A new hat for the ABJ(M) Matrix Model

[Exact computation of latitude Wilson loops]

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Localization & Matrix Models
Bremsstrahlung functions

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BPS Wilson loops

BPS Wilson Loops in Supersymmetric Gauge Theories: invariant non-local operators that preserve some global supercharges

The prototype example in 4D N = 4 SYM

$$\mathcal{W} = \mathrm{Tr}\left(P e^{-i\oint_\Gamma (A_\mu \dot{x}^\mu + i|\dot{x}|\theta_I \Phi^I) d\tau}\right)$$

It includes couplings with the six scalars (matter fields).[Maldacena, PRL80 (1998) 4859, Soo-Jong Rey, Jung-Tay Yee (1998)] [Drukker, Gross, Ooguri, PRD60 (1999) 125006]

The number of preserved supercharges depends on Γ and θ_I

They are in general non-protected operators and their expectation values

$$\langle WL \rangle \sim \int D[A, \hat{A}, C, \bar{C}, \psi, \bar{\psi}]\, e^{-S}\, \mathrm{Tr} P \exp\left[-i \int_{\Gamma} d\tau \mathcal{L}(\tau)\right]$$

undergo a non-trivial behavior.

- **Weak coupling** → Ordinary perturbation theory
- **Strong coupling** → Holographic methods: Dual description in terms of fundamental strings or M2-branes. The expectation value at strong coupling is **given** by the exponential of a minimal area surface ending on the WL contour.
- **Finite coupling** → Localization techniques (lower dimensional QFT/Matrix Model)

The Matrix Model provides an exact interpolating function to check the AdS/CFT correspondence

- They are related to physical quantities like Bremsstrahlung function and Cusp anomalous dimension. Therefore, they are also related to

↓

INTEGRABILITY IN AdS/CFT

- Parametric WL are related to correlation functions of the 1D defect CFT on the WL contour.

BPS Wilson loops in ABJ(M) theory

$\mathcal{N}=6$ **ABJ(M):** Chern-Simons-matter theory with gauge group $U(N_1)_k \times U(N_2)_{-k}$
[Aharony, Bergman, Jafferis, Maldacena, 0806.1218; Aharony, Bergman, Jafferis, 0807.4924]

Field content

- $U(N_1)_k \times U(N_2)_{-k}$ CS-gauge vectors A_μ, $\hat{A}_\mu$ minimally coupled to
- **SU(4)** complex scalars C_I, $\bar{C}^I$ and fermions $\bar{\psi}_I$, ψ^I, $I = 1,\dots,4$, in the (anti-)bifundamental representation of the gauge group. They also interact through a non-trivial potential **(φ^6)** and Yukawa-like interaction **($\varphi^2\psi^2$)**

$$S = S_{CS} + S_{mat} + S_{pot}^{bos} + S_{pot}^{ferm}$$

- Dual to M-theory on $AdS_4 \times S^7/Z_k$ for **N»k, N»k5** ($N_1=N_2=N$). The CFT describes the low energy dynamics of N M2-branes in M-theory probing a $\mathbf{C^4/Z_k}$ **singularity**
- Dual to Type II A on $AdS_4 \times CP^3$ for **k«N«k5**
- For $N_1 > N_2$ a two-form flux $B=N_1-N_2$ is turned on in the internal space
- The CFT describes the low energy dynamics of N_2 M2-branes probing a $\mathbf{C^4/Z_k}$ singularity, plus (N1 – N2) fractional branes sit at the singularity

Why BPS Wilson loops in SCSM in D=3?

We would like to test the AdS4/CFT3 version of the correspondence of course!.

BPS WL in 3D SCSM theories exhibit a richer spectrum of interesting properties compared to the 4D case. Among them:

- Due to dimensional reasons scalar ($M^{1/2}$) and fermions (M^{1}) can enter the definition of BPS WL. In general they increase the number of SUSY charges preserved by WL.
- Topological phases (framing factors) generally appear as overall complex phases in $\langle WL \rangle$. Choice of a SUSY preserving regularization scheme!
- Cohomological equivalence: different loops are actually equivalent (if SUSY preserved)
- Non-trivial relation with integrability: $h(\lambda)$

Prototype examples of WLs in ABJ(M)

Bosonic Wilson loops: 1/6 BPS (4 Supercharges) **v =1**

$$W_B(\nu) = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau(A_\mu \dot{x}^\nu - \frac{2\pi}{k}|\dot{x}|M_I{}^J(\nu)C_J\bar{C}^I)\right]$$

$$\hat{W}_B(\nu) = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau(\hat{A}_\mu \dot{x}^\nu - \frac{2\pi}{k}|\dot{x}|\hat{M}_I{}^J(\nu)\bar{C}^I C_J)\right]$$

The circuit is the usual circle $\Gamma = (\cos\tau, \sin\tau, 0)$.The matrices governing the coupling with scalars are equal and they are diagonal $M = \hat{M} = \mathrm{diag}(1, 1, -1, -1)$

—SU(2)XSU(2) local symmetry: not dual of the fundamental string

[Drukker, Plefka, Young, JHEP 0811 (2008) 019; Chen, Wu, NPB 825 (2010) 38;
Rey, Suyama, Yamaguchi, JHEP 0903 (2009)]

Fermionic Wilson loops: 1/2 BPS (12 Supercharges)

It is the holonomy of a **super-connection** living in the superalgebra $u(N_1|N_2)$ and built out of the fundamental fields, i.e.

$$W_F = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau \mathcal{L}(\tau)\right]$$

with

$$\mathcal{L}(\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}|M_I{}^J C_J \bar{C}^I & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I \bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I \bar{\eta}^I & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}|M_I{}^J \bar{C}^I C_J \end{pmatrix}$$

The circuit is the usual circle: $\Gamma = (\cos\tau, \sin\tau, 0)$. The fermionic couplings are given by

$$\eta_I^\alpha = e^{i\frac{\tau}{2}} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}_I (1, -ie^{-i\tau})^\alpha$$

The matrices governing the coupling with scalars are equal and they are diagonal $M = \hat{M} = \mathrm{diag}(1, 1, -1, -1)$

—SU(3)XU(1) local symmetry: dual of the fundamental string

[Drukker, Trancanelli, JHEP 02 (2010) 058]

N=2 Super-CS Theory on S^3

We are primarily interested in supersymmetric theories for which localization can be used. It has been shown (Kapustin, Willett, Yaakov, JHEP 1003 (2010) 089) that the Wilson loop

$$\langle \mathcal{W}_{SCS} \rangle = \langle \mathrm{Tr}\ \mathrm{P} e^{-i\int_\Gamma d\tau (\dot{x}^\mu A_\mu(x) - i|\dot{x}|\sigma)} \rangle$$

along a maximal circle of S^3 can be computed through localization. In absence of matter **(N=2 Super CS=Pure CS)** we obtain the known result for pure CS, but at framing χ**=1.**

This follows from requiring **consistency** between **contour-splitting regularization** and **supersymmetry** used to localize: The only point-splitting compatible with susy is the one where the contour and its frame wrap **two different Hopf fibers of S^3.**

Localisation is sensible to framing !

Adding Matter: the case of ABJM

1/6 BPS circular Wilson loops:

Matter contributes to framing. Exponentiation still works, so we can write (**λ**i ≡ Ni/k)

$$\langle \mathcal{W} \rangle = \underbrace{e^{\pi i \left(\lambda_1 - \frac{\pi^2}{2}\lambda_1 \lambda_2^2 + O(\lambda^5)\right)}}_{\text{“framing function”}} \left(1 - \frac{\pi^2}{6}(\lambda_1^2 - 6\lambda_1\lambda_2) + O(\lambda^4)\right)$$

Perturbative Framing function: $$\Phi_B = \lambda_1 - \frac{\pi^2}{2}\lambda_1\lambda_2^2 + O(\lambda^5)$$

Computed perturbatively at non-trivial framing [M. Bianchi, L. G., A. Mauri, S. Penati, D.Seminara (2016, 2018)]

In ordinary perturbation theory (framing = 0) there is **no contributions at odd orders** (Rey, Suyama, Yamaguchi, JHEP 0903 (2009)). Thus the additional term does not appear

The perturbative analysis agrees with the localization result encoded in the non-gaussian matrix model:

$$Z = \int \prod_{a=1}^{N_1} d\lambda_a \; e^{i\pi k \lambda_a^2} \prod_{b=1}^{N_2} d\mu_b \; e^{-i\pi k \mu_b^2} \times \left(\frac{1}{N_1} \sum_{a=1}^{N_1} e^{2\pi \lambda_a} \right)$$

$$\times \frac{\prod_{a<b}^{N_1} \sinh^2 \pi(\lambda_a - \lambda_b) \prod_{a<b}^{N_2} \sinh^2 \pi(\mu_a - \mu_b)}{\prod_{a=1}^{N_1} \prod_{b=1}^{N_2} \cosh^2 \pi(\lambda_a - \mu_b)}$$

[Kapustin, Willett, Yaakov, JHEP 1003; Drukker, Marino, Putrov, (2011); Klemm, Marino, Schiereck, Soroush, (2013)]

Cohomological Equivalence

The classical analysis of the supercharges shared by the 1/6 and 1/2 BPS circle allows us to show

$$W_F = \frac{N_1 W_B + N_2 \hat{W}_B}{N_1 + N_2} + \mathcal{Q}(\text{something})$$

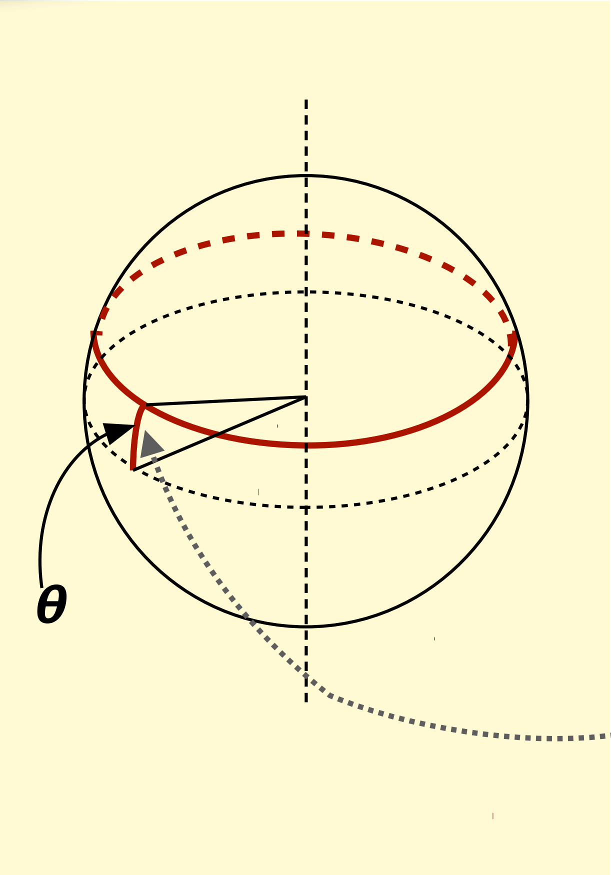
The charge **Q** can be used to localize in the approach of Kapustin Yaakov and Willett. We find

$$\langle W_F \rangle_1 = \frac{N_1 \langle W_B \rangle_1 + N_2 \langle \hat{W}_B \rangle_1}{N_1 + N_2}$$

namely, at the quantum level, the cohomological equivalence holds at framing one.

Supersymmetric Latitude in ABJ(M)

[V. Cardinali, L.G., G. Martelloni, D. Seminara. 2012]
[M. Bianchi, L.G., M. Leoni, S. Penati, D. Seminara. 2014]



It is possible to construct both **Bosonic** and **Fermionic** latitudes, depending on a real parameter

$$\nu = \sin 2\alpha \cos\theta \qquad \nu \in [0,1]$$

- **θ** measures the distance of the latitude from the equator.

The contour can be parameterized by

$$\Gamma(\tau) = (\cos\theta\cos\tau, \cos\theta\sin\tau, \sin\theta) \qquad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- **α** is the azimuthal angle on a S^2 in the internal (R-symmetry) space.
- It controls the bosonic couplings $M_I{}^J$ and the fermionic ones η_I η^I

Bosonic BPS Latitude

$$W_B(\nu) = \mathrm{Tr} P \exp\left[-i\int_\Gamma d\tau \left(A_\mu \dot{x}^\nu - \frac{2\pi}{k}|\dot{x}| M_I{}^J(\nu) C_J \bar{C}^I\right)\right]$$

where

$$M_I{}^J(\nu) = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Locally it is invariant under SU(2) X SU(2) ⊆ SU(4) (not dual to the fundamental string)
- It preserves two of the 24 superconformal charges (1/12 BPS)
- The usual 1/6 BPS circle is recovered for v =1.

Fermionic BPS Latitude: $$W_F(\nu) = \text{“Tr”}\left(P\exp\left[-i\int_\Gamma d\tau \mathcal{L}(\nu,\tau)\right]\right)$$

$\mathcal{L}(\nu,\tau)$ is a **super-connection** living in the super-Lie-algebra of U(N|M), built out of the fundamental fields of theory:

$$\boxed{\mathcal{L}(\nu,\tau) = \begin{pmatrix} A_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}|M_I{}^J(\nu) C_J \bar{C}^I & -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\eta_I(\nu)\bar{\psi}^I \\ -i\sqrt{\frac{2\pi}{k}}|\dot{x}|\psi_I \bar{\eta}^I(\nu) & \hat{A}_\mu \dot{x}^\mu - \frac{2\pi i}{k}|\dot{x}|M_I{}^J(\nu)\bar{C}^I C_J \end{pmatrix}}$$

where

$$M_I{}^J(\nu) = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \eta_I^\alpha = \frac{e^{i\frac{\nu\tau}{2}}}{\sqrt{2}} \begin{pmatrix} \sqrt{1+\nu} \\ -\sqrt{1-\nu}\, e^{i\tau} \\ 0 \\ 0 \end{pmatrix}_I (1, -ie^{-i\tau})^\alpha$$

“Tr” is actually a **super-trace** combined with a twisting matrix **T** which takes into account that the fermionic couplings are not periodic along the contour: $\text{“Tr”}(\cdot) \equiv \mathrm{STr}(\mathcal{T}\cdot)$

- Locally it is invariant under SU(3) X U(1) ⊆ SU(4) (dual to the fundamental string)
- It preserves 4 of the 24 superconformal charges (1/6 BPS)
- The usual 1/2 BPS circle is recovered for v=1.

Cohomological Equivalence: Fermionic vs Bosonic Latitude

There exists a common **supercharge $Q^{(\nu)}$** between fermionic and bosonic latitude such that

$$W_F(\nu) = \mathcal{R}\left[N_1 e^{-\frac{\pi i \nu}{2}} W_B(\nu) - N_2 e^{\frac{\pi i \nu}{2}} \hat{W}_B(\nu)\right] + \mathcal{Q}^{(\nu)}(\text{something})$$

If we were able to localize the path-integral by means of **$Q^{(\nu)}$**, we would get

$$\langle W_F(\nu)\rangle_\nu = \mathcal{R}\left[N_1 e^{-\frac{\pi i \nu}{2}} \langle W_B(\nu)\rangle_\nu - N_2 e^{\frac{\pi i \nu}{2}} \langle \hat{W}_B(\nu)\rangle_\nu\right]$$

The cohomological equivalence at quantum level is realized at non-integer framing ν. The possibility of non-integer framing is a characteristic of these Wilson loops. This relation was checked by explicit perturbative computations up to three loops

Matrix Model for the Latitude

In order to preserve cohomological equivalence and to obtain an exact expression for the latitude, we should perform the localization procedure with the supercharge Q(v). We expect a v-dependent Matrix Model.

We cannot directly apply the standard approach of Kapustin, Yaakov and Willett: Q(v) is not chiral and it cannot be embedded into the N=2 superspace formalism. It is not easy to localize

We have a non-trivial proposal (M.Bianchi, L.G., A.Mauri, S.Penati, D.Seminara, **arXiv:1802.07742**)

$$\langle W_B^m(\nu)\rangle_\nu = \left\langle \frac{1}{N_1} \sum_{1\leq i\leq N_1} e^{2\pi\, m\, \sqrt{\nu}\, \lambda_i} \right\rangle$$

$$\boxed{\begin{aligned} Z = \int \prod_{a=1}^{N_1} d\lambda_a\, e^{i\pi k \lambda_a^2} \prod_{b=1}^{N_2} d\mu_b\, e^{-i\pi k \mu_b^2} \\ \times \frac{\prod_{a<b}^{N_1} \sinh\sqrt{\nu}\pi(\lambda_a-\lambda_b)\sinh\frac{\pi(\lambda_a-\lambda_b)}{\sqrt{\nu}} \prod_{a<b}^{N_2} \sinh\sqrt{\nu}\pi(\mu_a-\mu_b)\sinh\frac{\pi(\mu_a-\mu_b)}{\sqrt{\nu}}}{\prod_{a=1}^{N_1}\prod_{b=1}^{N_2} \cosh\sqrt{\nu}\pi(\lambda_a-\mu_b)\cosh\frac{\pi(\lambda_a-\mu_b)}{\sqrt{\nu}}} \end{aligned}}$$

This Matrix Model can be solved in certain regimes and passes non-trivial tests both a weak and a strong coupling

Properties of the Matrix Model

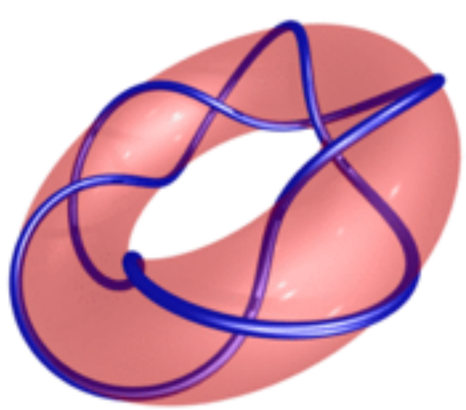
- One of the simplest deformation of the well-known matrix model of Kapustin, Willett and Yaakov [2010]. We recover it for v=1.

- **Consistency check on the partition function:**
  - For $N_1=N_2$ (ABJM) the partition function computed with the MM is independent of the parameter v
  - For $N_1\neq N_2$ (ABJ) the dependence is just a simple phase depending on N1-N2

$$\exp\left(\frac{\pi i}{12k}\left(\nu+\frac{1}{\nu}\right)((N_1-N_2)^3-(N_1-N_2))\right)$$

v-dependent framing anomaly.

- This matrix model resembles the one computing (P,Q) torus knot invariants in ordinary Chern-Simons theory for the supergroup $U(N_1|N_2)$



P e Q coprime integers

v=P/Q

Large N-limit (Planar) limit

For $N_1=N_2$ (ABJM) we can investigate the matrix model by reformulating the problem in terms of a one--dimensional ideal (non--interacting) quantum gas of particles with Fermi statistics [Fermi Gas].

- the coupling constant k plays the role of the Planck constant $\hbar$
- the number of colors N corresponds to the number of particles.

Large N is equivalent to the thermodynamic limit of the gas, whereas the $\hbar$ expansion encoding quantum corrections corresponds to an expansion at small k.

Bosonic Latitude:

$$\langle W_B(\nu)\rangle_\nu = \frac{1}{8\pi^2}\left(2\,\pi^2\,\csc\frac{2\pi}{k}\,\left(1+i\tan\frac{\pi\nu}{2}\right)\frac{\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{7}{3k}\right)\right)}{\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}\right.$$

$$\left.+2\,i\,\pi\,\nu\,\frac{\Gamma\left(\frac{\nu-1}{2}\right)\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma(\nu+1)}\csc\frac{2\pi\nu}{k}\,\frac{\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{6\nu+1}{3k}\right)\right)}{\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}\right)$$

Fermionic Latitude:

$$\langle W_F(\nu)\rangle_\nu = -\frac{\nu\,\Gamma\left(-\frac{\nu}{2}\right)\csc\left(\frac{2\pi\nu}{k}\right)\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{6\nu+1}{3k}\right)\right)}{2^{\nu+2}\sqrt{\pi}\,\Gamma\left(\frac{3-\nu}{2}\right)\mathrm{Ai}\left(\left(\frac{2}{\pi^2 k}\right)^{-1/3}\left(N-\frac{k}{24}-\frac{1}{3k}\right)\right)}$$

Fermi gas approach is suitable for studying the latitude expectation value **N→∞** and **k fixed**. [M—theory regime]. We can easily extract the usual strong-coupling (large N) limit.

- The weak coupling expansion of the matrix models matches a genuine perturbative computation performed at framing f=v up to three loops:

$$\langle W_B(\nu)\rangle_f = 1 + \frac{i\pi\nu N_1}{k} + \frac{\pi^2\left[3\left(\nu^2+1\right)N_1N_2 - \left(3\nu^2+1\right)N_1^2+1\right]}{6k^2}$$
$$-\frac{i\pi^3}{6k^3}\left[\nu\left(\nu^2+1\right)N_1^3 - 2N_1^2N_2\nu\left(\nu^2+2\right) + 3\nu N_1N_2^2 - \nu N_1 - N_2\nu\left(2+\nu^2\right)\right] + O\left(k^{-4}\right)$$

- The minimal string surface corresponding to the fermionic ABJM latitude was investigated in [Correa, Aguilera-Damia, Silvia 2014]. It yields the following leading strong coupling behavior: it's OK!

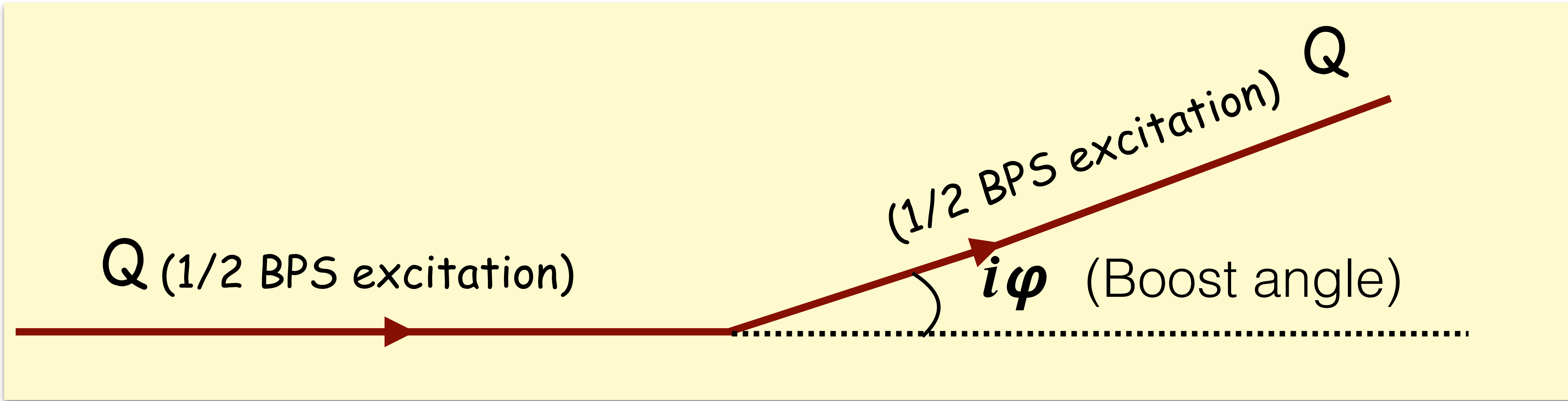
$$\langle W_F(\nu)\rangle \sim e^{\pi\nu\sqrt{2N/k}}$$

The sub-leading behaviour in λ=N/k (one-loop fluctuations for the sigma-model) for small θ have been recently investigated in [Aguilera-Damia, Faraggi, Pando Zayas, Rathee, Silva, arXiv:1805.00859]. There is again perfect agreement with our Matrix Model and its Fermi Gas expansion!

The Bremsstrahlung function B(λ)

[Correa, Henn, Maldacena, Sever, 2012]

We shall illustrate how the latitude WL can be used to compute the Bremsstrahlung function B(λ).

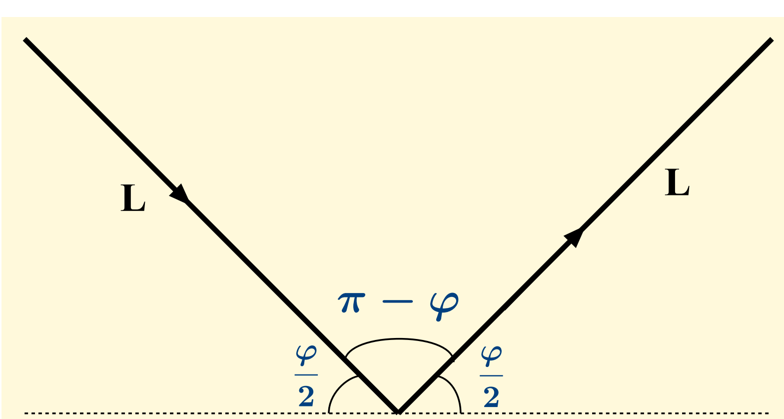


The function B(λ) is related to the amount of power radiated by a slowly moving probe (highly massive) charge.

$$\Delta E = 2\pi B(\lambda) \int dt\ \dot{v}^2$$

[In QED $2\pi B \mapsto \frac{2}{3} e^2$]

This function can be determined by computing a cusped Wilson loop



$$\mathcal{W} \simeq e^{-\log\left(\frac{L}{\epsilon}\right)\Gamma_{\text{cusp}}(\lambda,\varphi)}$$

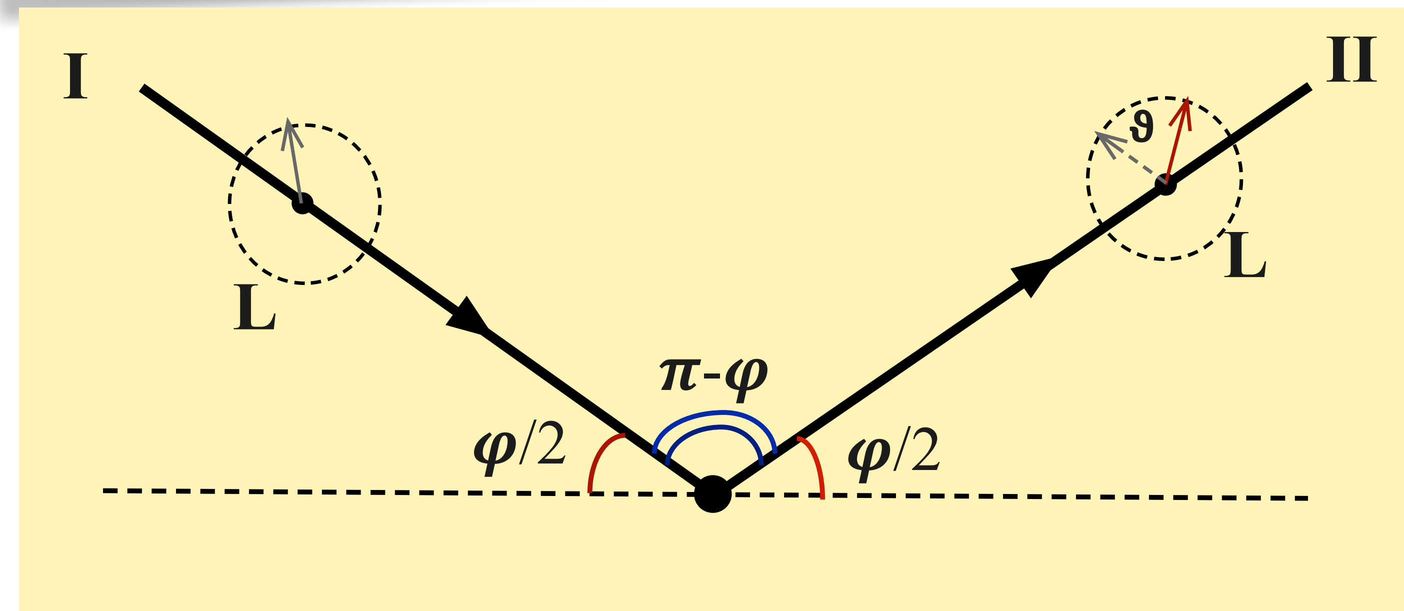
$$\Downarrow$$

$$\Gamma_{\text{cusp}}(\lambda,\varphi) \sim -\varphi^2 B(\lambda)$$

L= IR cut-off

ε= UV cut-off

The Bremsstrahlung function B(λ) in ABJM



In super-conformal field theories it is natural to add an angular cusp ϑ also in the R-symmetry internal space

$$\Gamma_{cusp}(\lambda, \varphi) \longrightarrow \Gamma_{cusp}(\lambda, \vartheta, \varphi)$$

In ABJM (N_1=N_2) we can define

$$W_F^{\angle}(\theta,\varphi) \sim \exp\left(-\log\frac{\Lambda}{\epsilon}\,\Gamma^{1/2}_{\mathrm{cusp}}(\lambda,\theta,\varphi)\right) \longrightarrow \Gamma^{1/2}_{\mathrm{cusp}}(\lambda,\theta,\varphi) \underset{\theta,\varphi\ll 1}{\simeq} B_{1/2}(\lambda)(\theta^2-\varphi^2)$$

BPS point: $\theta^2=\varphi^2$

$$W_B^{\angle}(\theta,\varphi) \sim \exp\left(-\log\frac{\Lambda}{\epsilon}\,\Gamma^{1/6}_{\mathrm{cusp}}(\lambda,\theta,\varphi)\right) \longrightarrow \Gamma^{1/6}_{\mathrm{cusp}}(\lambda,\theta,\varphi) \underset{\theta,\varphi\ll 1}{\simeq} B^{\theta}_{1/6}(\lambda)\theta^2 - B^{\varphi}_{1/6}(\lambda)\varphi^2$$

Can we relate the different Bremsstrahlung functions defined above to quantities exactly computable via localization ?

First attempt: 1/6 BPS circle with winding number m

$$B^{\theta}_{1/6}(\lambda) = \frac{1}{4\pi^2}\,\partial_m \log|\langle W_B^m\rangle|_{m=1}$$

[Lewkowycz, Maldacena, 2014]

It can be argued that the above Bremsstrahlung functions can be written as derivative of the latitude:

$$B_{1/2} = \frac{1}{4\pi^2}\,\partial_\nu \log|\langle W_F(\nu)\rangle|\,\Big|_{\nu=1} \qquad , \qquad B^{\theta}_{1/6} = \frac{1}{4\pi^2}\,\partial_\nu \log|\langle W_B(\nu)\rangle|\,\Big|_{\nu=1}$$

Perturbative tests: The relation for **$B_{1/2}$** was checked up to three loops [M. Bianchi, L. G., A. Mauri, M. Preti, S. Penati, D. Seminara (2017); M. Bianchi, L. G., A. Mauri, S. Penati, D. Seminara (2018)].

The relation for **$B^{\theta}_{1/6}$** was checked up to four loops [M. Bianchi, A. Mauri (2017)]. The perturbative analysis shows that

$$B^{\theta}_{1/6}(\lambda) = \frac{1}{2} B^{\varphi}_{1/6}(\lambda)$$

Strong coupling tests: The relation for **$B_{1/2}$** was checked at leading order [Correa, Aguilera-Damia, Silva (2014)].

Proof of the different relations: Using supersymmetric Ward identities on the two-point correlation functions in 1d defect CFT

- The relation for **$B_{1/2}$**: [L. Bianchi, L.G., M. Preti, D. Seminara (2017)]
- The relation for **$B^{\theta}_{1/6}$**: [Correa, Aguilera-Damia, Silva (2014)].
- The relation between **$B^{\theta}_{1/6}$** and **$B^{\varphi}_{1/6}$** was proven in [L. Bianchi, M. Preti, E. Vescovi (2017)]

Conclusions and Perspectives:

Matrix Model

1. We have constructed a Matrix Model which should determine the expectation value of the latitude WLs in ABJ(M). A solution for this Matrix Model in the limit $N \to \infty$ and **k fixed** has been constructed.
2. We found consistency both at weak and strong coupling: localization?
3. This result provides a new set of interpolating function that allows to test AdS4/CFT3 in different limits.

Bremsstrahlung functions

1. We have provided an exact prescription for computing the Bremsstrahlung functions in terms of latitude WL, which can be calculated at all orders.
2. If the exact B were also computed by a system of TBA equations exploiting integrability, as done in N = 4 SYM [Correa-Maldacena-Sever (2012)], this would be crucial tool for proving the conjectured form for interpolating function $h(\lambda)$ of ABJM [Gromov, Sizov (2014)]
3. Generalizing the relation Bremsstrahlung/latitude WL to the $N_1 \neq N_2$ case, we have an exact prediction for $B_{1/2}$ in the ABJ theory, which calls for some perturbative checks