Dipartimento di Fisica e Astronomia Galileo Galilei

Three-forms:

from Supergravity to Flux Compactification

Stefano Lanza
University of Padova

based on: arXiv:1706.09422, 1712.09366, 1803.01405
with I. Bandos, F. Farakos, L. Martucci, D. Sorokin

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Three-forms in Physics

Even though, in four dimensions, gauge three-forms

$$A_{\mu\nu\rho} \quad \text{with} \quad F_{\mu\nu\rho\sigma} = 4\partial_{[\mu}A_{\nu\rho\sigma]}$$

do not carry any propagating degrees of freedom, they can induce nontrivial physical effects.

Dynamical generation of the cosmological constant:

- Possible solution to the cosmological constant problem *(Brown, Teitelboim 1987 - Bousso, Polchinski 2000)*

Gauging the shift symmetry:

- Possible solution to the CP problem *(Dvali 2003)*
- New inflationary models *(Kaloper, Sorbo 2008)*

Plan of the talk

How gauge three-forms and membranes can be embedded into a four dimensional *N=1* Supergravity theory

↓

The UV completion:

- three-forms from flux compactification;
- membranes from wrapped higher dimensional branes.

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A simple model

Let us consider a simple model made up by a single chiral superfield

$$\Phi = \{\varphi, \psi^\alpha, f\}$$

with Lagrangian

$$\mathcal{L} = -\partial_\mu \bar{\varphi} \partial^\mu \varphi - i\psi\sigma^\mu\partial_\mu\bar{\psi} + \bar{f}f + \bar{b}f + b\bar{f}$$

where is b a complex constant. Integrating out f

$$f = -b$$

Plugging it back into the Lagrangian gives

$$\mathcal{L} = -\partial_\mu \bar{\varphi} \partial^\mu \varphi - i\psi\sigma^\mu\partial_\mu\bar{\psi} - b\bar{b}$$

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Positive potential: supersymmetry is spontaneously broken

A simple model: introducing three-forms

The previous model can also be recast in terms of gauge three-forms.

$$f \xrightarrow{\text{trade}} A_{\mu\nu\rho} \longrightarrow {}^*F_4 \equiv \frac{1}{4!}\varepsilon^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma}$$

We build a new Lagrangian

$$\mathcal{L} = -\partial_\mu \bar{\varphi}\partial^\mu \varphi - i\psi\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{24}\bar{F}^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} + \mathcal{L}_{\rm bd}$$

with the boundary terms $\mathcal{L}_{\rm bd} = \frac{1}{6}\partial_\mu\left(\bar{A}_{\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right) + {\rm c.c.}$

The solution to the three-form equation

$$\partial_\mu F^{\mu\nu\rho\sigma} = 0 \quad \Rightarrow \quad F^{\mu\nu\rho\sigma} = b\,\varepsilon^{\mu\nu\rho\sigma}$$

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Also boundary terms contribute to the on-shell potential.
The constant parameter b is here dynamically generated!

A simple model: three-forms in Superspace

$$\mathcal{L} = \int d^4\theta\, \Phi\bar{\Phi} + \left(\int d^2\theta\, \bar{b}\, \Phi + \text{c.c.}\right)$$

whose components are

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$$\Phi \to S \equiv -\frac{1}{4}\bar{D}^2\bar{\Sigma} \qquad \text{with} \qquad \bar{D}^2\Sigma = 0$$

Σ is a *complex linear multiplet*, with $\Sigma = \varepsilon^{\mu\nu\rho\sigma}(\theta\sigma_\mu\bar{\theta})A_{\nu\rho\sigma} + \ldots$

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$$\mathcal{L} = \int \mathrm{d}^4\theta\, S\bar{S} + \frac{1}{16}\left(\int \mathrm{d}^2\theta \bar{D}^2 - \int \mathrm{d}^2\bar{\theta} D^2\right)\left(\Sigma D^2 S - \bar{\Sigma}\bar{D}^2\bar{S}\right)$$

where no superpotential appears.

Its components are

$$\mathcal{L} = -\partial_\mu\bar{\varphi}\partial^\mu\varphi - i\psi\sigma^\mu\partial_\mu\bar{\psi} - \frac{1}{24}\bar{F}^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} + \mathcal{L}_{\text{bd}}$$

Three-forms and Membranes in Supergravity

Supergravity coupled with three-forms

How to insert gauge three-forms in Supergravity?

The field content is

Gravity Multiplet + (N+1) Chiral Multiplets

with N+2 (complex) scalar auxiliary fields $M, \; f^I$.

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The aim is to trade

$$\boxed{\begin{matrix} M \\ f^I \end{matrix}} \longrightarrow \boxed{\begin{matrix} {}^*F_4^I \\ {}^*\tilde{F}_{4J} \end{matrix}} \quad \text{which contain} \quad \boxed{\begin{matrix} A_3^I \\ \tilde{A}_{3J} \end{matrix}}$$

at the superspace level, this means

$$\boxed{\begin{matrix} \Phi^I \\ \text{with} \quad -\frac{1}{4}\mathcal{D}^2\Phi^I| \equiv f^I \end{matrix}} \longrightarrow \boxed{\begin{matrix} S^I \equiv \frac{1}{4}(\bar{\mathcal{D}}^2 - 8\mathcal{R})\mathcal{M}^{IJ}(\Sigma_J - \bar{\Sigma}_J) \\ \text{with} \quad -\frac{1}{4}\mathcal{D}^2 S^I| = \bar{M}s^I - \frac{\mathrm{i}}{2}\mathcal{M}^{IJ\,*}\mathcal{F}_{4J} \\ \text{and} \quad \mathcal{F}_{4I} \equiv \tilde{F}_{4I} - \bar{\mathcal{G}}_{IJ}F_4^J \end{matrix}}$$

Supergravity coupled with three-forms

The bosonic action is

$$S_{\rm bos} = \int \mathrm{d}^4x\, e \left(-\frac{R}{2} - K_{i\bar{\jmath}}\, \partial\phi^i \partial\bar{\phi}^{\bar{\jmath}} - V(A_3, \tilde{A}_3) \right)$$

with the off-shell potential expressed in terms of gauge three-forms

$$V(A_3, \tilde{A}_3) = \mathcal{T}^{IJ}\, {}^*\bar{\mathcal{F}}_{4I}\, {}^*\mathcal{F}_{4J} + e^{-1}\mathcal{L}_{\rm bd} \quad \text{with} \quad \mathcal{F}_{4I} \equiv \tilde{F}_{4I} - \bar{\mathcal{G}}_{IJ} F_4^J$$

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Setting the three-forms on-shell

$$2{\rm Re}(\mathcal{T}^{IJ}\, {}^*\mathcal{F}_{4J}) = m^I\,, \quad 2{\rm Re}(\mathcal{G}_{IJ}\mathcal{T}^{JK}\, {}^*\mathcal{F}_{4K}) = e_I\,,$$

gives an on-shell potential

$$V(A_3, \tilde{A}_3)|_{\text{on-shell}} = V(e, m)$$

This is the same potential that would have been obtained starting from

$$W(\Phi) = (e_A \mathcal{Z}^A + m^A \mathcal{G}_A(\mathcal{Z}))|_{\mathcal{Z}^0=1,\, \mathcal{Z}^I=\Phi^I}$$

and computing

$$V(e, m) = e^K (|DW|^2 - 3|W|^2)$$

Hence, the constants (e_A, m^A) are dynamically generated by the three-forms.

Membranes in Supergravity

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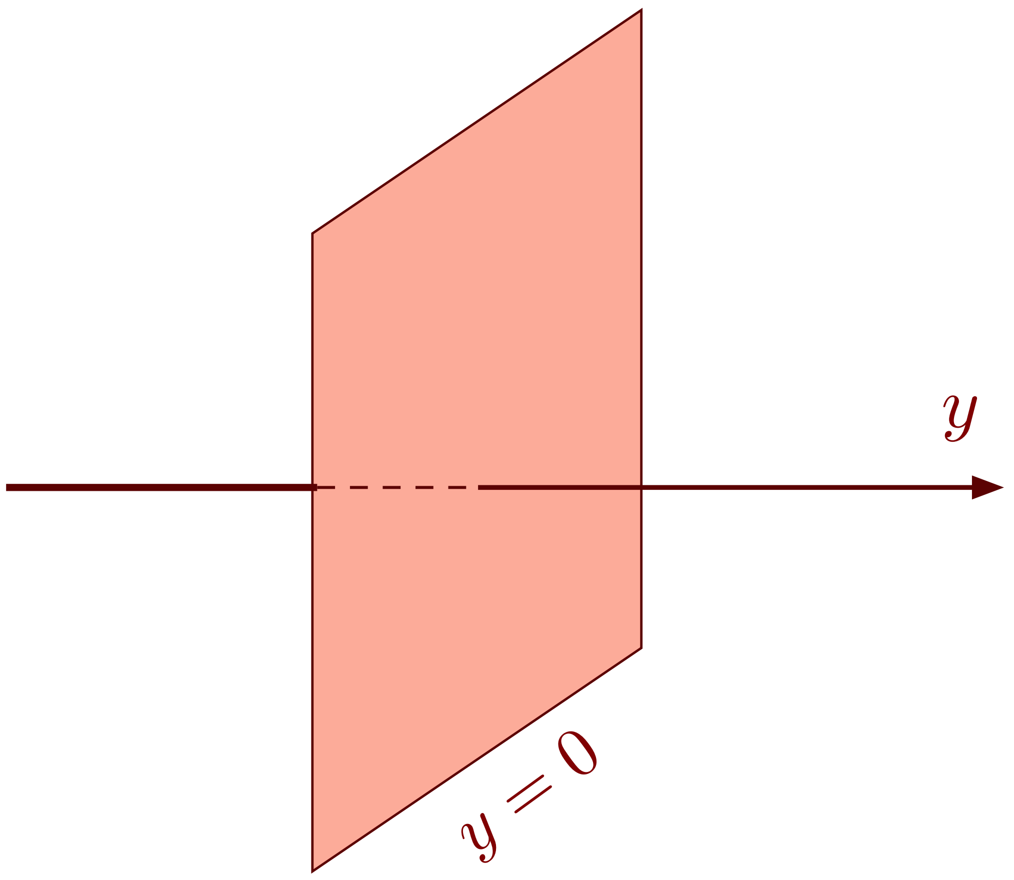
One of the main advantages of having three-forms in Supergravity is the natural coupling to membranes.

- We shall consider a single *flat* three-dimensional membranes localized at $y = 0$;
- These are *1/2-BPS* objects: on their worldvolume, they preserve one half of the *bulk* supersymmetry;
- At the bosonic level they can be coupled via some minimal coupling

$$q \int_{\mathcal{M}} A_3$$



Coupling Supergravity to Membranes

Membranes can be naturally embedded into a Supergravity theory by adding the following contribution

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$$S_{\mathrm{WZ}} = \int_{\mathcal{M}} \left(q_I \mathcal{A}_3^I - p^I \tilde{\mathcal{A}}_{3I} \right)$$

which is the Wess-Zumino term and dictates the coupling of the membrane. Here

$$(\mathcal{A}_3^I, \tilde{\mathcal{A}}_{3I})$$

are *super three-forms*, whose lowest components are ordinary three-forms

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$$S_{\rm NG} = -2 \int_{\mathcal{M}} {\rm d}^3\xi \sqrt{-h}\, \left| q_I S^I - p^I \mathcal{G}_I(S) \right|$$

which is the Nambu-Goto term and determines the dynamics of the membrane. Here, the chiral superfields are double three-form multiplets.
Its form is fully determined by *kappa-symmetry.*
The tension can be readily read off

$$T_{M2} = 2\, e^{\frac{K}{2}} \left| q_I S^I - p^I \mathcal{G}_I(S) \right| \big|_{\mathcal{M}}$$

Influence of the Membrane over the Effective Potential

The Wess-Zumino coupling term

$$S_{\rm WZ} = \int_{\mathcal{M}} \left(q_I \mathcal{A}_3^I - p^I \tilde{A}_{3I} \right)$$

modifies the solution to the equations of motion as

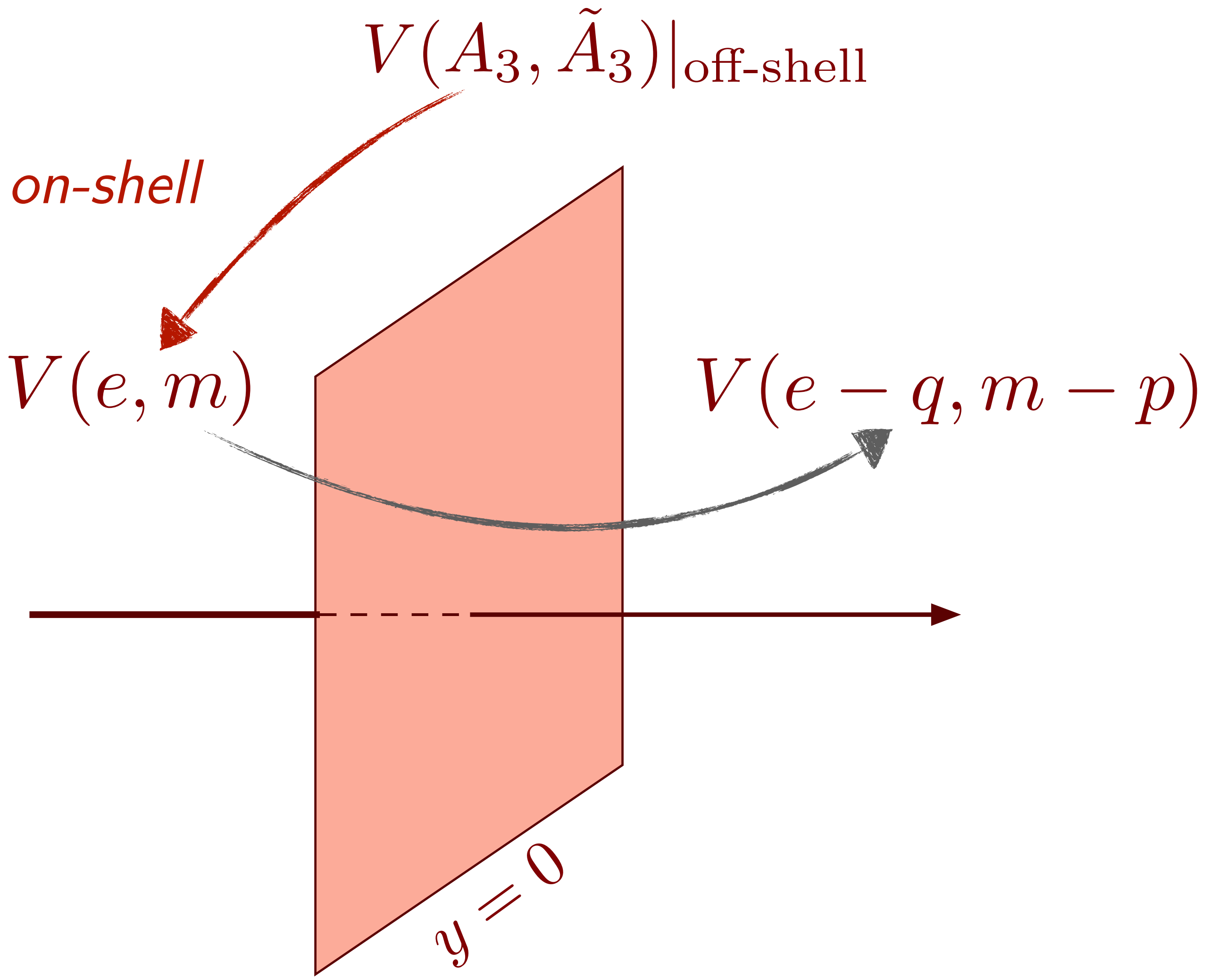
$$2{\rm Re}\left(\mathcal{T}^{IJ*}\mathcal{F}_{4J}\right) = m^I - p^I\Theta(y)\,, \quad 2{\rm Re}\left(\mathcal{G}_{IJ}\mathcal{T}^{JK*}\mathcal{F}_{4K}\right) = e_I - q_I\Theta(y)$$

This reflects on the on-shell expression of the potential

$$V(A_3, \tilde{A}_3)|_{\text{on-shell}} = \begin{cases} V(e, m) & \text{for} \quad y < 0 \\ V(e-q, m-p) & \text{for} \quad y > 0 \end{cases}$$

Scanning the Landscape

The potential changes once the membrane is crossed.



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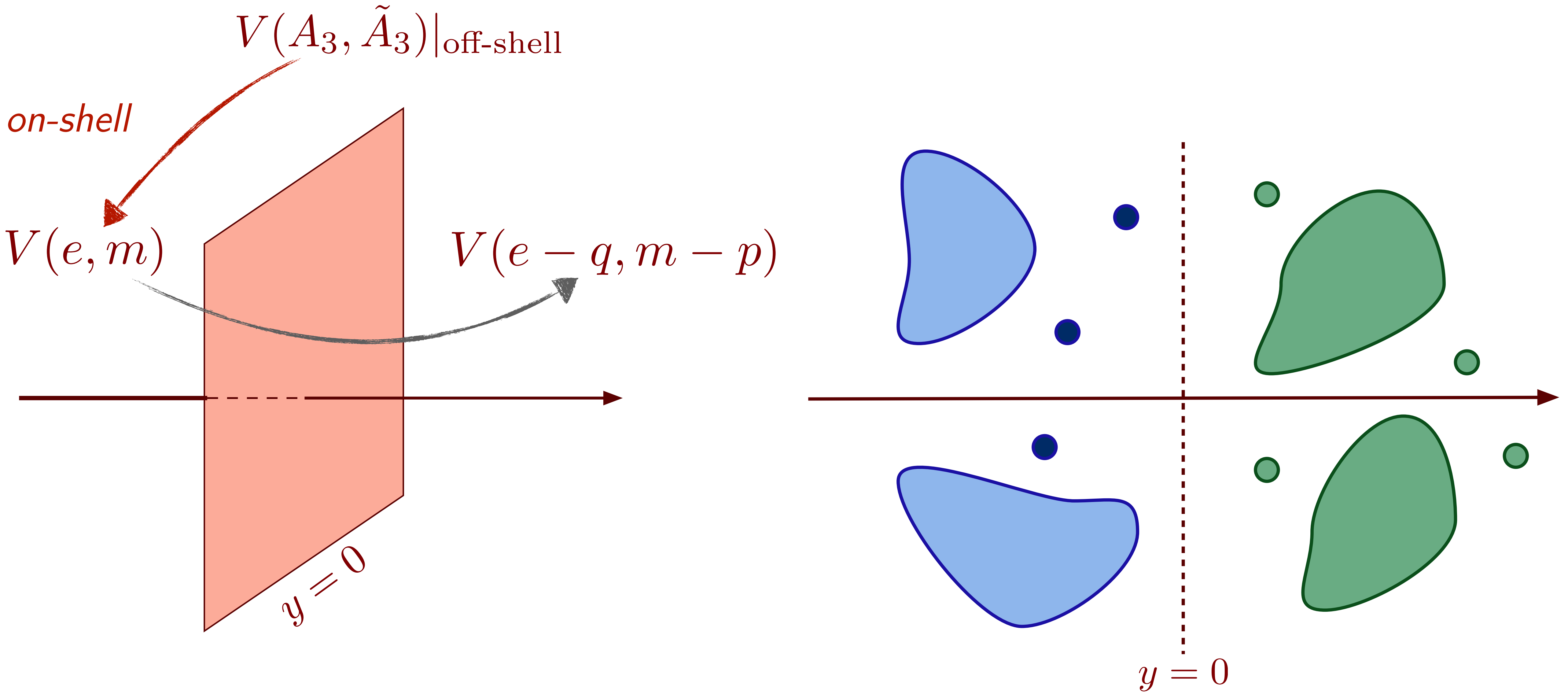

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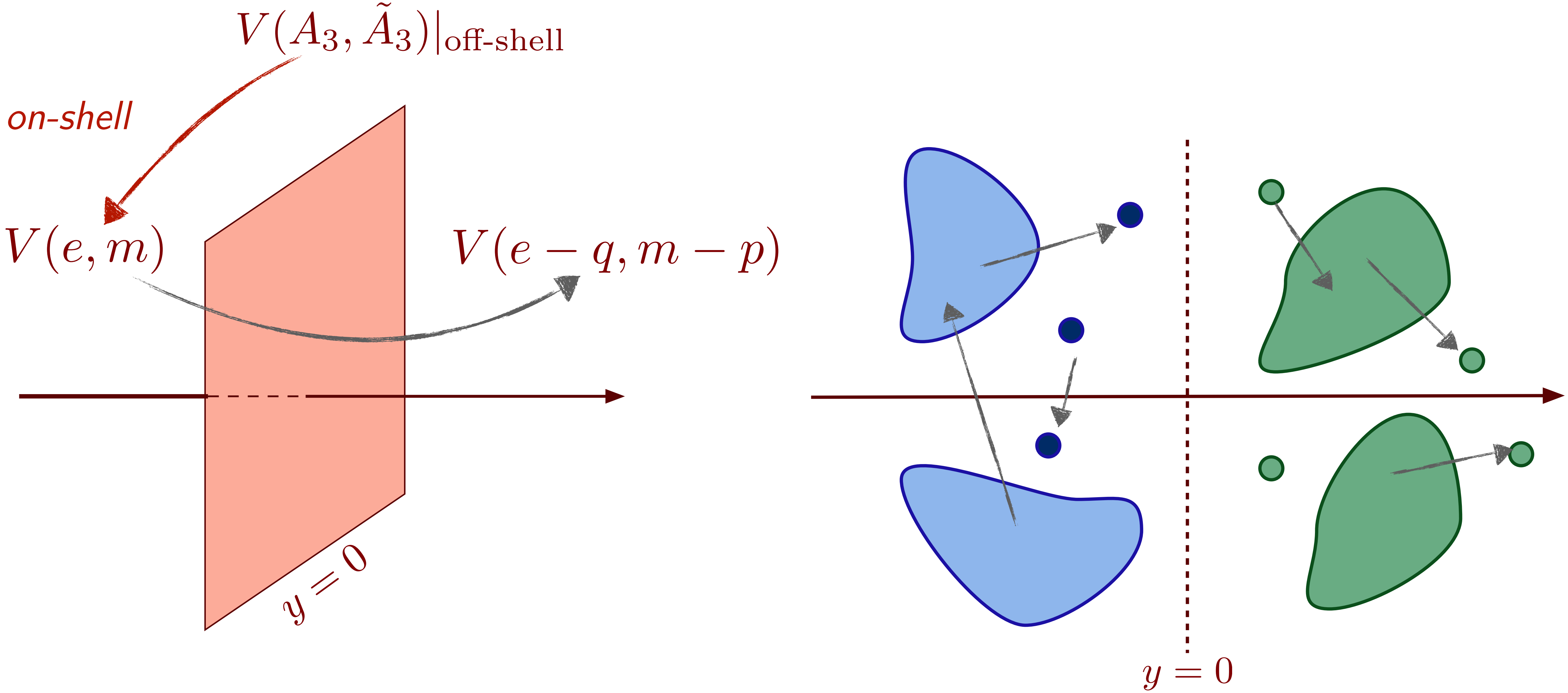

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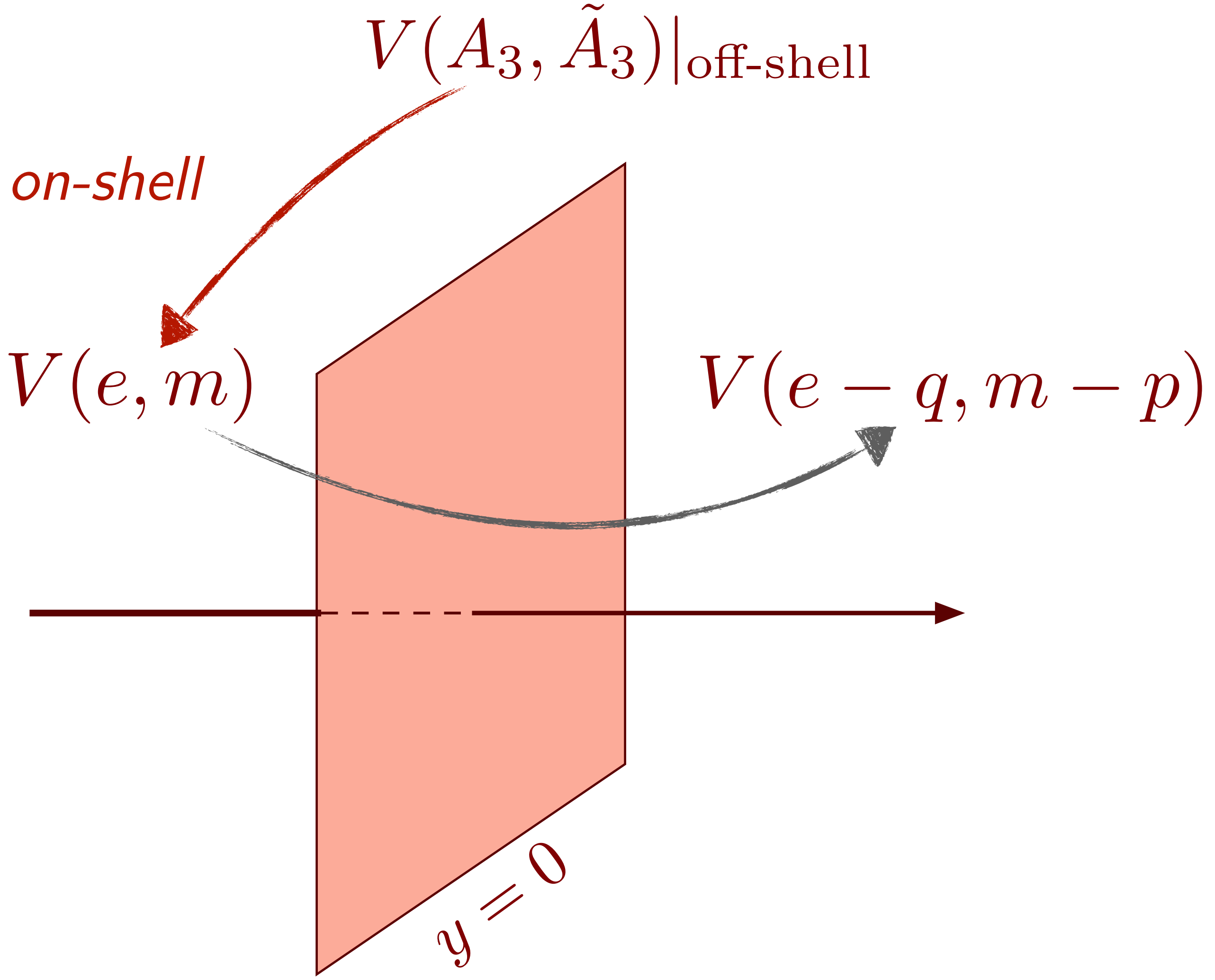
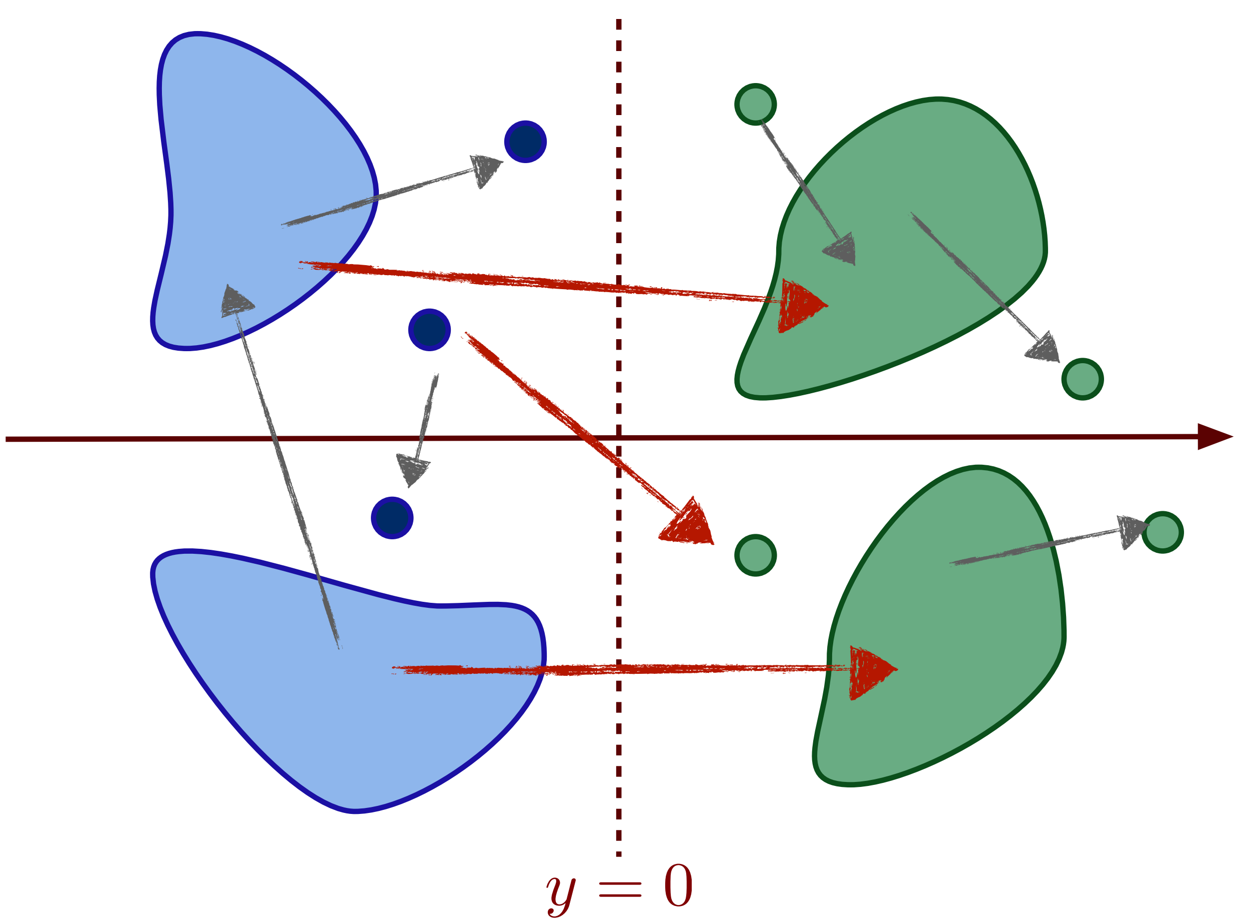


New domain wall solutions are generated, which connect the vacua on the two sides



The UV completion: three-forms and membranes from compactifications

Type IIA String Theory

We consider Type IIA String Theory compactified over a Calabi-Yau three-fold Y with orientifold

$$M_{10} = \mathbb{R}^{1,3} \times Y$$

$$\downarrow$$

The four dimensional effective field theory is a N=1 Supergravity

The field content of the closed string sector is

- Gravity
- Scalar Sector
  - Kahler moduli

    $$\varphi^i = v^i + ib^i \quad \text{with} \quad i = 1, \ldots, h_-^{1,1}(Y)$$

    These are the lowest components of some superfields $\Phi^i|_{\theta=\bar{\theta}=0} \equiv \varphi^i$
  - Complex structure moduli and axio-dilaton

    $$t^p \quad \text{with} \quad p = 1, \ldots, h^{2,1}(Y) + 1$$

    and they are $T^p|_{\theta=\bar{\theta}=0} \equiv t^p$

Type IIA String Theory

They determine:

- Kähler potential

  In the large volume and constant warping approximation, the Kähler potential is

$$K(\Phi, T, \bar{\Phi}, \bar{T}) = K(\Phi, \bar{\Phi}) + K(T, \bar{T})$$

  and the Kähler and complex structure moduli manifolds factorize

$$\mathcal{M}_{\text{moduli}} = \mathcal{M}_K \times \mathcal{M}_{cs}$$

- Superpotential

  The Superpotential originating from compactification is

$$W(\Phi, T) = e_0 + i e_i \Phi^i - \frac{1}{2} k_{ijk} m^i \Phi^j \Phi^k + \frac{i}{6} m^0 k_{ijk} \Phi^i \Phi^j \Phi^k + \hat{W}(\Phi, T)$$

  which is of the same as the one we considered before for the choice of prepotential

$$W(\Phi, T) = (e_A \mathcal{Z}^A + m^A \mathcal{G}_A(\mathcal{Z}) + \hat{W}(\mathcal{Z}, T))|_{\mathcal{Z}^0 = 1,\, \mathcal{Z}^i = i\Phi^i}$$

  with

$$\mathcal{G}(\mathcal{Z}) = \frac{1}{6\mathcal{Z}^0} k_{ijk} \mathcal{Z}^i \mathcal{Z}^j \mathcal{Z}^k$$

Type IIA String Theory

- Gauge Sector *(neglecting the NS sector)*

  In the *democratic formulation* all the even field strengths are taken into account

  $$F_0\,,F_2\,,F_4\,,F_6\,,F_8\,,F_{10}$$

  where $F_{2n}=\mathrm{d}A_{2n-1}$ and with the constraint $F_{2n}=*F_{10-2n}$

  We can also consider *internal fluxes*

  $$\bar{G}_0\,,\bar{G}_2\,,\bar{G}_4\,,\bar{G}_6\,,$$

  which thread the compactification space and are quantized

  $$\bar{G}_0=m^0\,,\int_{\tilde{\pi}^i}\bar{G}_2=m^i\,,\int_{\pi_i}\bar{G}_4=e_i\,,\int_Y\bar{G}_6=e_0$$

  with $m^0,m^i,e_i,e_0\in\mathbb{Z}$.

  The total fluxes can then be collected into a single ***polyform***

  $$\boxed{\mathbf{G}=\mathrm{d}\mathbf{A}+\bar{\mathbf{G}}}$$

Three-forms from Type IIA String Theory

Expanding the polyform $\mathbf{G} = \mathrm{d}\mathbf{A} + \bar{\mathbf{G}}$ in a basis of Calabi-Yau harmonic forms

$$G_0 = m^0, \qquad G_2 = m^i\,\omega_{2i} + \ldots,$$

$$G_4 = F_4^0 + e_i\,\tilde{\omega}_4^i + \ldots, \qquad G_6 = F_4^i \wedge \omega_{2i} + e_0\omega_6 + \ldots,$$

$$G_8 = \tilde{F}_{4i} \wedge \tilde{\omega}_4^i + \ldots, \qquad G_{10} = \tilde{F}_{40} \wedge \omega_6 + \ldots\ ,$$

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Plenty of four-form field strengths appear in the four dimensional effective field theory!

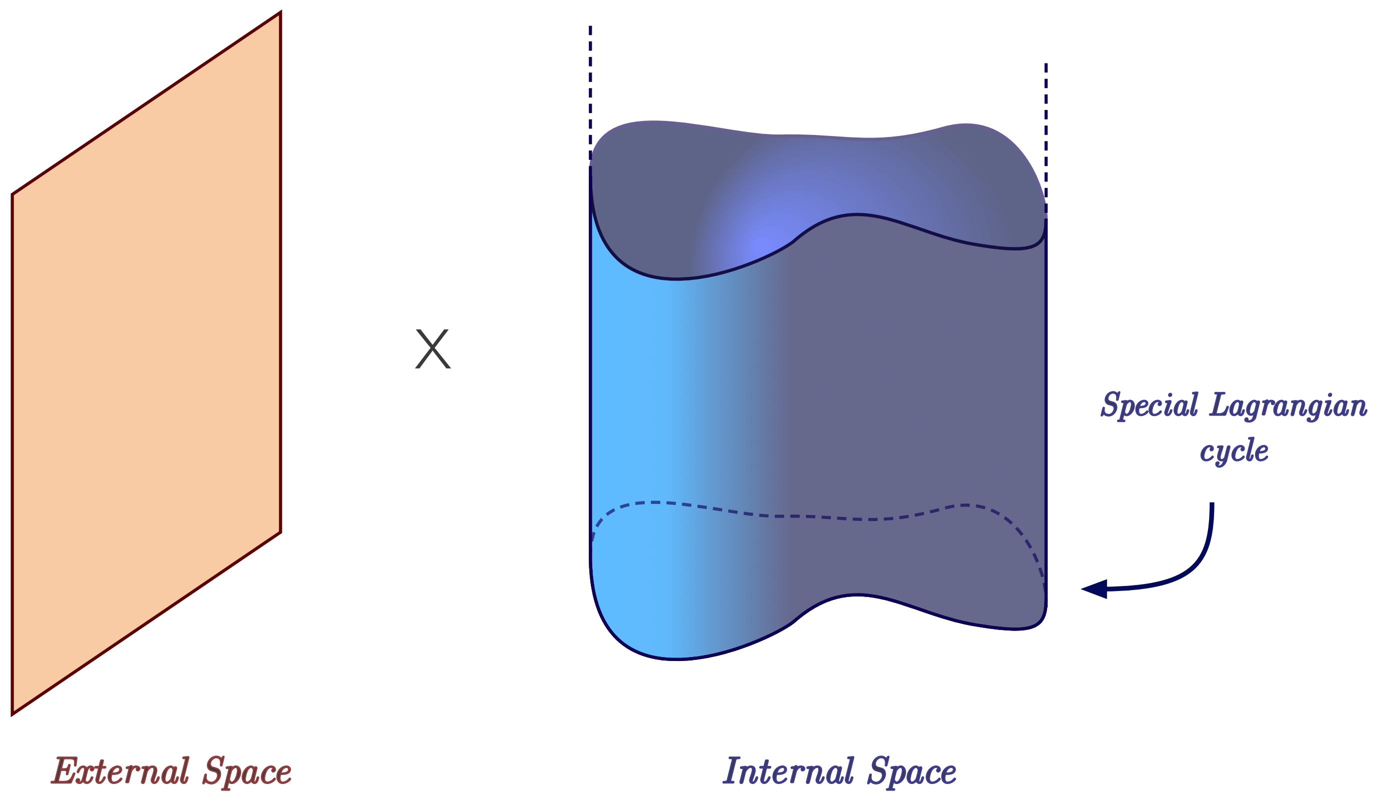
Along with the Kähler moduli, they organize into chiral multiplets

$$\Phi^I = \{\varphi^I, \psi^{I\alpha}, {}^*F_4^I, {}^*\tilde{F}_{4I}\}$$

which are endowed with gauge three-forms in their auxiliary components.
These are the multiplets which build up the four-dimensional EFT.

Tension from Branes

In Type IIB, consider a D5-brane wrapping a Special Lagrangian three-cycle in the internal space



The tension is given by the volume of the wrapped Lagrangian cycle

$$T_{M2} = e^{\frac{K}{2}} \int_{\Sigma} \mathrm{d}\,\mathrm{vol}_{\Sigma}$$

Upon using some calibration conditions and the expanding $\Sigma = q_I \Sigma^I - p^I \tilde{\Sigma}_I$ we get

$$T_{M2} = e^{\frac{K}{2}} \left| \int_{\Sigma} \Omega \right| = 2\, e^{\frac{K}{2}} \left| q_I S^I - p^I \mathcal{G}_I(S) \right|$$

coherently with its four dimensional counterpart.

Conclusion and future outlook

We have seen an effective four-dimensional *N=1* Supergravity theory which

- includes gauge three-forms as auxiliary fields;
- naturally allows for the coupling with membranes, enriching the landscape;
- is coherent with its ultraviolet completion.

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- naturally allows for the coupling with membranes, enriching the landscape;
- is coherent with its ultraviolet completion.

Further developments

- inclusion of NS and open string sector in the effective theory;
- implementing the tadpole cancellation condition directly in four-dimensions;
- extension to $\mathcal{N}=2$ case.

Thank you!