The ultraviolet behavior of quantum gravity with fakeons

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$$\Gamma^{(\mathrm{ct})}_{\mathrm{EH}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\big[R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + c_3R^{\mu\nu}_{\rho\sigma}R^{\rho\sigma}_{\alpha\beta}R^{\alpha\beta}_{\mu\nu} + \underbrace{\ldots}_{\infty}\big].$$

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Stelle action: renormalizable but not unitary theory

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\big[\gamma R + \alpha R_{\mu\nu}R^{\mu\nu} + \beta R^2 + 2\Lambda_{\mathrm{C}}\big].$$

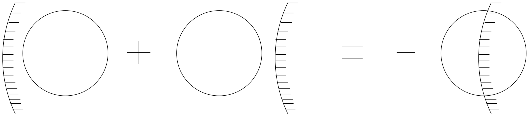
$$\Gamma^{(\mathrm{ct})}_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\big[a_\gamma R + a_\alpha R_{\mu\nu}R^{\mu\nu} + a_\beta R^2 + 2a_{\mathrm{C}}\big].$$

Unitarity

$$S^\dagger S = 1.$$

$$-i(T - T^\dagger) = T^\dagger T, \qquad S = 1 + iT.$$

Cutting equations (Cutkosky, 't Hooft and Veltman)



$$\mathrm{Disc}\mathcal{M} = 2i\mathrm{Im}\mathcal{M} = -\sum_j \mathcal{C}_j,$$

$$\mathcal{C}_j = \text{cut diagrams.}$$

$$T_{fi} = (2\pi)^4 \delta^{(4)}(p_i - p_f)\mathcal{M}(i \to f).$$

Higher-derivative theories cannot be defined in Minkowski spacetime.

U.G. Aglietti and D. Anselmi, *Inconsistency of Minkowski higher-derivative theories*, Eur. Phys. J. C 77 (2017) 84 and arXiv:1612.06510 [hep-th].

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Propagator in De Donder gauge (expansion $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$)

$$\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_0 = \frac{iM^4}{2(p^2+i\epsilon)}\frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{(p^2)^2 + M^4}.$$

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$$\begin{aligned}
&\langle h_{\mu\nu}(p)\, h_{\rho\sigma}(-p)\rangle_1^{\mathrm{nl\,d}} = \\
&\frac{\kappa^2 M^8}{240\pi^2(p^2)^2}\left[(68r+i)(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\nu\rho}\eta_{\mu\sigma}) + (373r-4i)\eta_{\mu\nu}\eta_{\rho\sigma}\right.\\
&-\frac{1}{8p^2}(125ir^2+544r+8i)\left(p_\mu p_\rho\eta_{\nu\sigma}+p_\mu p_\sigma\eta_{\nu\rho}+p_\nu p_\rho\eta_{\mu\sigma}+p_\nu p_\sigma\eta_{\mu\rho}\right)\\
&+\frac{1}{4p^2}(255ir^2-1522r+36i)\left(p_\mu p_\nu\eta_{\rho\sigma}+p_\rho p_\sigma\eta_{\mu\nu}\right)\\
&\left.-\frac{1}{2(p^2)^2}(185r^3+75ir^2-1048r+24i)p_\mu p_\nu p_\rho p_\sigma\right]\ln\left(\frac{\Lambda_{UV}^2}{M^2}\right), \quad r \equiv p^2/M^2.
\end{aligned}$$

In general, higher-derivative theories must be defined from Euclidean space.

A different formulation has been proposed by Lee and Wick.

T.D. Lee and G.C. Wick, *Negative metric and the unitarity of the S-matrix*, Nucl. Phys. B 9 (1969) 209.

T.D. Lee and G.C. Wick, *Finite theory of quantum electrodynamics*, Phys. Rev. D 2 (1970) 1033.

R.E. Cutkosky, P.V Landshoff, D.I. Olive, J.C. Polkinghorne, *A non-analytic S matrix*, Nucl.Phys. B12 (1969) 281-300.

B. Grinstein, D. O'Connell and M.B. Wise, *Causality as an emergent macroscopic phenomenon: The Lee-Wick $O(N)$ model*, Phys. Rev. D 79 (2009) 105019 and arXiv:0805.2156 [hep-th].

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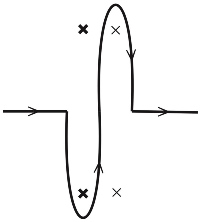
We reformulate the models as
nonanalytically Wick rotated Euclidean theories.
The new formulation solves the previous problems and makes the models both renormalizable and unitary.

D. Anselmi and M. Piva, *A new formulation of Lee-Wick quantum field theory*, JHEP 1706 (2017) 066 and arXiv:1703.04584 [hep-th].

D. Anselmi and M. Piva, *Perturbative unitarity of Lee-Wick quantum field theory*, Phys. Rev. D 96 (2017) 045009 and arXiv:1703.05563 [hep-th].

Average continuation

Pinching



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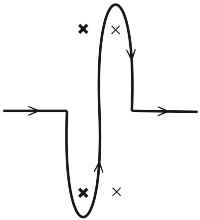
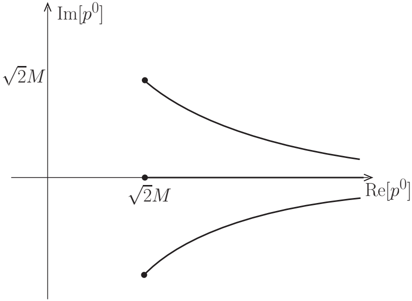


Branch cuts



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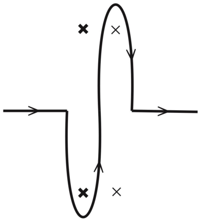

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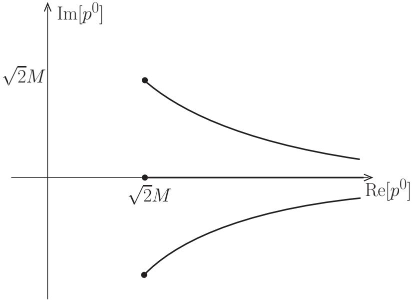

$$\mathcal{J}_{\mathrm{AV}}(p) = \frac{1}{2}\left[\mathcal{J}_{+}(p) + \mathcal{J}_{-}(p)\right].$$

D. Anselmi and M. Piva, *A new formulation of Lee-Wick quantum field theory*, JHEP 1706 (2017) 066, and arXiv:1703.04584 [hep-th].

D. Anselmi, *Fakeons and Lee-Wick models*, JHEP 02 (2018) 141, and arXiv:1801.00915 [hep-th].

Superrenormalizable quantum gravity

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QG}} \quad = \quad & -\frac{\sqrt{-g}}{2\kappa^2}\left[2\Lambda_C M^2 + \zeta R - \frac{\gamma}{M^2}R_{\mu\nu}R^{\mu\nu} + \frac{1}{2M^2}(\gamma-\eta)R^2\right. \\
& -\frac{1}{M^4}(D_\rho R_{\mu\nu})(D^\rho R^{\mu\nu}) + \frac{1}{2M^4}(1-\xi)(D_\rho R)(D^\rho R) \\
& +\frac{1}{M^4}\left(\alpha_1 R_{\mu\nu}R^{\mu\rho}R_\rho^\nu + \alpha_2 R R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^3 + \alpha_4 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right. \\
& \left.\left. + \alpha_5 R_{\mu\nu\rho\sigma}R^{\mu\rho}R^{\nu\sigma} + \alpha_6 R_{\mu\nu\rho\sigma}R^{\rho\sigma\alpha\beta}R_{\alpha\beta}^{\mu\nu}\right)\right].
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& - \frac{1}{M^4}(D_\rho R_{\mu\nu})(D^\rho R^{\mu\nu}) + \frac{1}{2M^4}(1-\xi)(D_\rho R)(D^\rho R) \\
& + \frac{1}{M^4}\Big(\alpha_1 R_{\mu\nu}R^{\mu\rho}R^\nu_\rho + \alpha_2 R R_{\mu\nu}R^{\mu\nu} + \alpha_3 R^3 + \alpha_4 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\
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Expansion $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$, $\qquad \eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

Propagator in De Donder gauge

$$
\langle h_{\mu\nu}(p)h_{\rho\sigma}(-p)\rangle^{\mathrm{free}}_{\eta=\xi=0} = \frac{iM^4}{2}\frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma}}{P(1,\gamma,\zeta,2\Lambda_{\mathrm{C}})},
$$

$$
P(a,b,c,d) = a(p^2)^3 + bM^2(p^2)^2 + cM^4p^2 + dM^6.
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\end{aligned}
$$

Counterterms (up to three loops)

$$
\mathcal{L}_{\mathrm{count}} = \frac{\sqrt{-g}}{(4\pi)^2\varepsilon}\left[2a_C M^4 + a_\zeta M^2 R - a_\gamma R_{\mu\nu}R^{\mu\nu} + \frac{1}{2}(a_\gamma - a_\eta)R^2\right].
$$

Unitarity cannot be proved at non vanishing Λ_{C}. Can be set to zero by solving a chain of RG conditions.

D. Anselmi, *On the quantum field theory of gravitational interactions*, JHEP 1706 (2017) 086 and arXiv:1704.07728 [hep-th].

The problem of uniqueness

$\mathcal{L}_{\mathrm{QG}}$ is the simplest representative of a general class

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$$\mathcal{L}'_{\mathrm{QG}} = -\frac{\sqrt{-g}}{2\kappa^2}\Big[2\Lambda_C M^2 + \zeta R - \frac{1}{M^2} R_{\mu\nu} P_n(\Box_c/M^2) R^{\mu\nu} + \frac{1}{2M^2} R Q_n(\Box_c/M^2) R + \mathcal{V}(R, M, \alpha_i)\Big].$$

P_n and Q_n polynomials of degree n, $\Box_c \equiv \nabla_\mu \nabla^\mu$.

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All these theories are superrenormalizable and of the Lee-Wick type (for suitable choices of the polynomials).

$$\begin{aligned} n &= 1 && \text{counterterms up to three loops.} \\ n &= 2 && \text{counterterms up to two loops.} \\ n &\geq 3 && \text{counterterms up to one loop.} \end{aligned}$$

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Problem of uniqness
Superrenormalizable models of quantum gravity are infinitely many.

New quantization prescription and fakeons

D. Anselmi, *On the quantum field theory of gravitational interactions*,
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$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{\lambda}{4!}\varphi^4.$$

Modified Euclidean propagator

$$\frac{p_E^2}{(p_E^2)^2 + \mathcal{E}^4}, \quad \mathcal{E} = \text{ ficticious LW scale.}$$

After the Wick rotation

$$\lim_{\mathcal{E}\to 0} \frac{p^2}{[(p^2)^2 + \mathcal{E}^4]_{\mathrm{AV}}}.$$

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One-loop bubble diagram (after renormalizing the UV divergence).

Feynman prescription

$$-\frac{i}{2(4\pi)^2} \ln \frac{-p^2 - i\epsilon}{\mu^2}.$$

New presciption

$$-\frac{i}{4(4\pi)^2} \ln \frac{(p^2)^2}{\mu^4}.$$

We can turn ghosts into “fake degrees of freedom”.

Quantum gravity with fakeons

$$S_{\mathrm{HD}} = -\frac{1}{2\kappa^2}\int\sqrt{-g}\left[2\Lambda_C + \zeta R + \alpha\left(R_{\mu\nu}R^{\mu\nu} - \frac{1}{3}R^2\right) - \frac{\xi}{6}R^2\right].$$

The action coincides with Stelle theory but we quantize it in a different way.

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With the new prescription it turns into

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Absorptive part of graviton self energy at $\Lambda_C = 0$

D. Anselmi and M. Piva, *The ultraviolet behavior of quantum gravity*, JHEP 05 (2018) 027, arXiv:1803.07777 [hep-th].

$$\mathcal{M}_{\text{abs}} = \text{Im}\mathcal{M} \qquad \mathcal{D}_{\text{abs}} = -\text{Re}\mathcal{D}, \qquad \mathcal{M} = \text{amplitude}, \quad \mathcal{D} = \text{diagram}.$$

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Possible cases in the diagram

i) **Fake-Fake** below the pinching threshold: $i \times$ real integral $\longrightarrow$ purely imaginary;

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The contributions of type iv) can be evaluated by using

$$\langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_0 = \langle h_{\mu\nu}(p) h_{\rho\sigma}(-p)\rangle_{0\text{grav}} \,.$$

We expand S_{HD} around the Hilbert term in powers of α and ξ.

$$\frac{1}{p^2(\zeta - up^2)} = \frac{1}{\zeta p^2} + \frac{u}{\zeta^2} + \frac{u^2 p^2}{\zeta^3} + \ldots, \qquad u = \alpha, \zeta.$$

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In the case of pure gravity the absorptive part can be written as

$$\Gamma_{\text{abs}} = -\int \frac{\delta S_{\text{HD}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu}, \quad \Delta g_{\mu\nu} \text{ piecewise local function of } h_{\mu\nu}.$$

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Adding (massless) matter fields of all types the absorptive part is

$$\begin{aligned} \Gamma_{\text{abs}} \quad &= \quad \frac{i\mu^{-\varepsilon}}{16\pi} \int \sqrt{-g} \left[c \left(R_{\mu\nu}\theta(-\Box_c)R^{\mu\nu} - \frac{1}{3}R\theta(-\Box_c)R \right) + \frac{N_s\eta^2}{36} R\theta(-\Box_c)R \right] \\ &\quad - \int \frac{\delta S_{\text{HD}}}{\delta g_{\mu\nu}} \Delta g_{\mu\nu}, \end{aligned}$$

$$c = \frac{1}{120}(N_s + 6N_f + 12N_v), \quad \eta \text{ nonminimal coupling for scalar fields.}$$

N_s = number of scalars;

N_v = number of vectors;

N_f = number of Dirac fermions plus 1/2 the number of Weyl fermions.

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- It is possible to compute physical quantities in order to discriminate our model from others.

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