Holographic Correlators and the Information Paradox

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(with A. Bombini, A. Galliani, E. Moscato, R. Russo)

Overview

Main goal

Study black hole microstates using string theory and holography

- A b.h. microstate is dual to a "heavy" operator ($\Delta_H \sim c$) and (in some cases) is described by a 10D classical geometry

$$O_H \Leftrightarrow ds^2_H$$

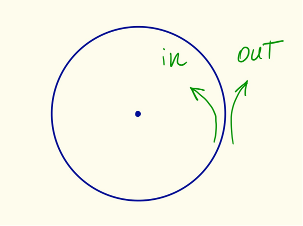
- Microstates can be probed by "light" operators ($\Delta_L \sim O(c^0)$)

$$\langle \bar{O}_H(\infty) O_L(z) \bar{O}_L(1) O_H(0) \rangle \equiv \langle O_L(z) \bar{O}_L(1) \rangle_{ds^2_H}$$

- HHLL correlators can diagnose information loss vs. unitarity

The semi-classical picture

- The classical b.h. has a central singularity and a smooth horizon
- For a large b.h. the curvature is small at the horizon and EFT should be valid
- EFT implies that the b.h. emits particles in an entangled state



$$|\psi\rangle_{\rm pair} = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_{\rm in}\,|\downarrow\rangle_{\rm out} - |\downarrow\rangle_{\rm in}\,|\uparrow\rangle_{\rm out}\right)$$

$$\rho_{\rm out} = {\rm Tr}_{\mathcal{H}_{\rm in}} |\psi\rangle_{\rm pair}\langle\psi|_{\rm pair}$$
$$= \frac{1}{2}\left(|\downarrow\rangle_{\rm out}\langle\downarrow|_{\rm out} + |\uparrow\rangle_{\rm out}\langle\uparrow|_{\rm out}\right)$$

- When the black hole has completely evaporated the outside radiation is entangled with nothing
  ⇒ Violation of unitarity!

Correlators and Information Loss

(Maldacena '01)

- The 2-point function in a b.h. background vanishes at large t

$$\langle O(t)\,\overline{O}(0)\rangle_{\mathrm{b.h.}} \sim e^{-4\pi\Delta T_H\, t}$$

with T_H the b.h. temperature

- This is not what we expect for the correlator in the thermal state in a unitary theory with finite entropy

Reminder: how to compute correlators in holography

- $\phi(r, x)$ is the bulk field dual to the CFT operator $O(x)$
- Solve the e.o.m for ϕ with

$$\phi(r, x) \xrightarrow{r\to\infty} \underset{\searrow\ \text{source for } \overline{O}(0)}{\delta(x)}\, r^{\Delta-d} + \overset{\nearrow\ \text{vev of } O(x)}{b(x)}\, r^{-\Delta}$$

- The correlator is

$$\boxed{\langle O(x)\, \overline{O}(0)\rangle = b(x)}$$

  - In the black hole background

    $b(t, \vec{x}) \sim e^{i\,\omega\,t}$ with $\omega =$ quasinormal frequency $\sim i\, T_H$

    $\Rightarrow$ large time decay!

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Correlators in unitary theories

(Dyson, Lindesay, Susskind; Barbon, Rabinovici)

- In a unitary theory with finite entropy and hence a discrete spectrum

$$\begin{aligned}\mathcal{C}_\beta(t) \equiv \langle O(t)\,\overline{O}(0)\rangle_\beta &= Z_\beta^{-1}\mathrm{Tr}\left[e^{-\beta H} O(t)\,\overline{O}(0)\right] \\ &= Z_\beta^{-1}\sum_{ij} e^{-\beta E_i}|\langle i|O(0)|j\rangle|^2 e^{i(E_i-E_j)t}\end{aligned}$$

- The long-time average of the correlator is

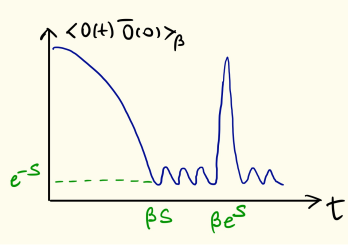
$$\lim_{T\to\infty}\frac{1}{T}\int_0^T dt\,|\mathcal{C}_\beta(t)|^2 \sim \frac{Z_{2\beta}}{Z_\beta^2} \sim e^{-S}$$

- Hence $\mathcal{C}_\beta(t)$ cannot be exponentially vanishing at late times

Qualitative behaviour of $\mathcal{C}_\beta(t)$

$$\mathcal{C}_\beta(t) = Z_\beta^{-1} \sum_{ij} e^{-\beta E_i} |\langle i | O(0) | j \rangle|^2 e^{i(E_i - E_j)t}$$

- For $t \ll \langle E_i - E_j \rangle^{-1} \sim \beta e^S$ the spectrum can be approximated as continuous: $\mathcal{C}_\beta$ is the Fourier transform of a function of width $\sim \beta^{-1}$ and hence $\mathcal{C}_\beta \sim e^{-t/\beta}$
- For $t \sim \beta S$ the correlator is of the order of its long-time average e^{-S}: it oscillates irregularly and no longer decreases
- For $t \sim \langle E_i - E_j \rangle^{-1}$ most of the phases are again of order 1 and hence $\mathcal{C}_\beta \sim O(1)$



A microscopic BH model: D1-D5-P

(Strominger, Vafa)

- The simplest BPS black hole with a finite-area horizon is

$$\boxed{\text{D1-D5-P on } \mathbb{R}^{4,1} \times S^1 \times T^4}$$

- We take $\mathrm{vol}(T^4) \sim \ell_s^4$ and $R(S^1) \gg \ell_s \Rightarrow$ 2D CFT
- The b.h. has a “near-horizon” limit: $\mathrm{AdS}_3 \times S^3 \times T^4$
- We take $G_N \to 0$ with R_{AdS} fixed $\Rightarrow c = 6 n_1 n_5 \equiv 6N \to \infty$
- The CFT has a 20-dim moduli space:
  - $g_s N \to 0$: free orbifold point $\iff R_{AdS} \ll \ell_s$
  - $g_s N \gg 1$: strong coupling point $\iff R_{AdS} \gg \ell_s$

Goal

Understand b.h. microstates at the strong coupling point

The D1-D5 CFT

- Symmetries:

  $(4,4)$ SUSY with $SU(2)_L \times SU(2)_R$ affine R-symmetry

- At the orbifold point

  sigma model on $(T^4)^N/S_N$

  one has free bosons and fermions

  $$(\partial X_r^{A\dot{A}}(z), \psi_r^{\alpha A}(z)) \ , \ (\bar{\partial} X_r^{A\dot{A}}(\bar{z}), \tilde{\psi}_r^{\dot{\alpha} A}(\bar{z})) \quad \text{with} \quad r = 1, \ldots, N$$

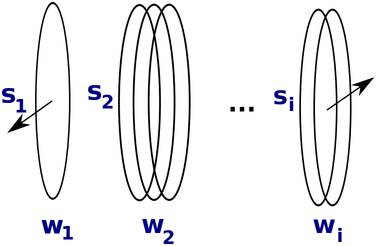
  $(\alpha \leftrightarrow SU(2)_L$, $\dot{\alpha} \leftrightarrow SU(2)_R$, $A \leftrightarrow SU(2)_1$, $\dot{A} \leftrightarrow SU(2)_2)$

- Spectral flow:

  $$\mathrm{NS} \rightarrow \mathrm{R} \ : \ j \rightarrow j + \frac{N}{2} \ , \ h \rightarrow h + j + \frac{N}{4}$$

2-charge states

- States carrying D1-D5 charges are RR ground states: $h = \bar{h} = \frac{N}{4}$
  Note: $h \sim c \Rightarrow$ these are “heavy” operators
- They are constructed from elementary “strands” characterised by winding w and $SU(2)_L \times SU(2)_R$ spin $|s\rangle$



$$\Leftrightarrow \quad \prod_i \left(|s_i\rangle_{w_i}\right)^{N_i}$$

with the constraint $\sum_i w_i N_i = N$

A small black hole

- The statistical ensemble of D1-D5 states is described by the "massless BTZ" geometry

$$\frac{ds^2}{R_{AdS}^2} = \frac{dr^2}{r^2} + r^2(-dt^2 + dy^2)$$

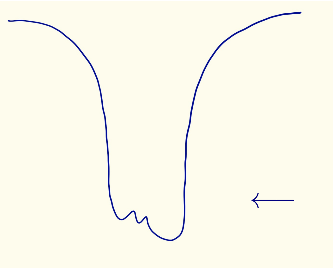
- It is a singular geometry with $A_{\mathrm{Hor}} = 0$
- Correlators in this geometry still display information loss

$$\langle O(t)\overline{O}(0)\rangle_{\mathrm{BTZ}_0} \sim t^{-\Delta}$$

The geometry of microstates

(Lunin, Mathur et al.)

- We can associate a 10D geometry to (coherent superpositions of) RR ground states
- All these geometries are smooth and horizonless
- They are asymptotically $\mathrm{AdS}_3 \times S^3 \times T^4$
  but in the interior AdS_3 and S^3 are non-trivially mixed
- They can be extended to asymptotically flat geometries



$\mathbb{R}^{4,1} \times S^1 \times T^4$

$\mathrm{AdS}_3 \times S^3 \times T^4$

$\longleftarrow$ $r \sim R_{\mathrm{Hor}}$ no horizon!

A note on 3-charge states

- We know the dual geometries for a class of D1-D5-P states, known as superstrata (Bena, SG, Martinec, Russo, Shigemori, Turton, Warner)

$$\prod_i \left[(J^+_{-1})^{m_i} \, (L_{-1} - J^3_{-1})^{n_i} \, |s_i\rangle_{w_i} \right]^{N_i}$$

Note: $J^+_{-1} \, , \, L_{-1} - J^3_{-1} \xrightarrow{\text{spectral flow}} J^+_0 \, , \, L_{-1}$
generate the global chiral algebra

- In this talk I will restrict to 2-charge states

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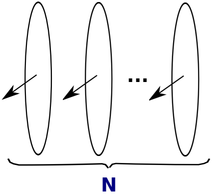
Example I: maximally rotating ground state

- The simplest D1-D5 state is the maximally rotating one

$$|+,+\rangle_1^N \qquad \leftrightarrow$$



- Spectral flow maps this state into the NS vacuum:

$$|+,+\rangle_1^N \quad \xrightarrow{\text{s.f.}} \quad |0\rangle_{NS}$$

- On the gravity side spectral flow is a change of coordinates mixing AdS$_3$ (t, y) and S^3 (ϕ, ψ) coordinates

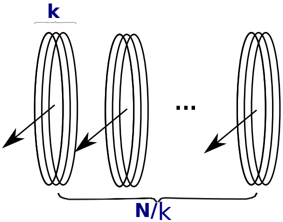
$$\phi \to \phi + \frac{t}{R} \quad , \quad \psi \to \psi + \frac{y}{R}$$

and maps the geometry dual to $|+,+\rangle_1^N$ into AdS$_3 \times S^3$

Example II: conical defects

- A state made of N/k identical copies of strands of winding k

$$|+,+\rangle_k^{N/k} \qquad \leftrightarrow$$



- The non-globally-defined change of coordinates

$$\phi \to \phi + \frac{t}{kR} \quad , \quad \psi \to \psi + \frac{y}{kR}$$

maps the geometry dual to $|+,+\rangle_k^{N/k}$ into $\mathrm{AdS}_3/\mathbb{Z}_k \times S^3$

- Generic states in the 2-charge ensemble have $k \sim \sqrt{N}$

The light operators

- We consider

$$O_L^{(1)} = \sum_{r=1}^{N} \frac{\epsilon_{AB}}{\sqrt{2N}} \psi_r^{-A} \tilde{\psi}_r^{-B} \, , \;\; O_L^{(2)} = \sum_{r=1}^{N} \frac{\epsilon_{AB}}{\sqrt{2N}} \partial X_r^{A\dot{1}} \, \bar{\partial} X_r^{B\dot{1}}$$

- $O_L^{(1)}$ is an anti-chiral-primary with $h_L = \bar{h}_L = -j_L = -\bar{j}_L = \frac{1}{2}$
- $O_L^{(2)}$ is a superdescendant

$$O_L^{(2)} = G^{+}_{-1/2} \tilde{G}^{+}_{-1/2} O_L^{(1)} \text{ with } h_L = \bar{h}_L = 1, \, j_L = \bar{j}_L = 0$$

- Since $G|O_H\rangle = \tilde{G}|O_H\rangle = 0$, correlators satisfy the Ward identity

$$\langle O_H | O_L^{(2)} \overline{O}_L^{(2)} | O_H \rangle = \partial \bar{\partial} \left[|z| \langle O_H | O_L^{(1)} \overline{O}_L^{(1)} | O_H \rangle \right]$$

The light operators: gravity picture

- $O_L^{(2)}$ is dual to a 6D minimally coupled scalar
- $O_L^{(1)}$ is dual to a scalar coupled to a 2-form in 6D

Solving the wave equation for $O_L^{(2)}$ is much simpler than for $O_L^{(1)}$

In the following we will use the notation:

$$\mathcal{C}_H^{(i)}(z,\bar{z}) \equiv \langle \bar{O}_H(\infty) O_L^{(i)}(z,\bar{z}) \bar{O}_L^{(i)}(1) O_H(0) \rangle$$

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Correlators in conical defects: $|O_H\rangle = (|++\rangle_k)^{N/k}$

$$\mathcal{C}_k^{(1)}(z,\bar{z}) = \frac{k^{-1}}{|z||1-z|^2}\,\frac{1-|z|^2}{1-|z|^{\frac{2}{k}}}$$

- The result is the same both in gravity and in the orbifold CFT
- The only operators in the OPE $O_L^{(1)}\overline{O}_L^{(1)}$ that have a non-zero vev in the state $(|++\rangle_k)^{N/k}$ are J^3-descendants of the identity

$$O_L^{(1)}(z)\overline{O}_L^{(1)}(1) = \frac{1}{|1-z|^2}\left[1 + (1-z)J^3 + (1-\bar{z})\tilde{J}^3 + \dots\right]$$

- In this case the correlator is completely determined by the chiral algebra and thus it is protected

Late time behaviour of $\mathcal{C}_k^{(1)}$

- Using $z = e^{i(\tau+\sigma)}\ ,\ \bar{z} = e^{i(\tau-\sigma)}$ with $\tau = t/R$, $\sigma = y/R$

$$\mathcal{C}_k^{(1)} \sim \frac{e^{-i\tau}\sin\tau}{\cos\tau - \cos\sigma}\ \frac{k^{-1}}{1 - e^{2\,i\,\tau/k}}$$

- For large τ ($\tau \gtrsim k$) $\mathcal{C}_k^{(1)}$ is an oscillating function of τ

  $\Rightarrow$ no information loss!

- For τ not so large ($\tau \ll k$)

$$\mathcal{C}_k^{(1)} \sim \tau^{-1}$$

  $\Rightarrow$ non-unitary b.h. behaviour

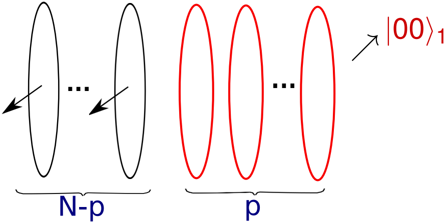
- Note that for typical states $k \sim \sqrt{N} \sim S$ as expected

A more generic state

- Consider the semiclassical RR ground state (Kanitscheider, Skenderis, Taylor)

$$|s_B\rangle = \frac{1}{N^{\frac{N}{2}}} \sum_{p=0}^{N} A^{N-p} B^{p}$$

$$(A^2 + B^2 = N)$$



- In the large N limit the sum is dominated by $p \sim B^2$
- The dual geometry is a deformation of $\mathrm{AdS}_3 \times S^3$ and cannot be mapped into $\mathrm{AdS}_3 \times S^3$ by any change of coordinates
- The deformation is essentially controlled by one scalar $w \sim C^{(0)}$

$$w = b \frac{a}{\sqrt{r^2 + a^2}} \sin\theta \cos\phi$$

with $a = a_0 \frac{A}{\sqrt{N}}$, $\frac{b}{\sqrt{2}} = a_0 \frac{B}{\sqrt{N}}$, $a_0 = \frac{\sqrt{Q_1 Q_5}}{R}$

A dynamical correlator: $|O_H\rangle = |s\rangle_B$

Free CFT

$$\mathcal{C}_B^{(1)} = \frac{1}{|z||1-z|^2} + \frac{B^2}{2N}\frac{|z|^2+|1-z|^2-1}{|z||1-z|^2} + \frac{A^2B^2}{N}\left(1-\frac{1}{N}\right)\frac{1}{|z|}$$

Gravity

$$\mathcal{C}_B^{(1)} = \frac{a}{a_0}e^{-i\tau}\sum_{l\in\mathbb{Z}} e^{il\sigma}\sum_{n=1}^{\infty}\frac{\exp\left[-i\frac{a}{a_0}\sqrt{(|l|+2n)^2+\frac{b^2 l^2}{2a^2}}\tau\right]}{\sqrt{1+\frac{b^2}{2a^2}\frac{l^2}{(|l|+2n)^2}}} + N\frac{b^2}{2a_0^2}\,e^{-i\tau}$$

Properties of $\mathcal{C}_B^{(1)}$

- For $b^2/a^2 \ll 1$ we can compute independently the correlators with $O_L^{(1)}$ and $O_L^{(2)}$ and check the Ward identity
- Non-BPS multiparticle operators contribute to $\mathcal{C}_B^{(1)}$

$$O_L^{(1)}(z)\overline{O}_L^{(1)}(1) = \sum_{n,\ell}(1-z)^{n+\ell-1}(1-\bar{z})^{\ell-1} : O_L^{(1)}\partial^{n+\ell}\bar{\partial}^{\ell}\overline{O}_L^{(1)} : + \dots$$

$\Rightarrow$ the correlator is non-protected

- In the light-cone limit $\bar{z} \to 1$ $(\tau \to \sigma)$ only the L_{-n} and J^3_{-n} descendants of the identity can contribute at strong coupling. Indeed the gravity correlator tends to

$$\mathcal{C}_B^{(1)} \sim \frac{1}{1-\bar{z}}\frac{\alpha\, z^{-1/2}}{1-z^{\alpha}} \quad \text{with} \quad \alpha = \frac{a^2}{a_0^2}$$

which is the affine block of the identity at large c (Fitzpatrick, Kaplan)

Late time behaviour of $\mathcal{C}_B^{(1)}$

- When $a \ll a_0$ the microstate geometry reduces to massless BTZ
- In this limit the $\mathcal{C}_B^{(1)}$ series is dominated by terms with $n \gg \frac{a_0}{2a}|l|$

$$\mathcal{C}_B^{(1)} \sim e^{-i\tau}\left[\frac{1}{1-e^{i(\sigma-\tau)}}+\frac{1}{1-e^{-i(\sigma+\tau)}}-1\right]\frac{\frac{a}{a_0}}{1-e^{-2i\frac{a}{a_0}\tau}}$$

- For $\tau \ll \frac{a_0}{a}$ one recovers the BTZ behaviour $\mathcal{C}_B^{(1)} \sim \tau^{-1}$
- For $\tau \gtrsim \frac{a_0}{a}$ $\mathcal{C}_B^{(1)}$ oscillates and stops decreasing with τ

No information loss even for non-protected correlators!

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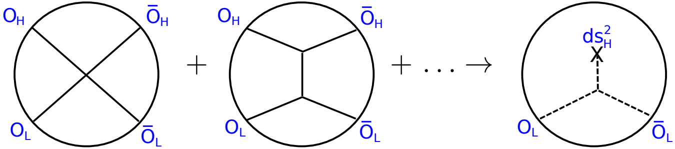
Connection with LLLL correlators?

- Our computation does not use Witten diagrams: $O_H \to ds^2_H$



- Witten diagrams in AdS_3 are subtle ... (D'Hoker, Freedman, Rastelli)
- When $b \ll a_0$ the state $|O_H\rangle$, spectrally flowed to the NS sector, is "light": $h_{NS} = \frac{N}{4}\frac{b^2}{a_0^2} \ll N$
- In this limit HHLL correlators reduce to LLLL ones?
- No! There is an order of limits problem:
  - HHLL: take $N \to \infty$ with b^2/N fixed, and then $b^2/N \ll 1$
  - LLLL: take $N \to \infty$ with b fixed and small

An overview of the fuzzball program

- We know families of smooth horizonless geometries with the same charges as 2, 3 (and 4)-charge black holes
- We can link these geometries to states of the D1-D5 CFT that is dual to the Strominger-Vafa black hole

Fuzzballs represent black hole microstates

- In some limits the microstate geometries are indistinguishable from the black hole if probed for a short time
- For sufficiently long times the microstate geometries deviate significantly from the black hole and produce correlators that are consistent with unitarity

Fuzzballs: open problems

- 2-charge: formally we know all the states but the sugra description becomes unreliable for typical ones (Chen, Michel, Polchinski, Puhm)
- 3-charge: the known geometris capture a parametrically small fraction of the entropy
- It is possible that most of the 3-charge states ("pure Higgs states") do not admit a description in supergravity

  (Bena, Berkooz, de Boer, El-Showk, Van den Bleeken; Sen)
- If sugra probes cannot distinguish typical states from the black hole, which tools do we have to describe them?