Hidden Symmetries of Supersymmetric Free Differential Algebras

Based on JHEP **1608** (2016) 095 and Phys. Lett. B **772** (2017) 578 in collaboration with L. Andrianopoli and R. D'Auria

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An old story...

An "almost" centrally extended superalgebra in $D = 11$ can be written:

$$\{Q, Q\} = -\mathrm{i}C\Gamma^a P_a - \frac{1}{2}C\Gamma_{ab} Z^{ab} - \frac{\mathrm{i}}{5!}C\Gamma_{a_1\dots a_5} Z^{a_1\dots a_5}$$

Relevant in the presence of $M2$ or $M5$ sources (M-algebra)

- First found by D'Auria-Fré in 1981 $\rightarrow$ It includes a nilpotent fermionic charge Q' ($\{Q', Q'\} = 0$):

$$[P_a, Q] \propto \Gamma_a Q', \quad [Z^{ab}, Q] \propto \Gamma^{ab} Q', \quad [Z^{a_1\dots a_5}, Q] \propto \Gamma^{a_1\dots a_5} Q'$$

$\{P_a, J_{ab}, Q, Z^{ab}, Z^{a_1\dots a_5}, Q'\}$ generate a hidden superalgebra (DF-algebra)
underlying the Free Differential Algebra of $D = 11$ supergravity

SOME OPEN QUESTIONS

- ☞ Which is the physical meaning of Q'? Is it really needed?
- ☞ Is the DF-algebra the fully extended superalgebra underlying $D = 11$ supergravity?

[L. Andrianopoli, R. D'Auria, LR, JHEP **1608** (2016) 095]

- ☞ Would a non-abelian charge deformation of the DF-algebra be possible?
  Which is the relation between the DF-algebra and $\mathfrak{osp}(1|32)$?

[L. Andrianopoli, R. D'Auria, LR, Phys. Lett. B **772** (2017) 578]

LIE SUPERALGEBRAS AND MAURER-CARTAN EQUATIONS

Lie superalgebra	Dual formulation	Maurer-Cartan equations
$[T_A, T_B\} = C_{AB}{}^C T_C$	$\longrightarrow$ $\sigma^A(T_B) = \delta^A_B$	$R^A \equiv d\sigma^A + \frac{1}{2} C_{BC}{}^A \sigma^B \wedge \sigma^C = 0$
T_A: Generators in the adjoint representation of the Lie group	σ^A: Differential 1-forms	$d^2 = 0 \leftrightarrow$ Jacobi identities

- R^A: Supercurvatures (super field-strengths), building blocks of supergravity in the geometric framework
- The Maurer-Cartan equations $R^A = 0$ define the vacuum of a supergravity theory
- Geometric formulation in superspace, spanned by the supervielbein $\{V^a, \Psi\}$

EXAMPLE: $N=1$, $D=4$ PURE SUPERGRAVITY IN SUPERSPACE

$$\sigma^A = \{V^a, \psi, \omega^{ab}\} \quad (a, b, \ldots = 0, 1, 2, 3)$$

- $\{V^a, \psi\}$: supervielbein
- ω^{ab}: Lorentz spin connection

Supercurvatures (super field-strengths):

$$R^{ab} \equiv d\omega^{ab} + \omega^{ac} \wedge \omega_c{}^b$$

$$T^a \equiv \left(dV^a + \omega^a_{\ b} \wedge V^b\right) - \frac{\mathrm{i}}{2}\bar{\psi} \wedge \gamma^a \psi = DV^a - \frac{\mathrm{i}}{2}\bar{\psi} \wedge \gamma^a \psi = 0$$

$$\rho \equiv \left(d\psi + \frac{1}{4}\omega^{ab} \wedge \gamma_{ab}\psi\right) = D\psi$$

They appear in the Einstein-Hilbert + Rarita-Schwinger Lagrangian 4-form in superspace:

$$\mathcal{L} = R^{ab} \wedge V^c \wedge V^c \epsilon_{abcd} + 4\bar{\psi} \wedge \gamma_5 \gamma_a \rho \wedge V^a = d^4x \left(\sqrt{g}R + \bar{\psi}_\mu \gamma_\nu D_\rho \psi_\sigma \epsilon^{\mu\nu\rho\sigma}\right)$$

The Maurer-Cartan equations $R^{ab} = 0$, $\rho = 0$ define the vacuum of the supergravity theory

Free Differential Algebras (FDAs) and Lie algebras cohomology

- *p*-index antisymmetric tensors naturally appear in supergravity theories in $D \geq 4$
- FDAs extend the Maurer-Cartan equations by incorporating *p*-form gauge potentials

[Sullivan (1977), D'Auria-Fré (1981)]

Free Differential Algebras (FDAs) and Lie algebras cohomology

- p-index antisymmetric tensors naturally appear in supergravity theories in $D \geq 4$
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[Sullivan (1977), D'Auria-Fré (1981)]

Steps for constructing a FDA:

1. Given set of Maurer-Cartan 1-forms $\{\sigma^A\}$, we can build up n-form cochains $\Omega^{(n)}$:

$$\Omega^{(n)} = \Omega_{A_1 \dots A_n} \sigma^{A_1} \wedge \cdots \wedge \sigma^{A_n}$$

2. If $\exists p$: $d\Omega^{(p+1)} = 0$ (cocycle), we can introduce $A^{(p)}$ (p-form gauge potential):

$$F^{(p+1)} \equiv dA^{(p)} + \Omega^{(p+1)} = 0$$

3. Consider $\left(\{\sigma^A\}, A^{(p)}\right)$ as a basis of Maurer-Cartan forms and look for new cocycles, iteratively $\Rightarrow$ FDA

$D = 11$ SUPERGRAVITY AND ITS FDA

Field content: $(g_{\mu\nu}, A_{\mu\nu\rho}, \Psi_{\mu\alpha})$ $\quad (\mu, \nu, \rho, \ldots = 0, 1, \ldots, 10,\ \alpha = 1, \ldots, 32)$

- Action built in 1978 [Cremmer-Julia-Scherk]
- Geometrically reformulated by D'Auria-Fré in terms of a supersymmetric FDA on superspace, with $A^{(3)} = A_{\mu\nu\rho} dx^\mu \wedge dx^\nu \wedge dx^\rho$ $\quad (\mu, \nu, \rho, \ldots = 0, 1, \ldots, 10,\ a = 0, 1, \ldots, 10)$

[R. D'Auria and P. Fré, Nucl. Phys. B **201** (1982)]

FDA of $D = 11$ supergravity introduced and investigated

$$\begin{cases} R^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega_c{}^b = 0 \\ T^a \equiv DV^a - \frac{i}{2} \overline{\Psi} \wedge \Gamma^a \Psi = 0 \\ \rho \equiv D\Psi = 0 \\ F^{(4)} \equiv dA^{(3)} - \frac{1}{2} \overline{\Psi} \wedge \Gamma_{ab} \Psi \wedge V^a \wedge V^b = 0 \end{cases}$$

(r.h.s. "vacuum") $\quad d^2 = 0 \quad \Leftrightarrow \quad 3\Psi$ Fierz identities $\quad (\Gamma_{ab}\Psi \wedge \bar{\Psi} \wedge \Gamma^a \Psi = 0)$

$D = 11$ SUPERSYMMETRIC FDA

- The d^2-closure of the supersymmetric FDA allows to include in the FDA

$$F^{(7)} \equiv dB^{(6)} - 15A^{(3)} \wedge dA^{(3)} - \frac{\mathrm{i}}{2}\overline{\Psi} \wedge \Gamma_{a_1 \dots a_5}\Psi \wedge V^{a_1} \wedge \dots \wedge V^{a_5} = 0$$

$$\left(\text{due to the Fierz identity } \Gamma_{[a_1 a_2}\Psi \wedge \bar{\Psi} \wedge \Gamma_{a_3 a_4]}\Psi + \frac{1}{3}\Gamma_{a_1 \dots a_5}\Psi \wedge \bar{\Psi} \wedge \Gamma^{a_5}\Psi = 0 \right)$$

- $F^{(7)}$ Hodge-dual to $F^{(4)}$ on spacetime

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- $F^{(7)}$ Hodge-dual to $F^{(4)}$ on spacetime
- The FDA is defined in terms of the p-form fields $(V^a, \Psi, A^{(3)}, B^{(6)})$
- It is invariant under p-form gauge transformations:

$$\begin{cases} \delta A^{(3)} = d\Lambda^{(2)} \\ \delta B^{(6)} = d\Lambda^{(5)} + 15\Lambda^{(2)} \wedge dA^{(3)} \end{cases}$$

with p-form gauge parameters $\Lambda^{(2)}$ and $\Lambda^{(5)}$

D'Auria-Fré investigation: *Can the supersymmetric FDA be traded for an ordinary Lie superalgebra?*

D'Auria-Fré recipe:

1. Associate $A^{(3)} \to B_{ab} = B_{[ab]}$ and $B^{(6)} \to B_{a_1 \dots a_5} = B_{[a_1 \dots a_5]}$ (B_{ab} and $B_{a_1 \dots a_5}$ are 1-forms)
2. Take as basis of Maurer-Cartan 1-forms $\sigma^A \equiv \{V^a, \Psi, \omega^{ab}, B_{ab}, B_{a_1 \dots a_5}, \dots\}$ with extra Maurer-Cartan equations:
$$\begin{cases} DB_{ab} = \frac{1}{2}\bar{\Psi} \wedge \Gamma^{ab}\Psi \\ DB_{a_1 \dots a_5} = \frac{\mathrm{i}}{2}\bar{\Psi} \wedge \Gamma^{a_1 \dots a_5}\Psi \end{cases}$$
3. Assume $A^{(3)} = A^{(3)}(\sigma)$ with all possible combinations:
$$A^{(3)}(\sigma) = T_0 B_{ab} \wedge V^a \wedge V^b + T_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} + T_2 B_{b_1 a_1 \dots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2 a_1 \dots a_4} +$$
$$+ T_3 \epsilon_{a_1 \dots a_5 b_1 \dots b_5 m} B^{a_1 \dots a_5} \wedge B^{b_1 \dots b_5} \wedge V^m + T_4 \epsilon_{m_1 \dots m_6 n_1 \dots n_5} B^{m_1 m_2 m_3 p_1 p_2} \wedge B^{m_4 m_5 m_6}{}_{p_1 p_2} \wedge B^{n_1 \dots n_5}$$
4. Require $dA^{(3)}(\sigma) = \frac{1}{2}\overline{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b$ (vacuum FDA on ordinary superspace)

D'AURIA-FRÉ INVESTIGATION: A SPINOR η MUST BE ADDED

Expressing the FDA with $A^{(3)} = A^{(3)}(\sigma)$ determines it $\rightarrow$ This requires to include a spinor 1-form η

$$D\eta = \mathrm{i}E_1\Gamma_a\Psi \wedge V^a + E_2\Gamma^{ab}\Psi \wedge B_{ab} + \mathrm{i}E_3\Gamma^{a_1\ldots a_5}\Psi \wedge B_{a_1\ldots a_5}$$

$$\begin{aligned}A^{(3)}(\sigma) = {} & T_0 B_{ab} \wedge V^a \wedge V^b + T_1 B_{ab} \wedge B^b{}_c \wedge B^{ca} + T_2 B_{b_1 a_1\ldots a_4} \wedge B^{b_1}{}_{b_2} \wedge B^{b_2 a_1\ldots a_4} + \\ & + T_3 \epsilon_{a_1\ldots a_5 b_1\ldots b_5 m} B^{a_1\ldots a_5} \wedge B^{b_1\ldots b_5} \wedge V^m + T_4 \epsilon_{m_1\ldots m_6 n_1\ldots n_5} B^{m_1 m_2 m_3 p_1 p_2} \wedge B^{m_4 m_5 m_6}{}_{p_1 p_2} \wedge B^{n_1\ldots n_5} + \\ & + \mathrm{i}S_1\bar{\Psi} \wedge \Gamma_a\eta \wedge V^a + S_2\bar{\Psi} \wedge \Gamma^{ab}\eta \wedge B_{ab} + \mathrm{i}S_3\bar{\Psi} \wedge \Gamma^{a_1\ldots a_5}\eta \wedge B_{a_1\ldots a_5}\end{aligned}$$

E_i, T_j, S_k fixed in terms of 1 free parameter [Bandos, de Azcarraga, Izquierdo, Picon, Varela (2004)]

- Basis of Maurer-Cartan 1-forms: $\sigma^A \equiv \{V^a, \Psi, \omega^{ab}, B_{ab}, B_{a_1\ldots a_5}, \eta\}$
- Dual set of generators: $T_A \equiv \{P_a, Q, J_{ab}, Z^{ab}, Z^{a_1\cdots a_5}, Q'\}$

The hidden superalgebra underlying $D = 11$ supergravity (DF-algebra) includes (besides super-Poincaré):

$$\{Q, Q\} = -\mathrm{i}C\Gamma^a P_a - \frac{1}{2}C\Gamma_{ab}Z^{ab} - \frac{\mathrm{i}}{5!}C\Gamma_{a_1\ldots a_5}Z^{a_1\cdots a_5}$$

$$\{Q', Q'\} = 0$$

$$[P_a, Q] \propto \Gamma_a Q', \quad [Z^{ab}, Q] \propto \Gamma^{ab}Q', \quad [Z^{a_1\cdots a_5}, Q] \propto \Gamma^{a_1\cdots a_5}Q'$$

- Z^{ab} and $Z^{a_1\cdots a_5}$ → Understood as M-brane charges, sources of $A^{(3)}$ and $B^{(6)}$

☞ What about Q'?

NEW RESULTS: ROLE OF η

DF-algebra = "Spinorial central extension" of the M-algebra with $Q' \leftrightarrow \eta$

[L. Andrianopoli, R. D'Auria, LR, JHEP **1608** (2016) 095]

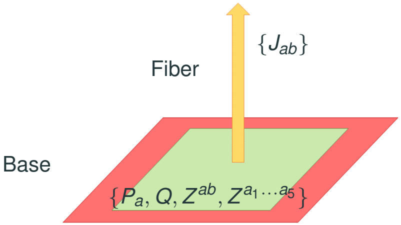
Role of the spinor 1-form η:

It allows to realize the M-algebra as a (hidden) symmetry of $D = 11$ supergravity...

- η behaves as a cohomological "ghost" for the *p*-form gauge invariance of the supersymmetric FDA(σ)
- It allows a fiber bundle structure $\mathcal{G} \rightarrow$ (superspace) on the group-manifold $\mathcal{G}$ generated by $\mathbb{G} =$ M-algebra, and intertwines between base and fiber
- *p*-form gauge transformations realized as diffeomorphisms in the fiber direction of the group-manifold $\mathcal{G}$

Visualizing the role of Q' (dual to η)

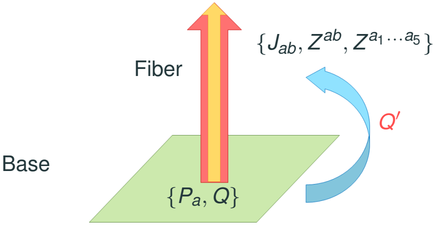
Without $Q' \Rightarrow$ Enlarged superspace



$\uparrow$

M-algebra

With $Q' \Rightarrow$ Projection on the fiber



$\uparrow$

DF-algebra

Fiber bundle structure

The hidden Lie superalgebra $\mathbb{G}$ generates a group-manifold $\mathcal{G}$ with a principal fiber bundle structure $\mathcal{G} \to K$, if $\eta \neq 0$:

- Base space K is superspace: $\{V^a, \Psi\} \in \mathbb{K}$

$$dA^{(3)}(\sigma) = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b \in \mathbb{K} \times \cdots \times \mathbb{K}$$

- Fiber generated by $\mathbb{H} = H_0 + \mathcal{H}$:

$$\{\omega^{ab}\} \in H_0, \qquad \{B_{ab}, B_{a_1\ldots a_5}\} \in \mathcal{H}$$

- η behaves like a cohomological "ghost" field: It allows to realize in a dynamical way the gauge invariance of $A^{(3)}$, guaranteeing that only the physical degrees of freedom appear in the FDA $\to$ FDA on ordinary superspace reproduced

GAUGE INVARIANCE AND THE ROLE OF η

- For $A^{(3)}(\sigma)$, the p-form gauge transformations of the FDA are realized through gauge transformations in $\mathcal{H}$:

$$\begin{cases} \delta B_{ab} = D\Lambda_{ab} \\ \delta B_{a_1\ldots a_5} = D\Lambda_{a_1\ldots a_5} \end{cases} \Rightarrow \begin{cases} \delta A^{(3)} = d\Lambda^{(2)} \\ \delta B^{(6)} = d\Lambda^{(5)} + 15\Lambda^{(2)} \wedge dA^{(3)} \end{cases}$$

- The gauge invariance of the FDA(σ) requires:

$$\delta_{\text{gauge}}\eta = -E_2\Lambda_{ab}\Gamma^{ab}\Psi - \mathrm{i}E_3\Lambda_{a_1\ldots a_5}\Gamma^{a_1\ldots a_5}\Psi$$

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- Consider the tangent vector in $\mathcal{H} \in \mathbb{G}$: $\quad \vec{z} \equiv \Lambda_{ab}Z^{ab} + \Lambda_{a_1\dots a_5}Z^{a_1\dots a_5}$
- We found: $\quad \bar{\Lambda}^{(2)} = \Lambda^{(2)}(\Lambda_{ab}, \Lambda_{a_1\dots a_5}; \sigma) = \imath_{\vec{z}}(A^{(3)}(\sigma))$, so that (since $\imath_{\vec{z}} dA^{(3)} = 0$)

$$\delta_{\bar{\Lambda}} A^{(3)}(\sigma) = d\bar{\Lambda}^{(2)} = \ell_{\vec{z}} A^{(3)}(\sigma)$$

where $\ell_{\vec{z}} = d\imath_{\vec{z}} + \imath_{\vec{z}} d$ is the Lie derivative in direction $\vec{z}$

$\Rightarrow$ For $\eta \neq 0$, $\delta A^{(3)}$ is realized as a diffeomorphism in the fiber direction of the group-manifold $\mathcal{G}$

[Assuming $\bar{\Lambda}^{(5)} = \imath_{\vec{z}}(B^{(6)}(\sigma))$, then $\delta_{\bar{\Lambda}} B^{(6)} = \ell_{\vec{z}} B^{(6)}(\sigma)$, whatever $B^{(6)}(\sigma)$ would be]

Summary and possible applications of the formalism

☞ Which is the physical meaning of Q'? Is it really needed?

- ✓ η (dual to Q') guarantees dynamically that the unphysical degrees of freedom of $A^{(3)}(\sigma)$ transform into each other and do not contribute to the FDA(σ)
- ✓ This framework allows to realize the M-algebra as a symmetry of (the vacuum of) $D = 11$ supergravity, in enlarged superspace $\{V^a, \Psi, B_{ab}, B_{a_1 \ldots a_5}\}$, including η
- ✓ In the enlarged superspace: $\delta_{\text{gauge}} A^{(3)}(\sigma) = \ell_{\mathcal{H}} A^{(3)}(\sigma)$
- Out of the vacuum? (The same is expected to work)
- The structure above relies on supersymmetry (3Ψ Fierz identities) $\rightarrow$ Extends to lower dimensions
  It could be a useful tool in generalized geometry (EFT): Dynamical way to implement the section constraints?

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☞ Is the DF-algebra the fully extended superalgebra underlying $D = 11$ supergravity?

✓ Some hint from minimal $D = 7$ supergravity: The full on-shell hidden symmetry involves two nilpotent fermionic charges $\leftrightarrow$ Two mutually dual p-forms

- Mutually dual p-forms associated to different spinors also in $D = 11$? $\rightarrow$ New extra 1-forms required for $B^{(6)}(\sigma)$?
  Interesting to calculate $B^{(6)}(\sigma)$

What about switching on non-abelian charges?

☞ Would a non-abelian charge deformation of the DF-algebra be possible?
Which is the relation between the DF-algebra and $\mathfrak{osp}(1|32)$?

How to switch on non-abelian charges, to come to more realistic cases?

- As in Exceptional Field Theory: By Scherk-Schwarz dimensional reduction to lower D
- And directly in $D = 11$? Massive theory problematic, but let's have a look closer anyhow:

The DF-algebra and $A^{(3)}(\sigma)$ depend on a free parameter

$\rightarrow$ What does it parametrize? Any relation with $\mathfrak{osp}(1|32)$?

ALGEBRAIC STRUCTURES IN $D = 11$ WITH 32 SUPERCHARGES

- M-algebra: $\{\omega^{ab}, V^a, B^{ab}, B^{a_1\cdots a_5}, \Psi_\alpha\}$, with $a = 0, 1, \ldots, 10$, $\alpha = 1, \ldots, 32$
- DF-algebra: $\{\omega^{ab}, V^a, B^{ab}, B^{a_1\cdots a_5}, \Psi_\alpha, \eta_\alpha\}$
- Simple superalgebra $\mathfrak{osp}(1|32)$: $\{\omega^{ab}, V^a, B^{a_1\cdots a_5}, \Psi_\alpha\}$, involving a scale parameter e
- "Torsion deformation" of $\mathfrak{osp}(1|32)$ constructed in [L. Castellani, P. Fré, F. Giani, K. Pilch and P. van Nieuwenhuizen (1983)]:

$$\omega^{ab} \to \omega^{ab} - eB^{ab}, \quad R^{ab} \to R^{ab} - eDB^{ab} + e^2 B^{ac} \wedge B_c{}^b$$

  - The authors tried to associate it with a gauge deformation of the DF-algebra, including a spinor 1-form η^e_α
    $\to$ Solution found for $\eta^e_\alpha = \frac{1}{e}\Psi$ (no free parameter)
  - In the $e \to 0$ limit it **does not** reproduce DF-algebra: $\eta^e_\alpha \neq \eta_\alpha$ for any value of the free parameter in the DF-algebra

Our contribution: Relation between the DF-algebra and $\mathfrak{osp}(1|32)$

[L. Andrianopoli, R. D'Auria, LR, Phys. Lett. B **772** (2017) 578]

- We found that $A^{(3)}(\sigma)$ and η admit the general decomposition

$$A^{(3)}(\sigma) = A^{(3)}_{(0)} + \alpha A^{(3)}_{(e)}; \qquad \eta = \eta_{(0)} + \alpha \eta_{(e)}$$

  - $A^{(3)}_{(0)}(V^a, \Psi, B^{ab}, \eta_{(0)})$ does not depend on $B^{a_1 \cdots a_5}$ (it explicitly breaks $\mathfrak{osp}(1|32)$)
  - $A^{(3)}_{(e)}(V^a, \Psi, B^{ab}, B^{a_1 \cdots a_5}, \eta_{(e)})$ is covariant under (the "torsion deformation" of) $\mathfrak{osp}(1|32)$

- In the vacuum FDA, we have

$$dA^{(3)}_{(0)} = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b$$

$$dA^{(3)}_{(e)} = 0$$

Only $dA^{(3)}_{(0)}$ is responsible for the 4-form cohomology of the supersymmetric FDA

The free parameter α parametrizes the cohomologically trivial deformations $dA^{(3)}_{(e)}$

Open directions left to a future investigation

- What happens out of the vacuum FDA? One would consider:

$$dA^{(3)} - \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b = F^{(4)} = F^{(4)}_{(0)} + \alpha F^{(4)}_{(e)}$$

$$dA^{(3)}_{(0)} = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b + F^{(4)}_{(0)}$$
$$dA^{(3)}_{(e)} = F^{(4)}_{(e)}$$

- $F^{(4)}$ appears in the topological term of $\mathcal{L}_{D=11}$: $\quad A^{(3)} \wedge F^{(4)} \wedge F^{(4)}$

  In [Hassaïne-Troncoso-Zanelli (2004)] a $D = 11$, M-algebra invariant super Chern-Simons action was considered and shown to depend on one free parameter. $\rightarrow$ Relations with the picture presented here?

Thank you!

Centrally extended superalgebra

- Super-Poincaré algebra $\mathbb{G}$ (in $D = 4$, $\mu, \nu, \ldots = 0, 1, 2, 3$, $\alpha = 1, \ldots, 4$): $\mathbb{G} = \{P_\mu, M_{\mu\nu}, T_M, Q_{\alpha A}\}$
  - $\{P_\mu, M_{\mu\nu}\}$: Poincaré
  - $\{T_M\}$: Gauge algebra generators
  - $\{Q_{\alpha A}\}$: Spinor generators ($A = 1, \ldots, N$: R-symmetry)

$$[P_\mu, Q_{\alpha A}] = (\gamma_\mu)_\alpha^{\ \beta} Q_{\beta A}, \quad [T_M, Q_{\alpha A}] = (C_M)_A^{\ B} Q_{\alpha B}$$

- Central extension:

$$\{Q_{\alpha A}, Q_{B\beta}\} = \gamma^\mu_{\alpha\beta} P_\mu \delta_{AB} + C_{\alpha\beta} Z_{[AB]}(T_M)$$

$$[Z_{AB}, \mathbb{G}] = 0$$

  - Z_{AB}: central charges [Haag-Lopusanski-Sohnius], associated to topological charges [Witten-Olive]

Centrally extended superalgebra

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$$\{Q_{\alpha A}, Q_{B\beta}\} = \gamma^\mu_{\alpha\beta} P_\mu \delta_{AB} + C_{\alpha\beta} Z_{[AB]}(T_M)$$

$$[Z_{AB}, \mathbb{G}] = 0$$

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“Almost” centrally extended superalgebra in $D = 11$:

$$\{Q, Q\} = -\mathrm{i} C\Gamma^a P_a - \frac{1}{2} C\Gamma_{ab} Z^{ab} - \frac{\mathrm{i}}{5!} C\Gamma_{a_1 \ldots a_5} Z^{a_1 \ldots a_5}$$

- Generalizing to $D = 11$ [Haag-Lopusansky-Sohnius, Witten-Olive]; analogous structure in lower D: “p-brane democracy” [Townsend]

SINGULAR LIMIT $\eta \to 0$

A singular limit $\eta \to 0$ exists, where a trivialization $A^{(3)}_{\text{lim}}(\sigma)$ can still be defined, with the same $\mathcal{G}$ but underlying a different FDA, with no gauge freedom:

$$dA^{(3)}_{\text{lim}}(\sigma) \in \mathbb{G} \times \cdots \times \mathbb{G}$$

(FDA lives in enlarged superspace)

Out of the vacuum FDA

$$dA^{(3)} - \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b = F^{(4)} = F^{(4)}_{(0)} + \alpha F^{(4)}_{(e)}$$

$$dA^{(3)}_{(0)} = \frac{1}{2}\bar{\Psi} \wedge \Gamma_{ab}\Psi \wedge V^a \wedge V^b + F^{(4)}_{(0)}$$

$$dA^{(3)}_{(e)} = F^{(4)}_{(e)}$$

$$q = \int_{\mathcal{M}_4} dA^{(3)} = q_{(0)} + \alpha q_{(e)}$$

- Possible connection with the analysis of 4-form cohomology of M-theory on spin manifolds [Witten (1997)]: Is $dA^{(3)}_{(0)}$ the contribution responsible for the canonical integral class of the spin bundle of $D = 11$ superspace? This would imply that $q_{(0)}$ could assume fractional values (in units of $q_{(e)}$)