Dynamical Generation of Fermion Mixing

Luca Smaldone
in collaboration with
Massimo Blasone, Petr Jizba and Nick Mavromatos

Dipartimento di Fisica "E.R.Caianiello"
and INFN Sezione di Napoli, Gruppo Collegato di Salerno

New Frontiers in Theoretical Physics
Cortona, 23th May 2018

Contents

Motivations

- Flavor mixing is a central ingredient in the Standard Model;
- Mixing transformations for fields have been shown to be non–trivial since they induce a condensate structure in the vacuum[1];
- This suggests the idea of dynamical generation of mixing in a similar way as it happens for the masses[2].

[1] M.Blasone, G.Vitiello, Ann. Phys., **244**,(1995), 283.
[2] M. Blasone, P. Jizba, N. E. Mavromatos, L. S., in preparation (2018).

Neutrino Mixing in QFT

Massive neutrinos

- Two-flavor neutrino mixing Lagrangian:

$$\mathcal{L} \;=\; \sum_{\sigma=e,\mu} \overline{\nu}_\sigma(x)\left(i\gamma^\mu\partial_\mu - m_\sigma\right)\nu(x) \;-\; m_{e\mu}\left(\overline{\nu}_e(x)\nu_\mu(x) + \overline{\nu}_\mu(x)\nu_e(x)\right)$$

- Mixing transformations define fields with definite mass ν_1, ν_2:

$$\begin{aligned} \nu_e(x) &= \cos\theta\,\nu_1(x) + \sin\theta\,\nu_2(x) \\ \nu_\mu(x) &= -\sin\theta\,\nu_1(x) + \cos\theta\,\nu_2(x) \end{aligned}$$

with

$$\tan 2\theta \;=\; \frac{2m_{e\mu}}{m_e - m_\mu}$$

Mass eigenstates

Fields with definite masses can be expanded as:

$$\nu_i(x) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k},r} \left[u^r_{\mathbf{k},i}(t)\alpha^r_{\mathbf{k},i} \;+\; v^r_{-\mathbf{k},i}(t)\beta^{r\dagger}_{-\mathbf{k},i} \right] e^{i\mathbf{k}\cdot\mathbf{x}} , \quad i = 1,2$$

- A mass-eigenstate neutrino is defined as:

$$|\nu^r_{\mathbf{k},i}\rangle = \alpha^{r\dagger}_{\mathbf{k},i}|0\rangle_{1,2} , \quad i = 1,2$$

where the mass vacuum satisfies:

$$\alpha^r_{\mathbf{k},i}|0\rangle_{1,2} \;=\; \beta^r_{\mathbf{k},i}|0\rangle_{1,2} , \quad i = 1,2$$

Flavor Charges

- Flavor charges[3]:

$$Q_{\nu_e}(t) = \int d^3\mathbf{x}\, \nu_e^\dagger(x)\nu_e(x)\,, \qquad Q_{\nu_\mu}(t) = \int d^3\mathbf{x}\, \nu_\mu^\dagger(x)\nu_\mu(x)$$

- Total flavor charge:

$$Q \;=\; Q_{\nu_e}(t) \;+\; Q_{\nu_\mu}(t)$$

Because of mixing, only this last is conserved!!

[3] M.Blasone, P.Jizba, G.Vitiello, Phys. Lett. B, **517**, (2001), 471.

Standard Flavor eigenstates

Flavor eigenstates, are usually taken as a simple combination of mass eigenstates:

$$|\nu^r_{\mathbf{k},e}\rangle_P \;=\; \cos\theta\,|\nu^r_{\mathbf{k},1}\rangle \,+\, \sin\theta\,|\nu^r_{\mathbf{k},2}\rangle$$

$$|\nu^r_{\mathbf{k},\mu}\rangle_P \;=\; -\sin\theta\,|\nu^r_{\mathbf{k},1}\rangle \,+\, \cos\theta\,|\nu^r_{\mathbf{k},2}\rangle$$

ACHTUNG! These are NOT eigenstates of the flavor charges:

$${}_P\langle\nu^r_{\mathbf{k},e}| :Q_{\nu_e}: |\nu^r_{\mathbf{k},e}\rangle_P \;=\; \cos^4\theta + \sin^4\theta + 2|U_{\mathbf{k}}|\sin^2\theta\cos^2\theta \;<\; 1$$

$${}_P\langle\nu^r_{\mathbf{k},e}| :Q_{\nu_\mu}: |\nu^r_{\mathbf{k},e}\rangle_P \;=\; 2\,(1-|U_{\mathbf{k}}|)\sin^2\theta\cos^2\theta \;>\; 0$$

with $U_{\mathbf{k}} = u^{r\dagger}_{2,\mathbf{k}} u^r_{1,\mathbf{k}}$.

Mixing generator

- At finite volume, mixing relations are rewritten as

$$\begin{aligned} \nu_e^\alpha(x) &= G_\theta^{-1}(t)\nu_1^\alpha(x)\, G_\theta(t) \\ \nu_\mu^\alpha(x) &= G_\theta^{-1}(t)\, \nu_2^\alpha(x) G_\theta(t) \end{aligned}$$

Mixing generator:

$$G_\theta(t) = \exp\left[\theta \int d^3\mathbf{x}\, \left(\nu_1^\dagger(x)\nu_2(x) - \nu_2^\dagger(x)\nu_1(x)\right)\right]$$

Decomposition of the mixing generator

- The mixing generator can be decomposed as[4]:

$$G_\theta \;=\; B(\Theta_1,\Theta_2)\;\; R(\theta)\;\; B^{-1}(\Theta_1,\Theta_2)$$

where $B(\Theta_1,\Theta) \equiv B_1(\Theta_1)\, B_2(\Theta_2)$,

$$R(\theta) \equiv \exp\left\{\theta \sum_{\mathbf{k},r}\Big[\left(\alpha^{r\dagger}_{\mathbf{k},1}\alpha^{r}_{\mathbf{k},2} + \beta^{r\dagger}_{-\mathbf{k},1}\beta^{r}_{-\mathbf{k},2}\right) e^{i\psi_k} - h.c.\Big]\right\}$$

$$B_i(\Theta_i) \equiv \exp\Big\{\sum_{\mathbf{k},r}\Theta_{\mathbf{k},i}\;\epsilon^r\Big[\alpha^{r}_{\mathbf{k},i}\beta^{r}_{-\mathbf{k},i}e^{-i\phi_{k,i}} - \beta^{r\dagger}_{-\mathbf{k},i}\alpha^{r\dagger}_{\mathbf{k},i}e^{i\phi_{k,i}}\Big]\Big\}, \quad i=1,2$$

and $\Theta_{\mathbf{k},i} = 1/2\ \cot^{-1}(|\mathbf{k}|/m_i)$, $\quad \psi_k = (\omega_{k,1} - \omega_{k,2})t, \quad \phi_{k,i} = 2\omega_{k,i}t.$

[4] M.Blasone, M.V.Gargiulo, G.Vitiello, Phys. Lett.B, **761**, (2016),104.

$B_i(\Theta_{\mathbf{k},i})$, $i = 1, 2$ are Bogoliubov transformations which induces a mass shift and $R(\theta)$ is a rotation.

Their action on the mass vacuum is:

$$
\begin{aligned}
|\widetilde{0}\rangle_{1,2} &\equiv B^{-1}(\Theta_1, \Theta_2)|0\rangle_{1,2} \\
&= \prod_{\mathbf{k},r} \Big[\cos\Theta_{\mathbf{k},i} + \epsilon^r \sin\Theta_{\mathbf{k},i}\alpha^{r\dagger}_{\mathbf{k},i}\beta^{r\dagger}_{-\mathbf{k},i}\Big]|0\rangle_{1,2} \\
R^{-1}(\theta)|0\rangle_{1,2} &= |0\rangle_{1,2}
\end{aligned}
$$

- A rotation of fields is not a rotation at the level of creation and annihilation operators!

Flavor Vacuum

- The flavor vacuum is defined by[5]:

$$|0(t)\rangle_{e,\mu} \equiv G_\theta^{-1}(t)\; |0\rangle_{1,2}$$

In the infinite volume limit:

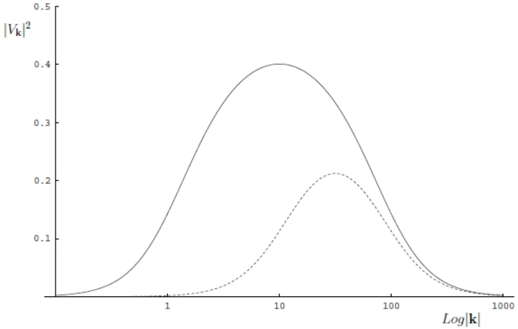
$$\lim_{V\to\infty}\; {}_{1,2}\langle 0|0(t)\rangle_{e,\mu} = \lim_{V\to\infty}\; e^{V\int \frac{d^3\mathbf{k}}{(2\pi)^3}\;\ln\left(1-\sin^2\theta\,|V_\mathbf{k}|^2\right)^2} = 0$$

where

$$|V_\mathbf{k}|^2 \;\;\equiv\;\; \sum_{r,s} |\; v^{r\dagger}_{-\mathbf{k},1} u^s_{\mathbf{k},2}\;|^2 \neq 0 \quad for \quad m_1 \neq m_2$$

[5] M.Blasone, G.Vitiello, Ann. Phys., **244**,(1995), 283.

Vacuum condensate



Solid line: $m_1 = 1$, $m_2 = 100$; Dashed line: $m_1 = 10$, $m_2 = 100$.

- Condensation density: ${}_{e,\mu}\langle 0|\alpha^{r\dagger}_{\mathbf{k},i}\alpha_{\mathbf{k},i}|0\rangle_{e,\mu} = \sin^2\theta\,|V_{\mathbf{k}}|^2$, with $i = 1, 2$. Same result for antiparticles.

- $|V_{\mathbf{k}}|^2 \simeq \frac{(m_2 - m_1)^2}{4k^2}$ for $k \gg \sqrt{m_1 m_2}$.

Flavor eigenstates

- A neutrino flavor-state can be defined:

$$|\nu^r_{\mathbf{k},\sigma}(t)\rangle = \alpha^{r\dagger}_{\sigma,\mathbf{k}}|0(t)\rangle_{e,\mu}\,, \qquad \sigma = e,\mu$$

- At any time these are eigenstates of the flavor charges:

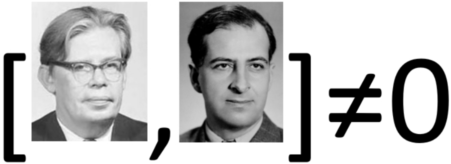
$$Q_\sigma(t)|\nu^r_{\mathbf{k},\sigma}(t)\rangle = |\nu^r_{\mathbf{k},\sigma}(t)\rangle$$

Note that $|\nu^r_{\mathbf{k},\sigma}(t)\rangle \neq |\nu^r_{\mathbf{k},\sigma}\rangle_P$ because

$$\left[B(m_1,m_2)\,,\ R^{-1}(\theta)\right]\ \neq\ 0$$

Bogoliubov vs Pontecorvo

- Bogoliubov and Pontecorvo do not commute!!



As a result, flavor vacuum gets a non-trivial term:

$$|0\rangle_{e,\mu} \equiv G_\theta^{-1}|0\rangle_{1,2} = |0\rangle_{1,2} + \left[B(m_1, m_2)\,,\; R^{-1}(\theta)\right]\; |\widetilde{0}\rangle_{1,2}$$

Flavor Vacuum and Condensate Structure

The flavor vacuum is characterized by a condensate structure:

$$|0\rangle_{e,\mu} = \prod_{\mathbf{k}} \prod_{r} \Big[(1 - \sin^2\theta \, |V_{\mathbf{k}}|^2) - \epsilon^r \sin\theta \, \cos\theta \, |V_{\mathbf{k}}| (\alpha^{r\dagger}_{\mathbf{k},1} \beta^{r\dagger}_{-\mathbf{k},2} + \alpha^{r\dagger}_{\mathbf{k},2} \beta^{r\dagger}_{-\mathbf{k},1})$$

$$+ \epsilon^r \sin^2\theta \, |V_{\mathbf{k}}| |U_{\mathbf{k}}| (\alpha^{r\dagger}_{\mathbf{k},1} \beta^{r\dagger}_{-\mathbf{k},1} - \alpha^{r\dagger}_{\mathbf{k},2} \beta^{r\dagger}_{-\mathbf{k},2}) + \sin^2\theta \, |V_{\mathbf{k}}|^2 \alpha^{r\dagger}_{\mathbf{k},1} \beta^{r\dagger}_{-\mathbf{k},2} \alpha^{r\dagger}_{\mathbf{k},2} \beta^{r\dagger}_{-\mathbf{k},1} \Big] |0\rangle_{1,2}$$

- SU(2) (Perelomov) coherent state.
- Condensate structure on vacuum as in systems with SSB (e.g. superfluids, superconductors).
- Exotic condensates: mixed pairs due to a non-diagonal Bogoliubov transformation.
- Note that $|0\rangle_{e\,\mu} \neq |a\rangle_1 \otimes |b\rangle_2 \Rightarrow$ entanglement.

Patterns of Dynamical Symmetry Breaking

Chiral symmetry

Let us consider a Lagrangian $\mathcal{L}$, invariant under the *chiral-flavor* group $SU(2)_A \times SU(2)_V \times U(1)_V$. Consider a flavor fermion-doublet:

$$\boldsymbol{\psi} \;=\; \begin{bmatrix} \tilde{\psi}_1 \\ \tilde{\psi}_2 \end{bmatrix} .$$

The chiral transformations act on it as

$$\mathbf{g}\boldsymbol{\psi} \quad = \quad \exp\left[i \left(\phi + \boldsymbol{\omega} \cdot \frac{\boldsymbol{\sigma}}{2} + \boldsymbol{\omega}_5 \cdot \frac{\boldsymbol{\sigma}}{2} \gamma_5 \right) \right] \boldsymbol{\psi} \,,$$

where $\sigma_i \,, i = 1, 2, 3$ are the Pauli matrices, ϕ is a real number and $\boldsymbol{\omega}$, $\boldsymbol{\omega}_5$ are vectors with real components.

Charges and Currents

- The vector and axial Noether currents are:

$$
\begin{aligned}
J^\mu &= \overline{\psi}\gamma^\mu \psi \\
\boldsymbol{J}^\mu &= \overline{\psi}\gamma^\mu \frac{\boldsymbol{\sigma}}{2}\psi \\
\boldsymbol{J}_5^\mu &= \overline{\psi}\gamma^\mu \gamma_5 \frac{\boldsymbol{\sigma}}{2}\psi
\end{aligned}
$$

and the conserved Noether charges:

$$
\begin{aligned}
Q &= \psi^\dagger \psi \\
\boldsymbol{Q} &= \int d^3\mathbf{x}\, \psi^\dagger \frac{\boldsymbol{\sigma}}{2}\psi \\
\boldsymbol{Q}_5 &= \int d^3\mathbf{x}\, \psi^\dagger \frac{\boldsymbol{\sigma}}{2}\gamma_5 \psi
\end{aligned}
$$

Explicit Symmetry Breaking

- Chiral symmetry is explicitly broken by a mass term:

$$\mathcal{L}_M = -\overline{\boldsymbol{\psi}}\, M\, \boldsymbol{\psi}$$

In fact

$$\begin{aligned} \partial_\mu J^\mu &= 0\,, \\ \partial_\mu \boldsymbol{J}^\mu &= \frac{i}{2}\overline{\boldsymbol{\psi}}\,[M, \boldsymbol{\sigma}]\,\boldsymbol{\psi}\,, \\ \partial_\mu \boldsymbol{J}_5^\mu &= \frac{i}{2}\overline{\boldsymbol{\psi}}\gamma_5\,\{M, \boldsymbol{\sigma}\}\,\boldsymbol{\psi} \end{aligned}$$

Explicit Breaking of Axial Symmetry

- If the mass matrix is proportional to the identity

$$M = m_0 \mathbb{1}$$

the axial symmetry is explicitly broken:

$$\begin{aligned} \partial_\mu J^\mu &= \partial_\mu \boldsymbol{J}^\mu = 0 \\ \partial_\mu \boldsymbol{J}_5^\mu &= i\, m_0\, \overline{\boldsymbol{\psi}}\, \gamma_5\, \boldsymbol{\psi} \end{aligned}$$

Isospin Symmetry Breaking

- Adding a mass-shift:

$$M \;=\; m_0\,\mathbb{1} \;+\; m_3\,\sigma_3$$

the isospin symmetry is broken to $U(1)_V \times U(1)_V^3$ (the subscript index indicates the generator)

$$\begin{aligned}
\partial_\mu J^\mu &= \partial_\mu J_3^\mu \;=\; 0 \\
\partial_\mu J_1^\mu &= \frac{m_3}{2}\,\overline{\boldsymbol{\psi}}\,\sigma_2\,\boldsymbol{\psi}\,, \\
\partial_\mu J_2^\mu &= -\frac{m_3}{2}\,\overline{\boldsymbol{\psi}}\,\sigma_1\,\boldsymbol{\psi}
\end{aligned}$$

Total Flavor Charge Conservation

- Adding an off-diagonal component:

$$M \;=\; m_0\,\mathbb{1} \;+\; m_3\,\sigma_3 \;+\; m_1\,\sigma_1$$

A residual phase symmetry survives:

$$\begin{aligned}
\partial_\mu J^\mu &= 0 \\
\partial_\mu J_1^\mu &= \frac{m_3}{2}\,\overline{\boldsymbol{\psi}}\,\sigma_2\,\boldsymbol{\psi} \\
\partial_\mu J_2^\mu &= -\frac{1}{2}\overline{\boldsymbol{\psi}}\,\left[m_1\sigma_3 + m_3\sigma_1\right]\,\boldsymbol{\psi} \\
\partial_\mu J_3^\mu &= \frac{m_1}{2}\,\overline{\boldsymbol{\psi}}\,\sigma_2\,\boldsymbol{\psi}
\end{aligned}$$

Conservation of the total flavor charge Q.

Dynamical Generation of Fermion Mixing

- Dynamical generation of mixing occurs if[6]

$$SU(2)_A \times SU(2)_V \times U(1)_V \longrightarrow U(1)_V$$

at the ground state level.

SSB is characterized by the existence of some (quasi)-local operators Φ_i so that

$$\langle\Omega| \left[Q_{j,(5)}(0), \Phi_\alpha(0)\right] |\Omega\rangle \; = \; \langle\Omega|\varphi_{(5),\alpha}(0)|\Omega\rangle \; \neq \; 0$$

on some *dressed* vacuum.

[6] M. Blasone, P. Jizba, N. E. Mavromatos, L. S., in preparation (2018).

Order parameters

We introduce the fermion bilinears

$$\begin{aligned}\Phi_0 &= \overline{\boldsymbol{\psi}}\boldsymbol{\psi} \\ \Phi_\alpha &= \overline{\boldsymbol{\psi}}\,\sigma_\alpha\,\boldsymbol{\psi}\,, \\ \Phi_\alpha^5 &= \overline{\boldsymbol{\psi}}\,\sigma_\alpha\,\gamma_5\boldsymbol{\psi}\,, \qquad \alpha = 1,2,3\,.\end{aligned}$$

and the scalar order parameters:

$$v_\alpha \;=\; \langle\Omega|\Phi_\alpha(0)|\Omega\rangle\,, \qquad \alpha = 0,1,2,3$$

The condition

$$\langle\Omega|\Phi_\alpha^5|\Omega\rangle \;=\; 0\,, \quad \alpha = 1,2,3$$

is imposed.

Dynamical Mass Generation

- Dynamical mass generation:

$$SU(2)_A \times SU(2)_V \times U(1)_V \longrightarrow SU(2)_V \times U(1)_V$$

when $v_0 \neq 0$. We add ε-term:

$$\mathcal{L}_\varepsilon \;=\; \varepsilon_0 \Phi_0$$

Ward-Takahashi identities are derived:

$$iv_0 \;=\; \lim_{\varepsilon_0 \to 0} \varepsilon_0 \int \mathrm{d}^4 y \langle \Omega | T \left[\Phi^5_\alpha(y)\, \Phi^5_\alpha(0) \right] | \Omega \rangle \,, \quad \alpha = 1, 2, 3$$

The r.h.s. contains a pole at zero mass: Goldstone theorem

Dynamical Generation of Different Masses

- Dynamical generation of different masses:

$$SU(2)_A \times SU(2)_V \times U(1)_V \longrightarrow U(1)_V^3 \times U(1)_V$$

when $v_0, v_3 \neq 0$. We add ε-term:

$$\mathcal{L}_\varepsilon \ = \ \varepsilon_0 \Phi_0 \ + \ \varepsilon_3 \Phi_3$$

Ward-Takahashi identities are derived:

$$iv_3 \ = \ -\lim_{\varepsilon_3 \to 0} \varepsilon_3 \int \mathrm{d}^4 y \langle \Omega | T \left[\Phi_\alpha(y) \, \Phi_\alpha(0) \right] |\Omega\rangle \, , \quad \alpha = 1, 2$$

Dynamical Generation of Mixing

- Dynamical generation of mixing:

$$SU(2)_A \times SU(2)_V \times U(1)_V \longrightarrow U(1)_V$$

when $v_0, v_3, v_1 \neq 0$. We add ε-term:

$$\mathcal{L}_\varepsilon \; = \; \varepsilon_0 \Phi_0 \; + \; \varepsilon_3 \Phi_3 \; + \; \varepsilon_1 \Phi_1$$

One of the previous Ward-Takahashi identities remains the same:

$$i v_3 \quad = \quad - \lim_{\varepsilon_3 \to 0} \varepsilon_3 \int \mathrm{d}^4 y \langle \Omega | T \left[\Phi_2(y) \, \Phi_2(0) \right] | \Omega \rangle$$

A NG boson appears in the spectrum.

Anomalous WT relations

- We get two anomalous WT identities:

$$
\begin{aligned}
i\,v_1 &= -\lim_{\varepsilon_1\to 0}\varepsilon_1\int \mathrm{d}^4y\,\langle\Omega|T\left[(\Phi_3(y)-v_3)\;(\Phi_3(0)-v_3)\right]|\Omega\rangle \\
&+ \lim_{\varepsilon_3\to 0}\varepsilon_3\int \mathrm{d}^4y\,\langle\Omega|T\left[(\Phi_1(y)-v_1)\;(\Phi_3(0)-v_3)\right]|\Omega\rangle
\end{aligned}
$$

$$
\begin{aligned}
i\,v_3 &= -\lim_{\varepsilon_3\to 0}\varepsilon_3\int \mathrm{d}^4y\,\langle\Omega|T\left[(\Phi_1(y)-v_1)\;(\Phi_1(0)-v_1)\right]|\Omega\rangle \\
&+ \lim_{\varepsilon_1\to 0}\varepsilon_1\int \mathrm{d}^4y\,\langle\Omega|T\left[(\Phi_3(y)-v_3)\;(\Phi_1(0)-v_1)\right]|\Omega\rangle
\end{aligned}
$$

Patterns of SSB

Here we have a summary of different patterns:

Symmetry Group	Mass Term	Order Parameters	Broken Charges
$U(2)_V \times SU(2)_A$			
$U(2)_V$	$m_0\overline{\boldsymbol{\psi}}\boldsymbol{\psi}$	v_0	$Q^5_\alpha\,,\ \alpha=1,2,3$
$U(1)^3_V \times U(1)_V$	$m_0\overline{\boldsymbol{\psi}}\boldsymbol{\psi}+m_3\overline{\boldsymbol{\psi}}\sigma_3\boldsymbol{\psi}$	v_0, v_3	$Q^5_\alpha\,,\ \alpha=1,2,3$ $Q_\alpha\,,\ \alpha=1,2$
$U(1)_V$	$m_0\overline{\boldsymbol{\psi}}\boldsymbol{\psi}+m_3\overline{\boldsymbol{\psi}}\sigma_3\boldsymbol{\psi}$ $+m_1\overline{\boldsymbol{\psi}}\sigma_1\boldsymbol{\psi}$	v_0, v_3, v_1	$Q^5_\alpha\,,\ \alpha=1,2,3$ $Q_\alpha\,,\ \alpha=1,2,3$

Vacuum Structure in Mean-Field Approximation

Mass vacuum: Equal Masses (1)

- In mean field approximation, the mass vacuum, in terms of bare vacuum $|0\rangle$, is

$$
\begin{aligned}
|0\rangle_m &= \prod_{j=1,2}\prod_{\mathbf{k},r}(\cos\Theta_{\mathbf{k}} - \epsilon^r \sin\Theta_{\mathbf{k}}\tilde{\alpha}^{r\dagger}_{\mathbf{k},j}\tilde{\beta}^{r\dagger}_{-\mathbf{k},j})|0\rangle \\
&= B(m)|0\rangle\,, \qquad \Theta_{\mathbf{k}} = \frac{1}{2}\cot^{-1}\left(\frac{|\mathbf{k}|}{m}\right)
\end{aligned}
$$

with $\epsilon^r = (-1)^r$. Here m is the physical mass and

$$
B(m) = B_1(m)\, B_2(m)
$$

Mass vacuum: Equal Masses (2)

where:

$$B_j(m) \;=\; \exp\left[\sum_r \int d^3\mathbf{k}\, \Theta_{\mathbf{k}}\, \epsilon^r \left(\tilde{\alpha}^r_{\mathbf{k},j}\tilde{\beta}^r_{-\mathbf{k},j} - \tilde{\beta}^{r\dagger}_{-\mathbf{k},j}\tilde{\alpha}^{r\dagger}_{\mathbf{k},j}\right)\right]$$

is the generator of a Bogoliubov transformation.

The order parameter is:

$$v_0 \;=\; 2\int d^3\mathbf{k}\, \sin 2\Theta_{\mathbf{k}}$$

Mass vacuum: Different Masses (1)

- The mass vacuum is:

$$
\begin{aligned}
|0\rangle_{1,2} &= \prod_{j=1,2}\prod_{\mathbf{k},r}(\cos\Theta_{\mathbf{k},j} - \epsilon^r \sin\Theta_{\mathbf{k},j}\,\tilde{\alpha}^{r\dagger}_{\mathbf{k},j}\tilde{\beta}^{r\dagger}_{-\mathbf{k},j})|0\rangle \\
&= B(m_1,m_2)|0\rangle\,, \qquad \Theta_{\mathbf{k},j} \;=\; \frac{1}{2}\cot^{-1}\left(\frac{|\mathbf{k}|}{m_j}\right), j=1,2
\end{aligned}
$$

$B(m_1, m_2)$ factorizes as the product of Bogoliubov transformations:

$$
B(m_1,m_2) \;=\; B_1(m_1)\,B_2(m_2)
$$

Mass vacuum: Different Masses (2)

The ladder operators of massive fields are:

$$\alpha^r_{\mathbf{k},j} \;=\; B(m)\,\tilde{\alpha}^r_{\mathbf{k},j}\,B^{-1}(m) \;=\; \cos\Theta_{\mathbf{k},j}\tilde{\alpha}^r_{\mathbf{k},j} + \epsilon^r \sin\Theta_{\mathbf{k},j}\tilde{\beta}^{r\dagger}_{-\mathbf{k},j}$$

$$\beta^r_{-\mathbf{k},j} \;=\; B(m)\,\tilde{\beta}^{r\dagger}_{-\mathbf{k},j}\,B^{-1}(m) \;=\; \cos\Theta_{\mathbf{k},j}\tilde{\beta}^r_{-\mathbf{k},j} - \epsilon^r \sin\Theta_{\mathbf{k},j}\tilde{\alpha}^{r\dagger}_{\mathbf{k},j}$$

The non-zero order parameters are

$$v_0 \;=\; \sum_{j=1,2}\int \mathrm{d}^3\mathbf{k}\,\sin 2\Theta_{\mathbf{k},j}$$

$$v_3 \;=\; \int \mathrm{d}^3\mathbf{k}\,\sin 2\Theta_{\mathbf{k},1} - \int \mathrm{d}^3\mathbf{k}\,\sin 2\Theta_{\mathbf{k},2}$$

Rotation Operator

Consider the chiral charge

$$Q_2 = -\frac{i}{2}\int d^3\mathbf{x}\, \Big(\tilde{\psi}_1^\dagger(x)\,\tilde{\psi}_2(x) - \tilde{\psi}_2^\dagger(x)\,\tilde{\psi}_1(x)\Big)$$
$$= -\frac{i}{2}\int d^3\mathbf{k}\, \Big(\tilde{\alpha}^{r\dagger}_{\mathbf{k},1}\tilde{\alpha}^{r}_{\mathbf{k},2} + \tilde{\beta}^{r\dagger}_{\mathbf{k},1}\tilde{\beta}^{r}_{\mathbf{k},2} - \tilde{\alpha}^{r\dagger}_{\mathbf{k},2}\tilde{\alpha}^{r}_{\mathbf{k},1} - \tilde{\beta}^{r\dagger}_{\mathbf{k},2}\tilde{\beta}^{r}_{\mathbf{k},1}\Big)$$

Exponentiating it, we obtain the generator of a rotation:

$$\tilde{R}(\theta) = \exp\left(2i\theta Q_2\right)$$

In terms of mass fields:

$$\tilde{R}(\theta) = \exp\left[\theta\int d^3\mathbf{x}\, \Big(\psi_1^\dagger(x)\,\psi_2(x) - \psi_2^\dagger(x)\,\psi_1(x)\Big)\right]\Bigg|_{t=0} \equiv G_\theta(0)$$

- This is the mixing generator at $t = 0$.

Flavor Vacuum and Massless Modes

- Because, classically, $\partial_\mu J_2^\mu(x) = 0$, it is natural to consider the state:

$$|0\rangle_{e,\mu} \;=\; \tilde{R}^{-1}(\theta) B(m_1, m_2)|0\rangle$$

that is formally like flavor vacuum at $t = 0$. The order parameters are:

$$\begin{aligned}
v_0 &= \sum_{j=1,2} \int \mathrm{d}^3\mathbf{k} \, \sin 2\Theta_{\mathbf{k},j} \\
v_3 &= \cos 2\theta \left(\int \mathrm{d}^3\mathbf{k} \, \sin 2\Theta_{\mathbf{k},1} - \int \mathrm{d}^3\mathbf{k} \, \sin 2\Theta_{\mathbf{k},2} \right) \\
v_1 &= \sin 2\theta \left(\int \mathrm{d}^3\mathbf{k} \, \sin 2\Theta_{\mathbf{k},1} - \int \mathrm{d}^3\mathbf{k} \, \sin 2\Theta_{\mathbf{k},2} \right)
\end{aligned}$$

Conclusions and Perspectives

Conclusions:

- The non-trivial condensate structure of the flavor vacuum suggests a dynamical origin of mixing. The same structure has been proved to be a mark of this phenomenon, in models which present chiral symmetry.
- Anomalous Ward-Takahashi identities characterizes dynamical generation of mixing
- The flavor vacuum structure naturally arises when studying vacuum properties in mean field approximation.

Perspectives

Perspectives:

- The application of FI methods has suggested the study of appearance of inequivalent representations in this framework[7].
- More general situations, eventually including Lorentz-Poincaré symmetry breaking were only slightly touched[8]. The use of FI's in studying these situations requires a generalization of the QM and QFT partition function[9].

[7] M. Blasone, P. Jizba, L. S., Ann. Phys., **383**, (2017), 205.
[8] M.Blasone, P. Jizba, G. Lambiase, N. Mavromatos, J. Phys. Conf. Series, **538** (2014), 012003.
[9] M. Blasone, P. Jizba, L. S., Phys. Rev. A, **96**, (2017), 052107.

Thank you for the attention!