Line defects and emitted radiation in $\mathcal{N} = 2$ theories

based on 1805.04111 with M. Meineri and M. Lemos

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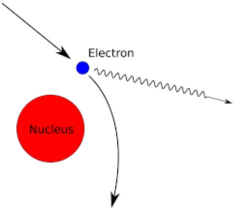

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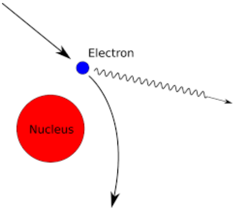

- For certain superconformal field theories this energy can be computed exactly. Examples are
  1. 1/2 BPS WL for $\mathcal{N} = 4$ SYM [Correa, Henn, Maldacena, Sever, 2012]
  2. 1/2 BPS WL for ABJM theory [M.S.Bianchi, Griguolo, Leoni, Penati, Seminara, 2014; M.S.Bianchi, Griguolo, Mauri, Penati, Preti, Seminara, 2017; LB, Griguolo, Preti, Seminara, 2017; LB, Preti, Vescovi, 2018; M.S.Bianchi, Griguolo, Mauri, Penati, Seminara, 2018]
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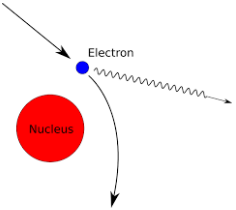

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- Superconformal defect approach proved very useful.
- Here we focus on $\mathcal{N} = 2$ theories in 4d, where a conjecture was put forward. [Fiol, Gerchkowitz, Komargodski, 2016]

Displacement operator

- A defect breaks translation invariance

$$\partial_\mu T^{\mu m}(x^\nu) = \delta_\Sigma(x)\; D^m(\tau),$$

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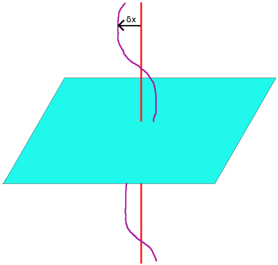

- $D^m(\tau)$ is the displacement operator
- It implements small modifications of the defect

$$\delta \langle X \rangle_W = - \int d\tau\, \delta x^m(\tau) \langle D^m(\tau) X \rangle_W$$

or, alternatively

$$\mathfrak{P}_m \,|W\rangle = -i \int d\tau\, \mathbb{D}_m(\tau)\,|W\rangle \equiv -i \int d\tau\, |\mathbb{D}_m\rangle$$

- Its two-point function is fixed by symmetry

$$\langle D^m(\tau)\, D^n(0) \rangle_W = C_D \,\frac{\delta^{mn}}{|\tau|^4}\,.$$

One-point functions

- A defect breaks translation invariance

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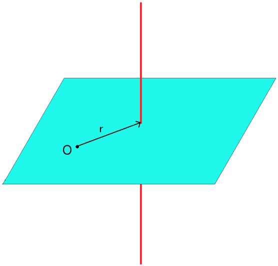


- Local operators acquire a non-vanishing one-point function.
- The kinematics is fixed by conformal invariance

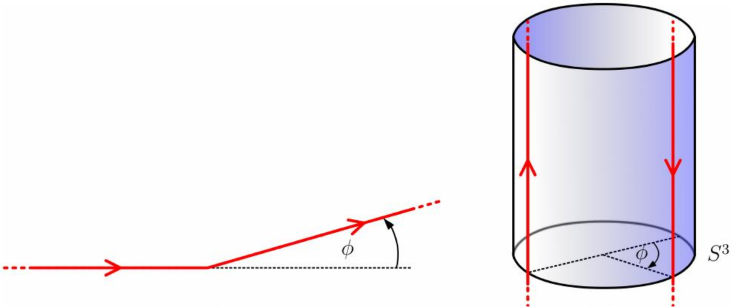
$$\langle O \rangle_W \equiv \frac{\langle W\, O \rangle}{\langle W \rangle} = \frac{C_O}{r^\Delta}$$

- For the stress tensor

$$\langle T_{\tau\tau} \rangle_W = \frac{h}{r^4}$$

$$\langle T_{ab} \rangle_W = -\frac{h}{r^4}\left(\delta_{ab} - 2n_a n_b\right)$$

Cusped Wilson lines

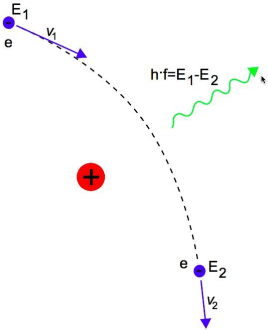


$$\langle W_{\text{cusp}} \rangle = e^{-\Gamma_{cusp}(\phi) \log \frac{L}{\epsilon}}$$

In the limit of small angle the expectation value is again controlled by the displacement operator.

$$\Gamma_{\text{cusp}}(\phi) \sim -B\phi^2 = -\frac{1}{2}\phi^2 \int d\tau \, \langle D(\tau) D(0) \rangle = -\frac{C_D}{12}\phi^2$$

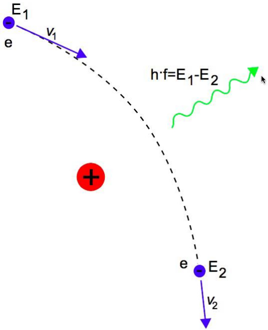
Emitted energy and C_D



Energy emitted by a moving electron (Larmor formula)

$$\Delta E = \frac{2e^2}{3} \int dt \dot{v}^2$$

Emitted energy and C_D



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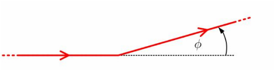
For a heavy probe in a CFT we can use conformal defect techniques.

Energy emitted by a heavy probe in a CFT

$$\Delta E = \frac{\pi}{6} C_D \int dt \dot{v}^2 = 2\pi B \int dt \dot{v}^2$$

In summary [Correa, Henn, Maldacena, Sever, 2012;]



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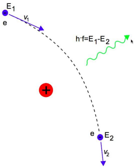

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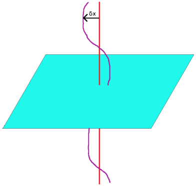
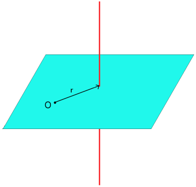
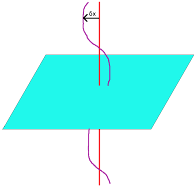


$$\langle D^a(x^i)\, D^b(0)\rangle_W = \frac{12\, B\, \delta^{ab}}{|x^i|^{2(d-1)}}$$





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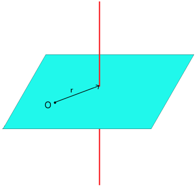

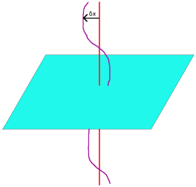

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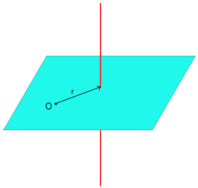
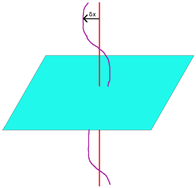
In summary





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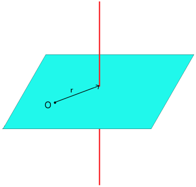

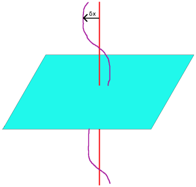

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- For $\mathcal{N}=4$ SYM and ABJM [Lewkowycz, Maldacena, 2013].

$$B = \frac{4\pi^{\frac{d+1}{2}}}{\Gamma(\frac{d-1}{2})} \frac{d-1}{d-2} \frac{h}{4\pi^2} = \begin{cases} 3h & d=4 \\ 2h & d=3 \end{cases}$$

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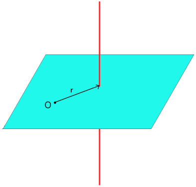

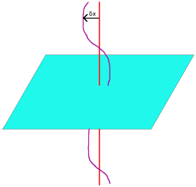

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- What about other theories/defects?

Wilson lines in $\mathcal{N} = 2$ theories

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Supergroup	$SU(2,2\|2)$
16 fermionic generators	$Q^a_\alpha, \bar{Q}^{a\dot{\alpha}}, S^a_\alpha, \bar{S}^{a\dot{\alpha}}$
Bosonic subgroup	$SO(1,5) \times SU(2)_R \times U(1)_r$

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1/2 BPS Wilson lines in $\mathcal{N} = 2$

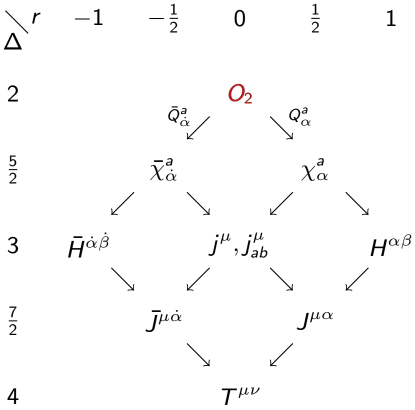
$$W = \text{Tr}\mathcal{P} \exp \left[i \int d\tau (A_\mu \dot{x}^\mu + |\dot{x}|\phi) \right]$$

Straight Wilson line in $\mathcal{N} = 2$	
Supergroup	$OSP(4^*|2)$
8 fermionic generators	$\mathcal{Q}^a_\alpha, \mathcal{S}^a_\alpha$
Bosonic subgroup	$SO(1,2) \times SU(2) \times SU(2)_R$

$$\mathcal{Q}^a_\alpha = Q^a_\alpha - i\sigma^4_{\alpha\dot{\alpha}} \bar{Q}^{a\dot{\alpha}} \qquad \mathcal{S}^a_\alpha = S^a_\alpha + i\sigma^4_{\alpha\dot{\alpha}} \bar{S}^{a\dot{\alpha}}$$

Stress tensor and displacement [LB, Meineri, Lemos, 2018]

$\Delta \backslash r$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
2			O_2		
		$\bar{Q}^a_{\dot{\alpha}}$ ↙		↘ Q^a_α	
$\frac{5}{2}$		$\bar{\chi}^a_{\dot{\alpha}}$		χ^a_α	
	↙		↘ ↙		↘
3	$\bar{H}^{\dot{\alpha}\dot{\beta}}$		j^μ, j^μ_{ab}		$H^{\alpha\beta}$
	↘		↙ ↘		↙
$\frac{7}{2}$		$\bar{J}^{\mu\dot{\alpha}}$		$J^{\mu\alpha}$	
			↘ ↙		
4			$T^{\mu\nu}$		



Scalar superprimary

$\bar{Q}^a_{\dot\alpha}$ ↙ ↘ Q^a_α

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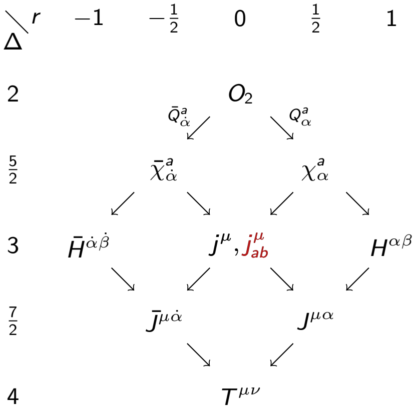
3	$\bar{H}^{\dot\alpha\dot\beta}$		j^μ, j^μ_{ab}		$H^{\alpha\beta}$	$U(1)_r$ current
$\frac{7}{2}$		$\bar{J}^{\mu\dot\alpha}$		$J^{\mu\alpha}$		
4			$T^{\mu\nu}$			

Stress tensor and displacement [LB, Meineri, Lemos, 2018]



Scalar superprimary

$U(1)_r$ current
$SU(2)_R$ current

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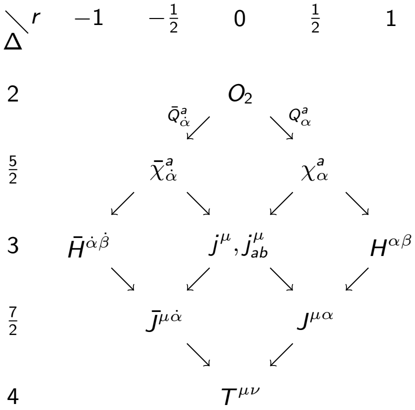

Symmetries are broken by the line

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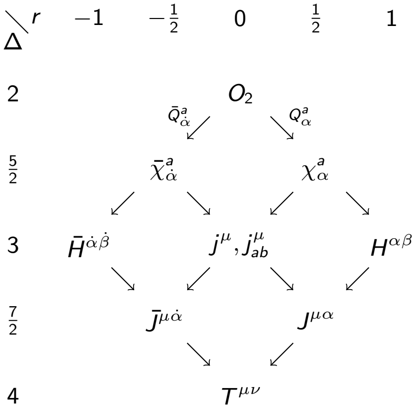

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$$\partial_\mu \mathfrak{J}^{\mu a}_\alpha = \delta^3(x)\Lambda^a_\alpha(\tau)$$

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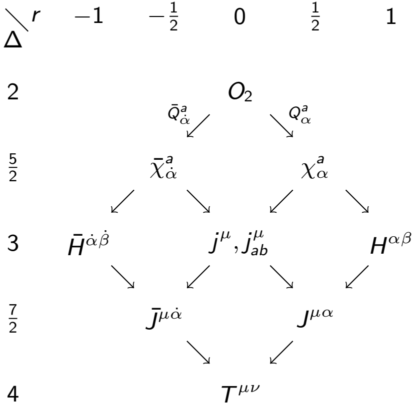

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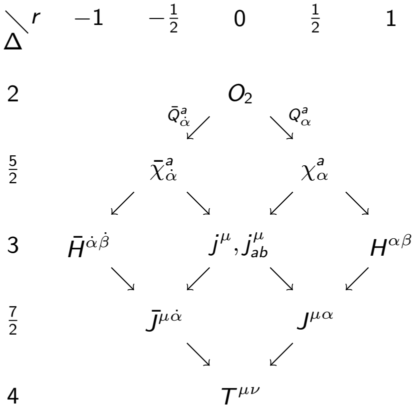

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$\mathcal{W}_2$

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$\Big\}$ $\mathbb{D}$ multiplet

$\hat{\mathcal{W}}_1$

Defect correlators [LB, Meineri, Lemos, 2018]

The symmetries preserved by the line allow to fix the kinematics of various correlators up to some constants [Billò, Goncalves, Lauria, Meineri, 2016]

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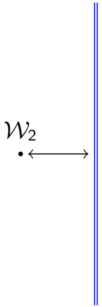
- Bulk one-point function of the full stress tensor multiplet

$$\langle \mathcal{W}_2 \rangle_W \sim h$$



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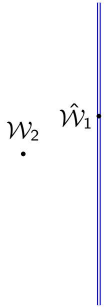
- Bulk to defect two-point function of the two multiplets using defect Ward identities

$$\langle \mathcal{W}_2 \hat{\mathcal{W}}_1 \rangle_W \sim B, h$$



Defect correlators [LB, Meineri, Lemos, 2018]

The symmetries preserved by the line allow to fix the kinematics of various correlators up to some constants [Billò, Goncalves, Lauria, Meineri, 2016]

- Bulk one-point function of the full stress tensor multiplet

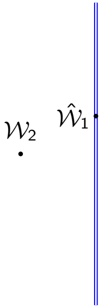
$$\langle \mathcal{W}_2 \rangle_W \sim h$$

- Defect two-point function of the full displacement multiplet

$$\langle \hat{\mathcal{W}}_1 \hat{\mathcal{W}}_1 \rangle_W \sim B$$

- Bulk to defect two-point function of the two multiplets using defect Ward identities

$$\langle \mathcal{W}_2 \hat{\mathcal{W}}_1 \rangle_W \sim B, h$$



Using $\langle \delta_{\text{susy}}(\mathcal{W}_2 \hat{\mathcal{W}}_1) \rangle_W$ we found several equations, all solved by

$$B = 3h$$

Conclusions and outlook

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- Second step needs better understanding. Stress tensor related to geometrical deformation, but connection to matrix model is not transparent. Perhaps one would need to understand the relation of the scalar superprimary with matrix model d.o.f. [Billò, Galvagno, Gregori, Lerda, 2018][see also Francesco's talk]

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THANK YOU