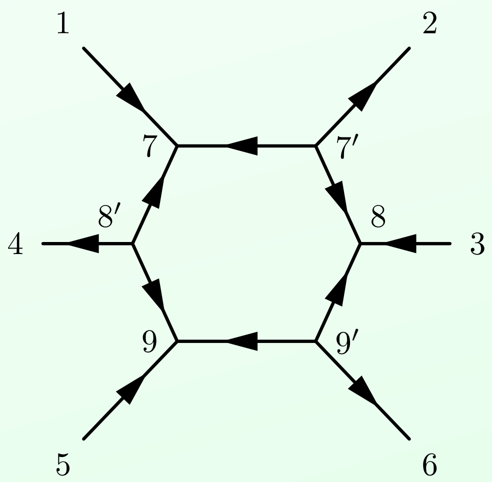
loop integrals



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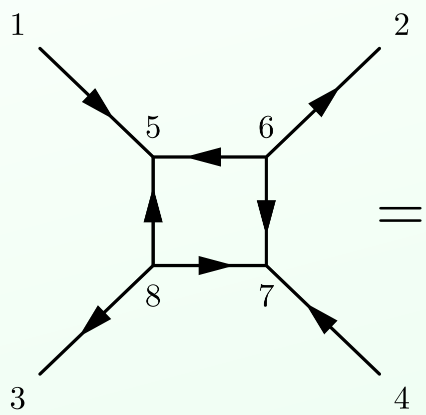
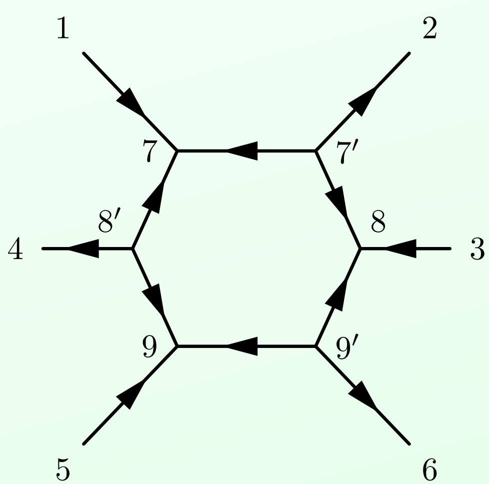


$$= -\frac{6\zeta(3)}{2}\left(\frac{g^2}{8\pi^2}\right)^2 \left[N_f\, \mathrm{tr}_{\mathrm{fund}}(\varphi\bar{\varphi})^2 - \mathrm{tr}_{\mathrm{adj}}(\varphi\bar{\varphi})^2\right]$$

loop integrals



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⟹ Match with matrix model vertex S_4 and S_2

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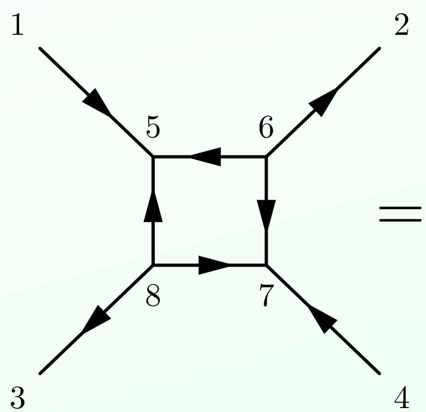

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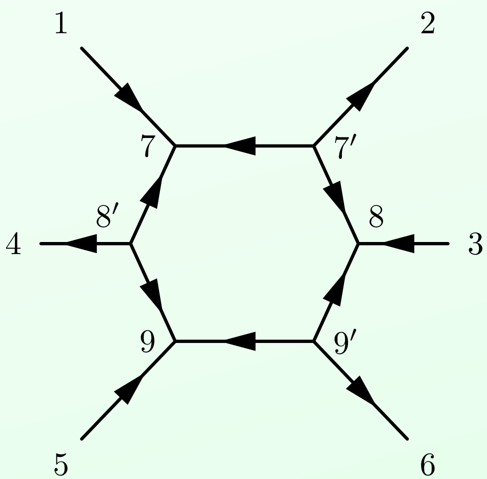

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- What about divergences ?

N=2* theory

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$$\mathcal{F}\left(a_D, -\frac{1}{\tau}\right) = \mathcal{F}\left(a, \tau\right) - \pi\,\mathrm{i}\,a \cdot a_D \qquad\qquad a_D = \frac{1}{2\pi i}\frac{\partial F}{\partial a}$$

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Classical — One-loop+instantons

$f_n(E_2, E_4, E_6)$

quasi-Modular functions of weight 2n-2

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- Other groups: $\tau \to -\frac{1}{2\tau} \quad SO \leftrightarrow Sp$!

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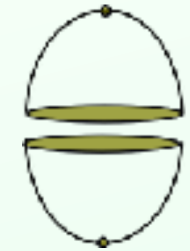

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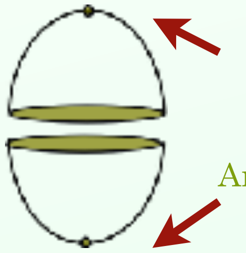

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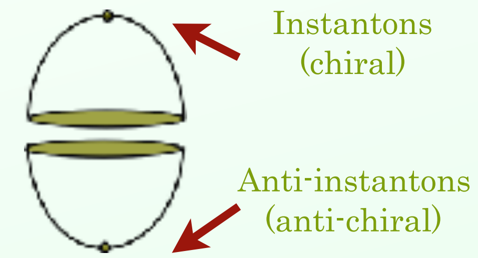

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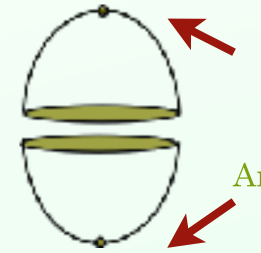

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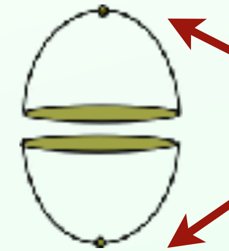

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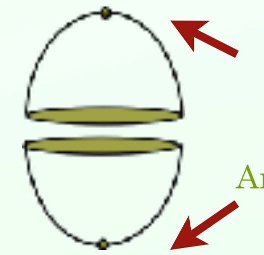

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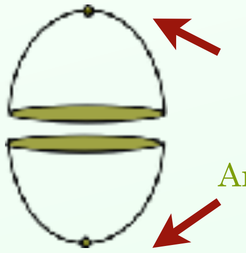

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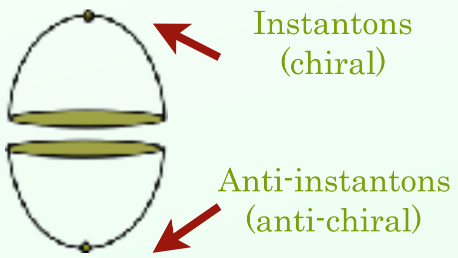

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  - Other areas of physics : Critical phenomena, condensed matter, gravity,

Thank you !!!