AdS_5 black string in the stu model of FI-gauged $N = 2$ supergravity

Matteo Azzola

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Outline

- The stu model of FI-gauged $\mathcal{N} = 2$, d = 5 supergravity
- Dimensional reduction and residual symmetries
- Black string with momentum along ∂z
- Dyonic black string with both momentum and rotation
- Solution with running scalars that generalize the previous of Maldacena-Nunez[1]

[1]J. M. Maldacena and C. Nunez, "Supergravity description of field theories on curved manifolds and a no go theorem," Int. J. Mod. Phys. A 16 (2001) 822 [hep-th/0007018].

The stu model of FI-gauged $N = 2$, $d = 5$ supergravity

Bosonic part of the lagrangian in the FI U(1) gauge, coupled to n_v vector multiplets

$$e^{-1}\mathscr{L}=\frac{1}{2}R-\frac{1}{2}\mathscr{g}_{ij}\partial_\mu\phi^i\partial^\mu\phi^j-\frac{1}{4}G_{IJ}F^I_{\mu\nu}F^{J\mu\nu}+\frac{e^{-1}}{48}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda-g^2V$$

$$V=\frac{1}{18}g_I g_J\left(\frac{9}{2}\mathscr{g}^{ij}\partial_i h^I\partial_j h^J-6h^I h^J\right)$$

where $h^I=h^I(\phi^i)$ are functions of the physical real scalar fields

$$h^1=\exp\left[-\left(\frac{\phi^1}{\sqrt{6}}+\frac{\phi^2}{\sqrt{2}}\right)\right]\qquad h^2=\exp\left[\frac{2\phi^1}{\sqrt{6}}\right]\qquad h^3=\exp\left[-\left(\frac{\phi^1}{\sqrt{6}}-\frac{\phi^2}{\sqrt{2}}\right)\right]$$

The susy variations are

$$\delta\psi_\mu=\left(D_\mu+\frac{i}{8}h_I\left(\Gamma_\mu^{\nu\rho}-4\delta_\mu^\nu\Gamma^\rho\right)F^I_{\nu\rho}+\frac{1}{6\sqrt{2}}\Gamma_\mu h^I g_I\right)\epsilon$$

$$\delta\lambda_i=\left(\frac{3}{8}\Gamma^{\mu\nu}F^I_{\mu\nu}\partial_i h_I-\frac{i}{2}\mathscr{g}_{ij}\Gamma^\mu\partial_\mu\phi^j+\frac{1}{2\sqrt{2}}g_I\partial_i h^I\right)\epsilon$$

Dimensional reduction: the *r*-map

Reducing the lagrangian to four dimensions using the *r*-map

$$ds_5^2 = e^{\frac{\phi}{\sqrt{3}}} ds_4^2 + e^{-\frac{2}{\sqrt{3}}\phi}(dz + K_\mu dx^\mu)^2 \qquad A^I = B^I(dz + K_\mu dx^\mu) + C_\mu^I dx^\mu$$

$$z^I = B^I + i e^{-\frac{\phi}{\sqrt{3}}} h^I \qquad e^k = \frac{1}{8} e^{\sqrt{3}\phi} \qquad gV_I = \frac{g_I}{3\sqrt{2}}$$

$$g_{I\bar{J}} = \frac{1}{2} e^{\frac{2\phi}{\sqrt{3}}} G_{IJ} \qquad F_{\mu\nu}^\Lambda = \frac{1}{\sqrt{2}}(K_{\mu\nu}, C_{\mu\nu}^I)$$

one ends up with the four-dimensional $N = 2$ FI-gauged supergravity with the cubic prepotential and the symplectic vector of FI parameters

$$F = -\frac{X^1 X^2 X^3}{X^0} \qquad G = (g^\Lambda, g_\Lambda)^t = (0,0,0,0,0, g_{1,} g_2, g_3)^t$$

This 4d theory has a residual symmetry that involves the stabilization of the symplectic vector of gauge couplings (FI parameters) under the action of the U-duality of the ungauged theory.

Residual Symmetry[2]

- The 4d gauged theory has a residual symmetry group U that comes from the ungauged U-duality symmetry group, i.e. the isometries of the non-linear sigma model

- The stabilizer of G must be considered in order to stay in the same theory

$$S_G = \{ g \in U \,|\, gG = G \}$$

- Another configuration can be constructed acting with $S \in S_G$ on a given solution $(\Omega, G, F_{\mu\nu})$ via the map

$$(\Omega, G, F_{\mu\nu}) \rightarrow (\Omega', G', F'_{\mu\nu}) := (S\Omega, SG, SF_{\mu\nu}) = (S\Omega, G, SF_{\mu\nu})$$

- Given a model (or the prepotential) and the gaugings, the stabilizer algebra is fixed. Then from a given solution the “rotated-dual” solution is unique.

[2]S.L.Cacciatori, D. Klemm and M. Rabbiosi, “Duality invariance in Fayet-Iliopoulos gauged supergravity,” JHEP 1609 (2016) 088 [arXiv:1606.05160 [hep-th]].

AdS_4 Cacciatori-Klemm black hole[3]

The starting point is one of the CK black hole solution in $N = 2$, d = 4 FI-gauged supergravity

$$F=-\frac{X^1X^2X^3}{X^0} \qquad ds^2=-4b^2dt^2+\frac{1}{b^2}\frac{ydy^2}{cy+2gp}+\frac{y^3}{b^2}\left(d\theta^2+\sinh^2\theta\, d\phi^2\right)$$

where

$$b^4=\frac{8g_1g_2g_3y^{9/2}}{H^0(cy+2gp)^{3/2}} \qquad H^0=\frac{2q_0}{3g^2p^2y^{3/2}}(cy+2gp)^{1/2}(cy-gp)+h^0$$

with field strengths and scalars

$$F^0=4\,dt\wedge d\left(H^0\right)^{-1} \qquad F^I=\frac{p^I}{2}\sinh\theta\, d\theta\wedge d\phi$$

$$z^I=i\frac{\sqrt{g_1g_2g_3}}{\sqrt{2}g_I}\frac{\sqrt{H^0}y^{3/4}}{(cy+2gp)^{1/4}}$$

The solution interpolates between $AdS_2\times H^2$ near the horizon and a curved domain wall for $y\rightarrow\infty$.

[3]S. L. Cacciatori and D. Klemm, “Supersymmetric AdS4 black holes and attractors,” JHEP 1001 (2010) 085 [arXiv:0911.4926 [hep-th]].

Black String with momentum along ∂z

The r-map can be used to uplift the CK solution to get a black string in $d = 5$

$$ds^2 = 2(H^0 b)^{-2/3}\left(\frac{1}{b^2}\frac{4r^4 dr^2}{cr^2+2gp} + \frac{r^6}{b^2}(d\theta^2 + \sinh^2\theta\, d\phi^2)\right) + \frac{1}{4}(H^0 b)^{4/3}\left(dz^2 - \frac{8\sqrt{2}}{H^0} dt dz\right)$$

where

$$b^4 = \frac{8 g_1 g_2 g_3 r^9}{H^0 (cr^2+2gp)^{3/2}} \qquad H^0 = \frac{2 q_0}{3 g^2 p^2 r^3}(cr^2+2gp)^{1/2}(cr^2-gp) + h^0$$

with field strengths and scalars

$$F^I = \frac{p^I}{\sqrt{2}} \sinh\theta\, d\theta \wedge d\phi \qquad h^I = \frac{(g_1 g_2 g_3)^{1/3}}{g_I}$$

The configuration satisfies the BPS equations with Killing spinor $\epsilon = Y(r)(1 + i\Gamma^{32})(1 - \Gamma^1)\epsilon_0$ so it is ¼ BPS.

Horizon in $r = 0$, the spacetime is AdS_3 x H^2. Asymptotically approaches AdS_5.

Dyonic black String with momentum and rotation

- Start from the lifted solution

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Dyonic black String with momentum and rotation

- Start from the lifted solution
- KK reduction along angular direction ϕ
- Duality transformation with the stabilizer
- Uplift to 5 dimensions

Dyonic black String with momentum and rotation

$$ds^2 = \frac{2\,dr^2}{g^2 r^2} + \frac{cr^2 + 2\,gp}{2\,g^2}\,d\,\Omega^2_{H^2} + \frac{2\sqrt{2}\,g\,r^3}{(cr^2 + 2\,gp)^{1/2}}\left(\frac{H^0}{4\sqrt{2}}\,dz - dt\right)dz$$

$$+4\sqrt{2}\,\omega\,\frac{cr^2 + 2\,gp}{g^2 H^0}\,\sinh^2\theta\left(d\,\phi + \frac{2\sqrt{2}\,\omega}{H^0}\,dt\right)dt$$

$$A^I = \frac{p^I}{\sqrt{2}}\cosh\theta\,d\,\phi + \frac{4}{H^0}\left(q_I + \omega\,s^I + \frac{\omega\,p^I}{\sqrt{2}}\cosh\theta\right)dt$$

Near-horizon limit $r \rightarrow 0$ is a deformation of AdS_3 x H^2

For $r \rightarrow \infty$ approaches to a magnetic AdS_5

Dyonic black String with momentum and rotation

$$ds^2 = \frac{2\,dr^2}{g^2 r^2} + \frac{cr^2 + 2gp}{2g^2}\, d\Omega^2_{H^2} + \frac{2\sqrt{2}\,g\,r^3}{(cr^2+2gp)^{1/2}} \left(\frac{H^0}{4\sqrt{2}}\, dz - dt \right) dz$$

$$+ 4\sqrt{2}\,\omega \frac{cr^2+2gp}{g^2 H^0} \sinh^2\theta \left(d\phi + \frac{2\sqrt{2}\,\omega}{H^0}\, dt \right) dt$$

$$A^I = \frac{p^I}{\sqrt{2}} \cosh\theta\, d\phi + \frac{4}{H^0} \left(q_I + \omega\, s^I + \frac{\omega\, p^I}{\sqrt{2}} \cosh\theta \right) dt$$

Near-horizon limit $r \to 0$ is a deformation of AdS_3 x H^2

For $r \to \infty$ approaches to a magnetic AdS_5

Is this solution still BPS?

Does the technique based on the stabilizer algebra preserves supersymmetry?

Running Scalars

- In [4] the problem of finding ¼ BPS strings with running scalars is reduced to solve a system of three first-order differential equations.

[4]S.L. Cacciatori, D. Klemm and W. A. Sabra, “Supersymmetric domain walls and strings in d = 5 gauged supergravity coupled to vector multiplets,” JHEP 0303 (2003) 023 [hep-th/0302218].

Running Scalars

- In [4] the problem of finding ¼ BPS strings with running scalars is reduced to solve a system of three first-order differential equations.
- Ansatz for the metric and for the magnetic fluxes

$$ds^2 = e^{2V}\left(-dt^2 + dz^2\right) + e^{2W}\left(du^2 + d\Omega_k^2\right) \qquad F^I_{\theta\phi} = kq^I F_k(\theta) \qquad F_k(\theta) = \begin{cases} \sin(\theta), k=1 \\ \sinh(\theta), k=-1 \end{cases}$$

- The warp factors are written in therms of the scalars $x^I(u)$

$$e^{2V} = \left(x^1 x^2 x^3\right)^{-\frac{1}{3}} e^{-g\int\left(x^1+x^2+x^3\right)du} \qquad e^{2W} = \left(x^1 x^2 x^3\right)^{\frac{2}{3}}$$

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- The supersymmetric flow equations for the scalars are

$$\begin{aligned} y^1{}' &= g\, y^2 y^3 + Q^1 \\ y^2{}' &= g\, y^1 y^3 + Q^2 \\ y^3{}' &= g\, y^1 y^2 + Q^3 \end{aligned}$$

Non-homogeneous version of the SU(2) Nahm equations

$$\frac{dT^I}{du} = \epsilon^{IJK}[T^J, T^K] + S^I \qquad T^I = y^I \sigma^I \qquad S^I = Q^I \sigma^I$$

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Generalizing Maldacena-Nunez

- A new solution can be found taking $Q^1 = Q^2 = 0$

$$ds^2=(x^1x^2x^3)^{-\frac{1}{3}}e^{g\int\frac{(x^1+x^2+x^3)}{y}du}(-dt^2+dz^2)+(x^1x^2x^3)^{\frac{2}{3}}\left(\frac{1}{(y^3)^2}dy^2+d\Omega_k^2\right)$$

- The solution of the Nahm system gives the scalar fields

$$x^1=\frac{1}{4}\left\{k_1e^{-gy}+k_2e^{gy}+\sqrt{k_1^2e^{-2gy}+k_2^2e^{2gy}+\frac{8ky}{g}}\right\}$$

$$x^2=\frac{1}{2}k_2e^{gy}$$

$$x^3=\frac{1}{4}\left\{-k_1e^{-gy}+k_2e^{gy}+\sqrt{k_1^2e^{-2gy}+k_2^2e^{2gy}+\frac{8ky}{g}}\right\}$$

Generalizing Maldacena-Nunez

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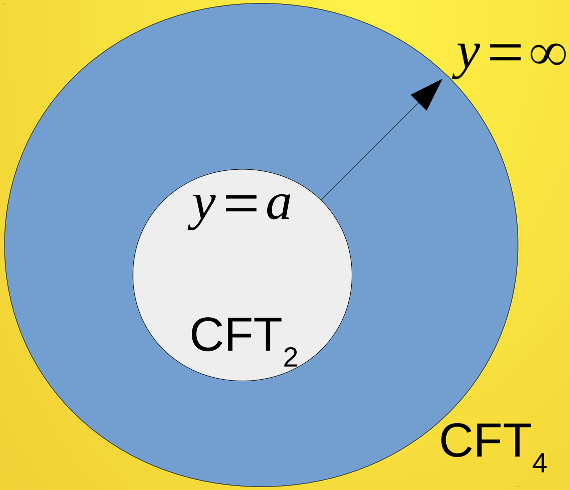
$$ds^2 = (x^1 x^2 x^3)^{-\frac{1}{3}} e^{g \int \frac{(x^1 + x^2 + x^3)}{y} du} (-dt^2 + dz^2) + (x^1 x^2 x^3)^{\frac{2}{3}} \left(\frac{1}{(y^3)^2} dy^2 + d\Omega_k^2 \right)$$

- This solution is an interpolation flow between AdS_5 and AdS_3 x H^2, for y that goes from infinity to the horizon $y = a$



- The value $k_1 = 0$ corresponds to the limit in which $x^1 = x^3$, i.e. the physical scalar field $\phi_2 = 0$. This truncation is the MN solution.

CFT dual picture

- The physical scalar fields are

$$\frac{2}{\sqrt{6}}\phi_1=\log\left(\frac{x^2}{\left(x^1x^2x^3\right)^{\frac{1}{3}}}\right) \qquad \sqrt{2}\,\phi_2=\log\left(\frac{x^3}{x^1}\right)$$

- Calculated on the conformal boundary of AdS

$$\frac{2}{\sqrt{6}}\phi_1\sim 2Qye^{-2gy} \qquad \sqrt{2}\,\phi_2\sim-\frac{k_1}{k_2}e^{-2gy}$$

- In the dual SCFT these are an expectation value of an operator and an insertion of dimension 2.

- The central charge of the 2d SCFT dual to the horizon configuration AdS_3 x H^2 is

$$c=\frac{6\pi(g-1)}{4G_5}=3N^2(g-1)$$ where g is the genus of the Riemann surface H^2

- Again, the truncation $k_1 = 0$ corresponds to the MN value.

Conclusion

- We have seen a very brief introduction to the N=2 d=5 supergravity model
- Two ways to find solutions have been presented
  - Using the residual symmetry of N=2 d=4 FI supergravity
  - Solving the spinning top equations
- New solutions that describe flows across dimensions have been found. One of these generalizes that of MN with two non zero running scalar fields.
- Open problem:
  - Does the technique based on the stabilizer algebra preserves supersymmetry?
  - Does the Nahm equations can be fully solved for all the $Q^I \neq 0$?