Localization of Effective Actions from Open String Field Theories

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Based on 1801.07607 (10.1007/JHEP03(2018)112) + a paper in preparation with **Carlo Maccaferri**.

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Summary of the presentation

- Motivations: Effective Actions from String Theory.
- Solution in the large Hilbert space (10.1007/JHEP03(2018)112).
- Yang-Mills instantons from $D3$ - $D(-1)$ branes system.
- Solution in the small Hilbert space (Paper in preparation).

Motivations

- String Theory is a consistent quantum theory of gravity whose spectrum contains **massless** particles and an infinite tower of massive particles.
- In first quantized approach, computing Effective Actions for light states is difficult and indirect.
- We would like to have an action for these (open) string excitations and just solve the equation of motion for the massive fields.

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- String Theory is a consistent quantum theory of gravity whose spectrum contains **massless** particles and an infinite tower of massive particles.
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Open string Field Theories for NS sector

- Open String Field Theory (OSFT) is the field theory for this tower of states and their interactions. So at least in principle ordinary QFT techniques, such as 1PI and Wilsonian approach, are available. Anyway it takes a lot of effort.
- We restrict ourself to a more simple task: the study of the algebraic part of the classical tree-level action. This can be done easily by solving the equations of motion for the massive fields $\delta_{M_i} S = 0$.
- We can choose between two approaches: the large Hilbert space (Berkovits WZW theory, two gauge structures) or the small Hilbert space (Erler-Konopka-Sachs A_∞ homotopy associative algebra, one single gauge structure).

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The large Hilbert perspective ('95, Berkovits)

- OSFT is an ordinary field theory.
- Berkovits theory is a WZW theory. Expanding all the exponentials:

Berkovits Action

$$S[\Phi] = -\frac{1}{2}\,\mathrm{Tr}[(\eta_0\Phi)(Q_B\Phi)] - \sum_{n=1}^{+\infty}\frac{(-1)^n}{(n+2)!}\,\mathrm{Tr}[(\eta_0\Phi)\,\mathrm{ad}_\Phi^n\,(Q_B\Phi)].$$

Here we recognize a kinetic term + interactions for the string field Φ in the large Hilbert space.

Berkovits Effective Action

- Integrating out the massive fields, the quartic level effective action for a massless field Φ is

$$S_4(\Phi) = \frac{1}{8}\mathrm{Tr}\,[[\eta_0\Phi, Q_B\Phi]\,\xi_0\frac{b_0}{L_0}\,[\eta_0\Phi, Q_B\Phi]] - \frac{1}{24}\mathrm{Tr}\,[[\eta_0\Phi, \Phi]\,[\Phi, Q_B\Phi]]\,.$$

- It comes with a natural assignment of the picture number. If one change the picture, possible anomalies arise due to the non trivial kernel of L_0 which is taken into account by a projector P_0 on the $L_0 = 0$ subspace:

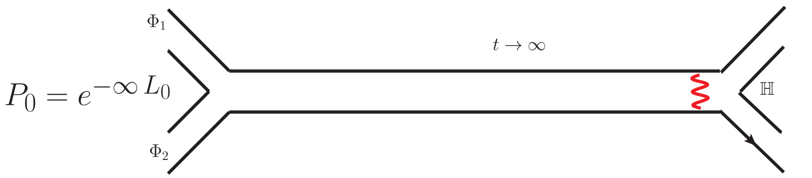
$$\left[Q_B, \frac{b_0}{L_0}\right] = 1 - P_0$$

- It turns out surprisingly that these anomalous terms alone compute the effective action (up to an overall factor)!

Moduli space boundary terms

$$S_{eff} \sim \mathrm{Tr}\,[[\Phi, \eta_0\Phi]\, P_0[\Phi, Q_B\Phi]]$$

Projector localizes at the boundary of moduli space



- Conformally equivalent to a degenerating Riemann surface $\sim$ boundary of the moduli space.
- Why such a drastic simplification?

$\mathcal{N} = 2$ On the Worldsheet

- We believe that such a simplification is possible only because some hidden symmetry is acting.

- For consistent and interesting string theory background we can always promote the natural worldsheet $\mathcal{N} = 1$ to $\mathcal{N} = 2$. (see Sen (1986), Banks and Dixon et al.(1987-88))

- So there is a $R-$symmetry current j in the matter sector. Assuming that the string field is the sum of opposite charged string fields

$$\Phi_{\mathcal{N}=1} = \Phi^{(+)} + \Phi^{(-)} \qquad , \qquad \Phi^{(\pm)} = c\gamma^{-1}\mathbb{V}^{(\pm)}_{\frac{1}{2}},$$

the action localizes because of the $R-$charge conservation in terms of unphysical auxiliary fields.

The Localized Action

The conservation of the $R-$charge implies that the effective action localizes in terms of unphysical "auxiliary fields" h, g:

$$S_{\rm eff}(\Phi) = \frac{1}{8}\left[\left\langle \widehat{h}^{(--)}\middle| h^{(++)}\right\rangle + \left\langle \widehat{g}^{(+-)}\middle| g^{(-+)}\right\rangle + \Big(+ \leftrightarrow -\Big)\right].$$

The explicit form of h, g is given by the leading OPE in the ghost/superghost CFT and between the matter superconformal primaries:

$$\mathbb{V}^{(\pm)}_{\frac{1}{2}}(z)\mathbb{V}^{(\pm)}_{\frac{1}{2}}(-z) = \mathbb{H}^{(\pm)}_{1}(0) + \cdots$$

$$\mathbb{V}^{(\mp)}_{\frac{1}{2}}(z)\mathbb{V}^{(\pm)}_{\frac{1}{2}}(-z) - \mathbb{V}^{(\pm)}_{\frac{1}{2}}(z)\mathbb{V}^{(\mp)}_{\frac{1}{2}}(-z) = \frac{1}{2z}\mathbb{H}_0 + \cdots .$$

The localized effective action

- Computing the universal ghost/superghost CFT, we remain with simple two-point functions of auxiliary fields:

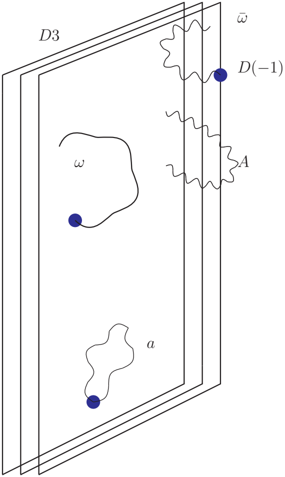
Localized action = two-point functions

$$S^{(4)}_{\text{eff}}(\Phi) = \mathrm{tr}\left[\langle \mathbb{H}_1^{(+)} | \, \mathbb{H}_1^{(-)} \rangle + \frac{1}{4} \langle \mathbb{H}_0 | \, \mathbb{H}_0 \rangle \right]$$

- This is the main result of 1801.07607: *Choose your string background; the effective action at the quartic level for the massless fields is given by the BPZ product of the matter part of the auxiliary field.*

- If not for this term, the effective action would be zero. This matter vertex depends on the chosen string background. So let us analyze one important illuminating example.

Y-M Instantons: the worldsheet theory of $D3$–$D(-1)$



First quantized computation by by Billò, Frau, Fucito, Lerda, Liccardo and Pesando, '02.

The string field is

$$\Phi(z) = c\gamma^{-1} \begin{pmatrix} A & \omega \\ \bar{\omega} & a \end{pmatrix} (z),$$

$$A(z) = A_\mu \psi^\mu(z) + \phi_p \psi^p(z),$$

$$\omega(z) = \omega_\alpha^{N\times k}\, \Delta S^\alpha(z),$$

$$\bar{\omega}(z) = \bar{\omega}_\alpha^{k\times N}\, \bar{\Delta} S^\alpha(z),$$

$$a(z) = a_\mu \psi^\mu(z) + \chi_p \psi^p(z).$$

- μ-indices label the directions along the D3 branes, p-indices label the transverse directions while $SO(4)$ spinor indices α are $(\pm\frac{1}{2}, \pm\frac{1}{2})$.

Benefits of the Localization

- Certainly we can compute this effective action using the Berkovits quartic effective action, but for the instanton potential it requires $4-$point functions of (excited) twist fields which are not even well known in the literature. The most challenging one is roughly given:

$$\mathrm{Tr}\left[[\eta_0\Phi, Q_B\Phi]\,\xi_0\frac{b_0}{L_0}\,[\eta_0\Phi, Q_B\Phi]\right] \sim \langle \tau^\mu(z_1)\bar{\tau}^\nu(z_2)\Delta(z_3)\bar{\Delta}(z_4)\rangle.$$

- So it is easy to appreciate how localization avoids this problems reducing all the computation to a $2-$point function.

Yang-Mills Instantons: Gauge Boson

We rearrange the worldsheet ψ^μ superconformal fields to a $U(n) \in SO(2n)$ decomposition with $(\jmath, \bar{\jmath}, j) = 1, ..., n$.

Bosonization procedure $\rightarrow$ complex representation

$$\psi^{\jmath} = \frac{1}{\sqrt{2}}(\psi^{2j-1} + i\psi^{2j}) = e^{ih_j} \qquad J_0\psi^{\jmath} = +\psi^{\jmath},$$

$$\psi^{\bar{\jmath}} = \frac{1}{\sqrt{2}}(\psi^{2j-1} - i\psi^{2j}) = e^{-ih_j} \qquad J_0\psi^{\bar{\jmath}} = -\psi^{\bar{\jmath}}.$$

The localizing current is $J(z) = -i\sum_{j=1}^{n}\partial h_j(z)$.

$$A = A^{(+)} + A^{(-)} + \phi^{(+)} + \phi^{(-)}, \qquad a = a^{(+)} + a^{(-)} + \chi^{(+)} + \chi^{(-)},$$

Here $\jmath, \bar{\jmath}$ denote respectively the $\mathbf{2}$ and $\bar{\mathbf{2}}$ of $SU(2) \subset SO(4)$. $m, \bar{m}$ transverse indices denote respectively the $\mathbf{3}$ and $\bar{\mathbf{3}}$ of $SU(3) \subset SO(6)$.

Yang-Mills Instantons: Stretched strings

- The worldsheet theory contains also the composite ΔS^{α} and $\bar{\Delta} S^{\alpha}$ which are superconformal primaries of weight $\frac{1}{2}$. Spin fields are defined through bosonization by the scalars h_1, h_2

Bosonization for the Spin Fields

$$S^{(\frac{1}{2},\frac{1}{2})} = e^{\frac{i}{2}(h_1+h_2)} \quad , \quad S^{(-\frac{1}{2},-\frac{1}{2})} = e^{-\frac{i}{2}(h_1+h_2)}$$

$$J_0\, S^{(\frac{1}{2},\frac{1}{2})} = +S^{(\frac{1}{2},\frac{1}{2})} \quad , \quad J_0\, S^{(-\frac{1}{2},-\frac{1}{2})} = -S^{(-\frac{1}{2},-\frac{1}{2})}.$$

We decompose the off-diagonal part of $\mathbb{V}_{\frac{1}{2}}$ as

$$\omega = \omega^{(+)} + \omega^{(-)} \quad , \quad \bar{\omega} = \bar{\omega}^{(+)} + \bar{\omega}^{(-)}$$

Y-M Instantons - Auxiliary fields and t'Hooft symbols

- After some algebra, it is possible to see that the auxiliary fields $\mathbb{H}_0, \mathbb{H}_1^{(+)}, \mathbb{H}_1^{(-)}$ along the D3 branes carry a SU(2) representation

$$\mathbb{H}_1^{(+)D3} = -\frac{i}{4}\,\eta_-^{\mu\nu}\;T_{\mu\nu}\,\psi_{1\,2}|0\rangle, \quad , \quad \mathbb{H}_1^{(-)D3} = +\frac{i}{4}\,\eta_+^{\mu\nu}\;T_{\mu\nu}\,\psi_{\bar{1}\,\bar{2}}|0\rangle,$$

$$\mathbb{H}_0^{D3} = -\frac{i}{2}\,\eta_3^{\mu\nu}\;T_{\mu\nu}\,|0\rangle.$$

$$\eta_+^{\mu\nu} \equiv \eta_1^{\mu\nu} + i\eta_2^{\mu\nu} \qquad , \qquad \eta_-^{\mu\nu} \equiv \eta_1^{\mu\nu} - i\eta_2^{\mu\nu},$$

- On the D3-branes' worldvolume, we get the quartic potential given by a perfect square

$$S_T = \mathrm{tr}\left[\langle\mathbb{H}_1^{(+)D3}|\,\mathbb{H}_1^{(-)D3}\rangle + \frac{1}{4}\langle\mathbb{H}_0^{D3}|\,\mathbb{H}_0^{D3}\rangle\right] = -\frac{1}{16}\,\mathrm{tr}\left[D_a D_a\right],$$

ADHM Constraint and the full effective action

- The auxiliary D_a contains the 3 ADHM constraints (on the $D(-1)$ slot):

$$D_a = \eta_a^{\mu\nu}\, T_{\mu\nu} = 0 \quad \rightarrow \quad \eta_a^{\mu\nu} \left[[a_\mu, a_\nu] - \frac{1}{2}\, \bar{\omega}_\alpha \, (\gamma_{\mu\nu})^{\alpha\beta}\, \omega_\beta \right] = 0$$

- The same computations are repeated for the transverse directions and the complete action is reproduced(see Billó, Frau, Fucito, Lerda, Liccardo and Pesando 2002):

$$\begin{aligned}
&-\frac{1}{8}\,\mathrm{tr}\left[[A_\mu, A_\nu]^2 + [a_\mu, a_\nu]^2 + [\phi_p, \phi_q]^2 + [\chi_p, \chi_q]^2 \right] \\
&-\frac{1}{4}\,\mathrm{tr}\left[[A_\mu, \phi_p]^2 + [a_\mu, \chi_p]^2 + \frac{1}{4}\,(\omega\gamma^{\mu\nu}\bar{\omega})^2 + \frac{1}{4}\,(\bar{\omega}\gamma^{\mu\nu}\omega)^2 \right] \\
&-\frac{1}{4}\,\mathrm{tr}\,[[A_\mu, A_\nu]\,\omega\gamma^{\mu\nu}\bar{\omega} - [a_\mu, a_\nu]\,\bar{\omega}\gamma^{\mu\nu}\omega] - \mathrm{tr}\,[\phi_p\,\omega\,\chi^p\,\bar{\omega}] \\
&+\frac{1}{2}\mathrm{tr}\left[\phi^2\omega\bar{\omega} - \chi^2\bar{\omega}\omega\right].
\end{aligned}$$

The small Hilbert space perspective: ('13, EKS)

- The large Hilbert space is natural for the NS sector, but a theory with two gauge structures is very hard to quantize (indeed, nobody can do that...). In some sense, the small Hilbert space alternative proposed by Erler, Konopka Sachs (EKS) seems simpler.

Erler-Konopka-Sachs Action

$$S_{EKS}(\Psi) = \frac{1}{2}\,\omega_S\,(\Psi\,,\,Q_B\Psi) + \sum_{n=2}^{+\infty} \frac{1}{n+1}\,\omega_S\,(\Psi\,,\,M_n(\Psi^n))$$

Here we recognize a kinetic term + interactions for the string field Ψ in the small Hilbert space.

A_∞ quartic effective action - Work in progress

- the small Hilbert space string field Ψ is related to the large Hilbert space string field Φ by partial gauge fixing.

$$\Psi = \eta_0 \Phi$$

- In an apparently different language, the effective action derived from A_∞ theory for a massless field in the small Hilbert space is:

$$S_{eff} = \frac{1}{2}\omega_S \left(M_2(\Psi^2), \frac{b_0}{L_0}\bar{P} M_2(\Psi^2) \right) + \frac{1}{4}\omega_S \left(\Psi, M_3(\Psi^3)\right)$$

which in form is similar to the Berkovits effective action. But when the explicit definitions for M_2, M_3 are considered, we end up with a not welcome surprise...

MORE THAN 40 CONTACT TERMS...

- All the capital multistring products are non-associative.

$$
\begin{aligned}
M_3(\Psi^3) \quad = \quad & +\frac{1}{2}\, M_2(\Psi, \bar{M}_2(\Psi,\Psi)) - \frac{1}{2}\, \bar{M}_2(\Psi, M_2(\Psi,\Psi)) \\
& +\frac{1}{2}\, M_2(\bar{M}_2(\Psi,\Psi),\Psi) - \frac{1}{2}\, \bar{M}_2(M_2(\Psi,\Psi),\Psi) \\
& +\frac{1}{2}\, Q_B \bar{M}_3(\Psi^3)
\end{aligned}
$$

where all the assignment of the picture are considered:

$$M_2(A,B) := \frac{1}{3}\left[X_0 m_2(A,B) + m_2(X_0 A, B) + m_2(A, X_0 B)\right]$$

$$\bar{M}_2(A,B) := \frac{1}{3}\left[\xi_0\, m_2(A,B) - m_2(\xi_0 A, B) - (-1)^{Deg(A)} m_2(A, \xi_0 B)\right]$$

$$m_2(A,B) = (-1)^{DegA} A * B$$

On-shell Equivalence - Work in progress

- Too many terms to perform explicit computations...
- The two theories must be equivalent on-shell. The amplitudes, and therefore the effective action, should be the same.
- This has been quiet formally demonstrated by Konopka ('15).
- But we have seen that

$$\left[Q_B, \frac{b_0}{L_0}\right] = 1 - P_0$$

so we want to be sure that such zero momentum anomalies do not occur.

Our results on the On-shell Equivalence - Work in progress

- Zero momentum anomalies cancels out completely.
- Decoupling of Exact states is proved.
- No correspondence between propagator terms or contact terms by themselves, but the sum of the two gives the correct action:

$$S^{Prop}_{EKS}(\Psi = \eta_0 \Phi) \neq S^{Prop}_{Ber}(\Phi) \qquad S^{Con}_{EKS}(\Psi = \eta_0 \Phi) \neq S^{Con}_{Ber}(\Phi)$$

$$BUT \qquad S_{EKS}(\Psi = \eta_0 \Phi) = S^{Prop}_{EKS} + S^{Con}_{EKS} = S_{Ber}(\Phi)$$

- The sum of the two terms cancels the extra terms, and the on-shell equivalence is demonstrated. Localization is available in the small Hilbert space, too.

Conclusions and future work

- We wanted a systematic way to compute effective actions from String theory.
- At least for the couplings we considered, this can be done easily both in the large/small Hilbert space thanks to localization on the boundary of moduli space.
- are α' corrections (in the higher orders) captured by this mechanism?
- Same techniques for closed strings? Ramond sector?
- Relation with topological strings ?

Thank you for your Attention!

Happy 50th Birthday String Theory!