Defect CFT set up

Matrix model approach

Perturbative checks

Conclusions

Correlators in presence of Wilson loops in superconformal gauge theories

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Introduction: Wilson loops in gauge theories

Wilson loops are a powerful tool to investigate non-abelian gauge theories also in non-perturbative regimes.

- **Definition**:

$$W_{\mathcal{R}}(C) = \frac{1}{N}\mathrm{Tr}_{\mathcal{R}}\,\mathcal{P}\exp\left[\mathrm{i}g\oint_C A_\mu(x)dx^\mu\right]$$

  where C is the closed loop, $\mathcal{R}$ the representation of the gauge group SU(N), $\mathcal{P}$ path ordering, g gauge coupling and A_μ gauge field.

- **Physical meaning**:
  - It represents the world line of a massive, charged particle in a gauge background.
  - Related to gauge invariant observables ($q\bar{q}$ potential, Bremsstrahlung, ...).

- Relevant in several areas of research (QCD, lattice QCD, AdS/CFT) of theoretical physics.

Introduction: superconformal gauge theories

Gauge theories enjoying extra space-time symmetries:

- **Supersymmetry**: bosons and fermions organized in supermultiplets $\rightarrow$ many cancellations to quantum corrections.
- **Conformal symmetry**: scale invariance also at the quantum level.

Motivation

Extra constraints $\rightarrow$ more tractable behaviour at the quantum level.

Framework:

- $\mathcal{N} = 2$ Yang-Mills theory on $\mathbb{R}^4$ with gauge group SU(N) and N_f fundamental flavours.
- For $N_f = 2N$ the beta function $\beta(g) = 0 \rightarrow$ **superconformal invariance**.
- **Goal**: insertion of Wilson loop and evaluation of correlators.

Main contents

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4 Conclusions

Supersymmetric Wilson loop and chiral operators

Supersymmetric circular Wilson loop (1/2 BPS):

$$W_{\mathcal{R}}(C) = \frac{1}{N}\mathrm{Tr}_{\mathcal{R}}\mathcal{P}\exp\left\{g\oint_C d\tau\Big[\mathrm{i}\,A_\mu(y)\,\dot{y}^\mu(\tau) + \frac{R}{\sqrt{2}}\,(\varphi(y)+\bar{\varphi}(y))\Big]\right\}$$

where R radius of C, $\varphi(y)$ adjoint complex scalar.

Chiral operators $O_n(x)$ of dimension n:

- gauge singlets of φ,
- annihilated by half of the supercharges → protected operators.

$$O_2(x) = \mathrm{Tr}\,\varphi^2(x) \qquad O_3(x) = \mathrm{Tr}\,\varphi^3(x)$$
$$O_4^{(1)}(x) = \mathrm{Tr}\,\varphi^4(x) \qquad O_4^{(2)}(x) = \big(\mathrm{Tr}\,\varphi^2(x)\big)^2$$

The quantity of interest is the 1/2 BPS 1-point function:

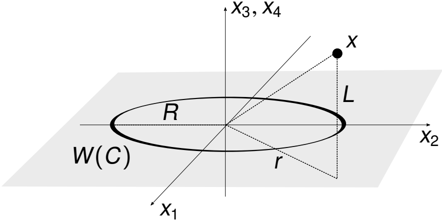
$$\Big\langle W(C)\,O_n(x)\Big\rangle$$

Line conformal defect

Geometrical set up and symmetry pattern

$W(C)$ in $(x^1, x^2) \subset \mathbb{R}^4$ $\qquad$ $O_n(x)$ in $x^\mu = (x^a; x^i)$

$$x^a\, x_a = r^2 \quad {}_{a=1,2}$$
$$x^i\, x_i = L^2 \quad {}_{i=3,4}$$
$$x^2 = r^2 + L^2$$



Average distance: $\|x\|_C = \dfrac{1}{R}\sqrt{(R^2 - x^2)^2 + 4L^2R^2}$

Residual Conformal symmetry[1]

The extended operator $W(C)$ breaks $\mathrm{SO}(1,5) \to \mathrm{SO}(1,2) \times \mathrm{SO}(3)$, sufficient to fix the spacetime dependence:

$$\langle\, W(C)\, O_n(x)\,\rangle = \frac{A_n(g, N)}{\left(2\pi\|x\|_C\right)^n}$$

[1] [Billò, Goncalves, Lauria, Meineri, 2016]

Exploiting the symmetries in field theories

$A_n(g, N)$ with gauge dependence only $\Rightarrow$ captured in QFT by *combinatorics of Feynman diagrams*.

Review of a **simpler problem**:

- $\mathcal{N} = 4$ theories (pure conformal SYM, stronger SUSY constraints).
- Pure $\langle W(C) \rangle$ vev (no operators insertion).
- No length scales $\rightarrow$ pure gauge dependence.

Using field theory techniques:
([2]) and ([3]) resummed the various contributions to the vev:

$$\langle\, W(C)\, \rangle = \frac{1}{N}\, L^{1}_{N-1}\Big(- \frac{g^2}{4} \Big)\, \exp\Big[\frac{g^2}{8} \Big(1 - \frac{1}{N} \Big) \Big]$$

L^n_m are Laguerre polynomials.

Remarks:
- Exact result for any value of the coupling g.
- Conjecture of an underlying **matrix model**.

[2] [Erikson, Semenoff, Zarembo, 2000]

[3] [Drukker, Gross, 2004]

Supersymmetric Localization: $\mathcal{N} = 4$ matrix model

With extended supersymmetry ($\mathcal{N} \geq 2$) Pestun[4] computed $\langle W(C) \rangle$ on a **sphere** S_4 using a **localization** procedure:

- Powerful technique, it requires a global invariance (SUSY).
- Idea: path integral localized to north/south poles contributions and so reduced to a finite-dimensional integral over the Cartan subalgebra of the gauge group.
- The result is a matrix model on S_4:

$$\mathcal{Z}_{S_4} = \int_h da \, | \, Z_{\mathbb{R}^4}(\mathrm{i}a, g) \, |^2 \ ,$$

where $\mathrm{i}a = \langle \varphi \rangle$ is a traceless $N \times N$ matrix.

- In $\mathcal{N} = 4$ the matrix model is Gaussian: $\mathcal{Z}_{S_4} = \int da \, \mathrm{e}^{-\mathrm{Tr}\, a^2}$.
- The Wilson loop operator has a matrix model equivalent:

$$\mathcal{W}(a) = \frac{1}{N} \mathrm{Tr} \exp\left(\frac{g}{\sqrt{2}} \, a \right)$$

- $\langle \mathcal{W}(a) \rangle_{S_4}$ matches the field theory results $\langle W(C) \rangle_{\mathbb{R}^4}$.

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[4] [Pestun, 2007]

Matrix model calculations

- The only variable a takes values in the gauge algebra:

$$a = a^b\, T^b, \quad b = 1, \dots, N^2 - 1.$$

- We only handle multitrace expressions like:

$$\langle (\mathrm{Tr}\, a^{n_1})(\mathrm{Tr}\, a^{n_2}) \dots \rangle$$

- We need:
  - color Wick contraction $\langle\, a^b\, a^c\,\rangle = \delta^{bc}$.
  - Matrix trace identities, e.g. $\mathrm{Tr}\, T^b\, T^c = \frac{1}{2}\delta^{bc}$.
- We use *recursive formulas*[5], with initial conditions:

$$\langle \mathrm{Tr}\, \mathbb{1} \rangle = N \qquad \langle \mathrm{Tr}\, a \rangle = 0 \qquad \langle \mathrm{Tr}\, a^2 \rangle = \frac{N^2 - 1}{2}$$

- The procedure returns **rational functions of N**.

[5] [Billò, Fucito, Lerda, Morales, Stanev, Wen, 2017]

$\mathcal{N} = 2$ interacting matrix model

Pestun's formula holds for $\mathcal{N} = 2$.

- Less amount of symmetry $\Rightarrow$ more complicated matrix model.
- Order by order in g it is reduced to Gaussian case.

$$\mathcal{Z}_{S_4} = \int da\, \mathrm{e}^{-\mathrm{Tr}\, a^2 - S_{\mathrm{int}}(a,g)}$$

where

$$S_{\mathrm{int}}(a,g) = 1 + \frac{g^2}{8\pi^2}\zeta(1)\, S_2 + \left(\frac{g^2}{8\pi^2}\right)^2 \zeta(3) S_4 + \left(\frac{g^2}{8\pi^2}\right)^3 \zeta(5) S_6 + \ldots$$

$$S_2 = (2N - N_f)\,\mathrm{Tr}\, a^2 \qquad S_4 = \frac{1}{2}\Big[(2N - N_f)\mathrm{Tr}\ a^4 + 6\left(\mathrm{Tr}\, a^2\right)^2\Big]$$

In $\mathcal{N} = 2$ a general correlator is:

$$\langle\, f(a)\,\rangle = \mathcal{Z}_{S_4}^{-1} \int da\, \mathrm{e}^{-\mathrm{Tr}\, a^2 - S_{\mathrm{int}}(a,g)}\, f(a) = \frac{\langle\, \mathrm{e}^{-S_{\mathrm{int}}(a,g)}\, f(a)\,\rangle_0}{\langle\, \mathrm{e}^{-S_{\mathrm{int}}(a,g)}\,\rangle_0}.$$

Comparison with field theory

([6]) verified $\langle \mathcal{W}(a)\rangle_{S_4}\Big|_{\mathcal{N}=2} = \langle W(C)\rangle_{\mathbb{R}^4}$ up to 2-loops order.

[6] [Andree, Young, 2010]

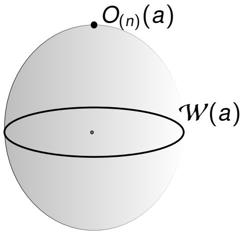
Chiral operators in the matrix model

Recap

- Circular $\langle W(C) \rangle$ has enough symmetries $\Rightarrow$ no space-time but only gauge dependence.
- The S_4-matrix model captures $\langle W(C) \rangle$ in both $\mathcal{N} = 2, 4$ theories.

Original problem $\langle\, W(C)\, O_n(x)\, \rangle$ also reduced to the evaluation of the gauge-dependent part $A_n(g, N)$.

- We need the equivalent of $O_n(x)$ in the matrix model.
- Naive idea: $\mathcal{O}_n(a) = O_n(x)\Big|_{\varphi \to a}$.
- E.g. $O_2(x) = \mathrm{Tr}\,\varphi^2 \to \mathrm{Tr}\, a^2$.



Issue: field theory propagator connects φ only with $\bar{\varphi}$ $\Rightarrow O_n(x)$ has **no self-contraction** and this is not true for $\mathcal{O}_n(a)$.

Dictionary between $\mathbb{R}^4$ and S_4

Normal ordered operators

We need $\mathcal{O}_n(a)$ with no self-contractions.

Definition: Given $\{O_p(a)\}$ matrix operators with dimension smaller than n, we define:

$$\mathcal{O}(a) = :O(a):_g = O(a) - \sum_{p,q} \langle O(a)\, O_p(a) \rangle (C_n^{-1})^{pq}\, O_q(a) \ .$$

where $\left(C_n\right)_{pq} = \langle O_p(a)\, O_q(a) \rangle$, $p+q \leq n$

Example: $\mathcal{O}_2(a) = :\mathrm{Tr}\, a^2:_g = \mathrm{Tr}\, a^2 - \frac{N^2-1}{2}$

$$+ \frac{3\,\zeta(3)\, g^4}{(8\pi^2)^2}\, \frac{(N^2-1)(N^2+1)}{2} + O(g^6) \ .$$

Remarks: • $\mathcal{O}_n(a)$ are g-dependent.

• we have a **map between $\mathbb{R}^4$ and S_4**[7] operators

$$O_n(x) \rightarrow \mathcal{O}_n(a) = \ :O_n(a):_g \ .$$

[7] See also: [Baggio, Niarchos, Papadodimas, 2014] and [Gerchkovitz, Gomis, Ishtiaque, Karasik, Komargodski, Pufu, 2016]

Computation of $\mathcal{A}_n = \left\langle \mathcal{W}(a)\, O_n(a) \right\rangle$:

- All the results in terms of Wilson loop vev: $\mathcal{W}(g,N)$.
- $\mathcal{A}_{(2)} = \left\langle \mathcal{W}(a) :\mathrm{tr}\, a^2: \right\rangle = \frac{1}{N} \sum_k \frac{g^k}{2^{k/2}\, k!} \left(\langle \mathrm{tr}\, a^k \mathrm{tr}\, a^2 \rangle - \frac{N^2-1}{2} \langle \mathrm{tr}\, a^k \rangle \right).$

$$= \frac{g}{2}\, \partial_g \mathcal{W}(g,N)$$

- $\mathcal{A}_{(3)} = \frac{g}{\sqrt{2}}\, \partial_g^2 \mathcal{W}(g,N) - \frac{g^2}{4\sqrt{2}N}\, \partial_g \mathcal{W}(g,N) - \frac{g(N^2-1)}{4\sqrt{2}N}\, \mathcal{W}(g,N)$

Remarks

- Always compact expressions of $\left\{ \partial_g^{(m)} \mathcal{W}(g,N) \right\}$;
- *Exact results* for any g and N.
- Powerful and efficient technique to compute gauge dependence of flat field theory.
- Results checked against $\mathcal{N}=4$ field theory[8].

[8] [Semenoff,Zarembo, 2001]

Results in $\mathcal{N}=2$ for each transcendentality order

Interacting matrix model

- Due to $S_{\text{int}}(a)$, $\mathcal{A}_n$ is computed perturbatively.
- In the conformal case
  $S_{\text{int}}(a,g)\Big|_{N_f=2N} = 1 - (\frac{g^2}{8\pi^2})^2 3\zeta(3)\left(\text{tr}\, a^2\right)^2 + O(g^6)$
- $S_2(a)\Big|_{N_f=2N} = 0 \quad \Rightarrow \quad \mathcal{A}_n\Big|^{\mathcal{N}=2}_{\text{tree},1-\text{loop}} = \mathcal{A}_n\Big|^{\mathcal{N}=4}_{\text{tree},1-\text{loop}}$.
- At each order $(\zeta(3), \zeta(5), \dots)$ we can get a compact polynomial in $\left\{\partial_g^{(m)}\mathcal{W}(g,N)\right\}$ as before.
- Examples:

$$\bullet\mathcal{A}_0\Big|_{\zeta(3)} = \langle\mathcal{W}(a)\rangle = \frac{3\,\zeta(3)\,g^4}{(8\pi^2)^2}\Big(-\frac{g^2}{4}\,\partial_g^2\mathcal{W} - \frac{g(2N^2+1)}{4}\,\partial_g\mathcal{W}\Big)$$

$$\bullet\mathcal{A}_2\Big|_{\zeta(3)} = \frac{3\,\zeta(3)\,g^4}{(8\pi^2)^2}\Big(-\frac{g^3}{8}\,\partial_g^3\mathcal{W} - \frac{g^2(2N^2+7)}{8}\,\partial_g^2\mathcal{W} - \frac{5g(2N^2+1)}{8}\,\partial_g\mathcal{W}\Big).$$

- No field theory results in literature → perturbative check needed.

Perturbative computation: approach

Goals

(**A**) Verify the factorization

$$\langle W(C)\, O_n \rangle = \frac{1}{(2\pi \|x\|_C)^n} A_n(g,N);$$

(**B**) prove the equality:

$$\mathcal{A}(g,N)\Big|_{S_4} = A(g,N)\Big|_{\mathbb{R}^4}$$

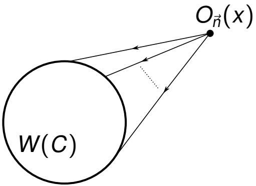


Figure: **Tree level**: $O_n(x)$ connected to $W(C)$ by n scalar propagators.

Issues

Diagrammatic evaluation of $\langle W(C) O_n(x) \rangle$ quite complicated (several diagrams, path-ordered integrals).

Tools

- $\mathcal{N} = 1$ superspace formalism: efficient way to implement Feynman rules with susy invariance.
- Diagrammatic difference $(\mathcal{N} = 2) - (\mathcal{N} = 4)$.

Diagrammatic difference $(\mathcal{N}=2)-(\mathcal{N}=4)$

- $\mathcal{N}=2$: **[vector]**: $V_{\mathcal{N}=2}=(\varphi,\lambda_\alpha^{(1,2)},A^\mu)$
  [hyper]: $Q=(q^{(1,2)},\psi_\alpha^{(1,2)})$
- $\mathcal{N}=4$: **[vector]** $V_{\mathcal{N}=4}=(\varphi^{(1,2,3)},\lambda_\alpha^{(1,2,3,4)},A^\mu)$
  $=V_{\mathcal{N}=2}+H$

  where $H=(\varphi^{(2,3)},\lambda_\alpha^{(3,4)})$ is an adjoint hypermultiplet.

If we split the actions as:

$\bullet S_{\mathcal{N}=2}^{(N_f)}=S_{\mathcal{N}=2}^{\text{gauge}}+S_Q\ ,$

$\bullet S_{\mathcal{N}=4}=S_{\mathcal{N}=2}^{\text{gauge}}+S_H\quad\Rightarrow\quad S_{\mathcal{N}=2}^{\text{gauge}}=S_{\mathcal{N}=4}-S_H$

we write the full $\mathcal{N}=2$ action as:

$$S_{\mathcal{N}=2}^{(N_f)}=S_{\mathcal{N}=4}+S_Q-S_H\ .$$

Then the amplitudes: $A_n^{\mathcal{N}=2}-A_n^{\mathcal{N}=4}=A_{n,Q}-A_{n,H}$.

In the difference we only consider diagrams with Q or H.

Tree level and one loop

Tree level

Only $\varphi - \bar{\varphi}$ propagators → it does not contribute to the difference.

One loop

Two possibilities:

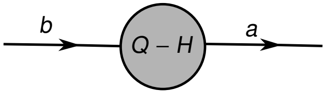


Figure: In the 1-loop correction to scalar propagator Q and H fields run, but its colour factor is $\propto (N_f - 2N)\delta^{ab}$

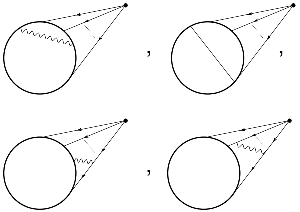


Figure: Only gluon/scalar exchanges, all these vanish in the difference

In full agreement with the matrix model result:

$$A_n\Big|_{\text{tree,1-loop}}^{\mathcal{N}=2} = A_n\Big|_{\text{tree,1-loop}}^{\mathcal{N}=4}$$

Two loops

- At g^4 order again several diagrams vanish due to colour factors or in the difference with $\mathcal{N}=4$.
- Efficient procedure: we are left with just 2 diagrams

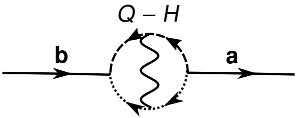


Figure: 2-loops propagator

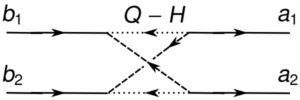


Figure: 2-loops effective vertex

- **Goal (A)**: Superdiagrams computations confirm $\langle W(C)\, O_n\rangle \propto \frac{1}{(2\pi\|x\|_C)^n}$.
- **Goal (B)**: The 2-loops result for the gauge dependence:

$$\delta A_{\vec{n}}\Big|_{2-\text{loop}} = -g^4\, \frac{3\,\zeta(3)}{(8\pi^2)^2}\left[\frac{g^n}{N\,2^{\frac{n}{2}}}\, R_{\vec{n}}^{b_1\dots b_n}\, \mathrm{tr}\Big(T^{a_1}\dots T^{a_n}\Big)\right]$$

$$\times\Big[n\,(N^2+1)\delta^{b_1 a_1}\dots\delta^{b_n a_n} - 2\sum_{p\in S_{n-1}} C_4^{b_1 b_2 a_{p(1)} a_{p(2)}}\,\delta^{b_3 a_{p(3)}}\dots\delta^{b_{n-1} a_{p(n-1)}}\,\delta^{b_n a_n}\Big].$$

which matches the (much simpler) matrix model computation.

Conclusions

In the present work:

- We discussed the implementation of **extra symmetries** in a field theory computation of the 1-point function of a chiral operator in presence of a line defect $\langle W(C)\, O_n(x) \rangle$.
- We exploited the residual conformal symmetry to **fix the space-time dependence** for any values of g.
- We verified up to 2-loops order that SUSY invariance allows the gauge dependent part $A_n(g, N)$ to be captured by the **matrix model** on a 4-sphere.
- We developed some **tools**:
  - To handle matrix model computations.
  - To simplify perturbative analysis.

Future developments

- Different operators insertion → Other observables.
  For example:
  $\langle W(C)\, T^{\mu\nu}(x) \rangle$ related to energy emitted by the heavy particle (**Bremsstrahlung function**).
- Pestun localization formula on S_4 is still valid in **non-conformal case**.
  - Correspondence with field theory is less immediate (renormalization, anomalous dimensions, . . .).
  - Can the matrix model still predict something?
- We have finite-N results → possible investigation on the **holographic dual** for large-N limit in AdS/CFT context.