Beyond Positivity Bounds

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based on arXiv:1710.02539 (B. Bellazzini, F. Riva, J.Serra, FS)

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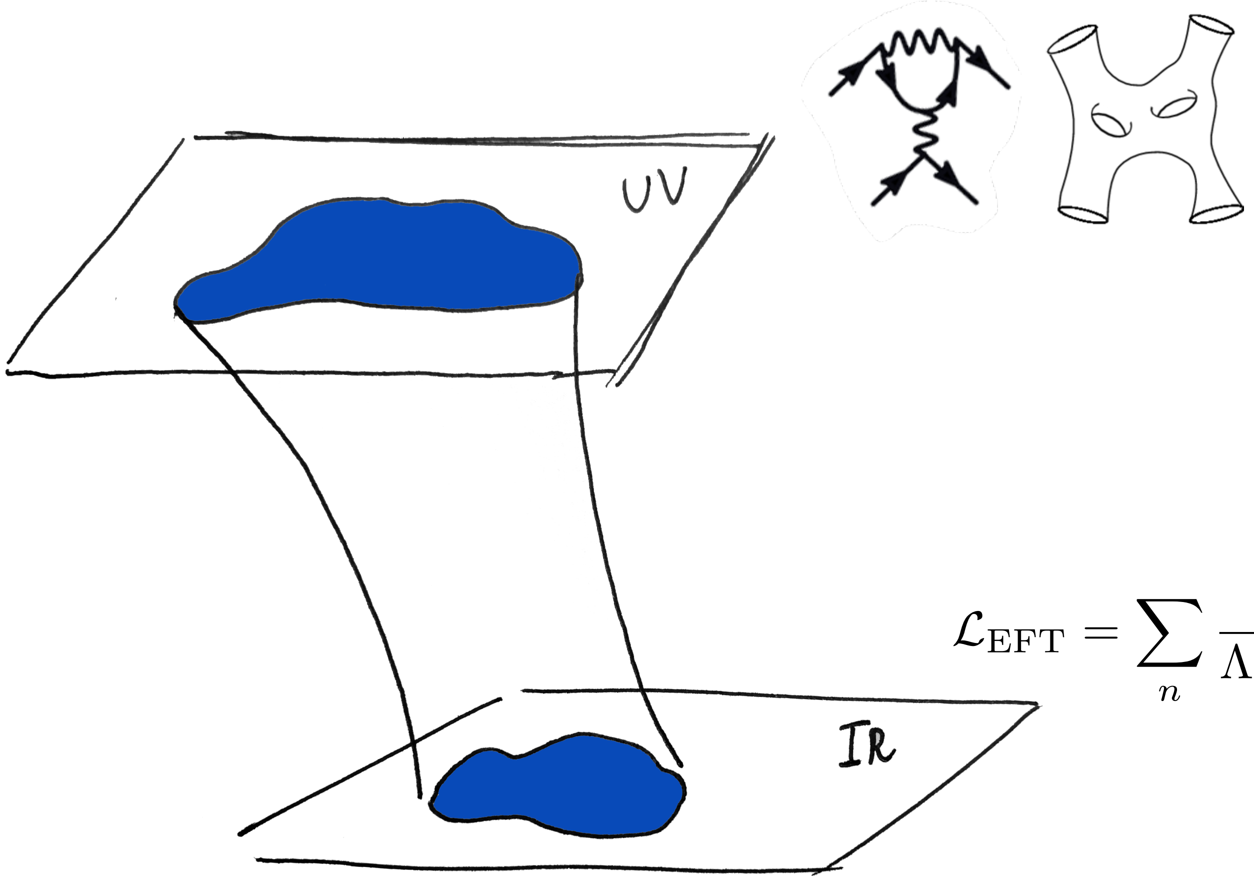
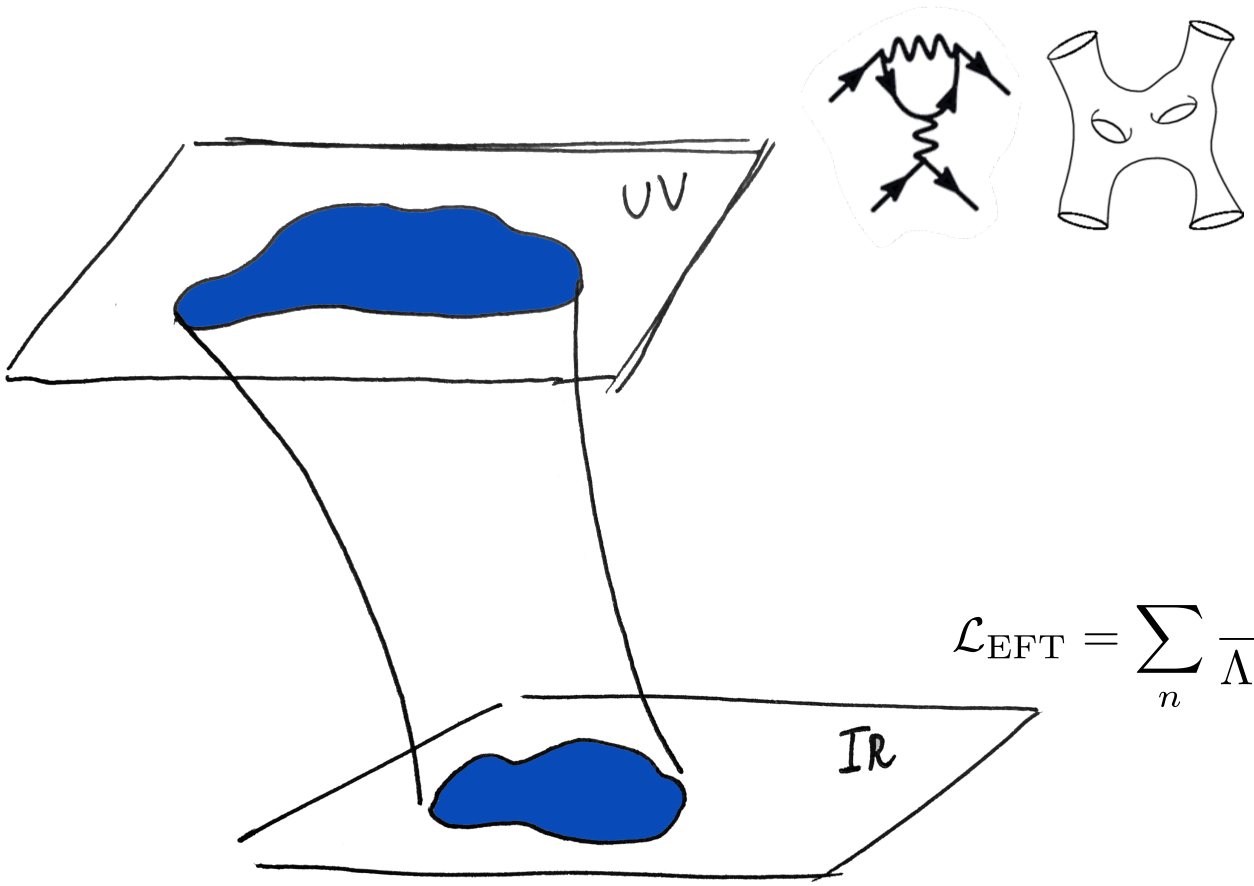
EFT paradigm



$$\mathcal{L}_{\mathrm{EFT}} = \sum_n \frac{c_n}{\Lambda^{\Delta_n - 4}} \mathcal{O}_n(x)$$

- Small parameter $E/\Lambda \ll 1$
- Symmetries
- Power counting

EFT paradigm



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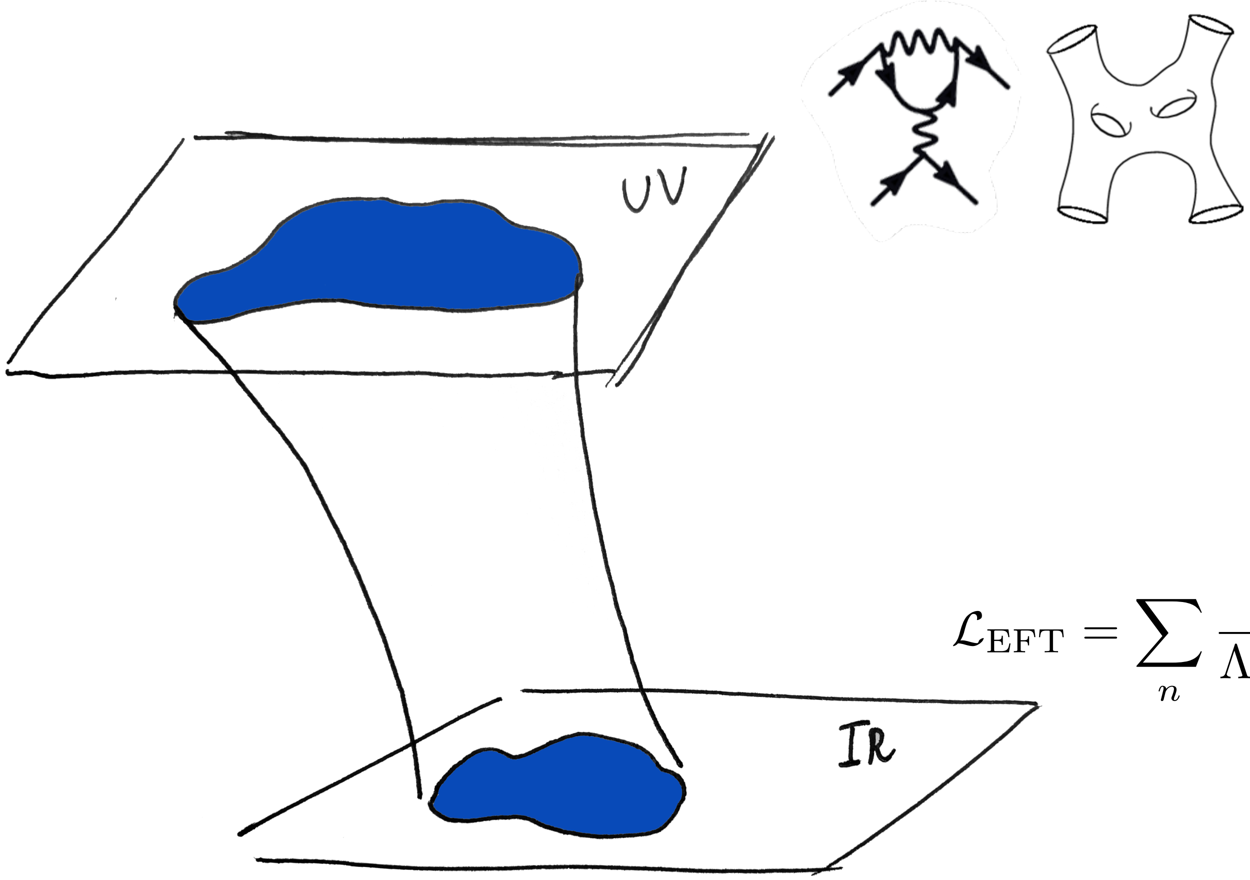
"Can any EFT be UV completed?"

hep-th/0602178

Unitarity, Locality, Analyticity and Crossing symmetry ⟶ Consistency conditions from dispersion relations

EFT paradigm



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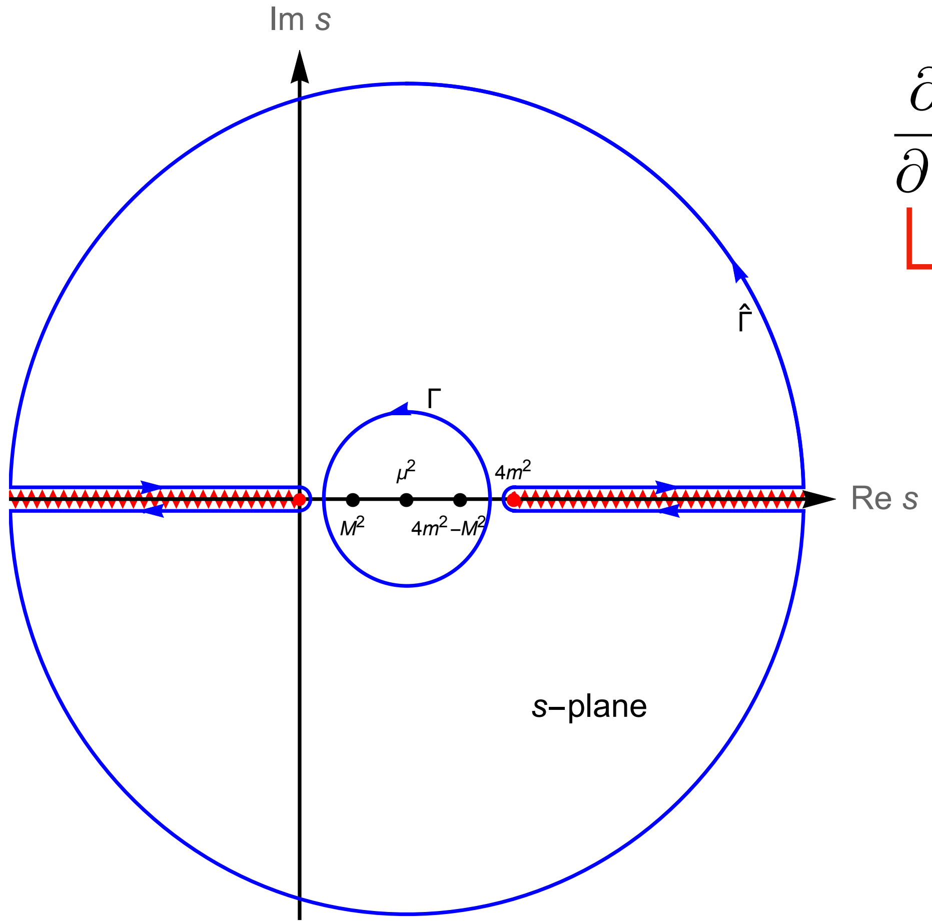
Unitarity, Locality, Analyticity and Crossing symmetry → Consistency conditions from dispersion relations

The answer is **NO**

Dispersion relations

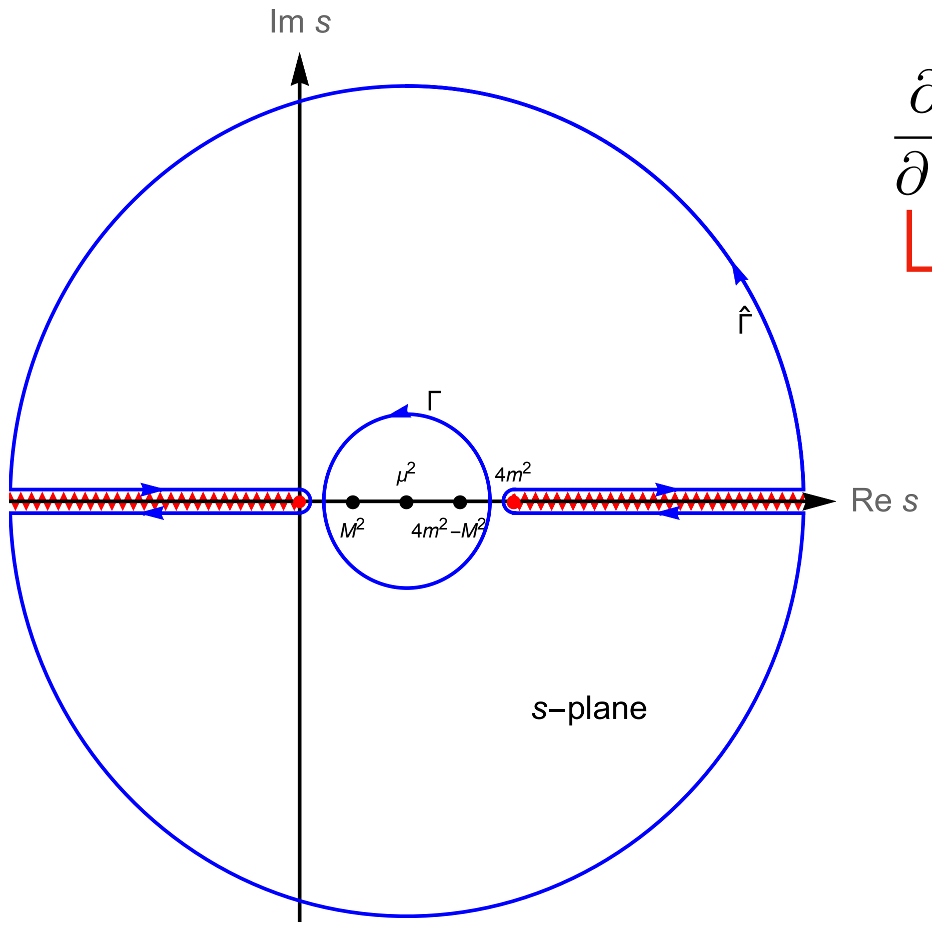


$$\underbrace{\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0)}_{\text{IR}} = \underbrace{\sum_X \int_0^\infty \frac{ds}{\pi s^2}\sigma_{12\to X}(s)}_{\text{UV}} > 0$$

- Analyticity → small circle = big circle + branch cuts
- Locality → Froissart Bound
- Crossing symmetry → only one branch cut
- Unitarity → positivity

Conclusion : s^2 coefficient is strictly positive

Dispersion relations



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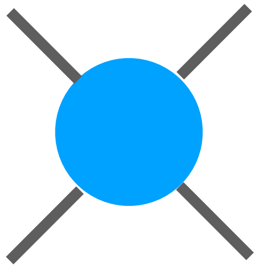
Example : $U(1)$ Goldstone EFT $\pi(x) \to \pi(x) + \text{const}$

hep-th/0602178

$$\mathcal{L} = \frac{1}{2}(\partial\pi)^2 + \frac{c}{\Lambda^4}(\partial\pi)^4 + \dots$$



$$\sim \frac{c}{\Lambda^4}s^2 + \dots$$

→ $c > 0$

Going beyond

$$\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0)=\sum_X\int_0^\infty \frac{ds}{\pi s^2}\sigma_{12\to X}(s)>0$$

IR calculable

Going beyond

UV contribution;

Unknown but positive

$$\frac{\partial^2}{\partial s^2}\mathcal{M}^{2\to 2}(s,t=0) = \sum_X \left(\int_0^{E^2 \ll \Lambda^2} + \int_{E^2 \ll \Lambda^2}^{\infty} \right) \frac{ds}{\pi s^2}\sigma_{12\to X}(s)$$

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IR calculable

integral over all values of t

away from the forward limit

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IR-residue > loop-factor x $\int_0^{E^2\ll\Lambda^2} ds\,[...]$

More strongly coupled, stronger bounds

Very useful when LHS suppressed > RHS unsuppressed

Example I : Galileon

$\pi \to \pi + c_\mu x^\mu + d$

$$-\frac{1}{2}(\partial\pi)^2\left[1 + \frac{c_3}{2\Lambda^3}\Box\pi + \frac{c_4}{2\Lambda^6}\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right) + \dots\right]$$

$$\mathcal{M}(\pi\pi \to \pi\pi) = -\frac{3}{4}(c_3^2 - 2c_4)\frac{stu}{\Lambda^6} \to 0$$

The theory is sick. We can add a tiny mass deformation $\mathcal{M}(s, t = 0) \sim \frac{c_3^2 {m_\pi}^2 s^2}{\Lambda^6}$

Usual positives give no new informations IR-residue $\sim m^2 > 0$

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Can the mass deformation be arbitrarily small?

IR-residue > loop-factor x $\int_0^{E^2 \ll \Lambda^2} ds\,[...]$

suppressed

$$m^2 > \Lambda^2\left(\frac{3}{320}\right)\frac{(c_3-2c_4/c_3)^2}{16\pi^2}\left(\frac{E}{\Lambda}\right)^8$$

The massless limit is not smooth. As $m \to 0$ the interactions switch off.

Example II : dRGT massive gravity

We explicitly break diff-invariance by adding a mass term to the Einstein Hilbert action

$$S_{\mathrm{dRGT}} = \int d^4\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{M_{Pl}^2 m^2}{8}V(g,h)\right]$$

1011.1232 de Rham, Gabadadze, Tolley

$h^2, h^3, h^4 ...$

- Coefficients fixed to propagate only 5 d.o.f.'s
- Only two independent coefficients c_3, d_5

We can reintroduce diff-invariance $\boxed{h_{\mu\nu} \to h_{\mu\nu}^{(\pm 2)} + \partial_{(\mu}A_{\nu)}^{(\pm 1)} + \partial_\mu\partial_\nu\pi^{(0)}}$

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Decoupling the scalar dof by taking $m \to 0$ and keeping $\Lambda_3^3 = m^2 M_{Pl}$ fixed → *Galileon structure*

Operators suppressed by Λ_3

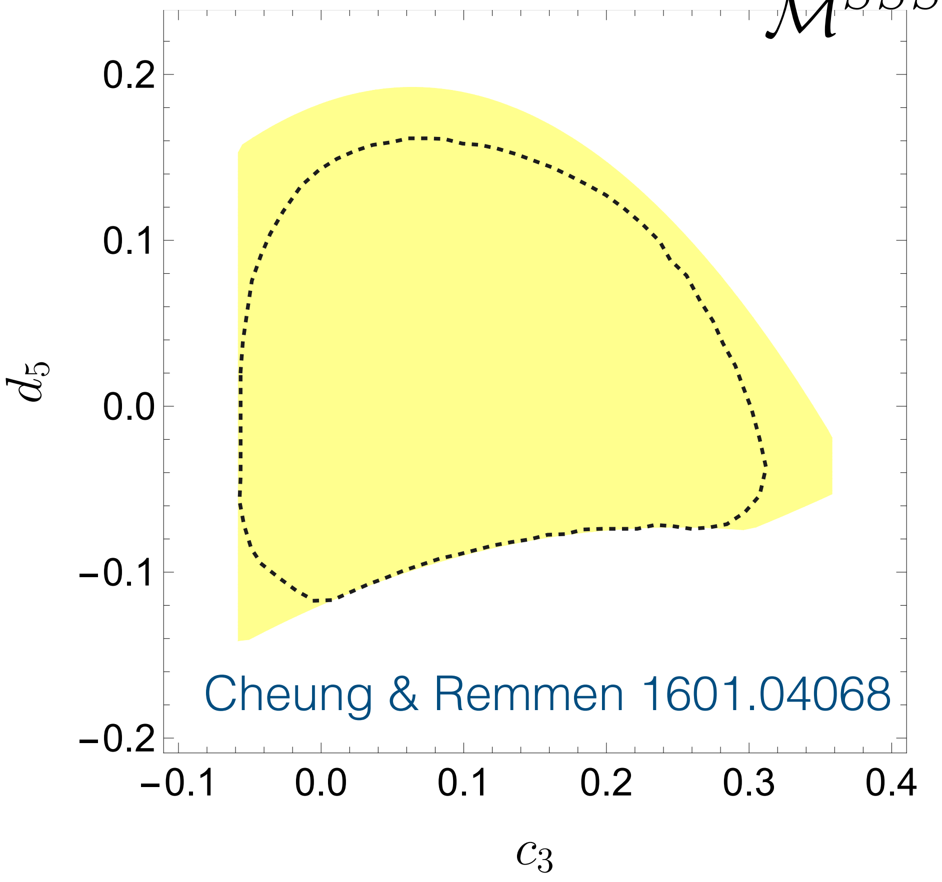
stu~galileon

$$\mathcal{M}^{SSSS} = \mathcal{M}(\pi\pi \to \pi\pi) \times [1 + o(\frac{m^2}{E^2})]$$

beyond decoupling contribution to the IR residue

$$\frac{m^2}{\Lambda_3^6}\alpha(c_3,d_5)s^2$$

$\alpha(c_3,d_5) > 0$



Cheung & Remmen 1601.04068

Can we do more? **YES!**

Example II : dRGT massive gravity

IR-residue > loop-factor x $\int_0^{E^2\ll\Lambda^2} ds\,[...]$

$$\Sigma_{\rm IR}^{SS} = \frac{2m^2}{9\Lambda_3^6}\left(7 - 6c_3(1+3c_3) + 48d_5\right) > 0$$

$$\Sigma_{\rm IR}^{VV} = \frac{m^2}{16\Lambda_3^6}\left(5 + 72c_3 - 240c_3^2\right) > 0$$

$$\Sigma_{\rm IR}^{VS} = \frac{m^2}{48\Lambda_3^6}\left(91 - 312c_3 + 432c_3^2 + 384d_5\right) > 0$$

$\cdots$

mass-suppressed

Hard scattering contribution $s\,,t \gg m^2$

$$\mathcal{M}^{SS} = \frac{st(s+t)}{6\Lambda_3^6}\left(1 - 4c_3(1-9c_3) + 64d_5\right)$$

$$\mathcal{M}^{VV} = \frac{9st(s+t)}{32\Lambda_3^6}(1-4c_3)^2$$

$$\mathcal{M}^{VS} = \frac{3t}{4\Lambda_3^6}\left(c_3(1-2c_3)(s^2+st-t^2) - \frac{5s^2+5st-9t^2}{72}\right)$$

$\cdots$

mass unsuppressed

We can derive a lower theoretical bound on the graviton mass

$$g_* \equiv \left(\frac{\Lambda}{\Lambda_3}\right)^3$$

$$\left(\frac{m}{4\pi M_{\rm Pl}}\right) > \frac{1}{F_i(c_3,d_5)}\left(\frac{g_*}{4\pi}\right)^4 \cdot \delta^6 \cdot [1 \pm \delta]$$

The most conservative bound is obtained by picking the maximum of minimums of $F_i(c_3, d_5)$

$$m > 10^{-32}\text{eV}\left(\frac{g_*}{4.5\cdot 10^{-10}}\right)^4\left(\frac{\delta}{1\%}\right)^6$$

1710.02539 B.Bellazzini, F.Riva, J.Serra, FS

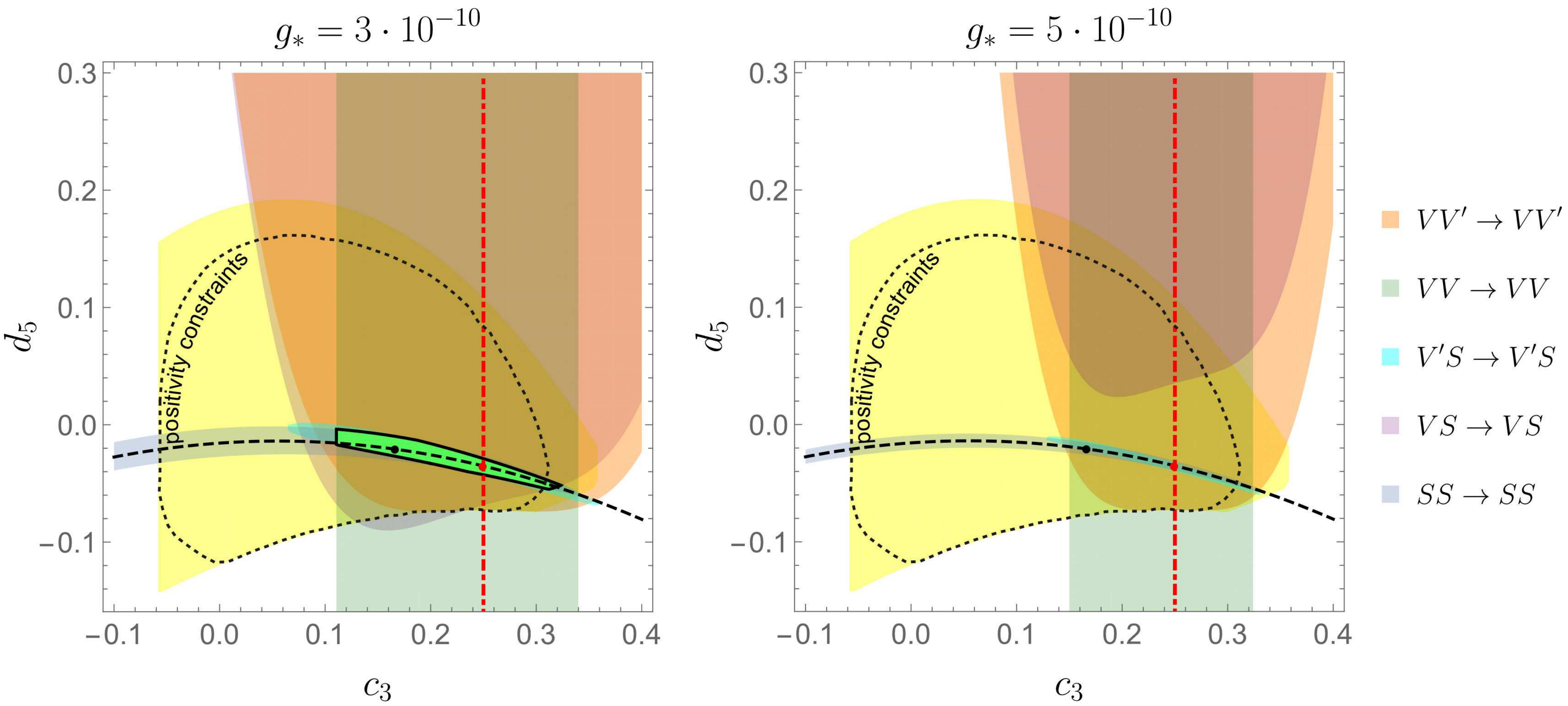
Graviton mass bound

$$m > 10^{-32}\mathrm{eV}\left(\frac{g_*}{4.5\cdot 10^{-10}}\right)^4\left(\frac{\delta}{1\%}\right)^6$$

The experimental bound on the graviton mass is $m < 10^{-32}\mathrm{eV} \longrightarrow g_* < 4.5\cdot 10^{-10}$

In the literature it is assumed O(1) coupling, or $\Lambda_3 = (m^2 M_{Pl})^{1/3} = \Lambda$ *Such scenario is ruled out!*

$\Lambda^3 = g_*\Lambda_3^3 \Rightarrow \Lambda \ll \Lambda_3$ Who cares about the strong coupling scale?

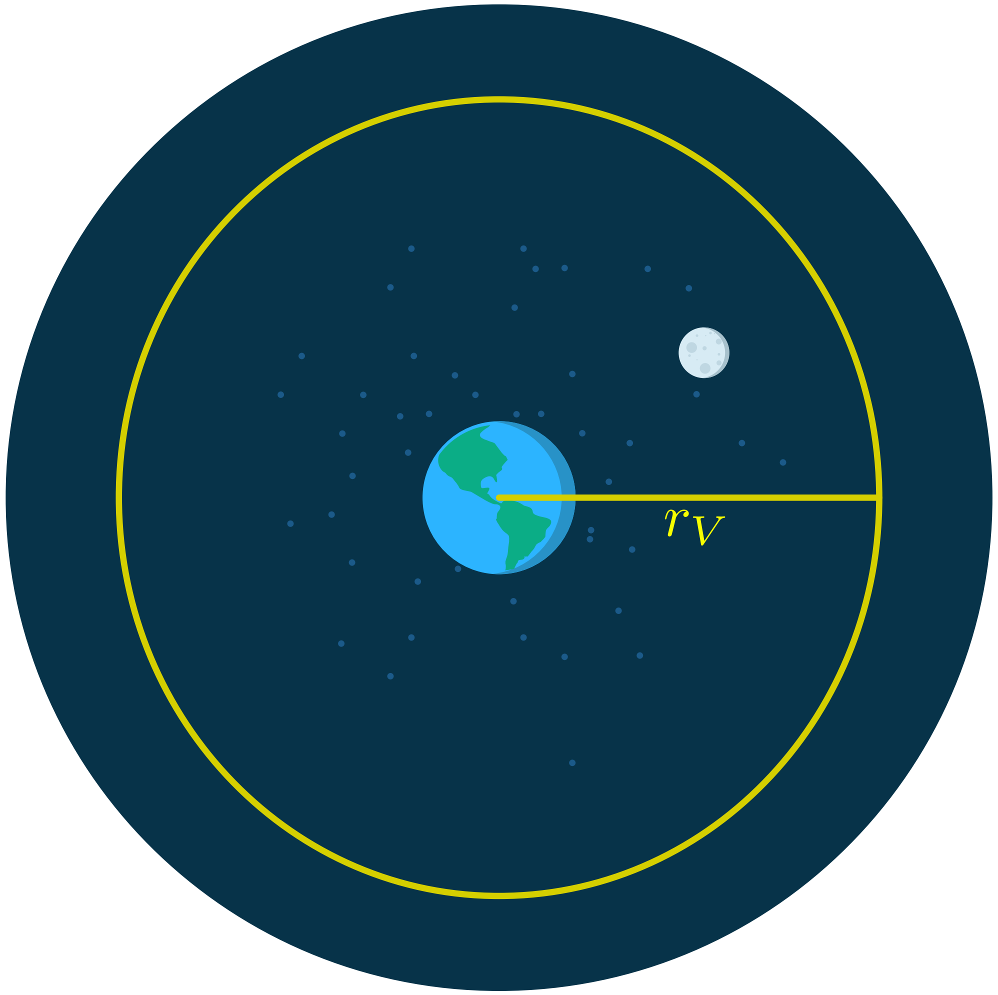


The fate of massive gravity

$$\frac{1}{\Lambda} \simeq r_{\rm moon} \left(\frac{g_*}{4.5 \cdot 10^{-10}} \right)^{-1/3} \left(\frac{m}{10^{-32}\,{\rm eV}} \right)^{-2/3}$$

The computation shown so far has been performed in flat space-time.

What about physics around massive bodies?



Non linearities $r_V = \frac{1}{\Lambda_3} \left(\frac{M_\oplus}{M_{Pl}} \right)^{1/3}$

Gravitational potential for a test massive body

$$\left(\frac{M_\oplus \, m_{\rm test}}{M_{\rm Pl}^2} \right) \frac{1}{r} \left[1 + \left(\frac{r_V}{r} \right)^3 + \ldots \right]$$

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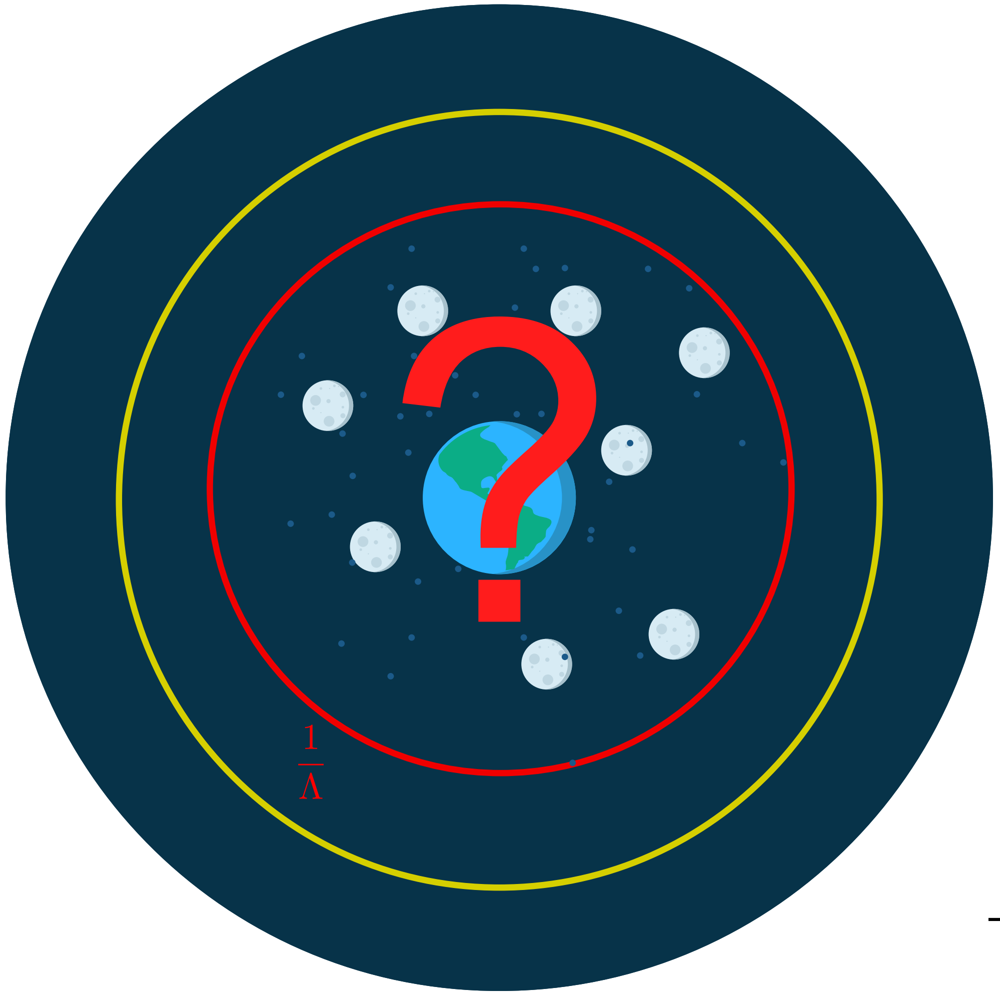

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quantum corrections

$$(\partial/\Lambda)^{2n}$$

Vainshtein screening breaks at $r \sim \frac{1}{\Lambda} \approx 10^{3\div4}\frac{1}{\Lambda_3} \approx (1\div 10)\, r_{\rm moon}$

The angular precession of the perihelion of the Moon gets modified

$$\left(\delta\phi^\pi\big|_{r=1/\Lambda}\sim\right)\pi\left(\frac{r}{r_V}\right)^{3/2}\sim 10^{-11}\div 10^{-10}$$

$$\delta\phi^{\rm exp}\big|_{moon}\sim 10^{-11}$$

Conclusions

- Dispersion relations provide a powerful method to establish the space of inconsistent EFT theories
- We can go beyond the forward limit $\int_0^{E^2 \ll \Lambda^2} \frac{ds}{\pi s^2} \sigma_{\text{tot}}(s) \longrightarrow$ Integral over all possible values of t
- In absence of ad-hoc assumptions dRGT-massive gravity as EFT is ruled out

Future applications (work in progress) : higher spins, conformal anomalies in 6d…

Consistent EFTs for massive higher spins : open problem

Gauge invariant self-interactions of higher spins are very soft $\sim \frac{1}{\Lambda^{4S}} \partial^{4S} h^4$

Mass term affects the IR residue of the 2-2 amplitude; expectation : $m < \Lambda_* < \Lambda$

Proof of the a-theorem in d=6 : open problem

Going beyond positivities one can prove the conformal anomaly cannot be arbitrarily negative;

Still the a-theorem lacks a proof

Thank you!

Backup slides

Crossing symmetry

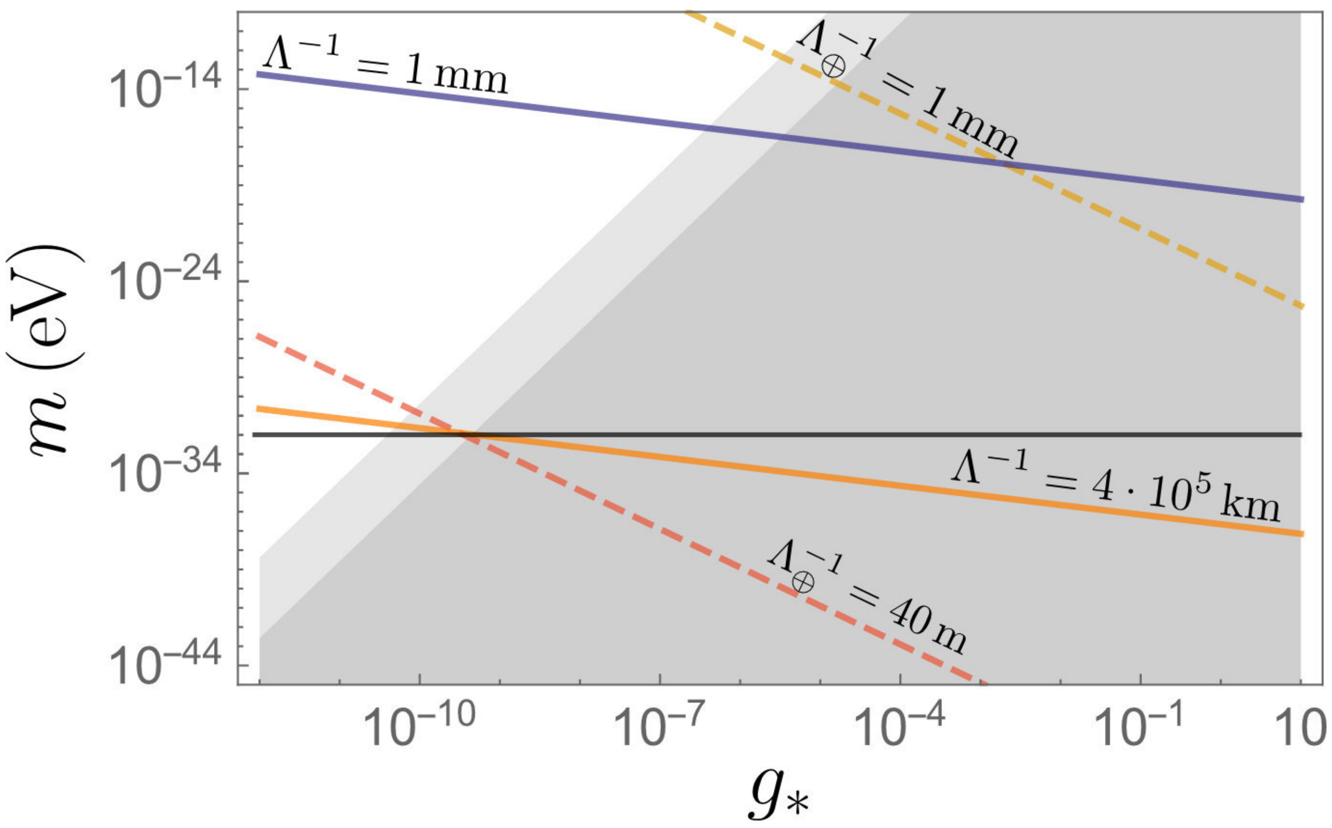
Crossing scalars is very simple $\mathcal{M}_{\pi\phi\to\pi\phi}(s,t,u) = \mathcal{M}_{\pi\bar{\phi}\to\pi\bar{\phi}}(u,t,s)$

What about spinning particles ?

- Polarizations are non-trivial and carry non-analyticities
- Crossing is not just $s \leftrightarrow t \leftrightarrow u$

Forward scattering is special $\mathcal{M}_{\text{particles}}(s,t=0) = \mathcal{M}_{\text{antiparticles}}(u,t=0)$

What if we assume ad-hoc Vainshtein screening?



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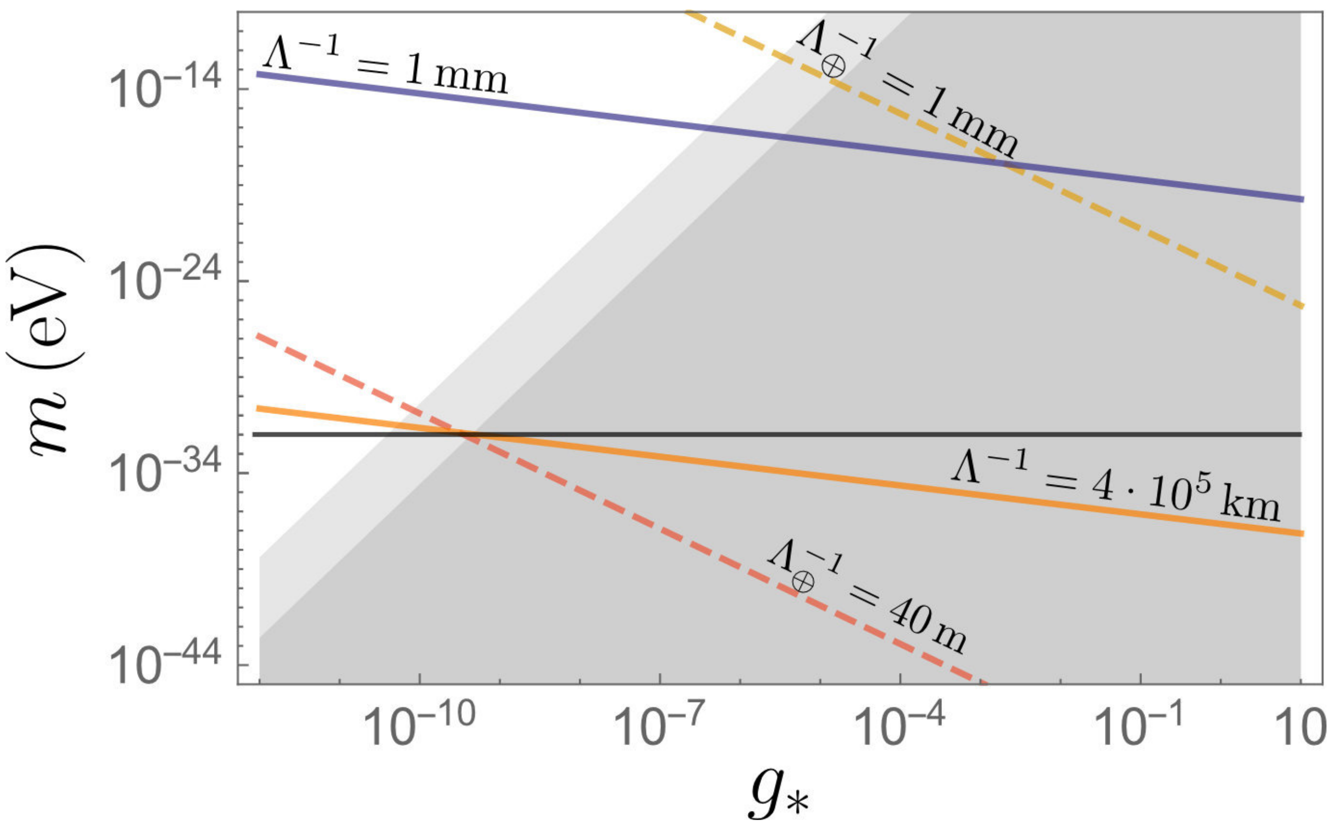

GR well

tested at mm