Curvature squared invariants in 6D $\mathcal{N} = (1, 0)$ supergravity

Gabriele Tartaglino-Mazzucchelli

ITF KU Leuven

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Based on:
Butter, Novak & GTM, JHEP 1705 (2017) 133;
Novak, Ozkan, Pang & GTM, PRL 119 (2017) no.11, 111602;
Butter, Novak, Ozkan, Pang & GTM *to appear*

See also:
Linch, III & GTM JHEP 1208, 075 (2012);

Why higher-derivatives invariants?

- String theory effective action: modified supergravity (SUGRA) by an infinite series of higher derivative quantum corrections

$$L^{low}_{string} = L_{SG} + \sum [\mathcal{D}^p \mathcal{R}^q_{\ldots}] + dilaton + forms + susy \;\; completion$$

SUSY higher-derivatives terms are poorly understood but, e. g.:

- important for (phenomenological) applications in string theory, see compactifications with fluxes; moduli stabilization; dualities .... [Antoniadis, Becker-Becker, Liu, Minasian, Polchinski, Sethi, Theisen ...]
- important for black-hole physics
  indeed for computing higher-order corrections to black-hole entropy
  microscopic vs macroscopic matching of entropy of SUSY black-holes
  [Strominger-Vafa, Sen, LopesCardoso-deWit, Murthy, ...]

even the simple SUSY $\mathcal{R}^2$ case is not fully understood in general
(see for example 6D) but

$\mathcal{R}^2$?

For instance $\mathcal{R}^2$ gravity attracted attention for over 50 years:

- renormalization of QFT in curved spacetime requires counterterms containing $\mathcal{R}^2$ [Utiyama & DeWitt (1962)]
- In 4D, $\mathcal{R}^2$ terms govern the structure of QFT conformal anomalies relevant in studying renormalization group flows, see 4D *a*-theorem [Komargodski and Schwimmer (2011)]
- In 4D, Renormalizable (not unitary) $\alpha(\mathcal{C}_{abcd})^2 + \beta(\mathcal{R}_{ab})^2 + \gamma\mathcal{R}^2$ [Stelle (1977)]
- In 3D ghosts-free higher-derivative theory of (massive) gravity, New-Massive-Gravity (NMG), plus other Generalized-Massive-Gravity (GMG) theories (TMG+NMG) [Bergshoeff-Hohm-Townsend (2009), $\cdots$] based on

$$\simeq \Lambda + \alpha_1\mathcal{R} + \alpha_2\varepsilon^{abc}\omega_a\mathcal{R}_{ab} + \alpha_3\mathcal{R}^2 + \alpha_4(\mathcal{R}_{ab})^2$$

  toy-model for quantum gravity with finite higher-derivatives series: AdS/CFT, black holes microstates... [Strominger (2008), $\cdots$]
- $\mathcal{R} + \mathcal{R}^2$ Starobinsky model of inflation [Starobinsky (1980)]
  Interestingly, $\mathcal{R} + \mathcal{R}^2$ SUGRA models are promising inflationary candidates for CMB data.

Gauss-Bonnet: $\mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2$

An important curvature squared combination is the Gauss-Bonnet one

- In 4D it is a topological term (Euler characteristic) arising as the Type A conformal anomaly.
- In D>4 it is involved in the definition of ghost free Lovelock gravities same spectrum of standard GR and second order EOM
- Governs the first α'-corrections in compactified string theory [Zweibach (1985), Deser-Redlich (1986)]......
- In general its structure for any space-time dimensions and amount of susy is not known. In particular, the dependence upon the extra sugra matter fields, see the NSNS b_2 gauge 2-form

The Gauss-Bonnet was constructed off-shell

4D $\mathcal{N} = 1$, [Cecotti-Ferrara-Girardello-Porrati (1985)]; [Theisen (1986)]...
4D $\mathcal{N} = 2$, [Butter-deWit-Kuzenko-Lodato (13)]
5D $\mathcal{N} = 1$, [Ozkan-Pang (13)]
6D ?

In 6D a full classification of the $\mathcal{R}^2$ invariants was missing and in particular the GB invariant has never been fully constructed.
We filled these gaps [Butter-Novak-Ozkan-Pang-GTM (17),(18)]

how higher-derivatives SUGRA?

If/once convinced about the importance of higher derivative supergravity invariants the question is: how to efficiently construct them?

- A powerful approach would be a formalism guaranteeing manifest supersymmetry in a model independent way (see e.g. $4D\ \mathcal{N}=1$)

  $\Downarrow$

  An off-shell approach to SUGRA, when available, can be used for general supergravity-matter couplings with model independent susy.
- Two possibilities:

  – component fields superconformal tensor calculus

  See "Supergravity" book by [Freedman & Van Proeyen (2012)]

  – superspace approaches See classic books:

  [Gates, Grisaru, Roček, Siegel (1983)] [Buchbinder, Kuzenko (1998)]

The two approaches can be used together through conformal superspace

- manifestly gauge entire superconformal algebra in superspace [Kugo-Uehara (1985)] and combine advantages of both approaches
- Details first by Butter 4D $\mathcal{N}=1$ in 2009 and $\mathcal{N}=2$ in 2011
- developed and extended to 3D $\mathcal{N}-$*extended* and 5D $\mathcal{N}=1$ SUGRA [Butter-Kuzenko-Novak-GTM (2013)-(2014)]
- 6D $\mathcal{N}=(1,0)$ [Butter-Kuzenko-Novak-Theisen-GTM (2016)-(2017)]

how higher derivatives off-shell SUGRA? outline

In superspace one can efficiently:

- Describe geometrically off-shell supermultiplets: SUGRA, matter
- Provide manifestly supersymmetric off-shell action principles
- Use powerful cohomological approach based on superforms to construct supersymmetric invariants.
  Rheonomic approach [Castellani-D'Auria-Fré (book-1991)];
  4D $\mathcal{N} = 1$ [Hasler (1996)];
  "Ectoplasm" [Gates(1996); Gates-Grisaru-Knutt–Wehlau-Siegel (1997)];
  Integral Forms [Castellani-Catenacci-Grassi (2014)]
- Reduce to components and derive superconformal tensor calculus

With these techniques, one can in principle have a systematic approach to study higher derivative invariants off-shell

Examples:

- 6D $\mathcal{N} = (1, 0)$ (four-derivatives) curvature squared terms

An interlude: Conformal gravity and Curvature squared terms in Poincaré gravity (GR)

$$\mathcal{L}_{R^2} \propto a\, C^{abcd} C_{abcd} + b\, \mathcal{R}^{abcd} \mathcal{R}_{abcd} + c\, \mathcal{R}^2$$

- Weyl tensor: $C_{ab}{}^{cd} = \mathcal{R}_{ab}{}^{cd} - \delta_{[a}{}^{[c} \mathcal{R}_{b]}{}^{d]} + \frac{1}{10} \delta_{[a}{}^{[c} \delta_{b]}{}^{d]} \mathcal{R}$ with $\mathcal{R}_{ab}{}^{cd}$ component Riemann tensor
- Ricci tensor: $\mathcal{R}_a{}^b := \mathcal{R}_{ad}{}^{bd}$
- Ricci scalar: $\mathcal{R} := \mathcal{R}_a{}^a$

A kinematic interlude: Conformal gravity

- Conformal gravity in six dimensions may be viewed as being based on gauging the entire conformal group SO(6,2), $X_{\tilde{a}} = \{P_a, M_{ab}, \mathbb{D}, K^a\}$
- Vielbein $e_m{}^a$, and its inverse $e_a{}^m$, associated with gauging P_a (diff.=local-translations)
  gauge connections are associated with the other generators which can be used to construct covariant derivatives

$$\nabla_a = e_a{}^m \partial_m - \frac{1}{2}\omega_a{}^{bc} M_{bc} - b_a \mathbb{D} - \mathfrak{f}_a{}^b K_b$$

- The covariant derivative algebra is constrained to be expressed entirely in terms of a tensor C_{abcd}

$$[\nabla_a, \nabla_b] = -F_{ab} = -\tfrac{1}{2} C_{ab}{}^{cd} M_{cd} - \tfrac{1}{6} \nabla^d C_{abcd} K^c$$

where C_{abcd} is the Weyl tensor a primary field of dimension 2

$$K_f C_{abcd} = 0 \ , \ \ \mathbb{D} C_{abcd} = 2 C_{abcd}$$
$$C_{abcd} = C_{[ab][cd]} \ , \ \ \eta^{ac} C_{abcd} = 0 \ , \ \ C_{[abc]d} = 0$$

A dynamical interlude: Poincaré gravity

- Poincaré gravity: conformal gravity coupled to a conformal primary $K^a\sigma = 0$ dimension 2 $\mathbb{D}\sigma = 2\sigma$ compensator scalar field $\sigma \neq 0$
- Kinematic: choose a gauge in which $b_a = 0$ and $\sigma = 1$, left: Poincaré gravity invariant only under diffeomorphisms and Lorentz.
- Dynamics: for example, the Einstein-Hilbert term $(e := \det e_m{}^a)$:

$$I = \int \mathrm{d}^6x\, e\, \sigma \nabla^a \nabla_a \sigma \ , \quad \textit{if} \ \ b_a = 0, \ \sigma = 1 \ \Longrightarrow \quad I_{EH} \propto \int \mathrm{d}^6x\, e\, \mathcal{R}$$

- Dynamics: for example, curvature squared terms:

$$I = \int \mathrm{d}^6x\, e\, \sigma^{-1} (\nabla^a \nabla_a \sigma)^2 \ , \quad \textit{if} \ \ b_a = 0, \ \sigma = 1 \ \Longrightarrow \quad I_{\mathcal{R}^2} \propto \int \mathrm{d}^6x\, e\, \mathcal{R}^2$$

$$I = \int \mathrm{d}^6x\, e\, \sigma^{-\frac{1}{2}} (\nabla^a \nabla_a)^2 \sigma^3 \ , \quad \textit{if} \ \ b_a = 0, \ \sigma = 1 \ \Longrightarrow \quad I_{\mathcal{R}^2_{ab}} \propto \int \mathrm{d}^6x\, e\, \mathcal{R}^{ab} \mathcal{R}_{ab}$$

$$I = \int \mathrm{d}^6x\, e\, \sigma C^{abcd} C_{abcd} \ , \quad \textit{if} \ \ b_a = 0, \ \sigma = 1 \ \Longrightarrow \quad I_{Weyl^2} \propto \int \mathrm{d}^6x\, e\, \mathcal{C}^{abcd} \mathcal{C}_{abcd}$$

Analogously, general off-shell Poincaré SUGRA: off-shell conformal SUGRA coupled to compensators see: superconformal tensor calculus

The standard Weyl multiplet of (1,0) conformal SUGRA

The minimal supersymmetric extension of conformal gravity
with Q_α^i and S_i^α supersymmetry generators, that are chiral fermions
$\mathcal{N} = (1,0)$, eight real supercharges each, the analogue of 4D $\mathcal{N} = 2$

Multiplet of local off-shell gauging of $\mathrm{OSp}(6,2|1)$, the $\mathcal{N} = (1,0)$ superconformal group in 6D. [Bergshoeff-Sezgin-VanProeyen (1986)]
$40 + 40$ off-shell physical multiplet composed by independent gauge fields

- P_a gauge connection: vielbein ${e_m}^a$;
- Q-susy gauge connection: the gravitino ${\psi_m}_i^\alpha$;
- $\mathrm{SU}(2)_R$ gauge field: ${\mathcal{V}_m}^{ij}$;
- dilatation gauge field: b_m (pure gauge);

and a set of covariant "auxiliary/matter" fields

- real anti-self-dual tensor T^-_{abc};
- a chiral fermion $\chi^{\alpha i}$;
- a real scalar field D.

fields necessary to close SUSY algebra off-shell (not unique set, see later)

Lorentz (${\omega_a}^{bc}$), K^a ($\mathfrak{f}_{a\,c}$), S-susy (${\phi_a}_\gamma^k$) connections are composite fields

How is this described in superspace?

6D conformal supergravity in conformal superspace

[Butter-Kuzenko-Novak-Theisen (2016)]

Take a $\mathcal{N} = (1,0)$ curved superspace $\mathcal{M}^{6|8}$ parametrised by coordinates

$$z^M = (x^m, \theta^\mu_{\rm i}) , \qquad m = 0,1,2,3,4,5 , \quad \mu = 1,2,3,4 , \quad {\rm i} = \underline{1}, \underline{2}$$

Choose the structure group X with generators $X_{\underline{a}} = (M_{cd}, J_{ij}, \mathbb{D}, S^\alpha_i, K^a)$ to contain $\mathrm{SO}(5,1) + \mathrm{SU}_R(2)$+(Dilatations)+($S$−susy) + ($K$−boosts). The superspace covariant derivatives $\nabla_A = (\nabla_a, \nabla^i_\alpha)$ are

$$\nabla_A = E_A{}^M \partial_M - \omega_A{}^{\underline{b}} X_{\underline{b}} = E_A{}^M \partial_M - \frac{1}{2}\Omega_A{}^{ab} M_{ab} - \Phi_A{}^{ij} J_{ij} - B_A \mathbb{D} - \mathfrak{F}_{AB} K^B$$

- $E_A{}^M(z)$ supervielbein associated with $P_A = (P_a, Q^i_\alpha)$, $\partial_M = \partial/\partial z^M$,
- $\Omega_A{}^{cd}(z)$ Lorentz connection, associated with M_{cd}
- $\Phi_A{}^{ij}(z)$ $\mathrm{SU}(2)_R$-connection, associated with J_{ij}
- $B_A(z)$ dilatation connection, associated with $\mathbb{D}$
- $\mathfrak{F}_{AB}(z)$ special superconformal connection, associated with $K^A = (K^a, S^\alpha_i)$
  - local covariant conformal SUGRA gauge transformations:

$$\mathcal{K} := \xi^A \nabla_A + \Lambda^{\underline{a}} X_{\underline{a}} = \xi^A \nabla_A + \frac{1}{2}\Lambda^{bc} M_{bc} + \Lambda^{ij} J_{ij} + \tau \mathbb{D} + \Lambda_A K^A$$

$$\delta_\mathcal{K} \nabla_A = [\mathcal{K}, \nabla_A] , \qquad \text{and on a tensor superfield } U\text{:} \qquad \delta_\mathcal{K} U = \mathcal{K} U$$

Should think of $\nabla_A \simeq P_A$: $\nabla_a \simeq P_a$ and $\nabla^i_\alpha \simeq Q^i_\alpha$

6D conformal supergravity in conformal superspace

One constrains the algebra:

$$[\nabla_A, \nabla_B\} = -T_{AB}{}^C \nabla_C - \frac{1}{2} R(M)_{AB}{}^{cd} M_{cd} - R(N)_{AB}{}^{kl} J_{kl} - R(\mathbb{D})_{AB}\mathbb{D} - R(S)_{AB}{}^k_\gamma S^\gamma_k - R(K)_{ABc} K^c$$

to be completely determined in terms of the super-Weyl tensor:

$$W^{\alpha\beta} = (\tilde{\gamma}^{abc})^{\alpha\beta} W_{abc} \qquad \text{[Linch-GTM (12)]}$$

describing in superspace the 6D $\mathcal{N} = (1,0)$ Weyl multiplet

- $W^{\alpha\beta}$ is a dimension-1 primary superfield

  $$K^A W^{\beta\gamma} = 0 \ , \quad \mathbb{D} W^{\alpha\beta} = W^{\alpha\beta}$$

  Jacobi/Bianchi Identities: differential constraints on $W^{\alpha\beta}$
- The standard Weyl multiplet of $6D\ \mathcal{N} = (1,0)$ conformal supergravity is encoded in the superspace geometry.
  The component fields can be readily identified as $\theta = 0$ projections of the superspace one-forms and descendants of $W^{\alpha\beta}$

  $$T^-_{abc} := -2W_{abc}|_{\theta=0} \ , \quad \chi^{\alpha i} := -\frac{3\mathrm{i}}{4} \nabla^i_\beta W^{\alpha\beta}|_{\theta=0} \ , \quad D := -\frac{3\mathrm{i}}{16} \nabla^k_\alpha \nabla_{\beta k} W^{\alpha\beta}|_{\theta=0}$$

  Other descendants of W_{abc} are composite superconformal curvatures: ex. $C_{abcd} \propto (\gamma_{ab})_\alpha{}^\beta (\gamma_{cd})_\gamma{}^\delta \nabla^k_\beta \nabla_{\delta k} W^{\alpha\gamma}$

The tensor multiplet and dilaton-Weyl multiplet

So far we have considered only the standard Weyl multiplet which possesses the covariant component fields: T^-_{abc}, $\chi^{\alpha i}$ and D

A variant description of the off-shell conformal supergravity multiplet:

- Dilaton-Weyl multiplet: obtained by coupling the standard Weyl multiplet to a (on-shell) tensor multiplet with scalar superfield Φ
- Φ is described by a gauge (NSNS) two-form B_2 in superspace. Its field strength is the closed super 3-form

$$H_3 = \mathrm{d}B_2 = \frac{1}{3!}\mathrm{d}z^P \wedge \mathrm{d}z^N \wedge \mathrm{d}z^M H_{MNP}(z), \qquad \mathrm{d}H_3 = 0$$

$$\begin{aligned} H_{\alpha}^{i}{}_{\beta}^{j}{}_{\gamma}^{k} &= 0 , \quad H_{a}{}_{\alpha}^{i}{}_{\beta}^{j} = 2\mathrm{i}\varepsilon^{ij}(\gamma_a)_{\alpha\beta}\Phi , \\ H_{ab}{}_{\alpha}^{i} &= (\gamma_{ab})_\alpha{}^\beta \psi_\beta^i , \qquad \psi_\alpha^i := \nabla_\alpha^i \Phi , \\ H_{abc} &= -\frac{\mathrm{i}}{8}(\tilde{\gamma}_{abc})^{\gamma\delta}\nabla_\gamma^k \psi_{\delta k} - 4W_{abc}\Phi , \end{aligned}$$

where Φ is primary ($K^A\Phi = 0$), $\mathbb{D}\Phi = 2\Phi$, satisfying $\nabla_\alpha^{(i}\nabla_\beta^{j)}\Phi = 0$

Assuming $\Phi \neq 0$ one can express the standard super-Weyl multiplet W_{abc} in terms of the tensor multiplet:

$$W_{abc} = -\frac{1}{4\Phi}H_{abc} - \frac{\mathrm{i}}{32\Phi}(\tilde{\gamma}_{abc})^{\gamma\delta}\nabla_\gamma^k \nabla_{\delta k}\Phi$$

The tensor multiplet and dilaton-Weyl multiplet

In components:

- define $\sigma := \Phi| \neq 0$ the conformal compensator
- the covariant component fields of the standard Weyl multiplet are:

$$
\begin{aligned}
T^{-}_{abc} &= \frac{1}{2\sigma} H^{-}_{abc} \\
\chi^{i} &= -\frac{15\mathrm{i}}{8\sigma} \nabla\!\!\!/ \psi^{i} - \frac{5\mathrm{i}}{32\sigma} T^{-}_{abc} \tilde{\gamma}^{abc} \psi^{i} \\
D &= \frac{15}{4\sigma} \left(\nabla^{a} \nabla_{a} \sigma + \frac{1}{3} T^{-abc} H_{abc} \right) + \text{fermion terms}
\end{aligned}
$$

- This means that in the dilaton-Weyl multiplet
  T^{-}_{abc}, $\chi^{\alpha i}$ and D
  are exchanged with the component fields of the tensor multiplet:
  σ, $\psi^{i}_{\alpha}|$ and $b_{mn} := B_{mn}|$ ($H_{abc} \simeq 3\nabla_{[a} b_{bc]}$)

6D $\mathcal{N} = (1,0)$ curvature squared invariants?

[Novak-Ozkan-Pang-GTM (17)]
[Butter-Novak-Ozkan-Pang-GTM (18)]
supersymmetric extensions of general curvature squared Lagrangian?

$$\mathcal{L}_{R^2} \quad \propto \quad a\, C^{abcd} C_{abcd} + b\, \mathcal{R}^{abcd} \mathcal{R}_{abcd} + c\, \mathcal{R}^2$$
$$+ \text{ SUSY completing terms}$$

- Weyl tensor: $C_{ab}{}^{cd} = \mathcal{R}_{ab}{}^{cd} - \delta_{[a}{}^{[c} \mathcal{R}_{b]}{}^{d]} + \frac{1}{10} \delta_{[a}{}^{[c} \delta_{b]}{}^{d]} \mathcal{R}$ with $\mathcal{R}_{ab}{}^{cd}$ component Riemann tensor
- Ricci tensor: $\mathcal{R}_a{}^b := \mathcal{R}_{ad}{}^{bd}$
- Ricci scalar: $\mathcal{R} := \mathcal{R}_a{}^a$

Here I will focus on the construction of the Gauss-Bonnet invariant

A new $B_2 \wedge H_4$ action principle

It turns out that we can construct all R^2 invariants by using an action corresponding to the supersymmetrization of a $B_2 \wedge H_4$ term:

- gauge 2-form B_2 of tensor multiplet, $H_3 = \mathrm{d}B_2$, hence dilaton-Weyl
- H_4 is a closed super 4-form $\mathrm{d}H_4 = 0$ based on $B_a{}^{ij}(z) = B_a{}^{(ij)}(z)$ a dimension-3 primary superfield satisfying the Bianchi identities

$$\nabla_\alpha^{(i} B^{\beta\gamma jk)} = -\frac{2}{3}\delta_\alpha^{[\beta}\nabla_\delta^{(i} B^{\gamma]jk)} \ , \quad [\nabla_\alpha^{(i}, \nabla_{\beta k}] B^{\alpha\beta j)k} = -8\mathrm{i}\nabla_{\alpha\beta} B^{\alpha\beta ij}$$

- By using the superform approach to construct SUSY invariants $\Longrightarrow$ locally superconformal invariant action principle:

$$\begin{aligned} S_{B_2\wedge H_4} &= \int \mathrm{d}^6 x\, e \left\{ \varepsilon^{abcdef} b_{ab} h_{cdef} - \frac{1}{4}\sigma C + \text{fermions} \right\} \\ h_{abcd} &= \frac{\mathrm{i}}{48}\varepsilon_{abcdef}(\tilde\gamma^e)^{\alpha\beta}\nabla_{\alpha k}\nabla_{\beta l} B^{f\,kl}|_{\theta=0} \ , \quad \nabla_{[a} h_{bcde]} \simeq 0 \\ C &:= \frac{\mathrm{i}}{12}(\tilde\gamma^a)^{\alpha\beta}\nabla_{\alpha k}\nabla_{\beta l} B_a{}^{kl}|_{\theta=0} \end{aligned}$$

$B_a{}^{ij}$ plays the role of a Lagrangian superfield

Riemann^2

It was first constructed by [Bergshoeff-Rakowski (1987)].
We can reproduce it by using the $B_2 \wedge H_4$ action principle with

$$B^{\alpha\beta ij} = -\frac{\mathrm{i}}{2}\Lambda^{\alpha(i}{}_{\gamma}{}^{\delta}\Lambda^{\beta j)}{}_{\delta}{}^{\gamma}$$

with a primary dimension 3/2

$$\begin{aligned}\Lambda^{\alpha i}{}_{\beta}{}^{\gamma} \quad = \quad & X^{i\,\alpha\gamma}_{\beta} - \frac{1}{3}\delta^{\alpha}_{\beta}X^{\gamma i} + \frac{1}{12}\delta^{\gamma}_{\beta}X^{\alpha i} + \frac{\mathrm{i}}{4}\Phi^{-1}\psi^{i}_{\beta}W^{\alpha\gamma} + \frac{\mathrm{i}}{12}\Phi^{-1}\delta^{\alpha}_{\beta}W^{\gamma\delta}\psi^{i}_{\delta} \\ & -\frac{\mathrm{i}}{12}\Phi^{-1}\delta^{\gamma}_{\beta}W^{\alpha\delta}\psi^{i}_{\delta} + \frac{\mathrm{i}}{12}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-1}\nabla_{\delta(\rho}\psi^{i}_{\beta)} - \frac{\mathrm{i}}{8}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-2}(\nabla_{\delta(\rho}\Phi)\psi^{i}_{\beta)} \\ & +\frac{\mathrm{i}}{32}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-2}H_{\rho\beta}\psi^{i}_{\delta} - \frac{1}{16}\varepsilon^{\alpha\gamma\delta\rho}\Phi^{-3}\psi^{i}_{\delta}\psi^{k}_{(\rho}\psi_{\beta)k}\end{aligned}$$

where

$$X^{\alpha i} := -\tfrac{\mathrm{i}}{10}\nabla^{i}_{\beta}W^{\alpha\beta}\ , \quad X^{k\,\alpha\beta}_{\gamma} = -\tfrac{\mathrm{i}}{4}\nabla^{k}_{\gamma}W^{\alpha\beta} - \delta^{(\alpha}_{\gamma}X^{\beta)k}$$

$$\psi^{i}_{\alpha} = \nabla^{i}_{\alpha}\Phi\ , \quad \nabla^{i}_{\alpha}\psi^{j}_{\beta} = -\tfrac{\mathrm{i}}{2}\varepsilon^{ij}(\gamma^{abc})_{\alpha\beta}H^{+}_{abc} - \mathrm{i}\varepsilon^{ij}(\gamma^{a})_{\alpha\beta}\nabla_{a}\Phi$$

$\Lambda^{\alpha i}{}_{\beta}{}^{\gamma}$ is a vector multiplet taking value in Lorentz algebra

Riemann^2

Then, in the gauge $\sigma = 1,\ b_m = 0$ reduced to components

$$\begin{aligned} e^{-1}\mathcal{L}_{\mathrm{Riem}^2} &= \mathcal{R}^{abcd}(\omega_-)\mathcal{R}_{ab\,cd}(\omega_-) - 4\mathcal{R}^{ab}{}_{ij}\mathcal{R}_{ab}{}^{ij} \\ &\quad -\tfrac{1}{4}\varepsilon^{abcdef}\, b_{ab}\,\mathcal{R}_{cd}{}^{gh}(\omega_-)\mathcal{R}_{ef\,gh}(\omega_-) + \text{fermions} \end{aligned}$$

It is only a functional of the Weyl and tensor (dilaton-Weyl) multiplets
Dependence on H_{abc} is in the torsionful Lorentz curvature $\mathcal{R}_{ab}{}^{cd}(\omega_-)$

$$\omega_{\pm\,m}{}^{cd} := \omega_m{}^{cd} \pm \tfrac{1}{2} e_m{}^a H_a{}^{cd}$$

such that

$$\mathcal{R}_{ab}{}^{cd}(\omega_\pm) = \mathcal{R}_{ab}{}^{cd} \pm \mathcal{D}_{[a} H_{b]}{}^{cd} - \tfrac{1}{2} H_{e[a}{}^{[c} H_{b]}{}^{d]e}.$$

A new curvature squared invariant

A new curvature squared invariant by using the $B_2 \wedge H_4$ action and the superfield [Butter-Kuzenko-Novak-Theisen (16)] $(Y_\alpha{}^{\beta ij} = -5/2 \nabla_\alpha^{(i} X^{\beta j)})$

$$B^{\alpha\beta\, ij} = -4 W^{\gamma[\alpha} Y_\gamma{}^{\beta]ij} - 32\mathrm{i}\, X_\gamma{}^{\alpha\delta(i} X_\delta{}^{\beta\gamma j)} + 10\mathrm{i}\, X^{\alpha(i} X^{\beta j)}$$

this leads to a new independent off-shell $\mathcal{R}^2$ invariant [Novak-Ozkan-Pang-GTM (17)]

$$\begin{aligned} S_{\rm new} \;=\; & \frac{1}{32}\int \mathrm{d}^6x\, e\Big\{\sigma C_{ab}{}^{cd} C_{cd}{}^{ab} + 3\sigma \mathcal{R}_{ab}{}^{ij}\mathcal{R}^{ab}{}_{ij} + \frac{4}{15}\sigma D^2 - 8\sigma T^{-dab}\nabla_d \nabla^c T^-_{abc} \\ & +4\sigma(\nabla_c T^{-abc})\nabla^d T^-_{abd} + 4\sigma T^{-abc} T^-_{ab}{}^d T^{-ef}{}_c T^-_{efd} - \frac{8}{45} H_{abc} T^{-abc} D \\ & -2H_{abc} C^{ab}{}_{de} T^{-cde} + 4H_{abc} T^-_d{}^{ab} \nabla_e T^{-cde} - \frac{4}{3} H_{abc} T^{-dea} T^{-bcf} T^-_{def} \\ & -\frac{1}{4}\varepsilon^{abcdef} b_{ab}\big(C_{cd}{}^{gh} C_{efgh} - \mathcal{R}_{cd}{}^{ij}\mathcal{R}_{ef\,ij}\big)\Big\} + \text{fermions} \end{aligned}$$

In the gauge $\sigma = 1$, $b_a = 0$

$$S_{\rm new} = \frac{1}{32}\int \mathrm{d}^6x\, e\Big\{\mathcal{R}_{abcd}\mathcal{R}^{abcd} - \mathcal{R}_{ab}\mathcal{R}^{ab} + \frac{1}{4}\mathcal{R}^2 + \cdots\Big\}$$

Application: Gauss-Bonnet $\mathcal{N} = (1,0)$ invariant

Constructed the new curvature squared invariant, we can describe an off-shell extension of the Gauss-Bonnet combination in six dimensions:

$$S_{\rm GB} = -3 S_{\rm Riem^2} + 128 S_{\rm new}$$

In the gauge $\sigma = 1$, $b_a = 0$

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm GB} \;\; = \;\; & \mathcal{R}_{abcd}\mathcal{R}^{abcd} - 4\mathcal{R}_{ab}\mathcal{R}^{ab} + \mathcal{R}^2 \\ & + \tfrac{1}{2}\mathcal{R}_{abcd}H^{abe}H^{cd}{}_e - \mathcal{R}^{ab}H^2_{ab} + \tfrac{1}{6}\mathcal{R}H^2 + \tfrac{1}{144}(H^2)^2 - \tfrac{1}{8}(H^2_{ab})^2 + \tfrac{5}{24}H^4 \\ & -\tfrac{1}{4}\epsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{gh}(\omega_+)\mathcal{R}_{ef\,gh}(\omega_+) + \epsilon^{abcdef}b_{ab}\mathcal{R}_{cd}{}^{ij}\mathcal{R}_{ef\,ij} + \text{fermions} \end{aligned}$$

where

$$H^2 := H_{abc}H^{abc} \;, \quad H^2_{ab} := H_a{}^{cd}H_{bcd} \;, \quad H^4 := H_{abe}H_{cd}{}^e H^{acf}H^{bd}{}_f$$

Advantages to know the off-shell (1,0) Gauss-Bonnet invariant:

- off-shell supersymmetry transformations completely under control (same as two derivative actions)
- complete off-shell descriptions of NSNS b_2-form which cannot be all recast in $\mathcal{R}_{ab}{}^{cd}(\omega_+)$ as tried in [Bergshoeff-Salam-Sezgin (1986-87)]
- possible to add the invariant to general sugra-matter couplings

Application: Einstein-Gauss-Bonnet supergravity

With $\mathcal{L}_{\mathrm{EH}}$ the 6D $\mathcal{N} = (1,0)$ off-shell Poincaré SUGRA constructed in [Bergshoeff-Sezgin-VanProeyen (1986)]

We can now consider the combination

$$\mathcal{L}_{\mathrm{EGB}} = \mathcal{L}_{\mathrm{EH}} + \frac{1}{16}\alpha' \mathcal{L}_{\mathrm{GB}}$$

off-shell extension of first order α'-corrected string theory effective action

- matches with on-shell string theory derivation of [Liu-Minasian (2013)] α'-corrected Type IIA reduced on K3, dual to Heterotic on T4.
- Action possesses an $\mathrm{AdS}_3 \times \mathrm{S}^3$ solution analogue of the $\mathrm{AdS}_5 \times \mathrm{S}^5$ solution in IIB string theory.
- By using the off-shel action, computed the α'-corrected KK spectrum of fluctuations around $\mathrm{AdS}_3 \times \mathrm{S}^3$ organized in short and long multiplets of isometry $\mathrm{SU}(1,1|2) \times \mathrm{SL}(2,R) \times \mathrm{SU}(2)$. see [de Boer (1999)]

Hints on dynamics of strings in $\mathrm{AdS}_3\times \mathrm{S}^3\times \mathrm{K3}(\mathrm{T}^4)$ backgrounds. [Novak-Ozkan-Pang-GTM (2017)]

Conclusion and Outlook

- We have constructed all off-shell 6D $\mathcal{N} = (1, 0)$ curvature-squared supergravity invariants
  [Novak-Ozkan-Pang-GTM (17)]
  [Butter-Novak-Ozkan-Pang-GTM (18)]
- The new 6D curvature-squared invariant complete an element missing since the 80s, see the Gauss-Bonnet
- Of importance in studying low energy String Theory and α'-corrected AdS/CFT ...
- Study properties of α'-corrections for solutions of GB and general curvature squared actions, e.g. Dyonic strings, ... (in progress)
- Extensions of $\mathcal{N} = (1, 0)$ curvature squared?
  general matter couplings and...
  How about $\mathcal{N} = (1, 1)$ (arising from Type IIA/Heterotic)?
  and $\mathcal{N} = (2, 0)$ (arising from Type IIB)?