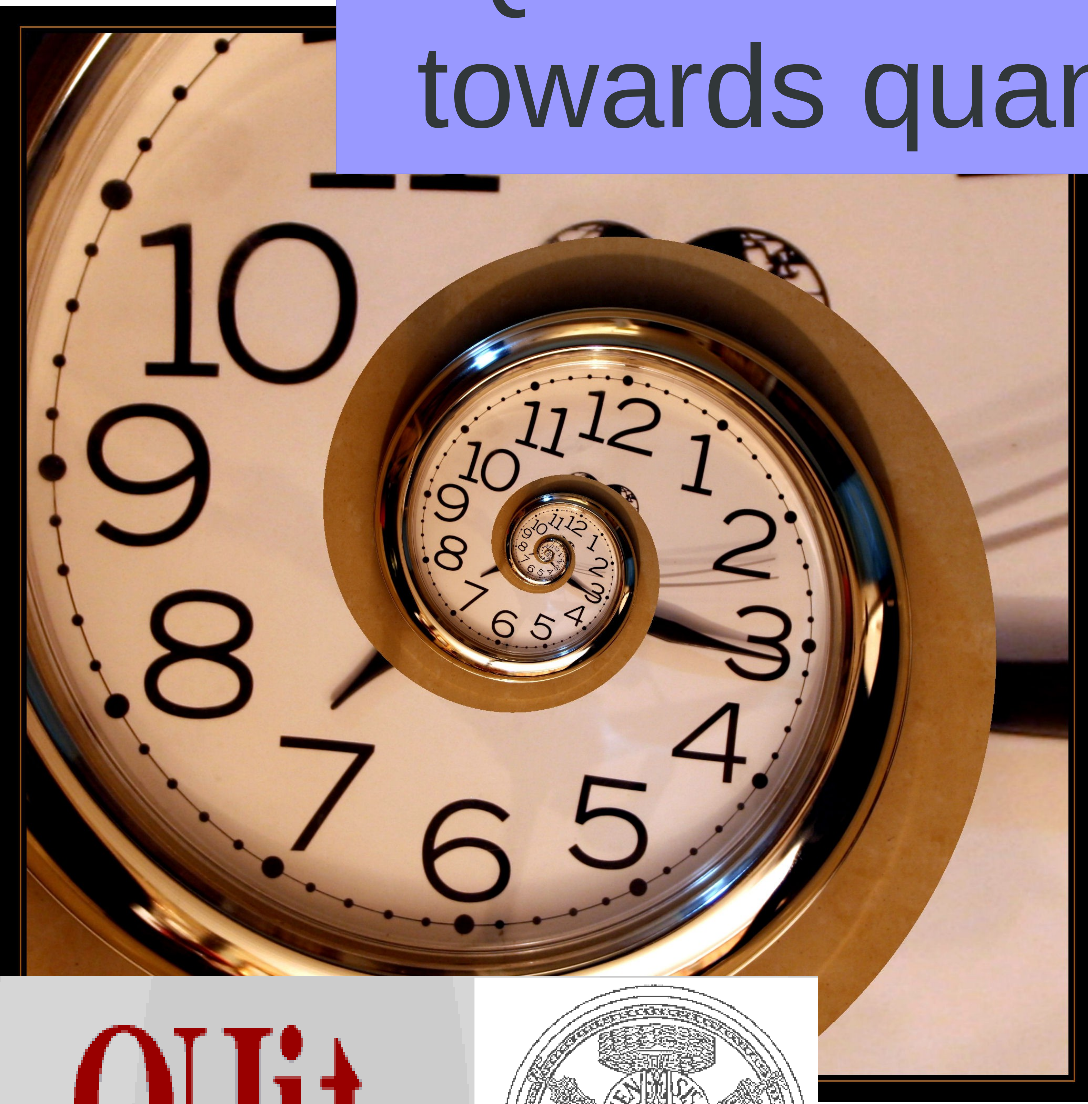
Quantum time mechanism, towards quantum spacetime



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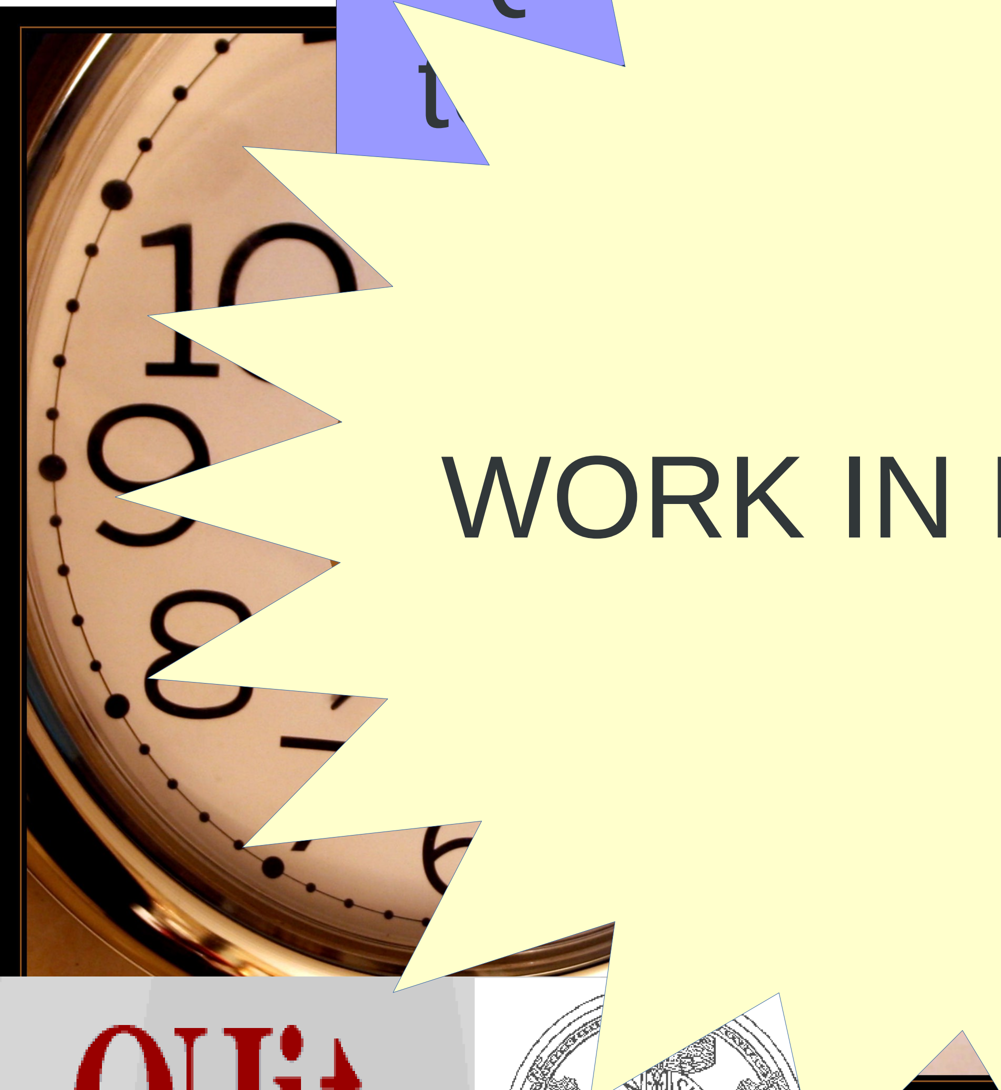
QUit quantum information theory group
www.qubit.it

FQXi Foundation,
"The physics of what happens"

Qu... ntu... ...echanism,
t... ...time



WORK IN PROGRESS!

...nnetti
...isa

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What I'm going to talk about

Time in quantum mechanics

a consistent formalization

based on conditional probability amplitudes

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... and some preliminary considerations about spacetime

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a classical parameter in the Schroedinger eq.

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BUT... **classical systems don't exist**

in a consistent theory of quantum mechanics
(they're just a limiting situation)

define: Time is
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then use a **quantum** system as a clock

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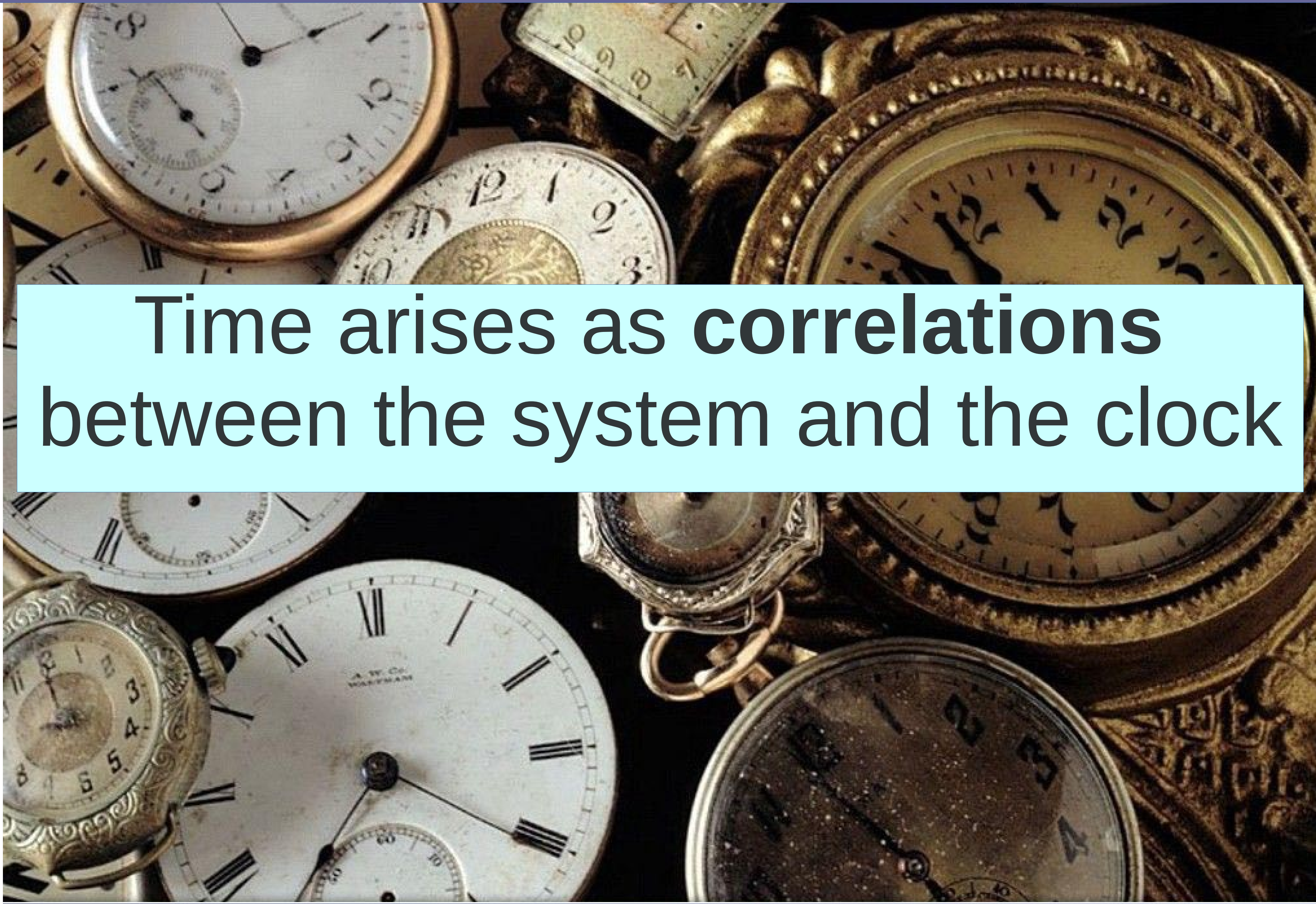
$$\mathcal{H} \equiv \mathcal{L}^2(\mathbb{R}) \quad \text{eigenbasis } \{|x\rangle\}$$

$$|t\rangle$$

Time and entanglement



Time arises as **correlations** between the system and the clock

The PWAK mechanism

Page and Wootters [PRD **27**,2885 (1983)]
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time Hilbert space

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clock “momentum”

$$\hat{\mathbb{J}} := \hbar\hat{\Omega} \otimes \mathbb{1}_S + \mathbb{1}_T \otimes \hat{H}_S\ ,$$

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This means that for physical states the Hamiltonian is the generator of time translations

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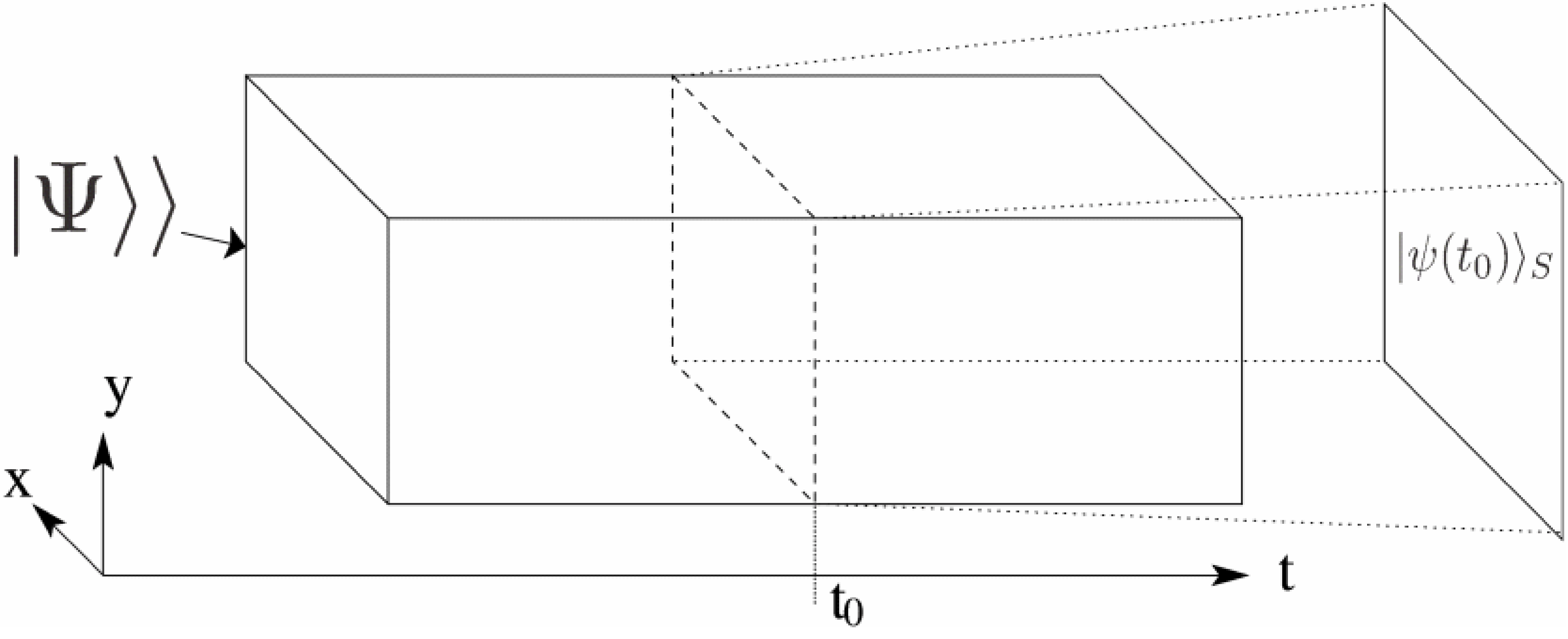
"momentum" representation=time indep. Schr eq.

what I've been saying is that

conventional qm arises in this framework through conditioning.



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is a **conditioned state**: the state *given that* the time was *t*

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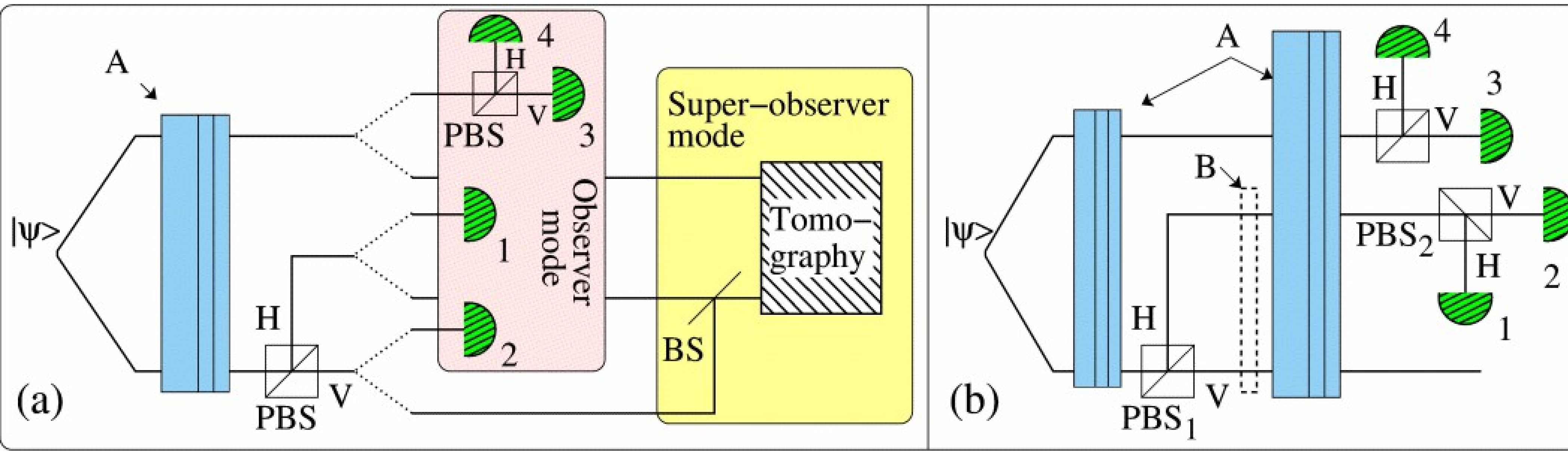
Other choices are possible!!

Experimental realization
(collaboration with the INRIM group)



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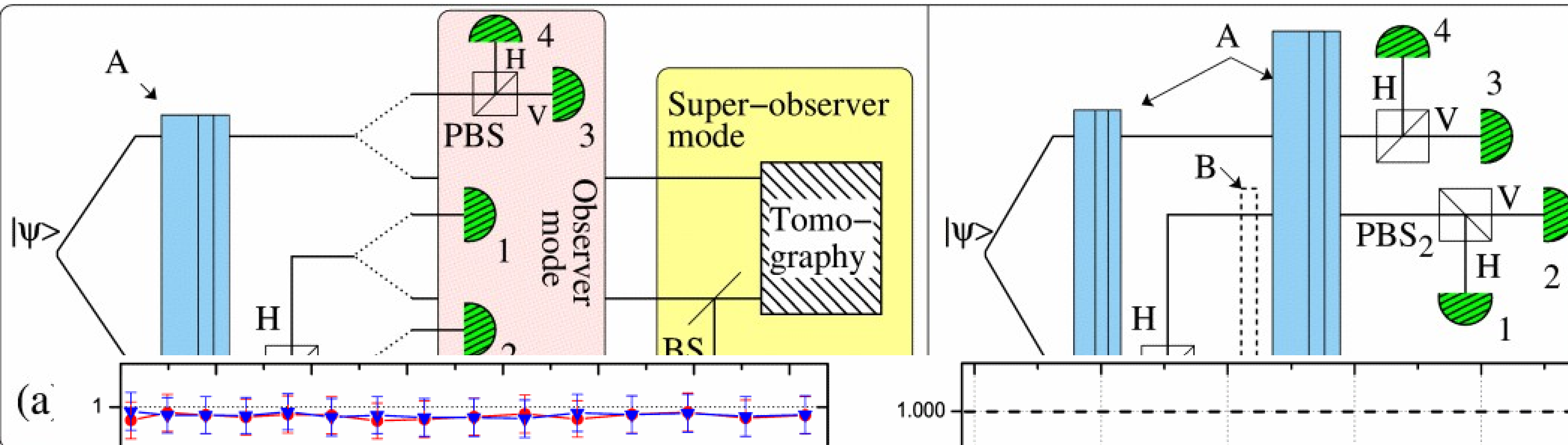

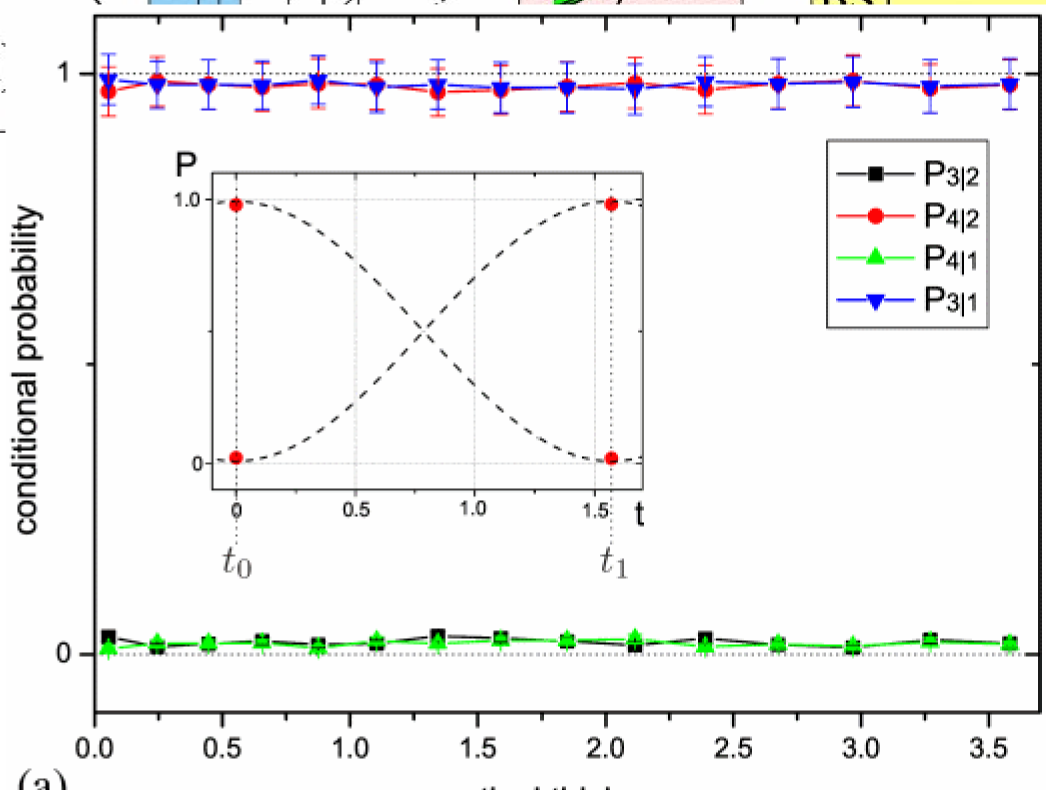

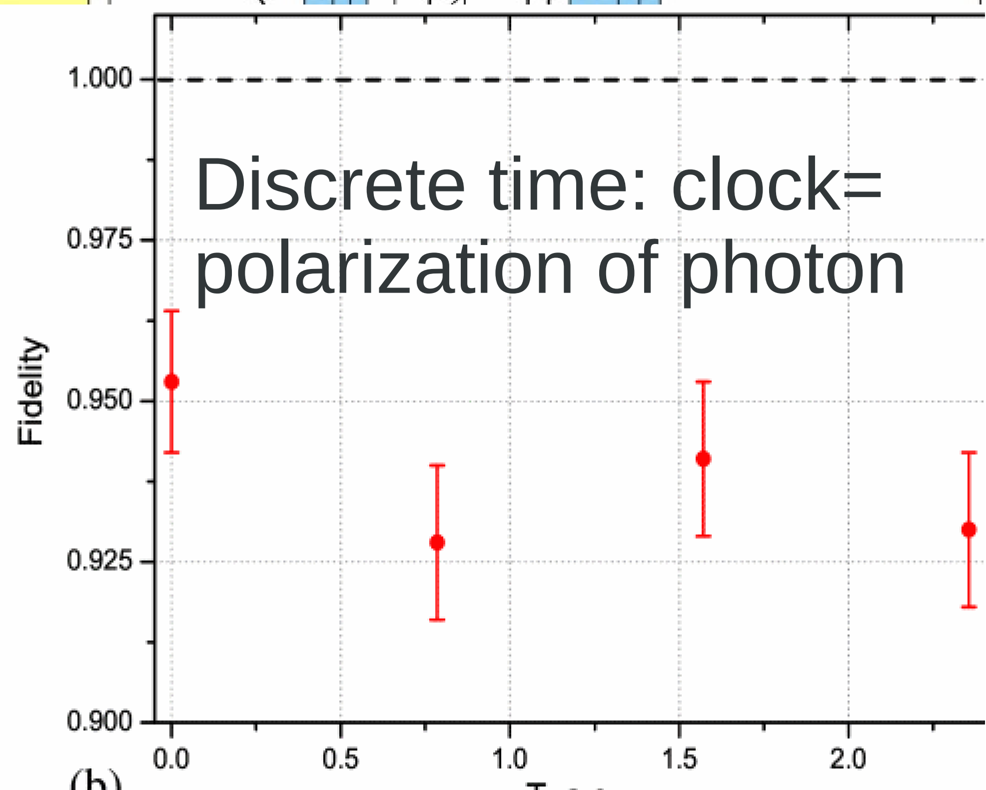

Discrete time: clock= polarization of photon

Experimental realization 2
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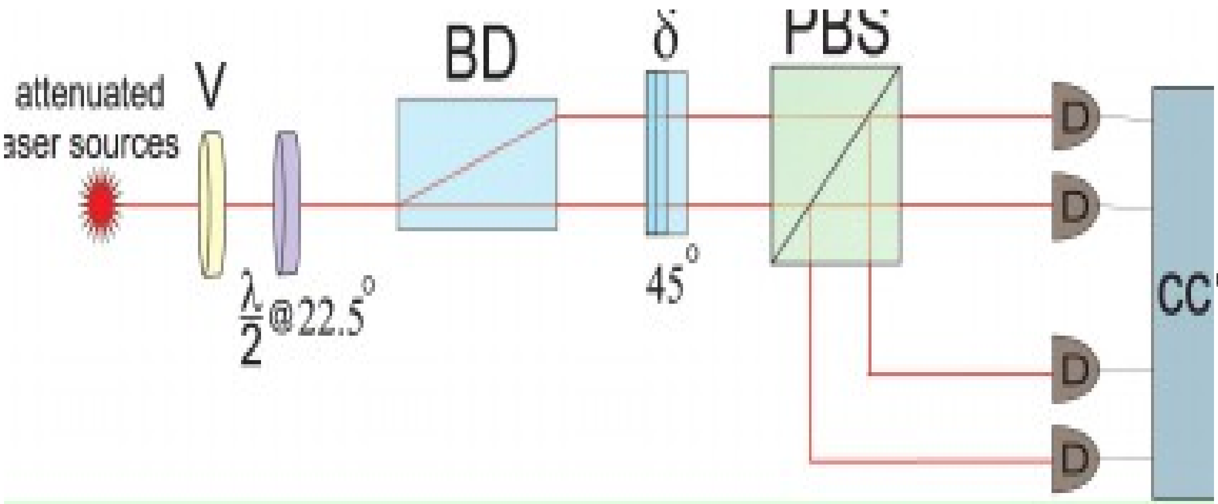

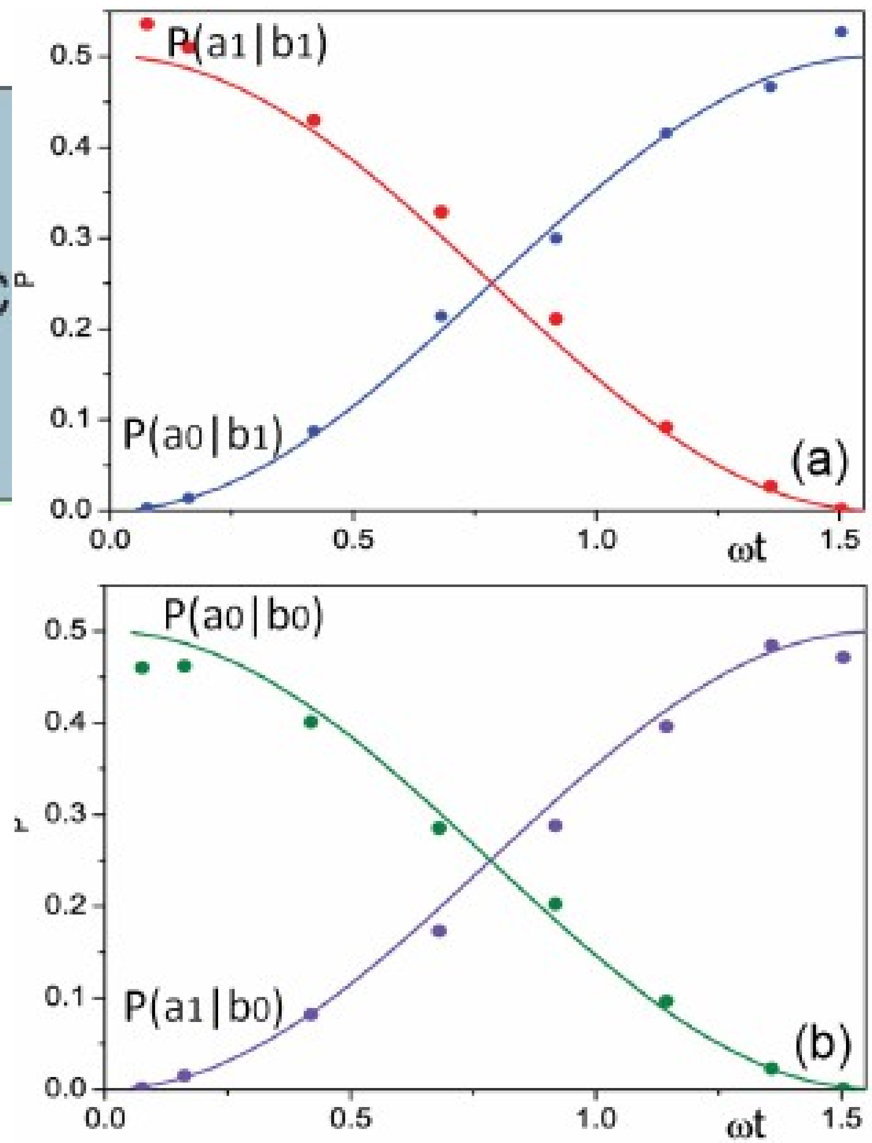

Continuous time case: the clock is a continuous d.o.f. (position of the photon)

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up to now ⟶ quantum time

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can we do the same for **space**?

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state of the system *given that* the **clock** shows *t*

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“vertical” QM:

$$|\theta(z)\rangle$$

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Quantum **spacetime**

use **quantum rods and clocks**

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now use a similar trick for quantum rods!

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However, the spatial state MUST contain the quantum clock!

A particle can be in the same place at different times, but it cannot be in different places at the same time!

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the construction presented here is a “sort” of *QM for events*:

by adding a clock for localizing events

Quantum space.

Same idea as before: introduce a **constraint** (WdW eq), and use **conditional probabilities!**

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conditioned states of the rest:

state of the system *given that* the **rod** shows *z*

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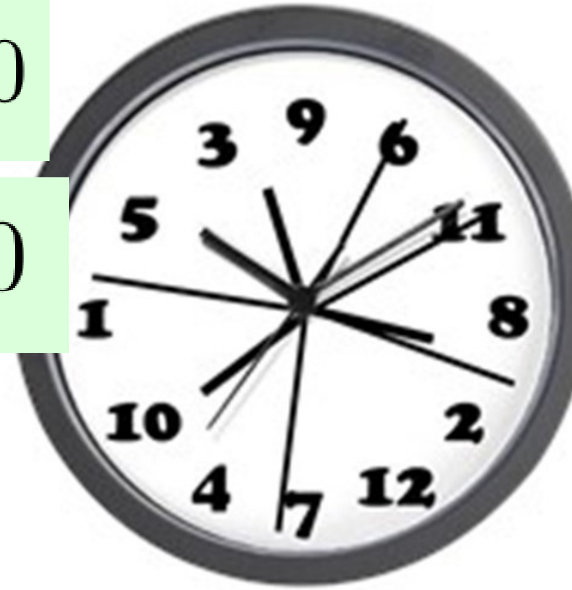

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Normalization:

$$\mathcal{N} = \langle\theta(z)|\theta(z)\rangle = \int dt\ \langle\psi(t)|z\rangle\langle z|\psi(t)\rangle$$

Conditional states both for space and time!

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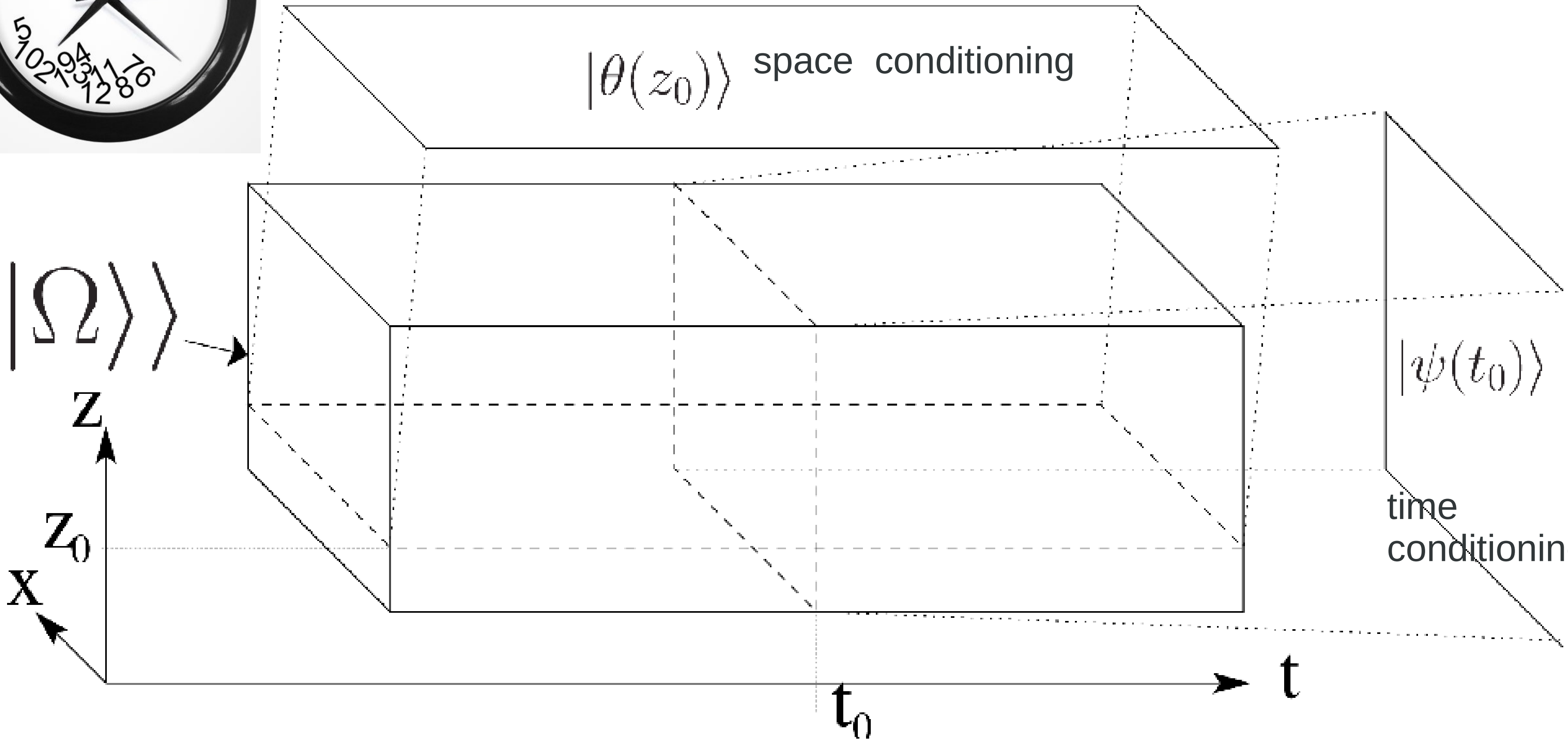

Comparison to Stuekelberg's qm

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prob. ampl. to find a particle at x,y and
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Conditional prob. ampl.
time (or space) are properties of an **EXTERNAL** system (clock or ruler), not **INTERNAL**

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$$|\Psi_{stu}\rangle = \int dt \int d^3x \, \Psi_{stu}(\vec{x},t) |\vec{x}\rangle |t\rangle$$

prob. ampl. to find a particle in **spacetime position** *x,y,z,t*.

$$\int dt \, d^3x |\Psi_{stu}|^2 = 1$$

(good only for qft?)

$$|\Psi\rangle\rangle = \int_{-\infty}^{+\infty} dt \, |t\rangle |\psi(t)\rangle$$

prob. ampl. to find a particle at x,y,z **given that** the time is t

$$|\Psi_{screen}\rangle\rangle = \int_{-\infty}^{+\infty} dz \, |z\rangle |\chi(z)\rangle$$

prob. ampl. to find a particle at x,y and time t **given that** the screen is at z

Conditional prob. ampl.

time (or space) are properties of an **EXTERNAL** system (clock or ruler), not **INTERNAL**

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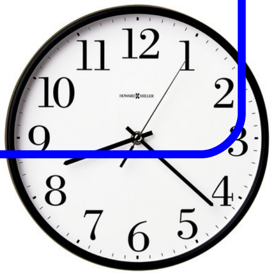
work in progress

Criticisms to time quantizations

The Pauli argument

Pauli: “time **cannot be quantized**, because a time operator that is a generator of energy translations implies that the energy is un-bounded (also from below)”



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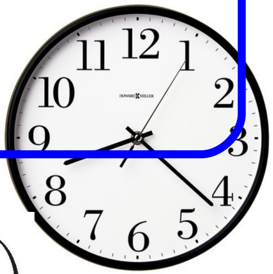

i.e. $[\hat{T}, \hat{H}_S] = i\hbar \Rightarrow \lambda(\hat{H}_S) \in (-\infty, +\infty)$

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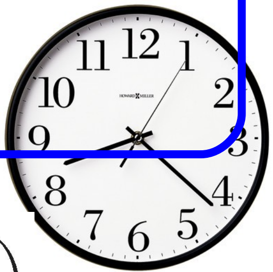

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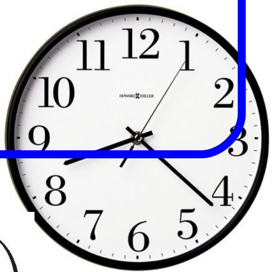

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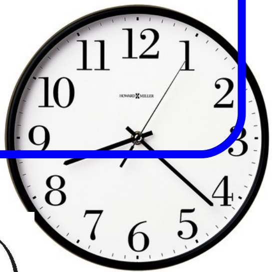

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In other words, the **Pauli argument fails** in our case because the energy-time connection is not enforced dynamically as

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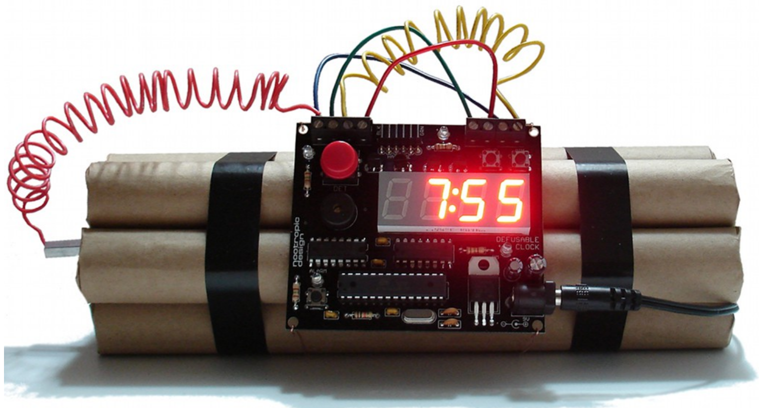

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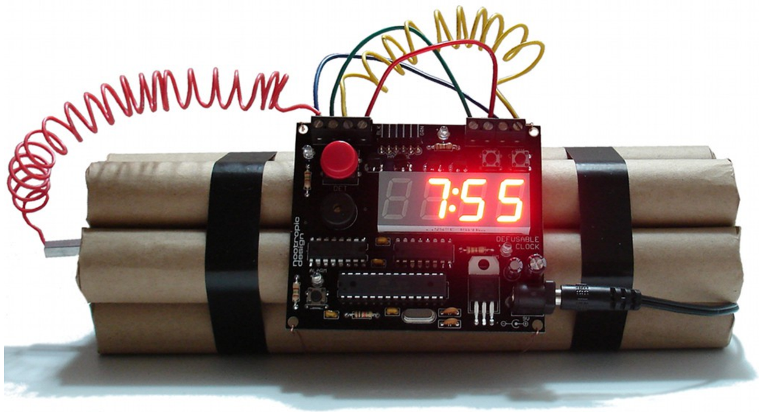

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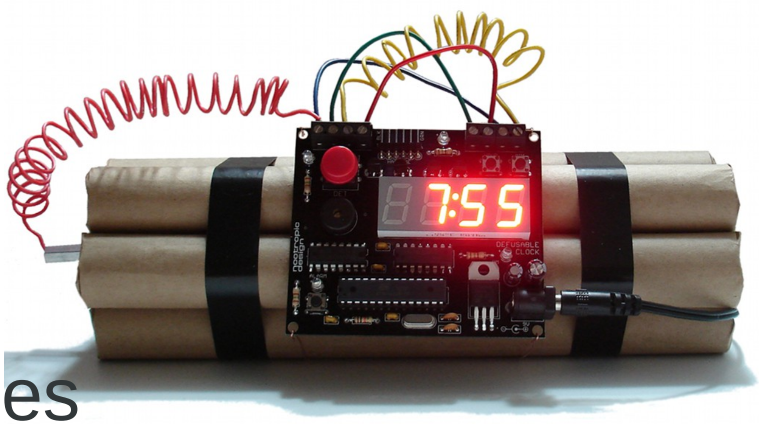
they act on different Hilbert spaces



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- in conventional qm, time is not a dynamical variable $\Rightarrow$ no problem.

- in our case, time *is* a dynamical variable, but its translations are NOT generated by $\hat{H}_S$ (but by $\hat{\Omega}$)

Conclusions

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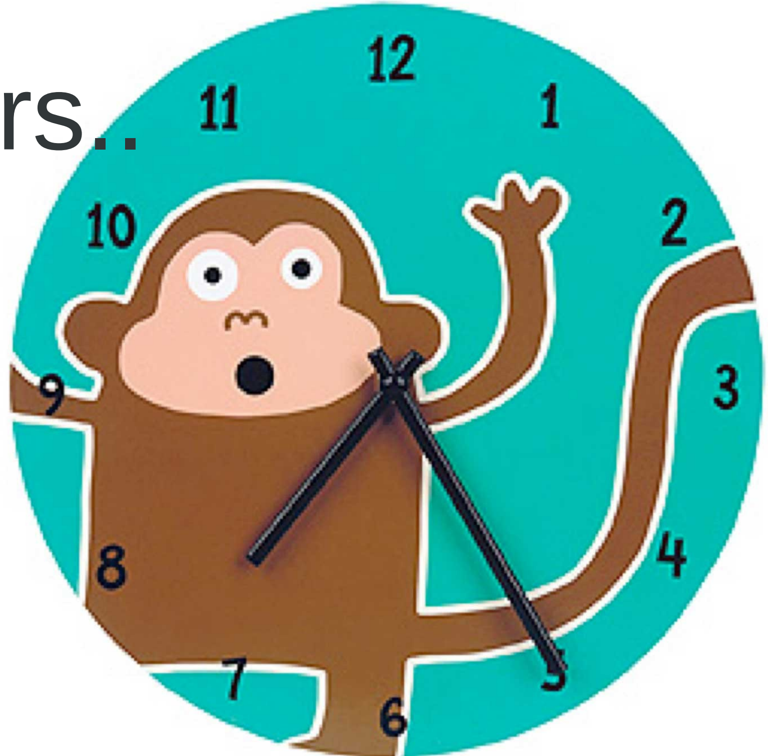
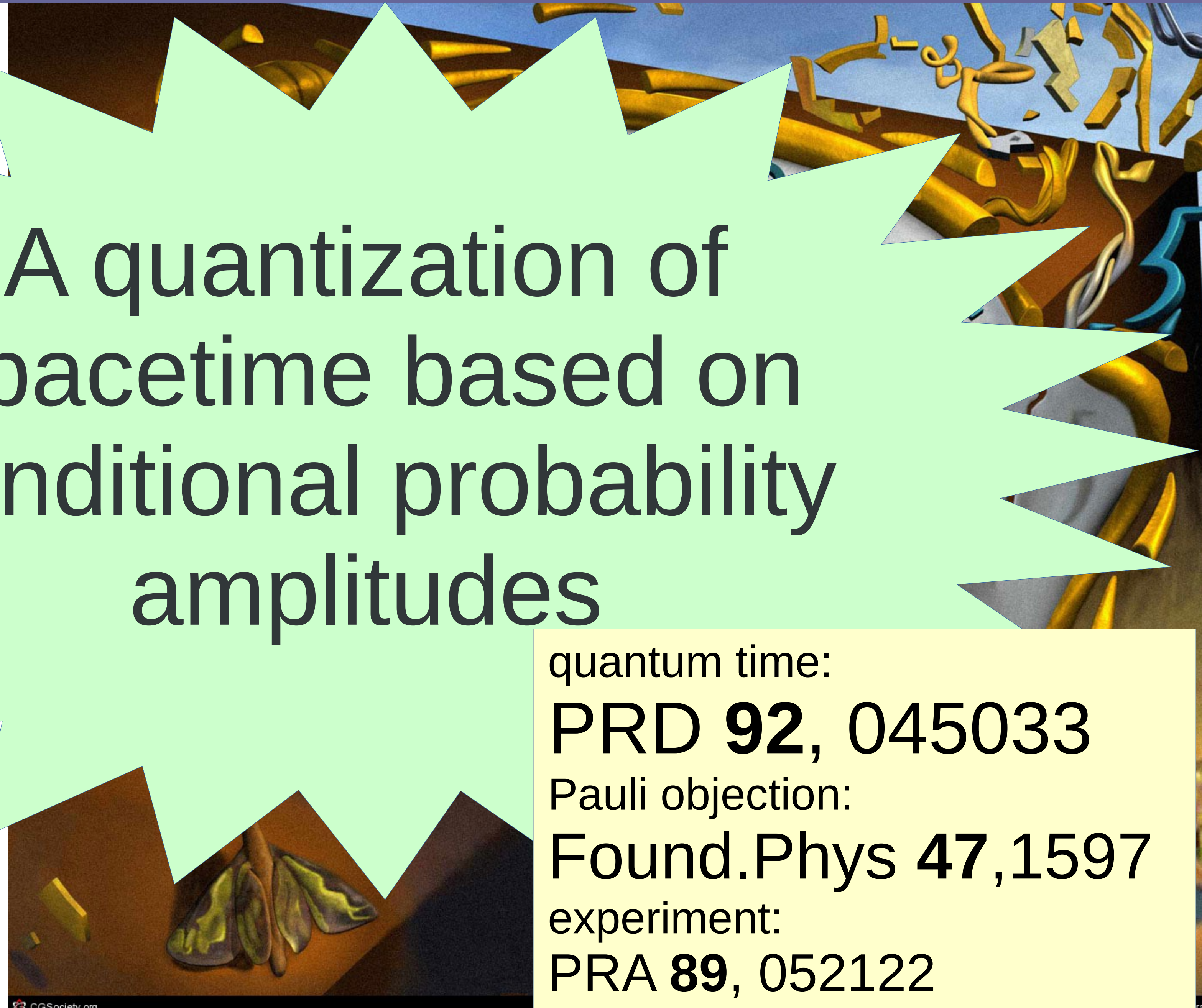
- Pauli objections and others...



A quantization of spacetime based on conditional probability amplitudes



quantum time:

PRD **92**, 045033

Pauli objection:

Found.Phys **47**,1597

experiment:

PRA **89**, 052122

PRD **96**, 102005

q spacetime: work in progress!

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