Holographic phase transition in $\mathcal{N} = 4$ defect theory

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Outline

- Motivations
- Description of the set-up
- Results in the field theory side (weak coupling)
- Results in the gravity side (strong coupling)
- Description of the phase transition (order,..)
- Conclusions

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AdS/CFT correspondence

- Important example of the conjecture: duality between

$\mathcal{N} = 4$ Super Yang-Mills (SYM) in 4 dimensions (all N and g_{YM})	$\Longleftrightarrow$	Type IIB String Theory on $AdS_5 \times S_5$

- **Weak/strong duality**: relates the non-perturbative strong coupling regime of one theory to the weak coupling perturbative regime of the other

  $\Rightarrow$ very interesting, but also difficult to check the correspondence

- We can consider the **Maldacena Wilson Loop** in particular setups in order to have non trivial check of the duality

Motivations

- dCFTs with holographic duals constitute an interesting new arena for precision tests of the AdS/CFT and for the search for integrable structures correspondence

- Non-vanishing one-point functions already at tree level

- **Double scaling limit**: $\frac{\lambda}{k^2}$
  - sugra computations (valid for large λ) $\rightarrow$ considered for **large k** in such a way that λ/k^2 is kept small
  - the results **on both side** of the correspondence are found to be expressible in powers of λ/k^2

    $\Rightarrow$ weak/strong computations are comparable

Defect Theory

Defect Conformal Field Theory

[DeWolfe, Freedman, Ooguri, 2003]

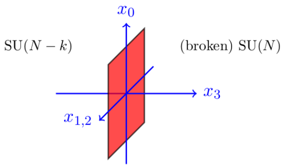
[Mortensen, de Leeuw, Ipsen, Kristjansen and Wilhelm, 2017]

- We consider a **codimension one defect**, inserted at $x_3 = 0$, connecting two $\mathcal{N} = 4$ SYM theories with gauge groups $SU(N)$ and $SU(N-k)$
- The interface reduces the total symmetry but preserves **conformal invariance**
- **Higgsing**: three of the scalar fields acquire an x_3-dependent vevs:

$$\langle \Phi_i(x) \rangle_{cl} = -\frac{1}{x_3} t_i \oplus 0_{(N-k)\times(N-k)} \qquad i = 1,2,3 \qquad x_3 > 0$$

$t_i :$ k-dimensional irreducible representation of the $SU(2)$ algebra

- The defect preserves 1/2 of the supersymmetries
- Original superconformal symmetry $PSU(2,2|4)$ of $\mathcal{N} = 4$ SYM $\rightarrow$ broken down to the subgroup $OSp(4|4)$



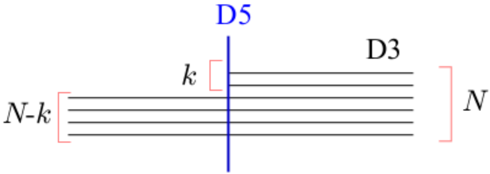
Holographic dual

- Realization in the String Theory side: the **D3-D5 brane configuration**
- D5 → probe brane in $AdS_5 \times S^5$

	0	1	2	3	4	5	6	7	8	9
D3	○	○	○	○	×	×	×	×	×	×
D5	○	○	○	×	○	○	○	×	×	×

- **D5 geometry** is $AdS_4 \times S^2$ and a certain background gauge field has a non vanishing flux $\kappa = \frac{\pi k}{\sqrt{\lambda}}$ on S^2

  ⇒ k out of the N D3 branes get dissolved in the D5 brane:



[Nagasaki, Tanida, Yamaguchi, 2012]

The Wilson loop in $\mathcal{N} = 4$ SYM

$$W(C) = \frac{1}{N} \mathrm{Tr}\, P \exp \left\{ \oint_C d\tau \, \left(i A_\mu \dot{x}^\mu + \Phi_i \, |\dot{x}| \, \theta^i \right) \right\}$$

- describes the phase factor linked to a massive quark in the fundamental repp-resentation of the gauge group $SU(N)$
- euclidean signature
- $x^\mu(\tau)$ is a parametrization of the loop
- θ_i is a unit six-vector that describes a point on S^5 $(\theta_i \theta^i = 1)$

Maldacena Wilson Loop in $\mathcal{N} = 4$ SYM

- **Holographic dual**: in the supergravity limit

$$\lambda \to \infty, \ \alpha' \to 0 \qquad (\lambda = g_{YM}^2 N \quad \text{'t Hooft coupling})$$

> $\langle W(C) \rangle =$ minimal area of the string worldsheet ending on the loop C

- Susy transformation on the Wilson loop:

$$\delta_\epsilon A^\mu = \bar{\Psi} \Gamma^\mu \epsilon$$

$$\delta_\epsilon \Phi^i = \bar{\Psi} \Gamma^i \epsilon$$

- Some of the supersymmetry will be preserved if:

$$\left(i \Gamma_\mu \dot{x}^\mu + \Gamma^i \left| \dot{x} \right| \theta_i \right) \epsilon(x) = 0$$

- $\epsilon(x) = \epsilon_0 + \Gamma^\mu x_\mu \epsilon_1$ 16-component Majorana-Weyl spinors
  ($\epsilon_0 \to$ Poincaré supersymmetries, $\epsilon_1 \to$ conformal supersymmetries)
- **Circular Wilson loop**: 1/2 BPS object

Circular Wilson loop in $\mathcal{N} = 4$ defect theory

- We consider a **circular Wilson Loop** of **radius R** placed on a plane parallel to the defect at a **distance L** from it:

$$W(C) = \frac{1}{N} \mathrm{Tr}\, P \exp \left\{ \oint_C d\tau \; (iA_\mu \dot{x}^\mu - |\dot{x}| \, (\sin\chi\Phi_3 + \cos\chi\Phi_6)) \right\}$$

$$x^\mu = (0, R\cos\tau, R\sin\tau, L)$$

$$|\dot{x}| = R$$

$\chi \in \; [0, \frac{\pi}{2}\,]$ angle on S^5

- $\chi = 0$ **BPS point**, the operator + the defect preserve **1/4** of the supercharges

- conformal invariance $\rightarrow$ $\langle W \rangle$ depends on R and L only through the ratio R/L

- We explore the interaction of W with the defect in two different regimes:
  - **weak coupling limit** $\rightarrow$ perturbative computations in the gauge theory side
  - **strong coupling limit** $\rightarrow$ non-perturbative computations in the string thoery side

Weak coupling limit

[Aguilera-Damia, Correa, Giraldo-Rivera, 2017]

- **Perturbative expansion**

$$\langle W \rangle = \langle W \rangle_0 + \langle W \rangle_1 + \langle W \rangle_2 + \ldots$$

- $\langle W \rangle_0$ is the **tree-level term** which receives contribution from the classical value of the massive scalar Φ_3

$$\langle W \rangle_0 = (N-k) + \frac{\sinh\left(\frac{\pi R \sin\chi}{L} k\right)}{\sinh\left(\frac{\pi R \sin\chi}{L}\right)}$$

Standard tree level contribution (with the substitution $N \to N-k$)

Interaction with the defect

- tadpole contribution: $\langle W \rangle_1 = 0$ since 1-point function at 1-loop vanishes after regularization

Weak coupling

- **Rainbow and ladder contributions** (planar limit):

$$\langle W\rangle_2 = \langle W\rangle_{(2)} + (N-k)\,\frac{g^2_{YM}R}{4\pi L}\int_0^\infty dr\,r\int_0^\pi d\delta\,\left(\frac{\sinh\left(\frac{(\pi-\delta)R\sin\chi}{L}k\right)}{\sinh\left(\frac{\pi(\pi-\delta)R\sin\chi}{L}\right)}+\right.$$

$$\left.+\ \frac{\sinh\left(\frac{(\pi+\delta)R\sin\chi}{L}k\right)}{\sinh\left(\frac{\pi(\pi+\delta)R\sin\chi}{L}\right)}\right)\left(\mathcal{J}_1+\sin^2\chi\,\mathcal{J}_2\right)$$

$$\mathcal{J}_1 = 2\cos\frac{\delta}{2}\sin\left(\frac{2\pi R}{L}\right) I_{\frac{k}{s}}(r)\,K_{\frac{k}{s}}(r)$$

$$\mathcal{J}_2 = \frac{\sin\left(\frac{2\pi R}{L}\cos\frac{\delta}{2}\right)}{\cos\frac{\delta}{2}}\left(\frac{k-1}{2k} I_{\frac{k+2}{2}}(r)\,K_{\frac{k+2}{2}}(r) + \frac{k+1}{2k} I_{\frac{k-2}{2}}(r)\,K_{\frac{k-2}{2}}(r) - I_{\frac{k}{2}}(r)\,K_{\frac{k}{2}}(r)\right)$$

- $\mathcal{J}_1$ and $\mathcal{J}_2$ complicated integrals $\to$ we can solve them in tow different limit:
  - $\frac{L}{R}\to 0$ circle very close to the defect
  - $\frac{R}{L}\to 0$ circle far from the defect

$\frac{L}{R} \to 0$ limit

- **tree level**:

$$\langle W \rangle^{I}_{(0)} = N - K \qquad \langle W \rangle^{II}_{(0)} = e^{\frac{(k-1)\pi R}{L}\sin\chi}$$

- **1-loop**:

$$\langle W \rangle^{I}_{(2)} = \frac{g^2_{YM}(N-k)^2}{8}$$

$$\langle W \rangle^{II}_{(2)} = \frac{\lambda R}{4\pi L k} e^{\frac{(k-1)\pi R}{L}\sin\chi} \frac{1}{\cos^3\chi}\left(\frac{\pi}{2} - \chi - \frac{1}{2}\sin 2\chi\right)\left(\sin^2\chi + \frac{L^2}{R^2}\right)$$

- $\langle W \rangle^{I}$ and $\langle W \rangle^{II}$ correspond to different configuration in the string theory side:
  - $\langle W \rangle^{I} \to$ **the string does not end on the D5**
  - $\langle W \rangle^{II} \to$ **the string do end on the D5**

$$\log \langle W \rangle^{II} \simeq \frac{k\pi R}{L}\left(\sin\chi + \frac{\lambda}{4\pi^2 k^2}\frac{1}{\cos^3\chi}\left(\frac{\pi}{2} - \chi - \frac{1}{2}\sin 2\chi\right)\left(\sin^2\chi + \left(\frac{L}{R}\right)^2\right)\right)$$

(large R/L and large k)

$\frac{R}{L} \to 0$ limit

$$\langle W \rangle^{II}_{(2)} = I_{BPS} + \sin^2\chi\, I$$

- Expanding for $\frac{R}{L} \to 0$:

$$\langle W \rangle^{II}_{(2)} = I^{(0)}_{BPS} + I^{(2)}_{BPS}\, x^2 \log x +$$
$$+ k(N-k)\frac{g^2_{YM} R}{L} \sin^2\chi \left[-\frac{x}{2}\log x + \frac{x}{4}\left[2\psi^{(0)}\left(\frac{k+1}{2}\right) + 3\gamma - 1 + \psi^{(0)}\left(\frac{3}{2}\right)\right]\right]$$

- The asymptotic expantion of I contains an $\boldsymbol{x}\,\mathbf{log}\boldsymbol{x}$ behavior:

$\Longrightarrow$ it could be an hint of an **anomalous dimension**

String Theory side

- AdS_5 **metric** (Poincaré patch):

$$ds^2_{AdS} = \frac{1}{y^2}\left(-dt^2 + dr^2 + r^2 d\phi^2 + dx_3^2 + dy^2\right)$$

- S^5 **metric**:

$$ds^2_{S^5} = d\theta^2 + \sin^2\,\theta d\Omega_2^2 + \cos^2\theta\, d\tilde{\Omega}_2^2$$

- Polyakov action in the conformal gauge:

$$S = \frac{\sqrt{\lambda}}{4\pi}\int d\tau d\sigma \frac{1}{y^2(\sigma)}(y'^2(\sigma) + r'^2(\sigma) + r^2(\sigma) + x_3'^2(\sigma) + y^2(\sigma)\theta'^2(\sigma))$$

- Using the **Virasoro constraint** (VC)

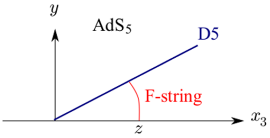
$$y'^2(\sigma) + r'^2(\sigma) + x_3'^2(\sigma) + y^2(\sigma)\theta'^2(\sigma) = r^2(\sigma)$$

- The action becomes:

$$S = \frac{\sqrt{\lambda}}{2\pi}\int d\tau d\sigma \frac{r^2(\sigma)}{y^2(\sigma)} = \sqrt{\lambda}\sqrt{\frac{m^2-1}{k^2+1}}\left(\left(1 - \frac{E(k^2)}{K(k^2)}\right)\tilde{s} - \frac{\pi}{2K(k^2)}\frac{\vartheta_1'\left(\frac{\pi\,\tilde{s}}{2K(k^2)}\right)}{\vartheta_1\left(\frac{\pi\,\tilde{s}}{2K(k^2)}\right)}\right)$$

Boundary conditions

- **Fundamental string**
- $\rightarrow$ stretched from the boundary $(\sigma = 0)$ to the D5 $(\sigma = \tilde{\sigma})$

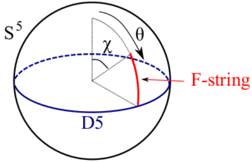


- **boundary conditions** in $\sigma = 0$:

$$r(0) = R \qquad y(0) = 0$$
$$x_3(0) = L \qquad \theta(0) = \chi$$

- **boundary conditions** in $\tilde{\sigma}$:

$$y'(\tilde{\sigma}) - \kappa c y^2(\tilde{\sigma}) = 0 \qquad r'(\tilde{\sigma}) = 0$$
$$y(\tilde{\sigma}) - \frac{1}{\kappa} x_3(\tilde{\sigma}) = 0 \qquad \theta(\tilde{\sigma}) = \frac{\pi}{2}$$



Double scaling limit

- We want to **match the string computation with the gauge result**: $\frac{L}{R} \to 0$ limit. Possibile because of $k \Rightarrow$ we can organize the espression for S as a series in $\frac{\lambda}{k^2}$

- **Wilson loop close to the defect** $\Rightarrow$ **string attached to the D5** $\Rightarrow$ m **large** $(m \to$ growing rate for $\theta)$

- large m corresponds also to large value for the flux κ

- taking also c (appears in the EOM for x_3) and small χ (close to the BPS point):

$$\log \langle W \rangle = \frac{k\pi R}{L}\left[\chi + \frac{\lambda}{8\pi}\left(\frac{L}{Rk}\right)^2\left(1 - \frac{4\chi}{\pi} + \chi^2\left(\frac{R^2}{L^2} + \frac{5}{2}\right) + \mathcal{O}\left(\chi^3\right)\right)\right]$$

- agreement with the perturbative computation

- Boundary conditions (BC) in $\sigma = 0$ are quite easy to solve, **BUT** BC in $\tilde{\sigma}$ are a non trivial issue!!
- After having imposed all the BC, there are only three indipendent parameters left:

$$\tilde{s} \ , \ m^2 \ , \ k^2$$

- As seen in the perturbative computation, we have two different phases:

1. **Disconnected phase** → Wilson loop far from the defect, the string does not reach the brane.
   This solution exists only in a certain range of the parameters
2. **Connected phase**→ Wilson loop close to the defect, string attached to the D5

There is a phase transition

Disconnected Phase

- x_3 is constant $(x_3 = L)$ $\Rightarrow$ $\mathbf{c = 0}$
- θ is costant $\Rightarrow$ $\mathbf{m = 0}$ $(\theta' = m)$

- In this limit, we have a particular reparametrization for the circle:

$$y(\sigma) = R\,\tanh(\sigma) \qquad r(\sigma) = R\,\mathrm{sech}(\sigma)$$

$$r(\sigma)^2 + y(\sigma)^2 = R^2 \quad \Rightarrow \quad \text{spherical cap equation}$$

- Equation for the D5: $y(\tilde{\sigma}) = \frac{L}{\kappa}$

$$\frac{L^2}{R^2\kappa^2} = 1 \quad \Rightarrow$$ **The brane and the D5 touch each other only in one point**

$$\frac{L^2}{R^2\kappa^2} < 1 \quad \Rightarrow$$ **string and D5 attached, CONNECTED SOLUTION**

Order of the phase transition

- **Determine the order** of the phase transition $\rightarrow$ look at the value of the area of the connected phase and at its derivatives for the critical values of the parameters

- $c = 0 \quad (\Rightarrow \; k^2 = -m^2, \;\; k^2 = -1/m^2)$

$$S = \frac{\sqrt{\lambda}}{\sqrt{m^2+1}} \left(m^2 \, K\left(\frac{1}{m^2+1}\right) - (m^2+1)\, E\left(\frac{1}{m^2+1}\right) \right)$$

- For $m \rightarrow 0, \;\; S = -\sqrt{\lambda} \quad \Rightarrow$ is the value of area of the disconnected solution, **no zero order phase transition**

Order of the phase transition

- Consider now the first derivative of the area respect to the parameter of the transition $x = \frac{L^2}{R^2 \kappa^2}$
- $c = 0 \ \rightarrow \ m^2 = \frac{1}{x} - 1$:

$$\Longrightarrow \quad \frac{\partial S}{\partial\, m^2} \frac{\partial\, m^2}{\partial x} \text{ shows a logarithmic divergence when } \ m \to 0$$

$$\frac{1}{4}\left(2 - 4\log 2 + \log m^2\right)$$

First order phase transition

Conclusion

- We analyzed the Circular Wilson loop operator in the $\mathcal{N} = 4$ SYM theory with the insertion of a defect
- **Field Theory side**: we can study the problem in two different limit and we found that when it is far from the defect, it receives a contribution that could be linked to the anomalous dimension
- **String Theory side**: non trivial boundary conditions problem, we are left with three indipendent parameter

  $\Longrightarrow$ we have a phase transition regulated by the parameter $\frac{L^2}{R^2 \kappa^2}$

  This is a first order phase transition
- **Outlook**: Look at what happens if we consider, for example, a Zarembo Wilson Loop

Thank you for the attention!!