On the Lagrangian formulation of Gravity as a double-copy of two Yang-Mills theories

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Plan

1. Scattering amplitudes in Yang-Mills theory and Gravity
2. Lagrangian approach: attempts
3. Lagrangian approach: our results
4. Conclusions and outlook

Scattering amplitudes in Yang-Mills theory and Gravity

From KLT relations to the double copy

KLT relations

Two different theories, stricly related amplitudes

YM theory

$$\mathcal{L}_{YM} \sim A\partial^2 A + (\partial A)A^2 + A^4$$

General Relativity

$$\mathcal{L}_{EH} \sim h\partial^2 h + \sum_{n=3}^{+\infty} \partial^2 h^n$$

KLT relations

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KLT relations:

[Kawai, Lewllen, Tye 1986]

$$\mathcal{M}_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)\tilde{A}_4^{\text{tree}}(1,2,3,4),$$

$$\mathcal{M}_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\text{tree}}(1,2,3,4,5)\tilde{A}_5^{\text{tree}}(2,1,4,3,5)+$$

$$+is_{13}s_{34}A_5^{\text{tree}}(1,3,2,4,5)\tilde{A}_5^{\text{tree}}(3,1,4,2,5)$$

$$\ldots$$

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$$\dots$$

Two different YM partial amplitudes (no color, gauge-invariant)

Color-kinematics duality (tree-level)

[Bern, Carrasco, Johansson 2008]

A hidden symmetry of YM scattering amplitudes.

$$\mathcal{A}_n = g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i \quad n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

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diagrams with cubic vertices

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Invariant under $n_i \to n_i' = n_i + \Delta_i$, if $\displaystyle\sum_{i \in \Gamma_3} \frac{c_i \Delta_i}{\prod_{\alpha_i} s_{\alpha_i}} = 0 \longrightarrow$ generalized gauge transformation

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Color-kinematics duality (BCJ):

$\boxed{c_i + c_j + c_k = 0 \Rightarrow n_i + n_j + n_k = 0}$ and $\boxed{c_i = -c_j \Rightarrow n_i = -n_j}$

(not evident from Feynman diagrams expansion!)

Proven at tree-level.

Double copy relations (tree-level)

A new implementation of GR=(YM)^2

Yang-Mills amplitude:

$$\mathcal{A}_n = g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

(in CK-dual form)

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$$g^{n-2}\sum_{i\in\Gamma_3}\frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

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$$i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{i\in\Gamma_3}\frac{\tilde{n}_i\, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Double copy relations (tree-level)

[Bern, Carrasco, Johansson 2008]

A new implementation of GR=(YM)^2

(Super-)Gravity amplitude:

$$\mathcal{M}_n = i\left(\frac{\kappa}{2}\right)^{n-2} \sum_{i\in\Gamma_3} \frac{\tilde{n}_i\, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

[Bern, Dennen, Huang, Kiermaier 2010]

Proven at tree-level.

Particle content of the double copy

Not only Gravity

We consider the case (Pure YM)^2 $=$ ($\mathcal{N} = 0$ SUGRA):

$$\square \otimes \square = \underbrace{\square\square\Big|_T}_{\text{graviton}} \oplus \underbrace{\begin{array}{c}\square\\ \square\end{array}}_{\text{two}-\text{form}} \oplus \underbrace{\bullet}_{\text{scalar}}$$

Proposed action (low-energy effective action of closed bosonic string, here any D):

$$\mathcal{S}_{\mathcal{N}=0} = \int d^D x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{6} e^{-\frac{4\kappa\varphi}{D-2}} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{2(D-2)} \partial_\mu \varphi \partial^\mu \varphi \right\}$$

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non-minimal coupling

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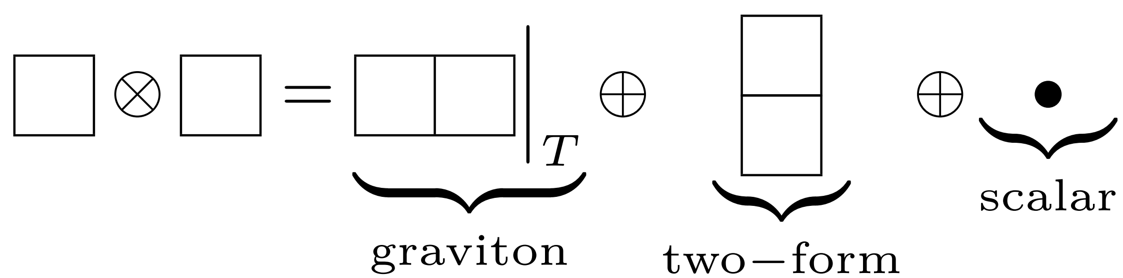

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Double copy relations also hold in the supersymmetric case:

$$[\mathcal{N}_L \; SYM] \otimes [\mathcal{N}_R \; SYM] = [\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R \; SUGRA]$$

CK duality and DC at loop-level

[Bern, Carrasco, Johansson 2010]

An appealing conjecture

Difficult part: obtaining CK dual representations of YM amplitudes (only conjectured but several non-trivial examples). If this is possible:

$$\mathcal{A}_n^{L-\text{loop}} = i^L g^{n-2+2L} \sum_{i\in\Gamma_3} \int \Big(\prod_{k=1}^{L} \frac{d^D l_k}{(2\pi)^D} \Big) \frac{1}{S_i} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

DC follows from generalized unitarity cuts and the validity of the DC at tree-level.

$$\mathcal{M}_n^{L-loop} = i^{L+1} \Big(\frac{\kappa}{2}\Big)^{n-2+2L} \sum_{i\in\Gamma_3} \int \prod_{j=1}^{L} \Big(\frac{d^L l_j}{(2\pi)^D} \Big) \frac{1}{S_i} \frac{\tilde{n}_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Open questions

CK duality and DC (can be made) manifest in scattering amplitudes

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What is their Lagrangian origin?

What is their geometrical meaning?

(if any)

Lagrangian approach: attempts

A (short) review of the contributions to understand the double copy at the Lagrangian level

Relating Yang-Mills and gravitational symmetries (1)

[Anastasiou, Borsten, Duff, Hughes, Nagy 2014]

Off-shell product of YM fields:

$$H_{\mu\nu}(x) \equiv \left[A^a_\mu \circ \Phi^{-1}_{aa'} \circ \tilde{A}^{a'}_\nu\right](x) \equiv \left[A^a_\mu \star \tilde{A}^{a'}_\nu\right](x)$$

convolution

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biadjoint (spectator) scalar field

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Yang-Mills linearized gauge symmetry $[\varepsilon(x)]$

$$\begin{cases} \delta A^a_\mu = \partial_\mu \varepsilon^a \\ \delta \tilde{A}^a_\mu = \partial_\mu \tilde{\varepsilon}^{a'} \\ \delta \Phi^{-1}_{aa'} = \end{cases}$$

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Yang-Mills linearized gauge symmetry [$\varepsilon(x)$] + global symmetry [constant ϑ]:

$$\begin{cases} \delta A^a_\mu = \partial_\mu \varepsilon^a + f^{abc} A^b_\mu \vartheta^c, \\ \delta \tilde{A}^a_\mu = \partial_\mu \tilde{\varepsilon}^{a'} + f^{a'b'c'} \tilde{A}^{b'}_\mu \tilde{\vartheta}^{c'}, \\ \delta \Phi^{-1}_{aa'} = f^{abc} \Phi^{-1}_{ba'} \vartheta^c + f^{a'b'c'} \Phi^{-1}_{ab'} \tilde{\vartheta}^{c'}, \end{cases}$$

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Linearized (extended) gravitational symmetries: $[H_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} + \gamma \eta_{\mu\nu} \varphi]$

$$\begin{cases} h_{\mu\nu} = H^S_{\mu\nu} - \gamma \eta_{\mu\nu} \varphi \to \delta h_{\mu\nu} = \partial_{(\mu} \xi_{\nu)}, \\ B_{\mu\nu} = H^A_{\mu\nu} \to \delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}, \\ \varphi = H - \frac{\partial \cdot \partial \cdot H}{\Box} \to \delta\varphi = 0, \quad \left[H \equiv H^\alpha_\alpha\right] \end{cases} \qquad \xi_\mu = \frac{1}{2}(\alpha + \tilde{\alpha})_\mu, \Lambda_\mu = \frac{1}{2}(\alpha - \tilde{\alpha})_\mu.$$

Comment: definition of the scalar field

Scattering amplitudes are on-shell quantities, with physical polarizations:

$$p_\mu = (p, 0, ..., 0, p): \quad \varepsilon_\mu(p)\tilde{\varepsilon}_\nu(p) \equiv H_{\mu\nu}(p) = \begin{pmatrix} 0 & \dots & 0 \\ \dots & H_{ij}^{(D-2)\times(D-2)} & \dots \\ 0 & \dots & 0 \end{pmatrix}$$

$\Rightarrow$ the actual scalar degree of freedom is the trace of H_{ij}.

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The covariant formulation of theories with massless particles calls for gauge invariance, while $\delta H \neq 0 \rightarrow$ introduce a pure gauge term to compensate:

$$\varphi \equiv H - \frac{\partial \cdot \partial \cdot H}{\Box} \quad \text{such that} \quad \delta\varphi = 0$$

the non-local term is due to the necessity of extracting covariantly a gauge-invariant scalar. It is identically zero on-shell.

The theory is local in terms of $h_{\mu\nu}$, $B_{\mu\nu}$, φ.

Relating Yang-Mills and gravitational symmetries (2)

Towards a free Lagrangian

From the definition of the "product" $H_{\mu\nu} = A_\mu \star \tilde{A}_\nu$:

[Anastasiou, Borsten, Duff, Hughes, Nagy 2014]

- Linearized Riemann tensor (with torsion): $R^*_{\mu\nu\rho\sigma} \equiv -\frac{1}{2} F_{\mu\nu} \star \tilde{F}_{\rho\sigma}$

  $\rightarrow$ from $H^S_{\mu\nu}$: purely metric part $[g_{\mu\nu} = \eta_{\mu\nu} + H^S_{\mu\nu}]$.

  $\rightarrow$ from $H^A_{\mu\nu}$: purely torsion part $[-T_{\mu\alpha\beta} = H_{\mu\alpha\beta} = (dB)_{\mu\alpha\beta}]$.

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  $\rightarrow$ from $H^A_{\mu\nu}$: purely torsion part [$-T_{\mu\alpha\beta} = H_{\mu\alpha\beta} = (dB)_{\mu\alpha\beta}$].

- Linearized YM e.o.m. $\Rightarrow$ linearized e.o.m. for $h_{\mu\nu}$, $B_{\mu\nu}$, φ.

  $\rightarrow$ without sources. [Cardoso, Inverso, Nagy, Nampuri 2018]

  $\rightarrow$ with sources, introducing non-localities.

DC of BDHK Lagrangian

[Bern, Dennen, Huang, Kiermaier 2010]

Lagrangian DC at fixed gauge for $\mathcal{A}_4$

At four points (Feynman gauge) CK-dual rules from:

$$\mathcal{L}_{\mathrm{BDHK}} = \frac{1}{2} A^{a\mu} \Box A^{a}_{\mu} - B^{a\rho\mu\nu} \Box B^{a}_{\rho\mu\nu} - g f^{abc} (\partial_{\mu} A^{a}_{\nu} + \partial^{\rho} B^{a}_{\rho\mu\nu}) A^{b\mu} A^{c\nu}$$

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auxiliary field

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DC in momentum space: $A_\mu \tilde{A}_\nu \to H_{\mu\nu}$.

- $-\frac{1}{2} \int dk_{1,2} \delta(k_1 + k_2) k_1^2 \left[A^1_\mu A^\mu_2 - 2 B_1^{a\rho\mu\nu} B^2_{a\rho\mu\nu} \right]$
- $\int dk_{1,2,3} \delta(k_1 + k_2 + k_3) \sum_{\sigma \in S_3} (-1)^{sgn(\sigma)} \Big[\Big(k_\mu^{\sigma(1)} A_\nu^{\sigma(1)} + k^\rho_{\sigma(1)} B^{\sigma(1)}_{\rho\mu\nu} \Big)$
$A^\mu_{\sigma(2)} A^\nu_{\sigma(3)} \Big]$

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DC in momentum space: $A_\mu \tilde{A}_\nu \to H_{\mu\nu}$.

- $-\frac{1}{2} \int dk_{1,2} \delta(k_1 + k_2) k_1^2 \left[A^1_\mu A^\mu_2 - 2 B_1^{\alpha\rho\mu\nu} B^2_{\alpha\rho\mu\nu} \right] \left[\tilde{A}^1_\alpha \tilde{A}^\alpha_2 - 2 \tilde{B}_1^{a\gamma\alpha\beta} \tilde{B}^2_{a\gamma\alpha\beta} \right]$
  $\to -\frac{1}{2} \int d^D x \left[H^{\mu\alpha} \Box H_{\mu\alpha} - 2 g^{\mu\gamma\alpha\beta} \Box g_{\mu\gamma\alpha\beta} + \ldots \right.$ auxiliary fields
- $\int dk_{1,2,3} \delta(k_1 + k_2 + k_3) \sum_{\sigma \in S_3} (-1)^{sgn(\sigma)} \left[\left(k_\mu^{\sigma(1)} A_\nu^{\sigma(1)} + k^\rho_{\sigma(1)} B^{\sigma(1)}_{\rho\mu\nu} \right) \right.$
  $\left. A^\mu_{\sigma(2)} A^\nu_{\sigma(3)} \right] \cdot \sum_{\tau \in S_3} (-1)^{sgn(\tau)} \left[\left(k_\alpha^{\tau(1)} \tilde{A}_\beta^{\tau(1)} + k^\gamma_{\tau(1)} \tilde{B}^{\tau(1)}_{\gamma\alpha\beta} \right) \tilde{A}^\alpha_{\tau(2)} \tilde{A}^\beta_{\tau(3)} \right]$

DC of BDHK Lagrangian

[Bern, Dennen, Huang, Kiermaier 2010]

Lagrangian DC at fixed gauge for $\mathcal{A}_4$

At four points (Feynman gauge) CK-dual rules from:

$$\mathcal{L}_{\mathrm{BDHK}} = \frac{1}{2} A^{a\mu} \Box A^a_\mu - B^{a\rho\mu\nu} \Box B^a_{\rho\mu\nu} - g f^{abc} (\partial_\mu A^a_\nu + \partial^\rho B^a_{\rho\mu\nu}) A^{b\mu} A^{c\nu}$$

DC in momentum space: $A_\mu \tilde{A}_\nu \to H_{\mu\nu}$.

- $-\frac{1}{2} \int dk_{1,2} \delta(k_1 + k_2) k_1^2 \left[A^1_\mu A^\mu_2 - 2 B_1^{a\rho\mu\nu} B^2_{a\rho\mu\nu} \right] \left[\tilde{A}^1_\alpha \tilde{A}^\alpha_2 - 2 \tilde{B}_1^{a\gamma\alpha\beta} \tilde{B}^2_{a\gamma\alpha\beta} \right]$
  $\to -\frac{1}{2} \int d^D x \left[H^{\mu\alpha} \Box H_{\mu\alpha} - 2 g^{\mu\gamma\alpha\beta} \Box g_{\mu\gamma\alpha\beta} + \ldots \right.$
- $\int dk_{1,2,3} \delta(k_1 + k_2 + k_3) \sum_{\sigma \in S_3} (-1)^{sgn(\sigma)} \left[\left(k_\mu^{\sigma(1)} A_\nu^{\sigma(1)} + k^\rho_{\sigma(1)} B^{\sigma(1)}_{\rho\mu\nu} \right) \right.$
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Generalized up to $\mathcal{A}_6$ at tree-level:

non-local CK dual Lagrangian $\to$ auxiliary fields with cubic interactions $\to$ DC

To summarize

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$\boxed{\text{Duff et al.}}$

To summarize

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- non-localities beyond $\mathcal{A}_4$
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Lagrangian approach: our results

A step forward towards an off-shell double copy

Quadratic Lagrangian

($\mathcal{N} = 0$ SUGRA)=(YM)^2!

$$\mathcal{L}_H = -\frac{1}{8}(F_{\mu\alpha} \star \tilde{F}_{\nu\beta})\frac{1}{\Box}(F^{\mu\alpha} \star \tilde{F}^{\nu\beta}) = -\frac{1}{2}R^*_{\mu\alpha\nu\beta}\frac{1}{\Box}R^{*\mu\alpha\nu\beta} =$$
$$= \frac{1}{2}H^{\alpha\beta}\Big(\eta_{\alpha\mu}\eta_{\beta\nu}\Box - \eta_{\beta\nu}\partial_\alpha\partial_\mu - \eta_{\alpha\mu}\partial_\beta\partial_\nu + \frac{\partial_\alpha\partial_\beta\partial_\mu\partial_\nu}{\Box}\Big)H^{\mu\nu}.$$

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- $\gamma = 0$: $\mathcal{L}_H = \mathcal{L}_h + \mathcal{L}_B - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi$,
- $\gamma = \frac{1}{D-2}$: $\mathcal{L}_H = \mathcal{L}_h + \mathcal{L}_B - \frac{1}{2(D-2)}\partial_\mu\varphi\partial^\mu\varphi$.

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Possible Faddeev-Popov gauge-fixing such that:

$\mathcal{L}_H + \mathcal{L}_{\rm GF} = \frac{1}{2}H_{\mu\alpha}\Box H^{\mu\alpha} \rightarrow$ "square" of Feynman gauge: $\mathcal{P}_{\mu\alpha\nu\beta} = \frac{i}{p^2}\eta_{\mu\nu}\eta_{\alpha\beta}$.

Noether procedure

A powerful tool to build interacting gauge theories

From free theory ($\mathcal{S}_0$) + free gauge invariance ($\delta_0\varphi$) we add vertices with a perturbative expansion. The gauge transformation must be deformed consistently:

$$\mathcal{S} = \mathcal{S}_0 + g\mathcal{S}_1 + g^2\mathcal{S}_2 + \ldots \Rightarrow \delta\varphi = \delta_0\varphi + g\delta_1\varphi + g^2\delta_2\varphi + \ldots$$

$$\delta\mathcal{S} = 0 \Rightarrow \begin{cases} \delta_0\mathcal{S}_0 = 0, \\ \delta_1\mathcal{S}_0 + \delta_0\mathcal{S}_1 = 0, \\ \delta_2\mathcal{S}_0 + \delta_1\mathcal{S}_1 + \delta_0\mathcal{S}_2 = 0, \\ \ldots \end{cases}$$

$\rightarrow$ order by order we can find $\mathcal{S}_n$ and then $\delta_n\varphi$.

Cubic interactions (1)

Cubic vertices for $H_{\mu\nu}$

$$[\mathcal{D}_\beta \equiv \partial^\mu H_{\mu\beta} - \tfrac{1}{2}\partial_\beta \tfrac{\partial\cdot\partial\cdot H}{\Box}, \qquad \tilde{\mathcal{D}}_\alpha \equiv \partial^\mu H_{\alpha\mu} - \tfrac{1}{2}\partial_\alpha \tfrac{\partial\cdot\partial\cdot H}{\Box}]$$

$$\begin{aligned}\mathcal{L}_1 = a\{ & H^{\mu\nu}\partial_\mu\partial_\nu H_{\alpha\beta}H^{\alpha\beta} + H^{\mu\nu}\partial_\mu H^{\alpha\beta}\partial_\beta H_{\alpha\nu} + H^{\mu\nu}\partial_\nu H^{\alpha\beta}\partial_\alpha H_{\mu\beta} + \\ & -\frac{1}{2}\partial\cdot\mathcal{D}H_{\alpha\beta}H^{\alpha\beta} - \mathcal{D}^\beta\partial^\alpha\frac{\partial\cdot\mathcal{D}}{\Box}H_{\alpha\beta} - \tilde{\mathcal{D}}^\alpha\partial^\beta\frac{\partial\cdot\mathcal{D}}{\Box}H_{\alpha\beta}\} + \\ +b\{ & H^{\mu\nu}\partial_\mu\partial_\nu H_{\alpha\beta}H^{\alpha\beta} + 2H^{\mu\nu}\partial_\mu\partial_\nu H_{\alpha\beta}H^{\beta\alpha} + H^{\mu\nu}\partial_\mu H^{\alpha\beta}\partial_\alpha H_{\beta\nu} + \\ & +H^{\mu\nu}\partial_\nu H^{\alpha\beta}\partial_\alpha H_{\beta\mu} + H^{\mu\nu}\partial_\nu H^{\alpha\beta}\partial_\beta H_{\alpha\mu} + H^{\mu\nu}\partial_\mu H^{\alpha\beta}\partial_\alpha H_{\nu\beta} + \\ & +H^{\mu\nu}\partial_\mu H^{\alpha\beta}\partial_\beta H_{\nu\alpha} + H^{\mu\nu}\partial_\nu H^{\alpha\beta}\partial_\beta H_{\mu\alpha} + \\ & -\frac{1}{2}\partial\cdot\mathcal{D}H_{\alpha\beta}H^{\alpha\beta} - \partial\cdot\mathcal{D}H_{\alpha\beta}H^{\beta\alpha} - \mathcal{D}^\beta\partial^\alpha\frac{\partial\cdot\mathcal{D}}{\Box}H_{\alpha\beta} + \\ & -\tilde{\mathcal{D}}^\alpha\partial^\beta\frac{\partial\cdot\mathcal{D}}{\Box}H_{\alpha\beta} - 2\mathcal{D}^\beta\partial^\alpha\frac{\partial\cdot\mathcal{D}}{\Box}H_{\beta\alpha} - 2\tilde{\mathcal{D}}^\alpha\partial^\beta\frac{\partial\cdot\mathcal{D}}{\Box}H_{\beta\alpha}\}\end{aligned}$$

abritrary coefficients

Cubic interactions (2)

Comments on the cubic vertices

- The choice $a = 1, b = 0$ reproduces the results of the DC and matches (modulo local field redefinitions) the vertices of $\mathcal{N} = 0$ **SUGRA** if $H_{\mu\nu} = h_{\mu\nu} + B_{\mu\nu} + \frac{1}{D-2}\eta_{\mu\nu}\varphi$ $\quad(\gamma = \frac{1}{D-2})$.

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- The choice $a = b$ eliminates the two-form field.
- Other choices produce the same type of amplitudes but with different relative coefficients.

Gauge transformations and correction to the dilaton

Hints of geometry

From the Noether procedure:

$$\begin{cases} \delta_1 H^S_{\mu\nu} = \xi\cdot\partial H^S_{\mu\nu} + \partial_\mu\xi^\alpha H^S_{\alpha\nu} + \partial_\nu\xi^\alpha H^S_{\mu\alpha}, \\ \delta_1 H^A_{\mu\nu} = \xi\cdot\partial H^A_{\mu\nu} + \partial_\mu\xi^\alpha H^A_{\alpha\nu} - \partial_\nu\xi^\alpha H^A_{\alpha\mu}, \end{cases}$$

but $\delta_1\varphi \neq \xi\cdot\partial\varphi$!

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Solution: the actual scalar is $\psi = \psi^{(1)} + \psi^{(2)} + \dots$, with $\psi^{(1)} = \varphi = \frac{R_H^{(1)}}{\Box}$.

Ricci scalar
in terms of H

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Laplace-Beltrami operator for a scalar

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Also the graviton must be modified: $h_{\mu\nu} = H^S_{\mu\nu} - \gamma X_{\mu\nu}\psi$, $X^{(0)}_{\mu\nu} = \eta_{\mu\nu}$.

$\Rightarrow X^{(1)}_{\mu\nu} = H^S_{\mu\nu}$. Doesn't generate new cubic vertices $\leftrightarrow \gamma = \frac{1}{D-2}$.

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Thank you for your attention.