From the Sakai-Sugimoto Model to the Generalized Skyrme Model

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25 May 2018

Contents

Baryons as Solitons

Witten/'t Hooft: study QCD in $N_c \to \infty$ limit

Greatly simplified

- Description as an effective field theory of mesons and glueballs
- Meson coupling $\sim \mathcal{O}\left(N_c^{-1}\right)$
- Baryons are solitons of this large N_c Lagrangian
- Baryon mass $\sim \mathcal{O}\left(N_c\right)$
- Baryon size $\sim \mathcal{O}\left(1\right)$

The Skyrme Model

Degrees of freedom: chiral field U (map $R^3 \to S^3$)
Action: NLσ Model with quartic "Skyrme Term"

$$\mathcal{L} = \frac{f_\pi^2}{16} \mathrm{Tr}\left(\partial_\mu U \partial_\mu U^\dagger\right) + \frac{1}{32e^2} \mathrm{Tr}\left[\partial_\mu U U^\dagger, \partial_\nu U U^\dagger\right]^2$$

Hedgehog ansatz:

$$U = \exp\left[i f(r) \hat{x} \cdot \vec{\tau}\right]$$

The Skyrme Model

Remarks:

- Admits solitonic solutions ("Skyrmions")
- Winding number $\leftrightarrow$ Baryon number
- Solitons can be quantized as fermions
- Accuracy within 30%
- Can (and SHOULD) be extended with other terms

Holographic QCD

1997: Maldacena introduces AdS/CFT correspondence

1998: Witten develops the confining background geometry

$\Downarrow$

Bulk geometry fixes a length scale dual to confinement scale

Holographic QCD models become feasible

Why holography?

Strong/weak duality

$\Downarrow$

Perturbative QFT $\leftrightarrow$ Strongly coupled gravity

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Great for studying the rich non-perturbative sector of strongly coupled QFTs

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WSS Model

Key elements:

- COLOR: Background $10d$ geometry from string theory (stack of N_c $D4-$Branes)
- FLAVOR: $U(N_f)$ Yang-Mills/Chern-Simons theory on $D8-$Branes

Compactify to $5d$

Flavor gauge field:

$$\mathcal{A}_\alpha = \widehat{A}_\alpha \frac{\mathbb{I}}{N_f} + A^a_\alpha \frac{\tau^a}{2}$$

$$\alpha = 0, i, z$$

SUGRA Background

Geometry

$$\mathrm{ds}^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\tau^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$f(u) = 1 - \frac{u_{KK}^3}{u^3}$$

Subspace (u, τ): "Cigar"

$\Downarrow$

$u = u_{KK}$ The geometry ends here

$\Downarrow$

$$R_\tau = \frac{4\pi}{3}\frac{R^{3/2}}{u_{KK}^{1/2}} \Rightarrow M_{KK} = \frac{3}{2}\frac{u_{KK}^{1/2}}{R^{3/2}}$$

Mass scale of the theory (glueballs)

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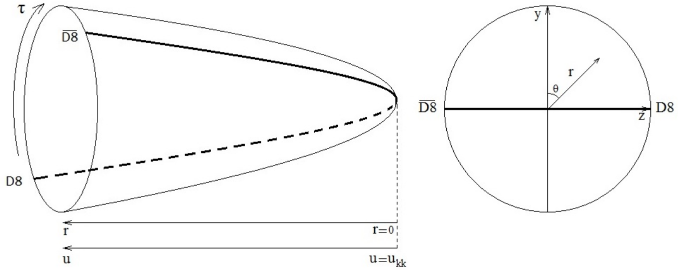
Mass scale of the theory (glueballs)

Flavor D8-Branes

N_f couples of antipodal D8/$\overline{\text{D8}}$-Branes on S^1



The branes join at $u = u_{KK}$

$$\Downarrow$$

$$U(N_f) \times U(N_f) \to U(N_f)$$

$$\Downarrow$$

Geometrical realization of SSB of chiral symmetry

WSS $5d$ Effective Action

$5d$ effective action: $S = S_{YM} + S_{CS}$

$$S_{YM} = -\frac{N_c \lambda}{216\pi^3} \mathrm{tr} \int d^4x dz \left[k(z) \mathcal{F}_{\mu z}^2 + \frac{1}{2} h(z) \mathcal{F}_{\mu\nu}^2 \right]$$

$$S_{CS} = \frac{N_c}{384\pi^2} \epsilon_{\alpha_1 \cdots \alpha_5} \int d^4x dz \widehat{A}_{\alpha_1} \left[3 F^a_{\alpha_2\alpha_3} F^a_{\alpha_4\alpha_5} + \widehat{F}_{\alpha_2\alpha_3} \widehat{F}_{\alpha_4\alpha_5} \right]$$

$$\boxed{k(z) = 1 + z^2 \ ; \ h(z) = k(z)^{-1/3}}$$

Meson modes expansion:

$$\mathcal{A}_\mu = \mathcal{U}^{-1} \partial_\mu \mathcal{U} \psi_+(z) + \sum_1^{+\infty} B_\mu^{(n)}(x) \psi_{(n)}(z)$$

$$-h(z) \partial_z (k(z) \partial_z \psi_n) = \lambda_n \psi_n$$

WSS 5d Effective Action

Remark: if we only include pions

$$\mathcal{A}_\mu = \mathcal{U}^{-1}\partial_\mu \mathcal{U}\psi_+(z)$$

And drop the CS term, we find that:

We obtain a Skyrme model in the low energy regime

$$f_\pi = \sqrt{\frac{\kappa}{\pi}} \quad ; \quad e \sim -\frac{1}{2.5\kappa}$$

But to set to zero vector mesons by brute force is not the correct way of deriving the effective action

$$\Downarrow$$

We should instead integrate away higher energy states from the action

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The Sextic term

A potential which is sextic in the derivatives can be added to the Skyrme model:

$$\mathcal{L} = \frac{\gamma^2}{24^2}\left[\epsilon^{\mu\nu_1\nu_2\nu_3}\left(R_{\nu_1}R_{\nu_2}R_{\nu_3}\right)\right]^2$$

With $R_\mu = \mathcal{U}^{-1}\partial_\mu\mathcal{U}$

A sextic term can be generated by extending the Skyrme model with ω-mesons and integrating them away

$\Downarrow$

We will use this guideline to derive such term from the holographic model

The Sextic term

- Abelian ansatz and factorization for the vector meson part:

$$\mathcal{A}_\mu = \begin{cases} \widehat{A}_\mu = B_\mu(x)\chi(z) \\ A_\mu = \mathcal{U}^{-1}\partial_\mu\mathcal{U}\psi_+(z) \end{cases}$$

- Field strength becomes

$$F_{\mu\nu} = [R_\mu, R_\nu]\,\psi_+\,(i\psi_+ - 1)$$
$$F_{z\mu} = R_\mu\psi'_+$$
$$\widehat{F}_{\mu\nu} = f_{\mu\nu}\chi$$
$$\widehat{F}_{z\mu} = B_\mu\chi'$$

$$f_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Equation of motion for $\widehat{A}_\mu$

Neglecting higher derivative terms $(\dots)$ we obtain:

$$2\kappa z B_\mu \chi' + \kappa(1+z^2)B_\mu \chi'' + \ldots +$$

$$+\frac{N_c}{16\pi^2}\epsilon^{\mu z \mu_1 \mu_2 \mu_3}\mathrm{Tr}\left(R_{\mu_1}\left[R_{\mu_2}, R_{\mu_3}\right]\psi_+\psi'_+(i\psi_+ - 1)\right) + \ldots = 0$$

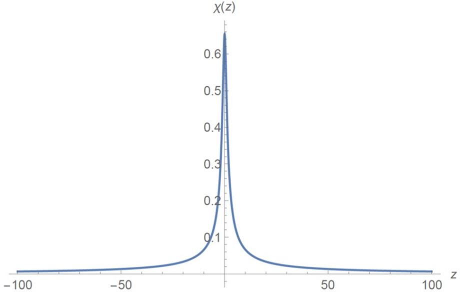
Which is nicely decoupled as

$$2z\chi' + k(z)\chi'' = \frac{N_c}{16\kappa\pi^2}\psi_+\psi'_+(i\psi_+ - 1)$$

$$B_\mu(x) = -\epsilon_\mu{}^{z\nu_1\nu_2\nu_3}\mathrm{Tr}\left(R_{\nu_1}\left[R_{\nu_2}, R_{\nu_3}\right]\right)$$

Profile in the holographic direction

$$\chi = -\frac{N_c}{64\pi^3\kappa}\left(\frac{5\pi^2}{48} - \frac{1}{2}\arctan^2(z) + \frac{1}{3\pi^2}\arctan^4(z)\right)$$



$$a_n \equiv |\langle \chi^{(norm)}, \psi_n \rangle|^2$$

$$a_1 = 0,988 \quad ; \quad a_3 = 0,0115 \quad ; \quad a_5 = 0,00029$$

Resulting sextic term

Plug configuration in $S_{YM} + S_{CS}$ and integrate bulk direction

$$S_6 = S_6^{YM} + S_6^{CS} = \frac{51 N_c}{8960\lambda} \int d^4x \left[\epsilon^{\mu z \nu_1 \nu_2 \nu_3} \mathrm{Tr}\left(R_{\nu_1} R_{\nu_2} R_{\nu_3}\right)\right]^2$$

We can also add a quark mass term via Aharony-Kutasov action

$$S_{AK} = mc \int d^4x \mathrm{Tr}\left[(\mathcal{U} - 1) + \mathrm{c.c.}\right]$$

Which produces a pion mass potential

$$S_0 = 4mc \int d^4x \left(\sigma - 1\right)$$

$$\mathcal{U} \equiv \sigma + i\vec{\pi} \cdot \vec{\tau}$$

The SSM "contains" a Generalized Skyrme Model

$$\mathcal{L}_{GSM} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \mathcal{L}_0$$

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The small λ limit

We already know the picture in the large λ regime:

$$\boxed{\text{BPST instanton of size } \mathcal{O}(\lambda^{-1/2})}$$

We have derived a GSM as a low energy effective field theory: what happens to this picture in the new SMALL λ limit?

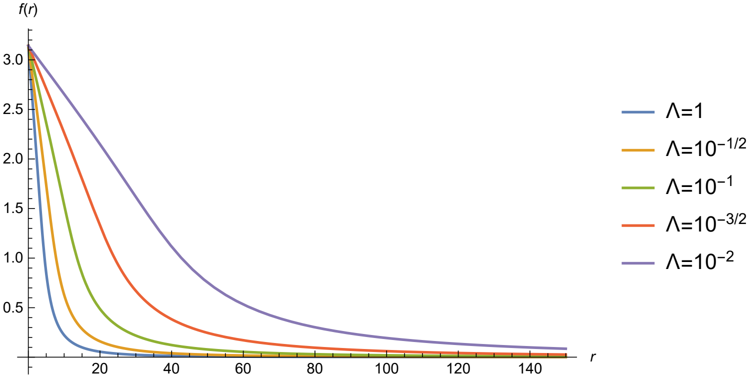
make use of Derrick's theorem

$$\Downarrow$$

Different outcomes for massive or massless quarks

Massless quarks case

$\mathcal{L}_2$ and $\mathcal{L}_6$ are dominant



Skyrmion profiles for various values of λ. Size $R \sim \lambda^{-1/2}$

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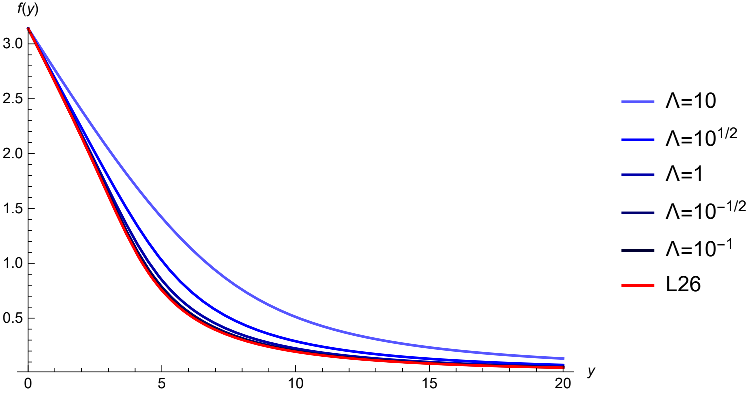
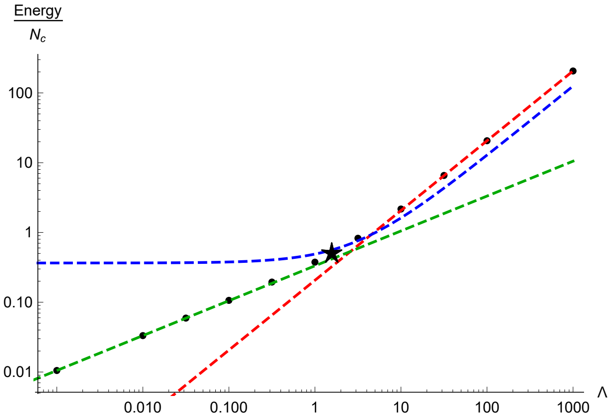


Skyrmion profiles for various values of λ rescaled to the same size. In red the solution of the model $\mathcal{L}_2 + \mathcal{L}_6$

Massless quarks case

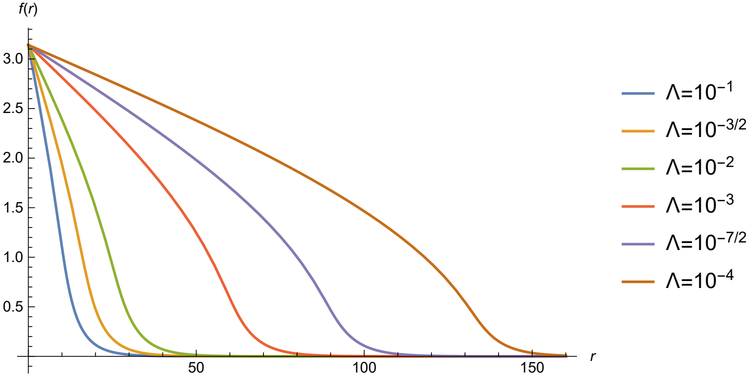
Energy as a function of λ



Green and red lines: asymptotic power laws (Derrick)
Blue line: correct behaviour for large λ
Black star: phenomenological λ

Massive quarks case

$\mathcal{L}_0$ and $\mathcal{L}_6$ are dominant



Size $R \sim \lambda^{-1/3}$ in the small λ regime
but more interesting things happen...

Massive quarks case

$$\boxed{\mathcal{L}_0 + \mathcal{L}_6 \text{ is the "BPS Skyrme Model"}}$$

$$\Downarrow$$

It admits an analytic solution of the compacton type

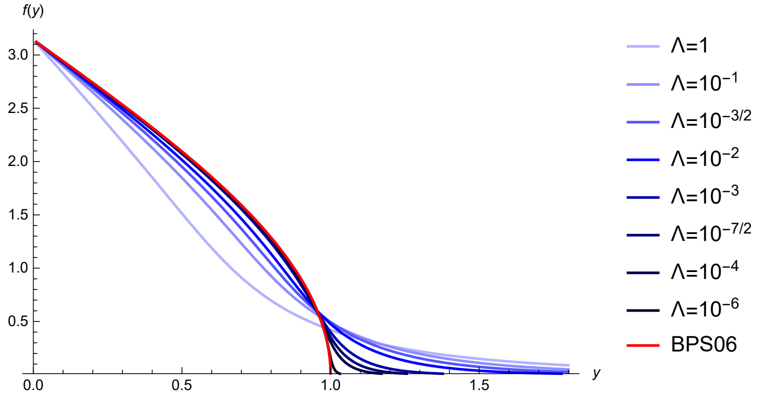
$$f(r) = \begin{cases} 2\arccos(Ar) & \text{for} \quad r \in \left[0, A^{-1}\right] \\ 0 & \text{for} \quad r \geq A^{-1} \end{cases}$$

$$A^{-1} = \sqrt[3]{\frac{4\sqrt{\alpha}}{\Lambda m_\pi}}$$

$$\alpha = 76,701 \quad ; \quad \Lambda = \frac{8\lambda}{27\pi}$$

Massive quarks case

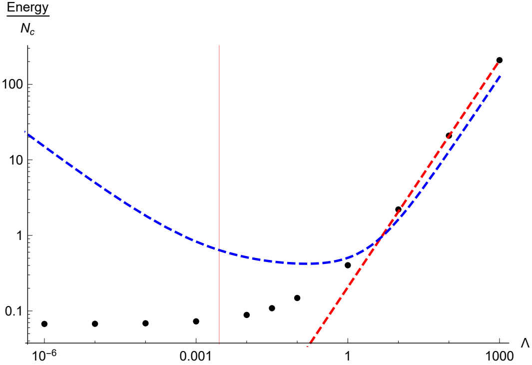
Rescale again with A^{-1}



In red the analytic compacton solution. Again, numerical solutions confirm the expected behaviour

Massive quarks case

Energy as a function of λ



Red dashed line: asymptotic linear law

Red vertical line: size corresponding to $R \sim m_\pi^{-1}$

Blue line: correct behaviour at large λ

Summing up our results

- We obtained a Generalized Skyrme model within the holographic model of Sakai-Sugimoto.
- The mechanism with which it is obtained resembles of the old idea of integrating out the ω meson, extending it to the whole tower of states with the same quantum numbers.
- In the small λ regime a BPS Skyrme model is obtained

Large $\lambda \Rightarrow$ Self-dual BPST instanton

The SSM interpolates with λ between two BPS models

- Phenomenological λ is not in any of the two regimes: can small nuclear binding energies be thought as a consequence of (inevitable?) closeness to a BPS model?

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Thanks for your attention