Non-perturbative treatment of non-Markovian dynamics of open quantum systems

Andrea Smirne, Susana Huelga and Martin Plenio

Institute for Theoretical Physics, Ulm Universität

In collaboration with Dario Tamascelli

Universita' degli Studi di Milano

Quantum Foundations: New frontiers in testing quantum mechanics from underground to the space

Frascati, 29 November 2017

Complex open quantum systems

- Biological molecular complexes

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LETTERS

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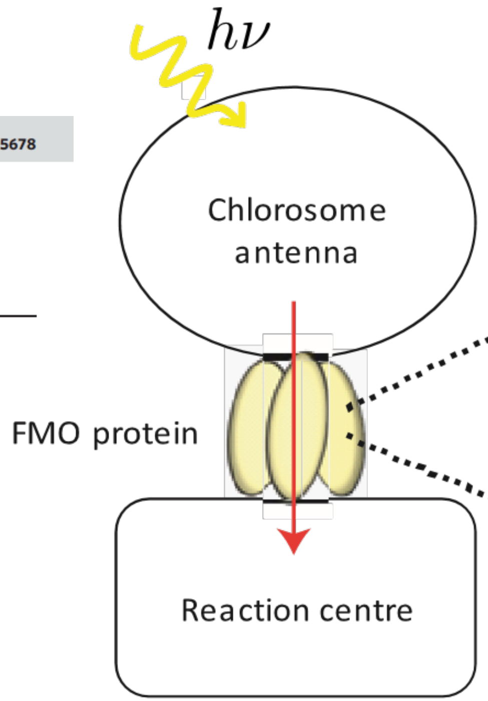
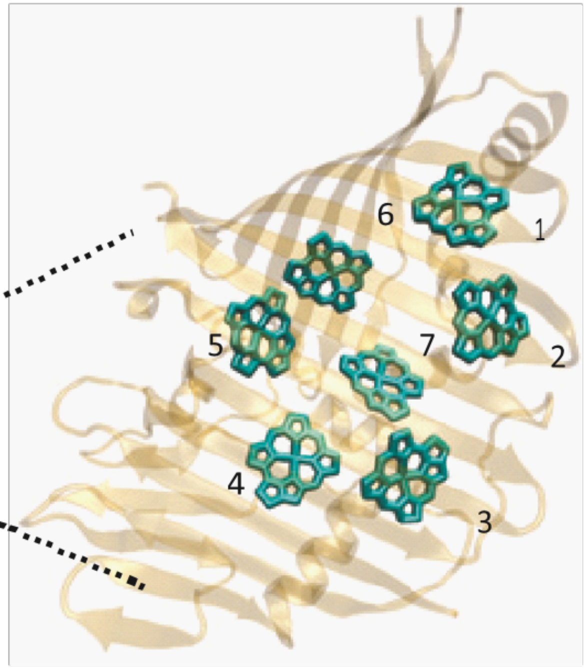


Fenna-Matthews-Olsen (FMO) Complex



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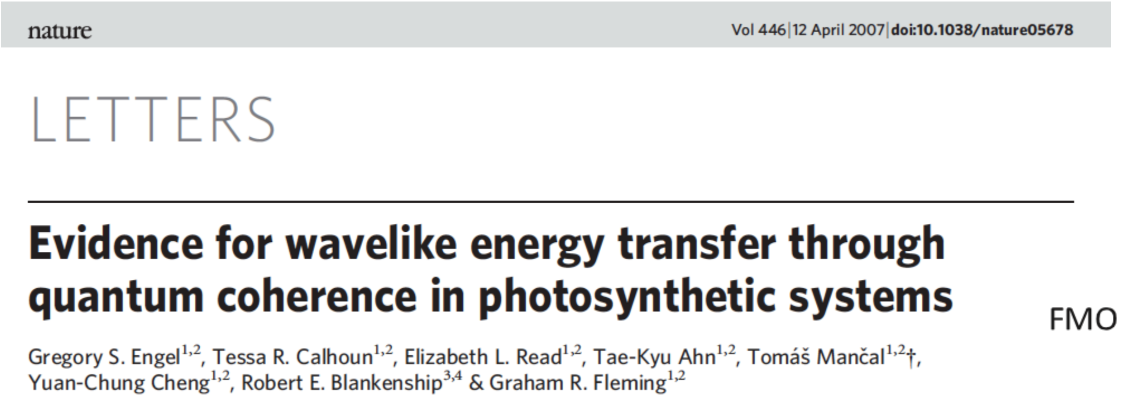

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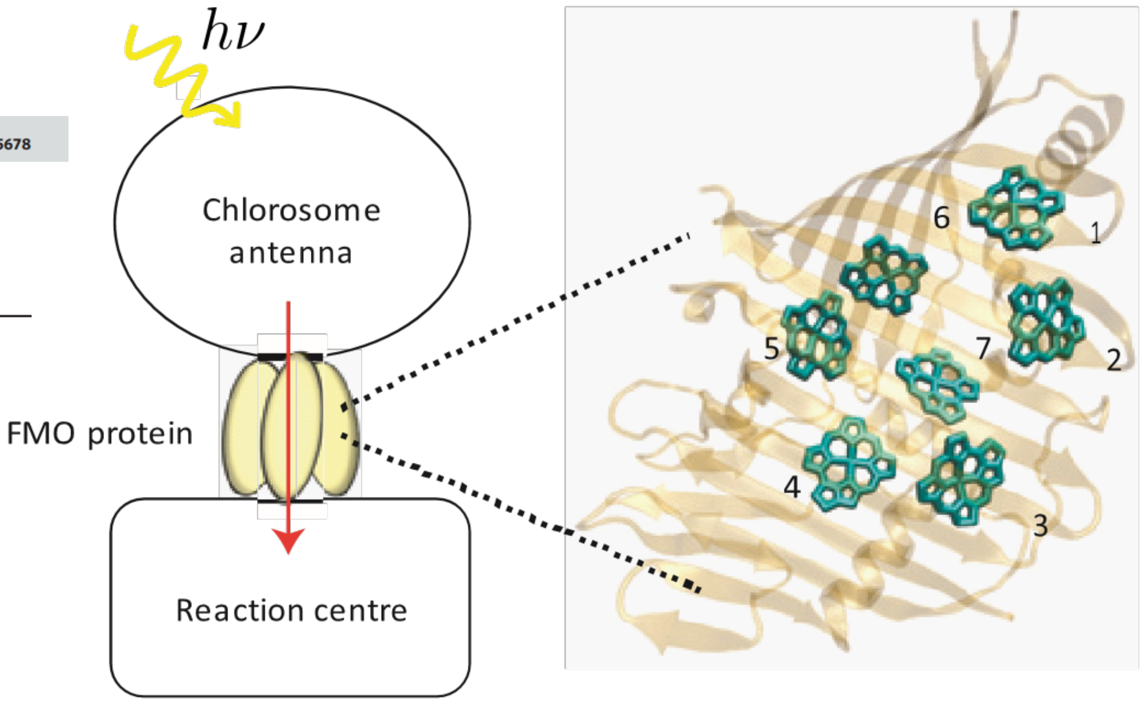

- Nanoscale quantum thermal machines
- Solid-state implementations of quantum protocols
- Possibly structured systems subjected to the collapse noise
- and many more…

Non-Markovian dynamics

- Different non-equivalent definitions of quantum non-Markovianity

REVIEWS OF MODERN PHYSICS, VOLUME 88, APRIL–JUNE 2016

***Colloquium*: Non-Markovian dynamics in open quantum systems**

Heinz-Peter Breuer*

Physikalisches Institut, Universität Freiburg, Hermann-Herder-Straße 3, D-79104 Freiburg, Germany

Elsi-Mari Laine

QCD Labs, COMP Centre of Excellence, Department of Applied Physics, Aalto University, P.O. Box 13500, FI-00076 AALTO, Finland and Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 Turun yliopisto, Finland

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IOP Publishing

Rep. Prog. Phys. **77** (2014) 094001 (26pp)

Review Article

Quantum non-Markovianity: characterization, quantification and detection

Ángel Rivas[1], Susana F Huelga[2,3] and Martin B Plenio[2,3]

[1] Departamento de Física Teórica I, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain
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Non-Markovian dynamics

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- Different non-equivalent definitions of quantum non-Markovianity

- Lack of a general characterization which i) guarantees a well-defined (CP) evolution

  ii) encloses all the relevant information about the environment, simplifying the description

Non-Markovian dynamics

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- Lack of a general characterization which i) guarantees a well-defined (CP) evolution
  ii) encloses all the relevant information about the environment, simplifying the description

Compare with the Lindblad equation

$$\dot{\rho} = -i\,[H,\rho] + \sum_{i=1}^{N^2-1} \gamma_i \left(L_i \rho L_i^\dagger - \frac{1}{2}\left\{ L_i^\dagger L_i, \rho \right\} \right)$$

$$\gamma_j \geq 0$$

G. Lindblad Comm. Math. Phys. 48, 119 (1976)
V. Gorini, A. Kossakowski and E.C.G. Sudarshan, J. Math. Phys. 17, 821(1976)

Analytical and numerical methods

➢ Perturbative methods

- Projection super-operators (TCL, Nakajima-Zwanzig,…)

  Breuer & Petruccione, *The theory of open quantum systems* (2002)

- Expansions on the average of stochastic equations

  Adler & Bassi, J. Phys. A 40, 15083 (2007)
  Barchielli, EPL 91, 24001 (2010)

- Recursive approaches — Luca's talk

  Gasbarri & Ferialdi, arXiv: 1707.06540 (2017)

- Hierarchical equations of motions

  Tanimura & Kubo, J. Phys. Soc. Jap. 58, 101 (1989)

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Non-perturbative methods

- Non-Markovian piecewise quantum dynamics

  Vacchini, Phys. Rev. Lett. 117, 230401 (2016)

- Collisional models

  Lorenzo, Ciccarello & Palma, PRA 93, 05211 (2016)

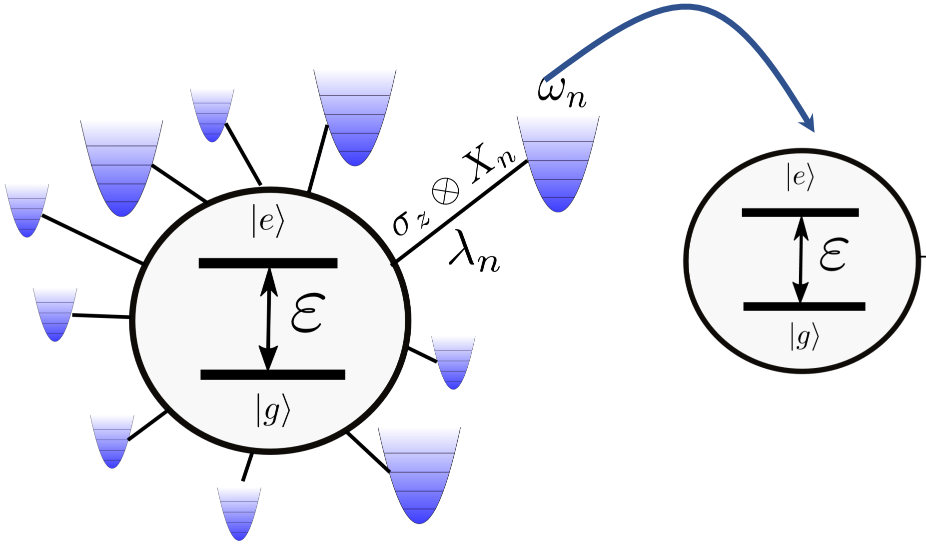
  (both: Bassano's talk)

- TEBD and other numerical techniques relying on the Trotter decomposition

Auxiliary models: unitary equivalence

Prior et al., Phys. Rev. Lett. 105, 050404 (2010)

Unitary equivalence



TEDOPA

Time
Evolving
Density operator
Orthogonal
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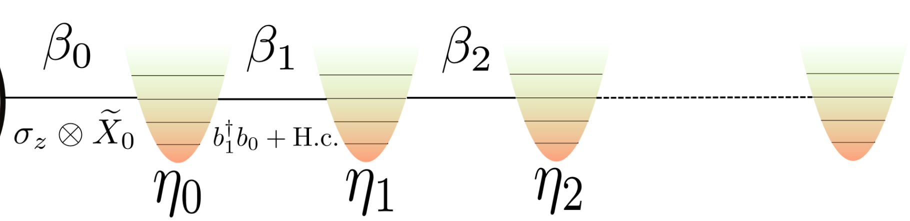

Nearest-neighbor interactions

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MPS/MPO: Efficient parametrization State-Space

↓

Evolution: TEBD

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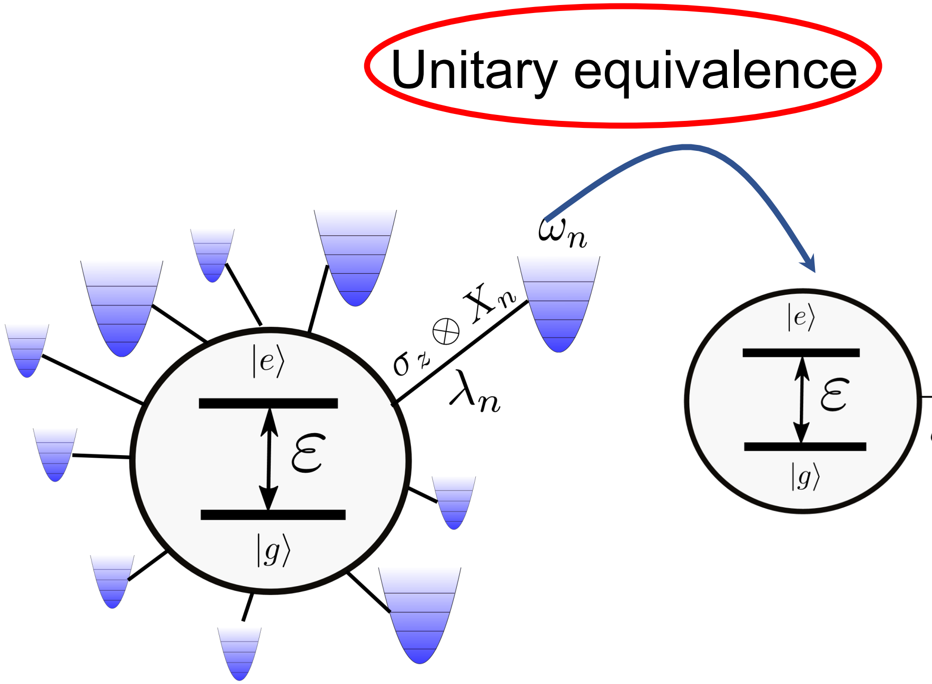
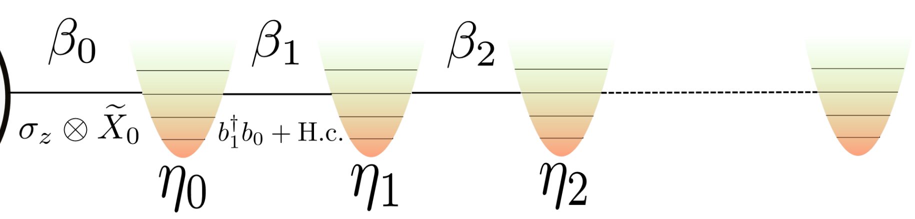


TEDOPA: Time Evolving Density operator Orthogonal PloynomiAls



Nearest-neighbor interactions

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MPS/MPO: Efficient parametrization State-Space

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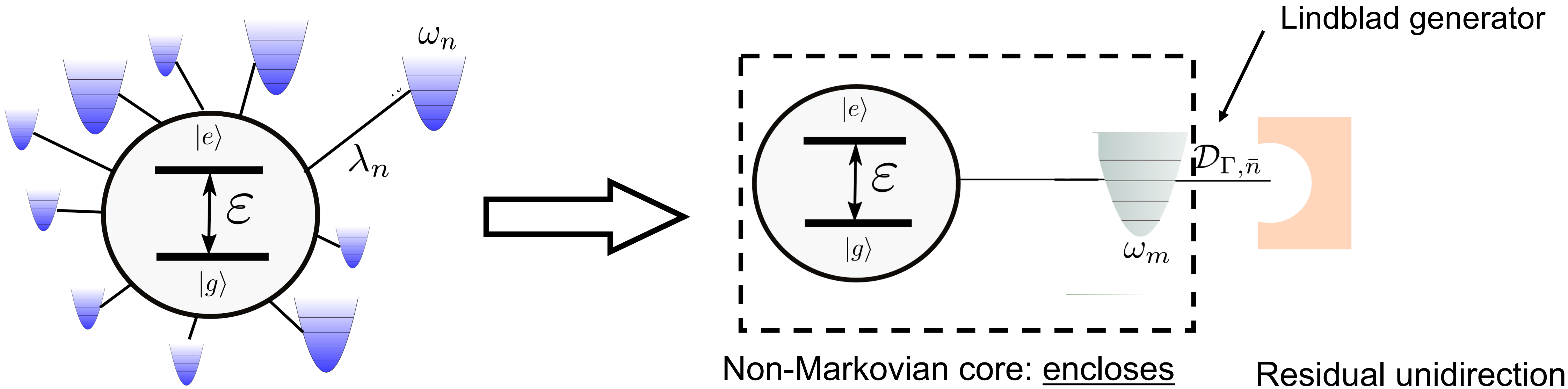
Evolution: TEBD

- Chin et al, Nat. Phys. 2013: The role of non-equilibrium vibrational structures for long-lasting electronic coherences in pigment–protein complexes
- The auxiliary system is more suitable for numerical simulation
- BUT the number of degrees of freedom involved is still very high (one needs to cut the chain, …)

Auxiliary models: non-Markovian core

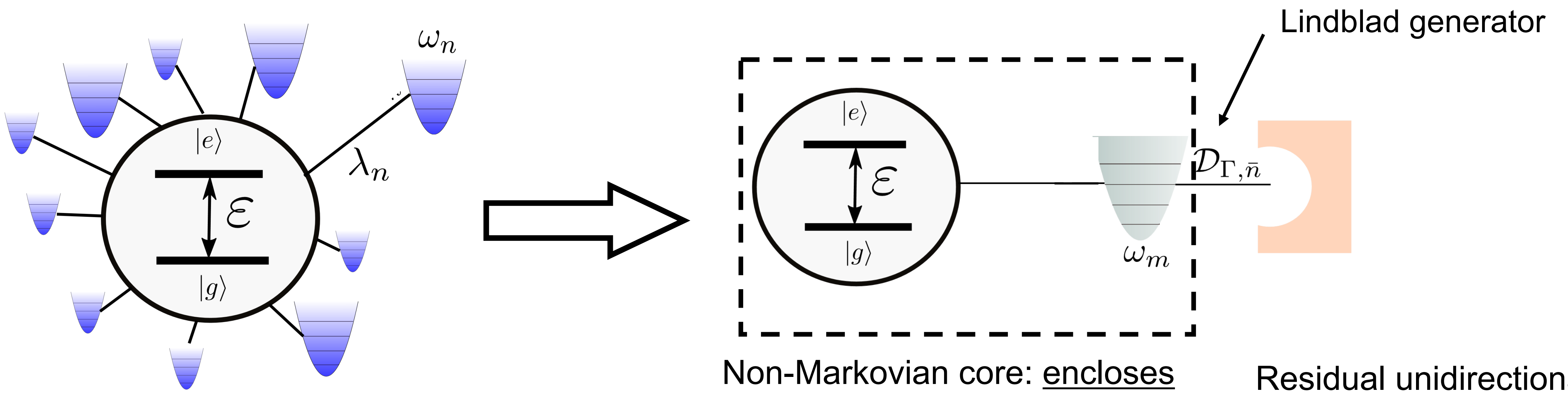


Non-Markovian core: encloses all the memory effects

Residual unidirectional leaking of information

AIM: same open-system dynamics, we do not necessarily need the same overall dynamics!!

Auxiliary models: non-Markovian core



Non-Markovian core: encloses all the memory effects

Residual unidirectional leaking of information

AIM: same open-system dynamics, we do not necessarily need the same overall dynamics!!

- The resulting configuration is much simpler than the original one: reduced number of d.o.f.
- Usually motivated on the ground of numerical simulations or approximate arguments

Imamoglu, Phys. Rev. A 50, 3650 (1994)
Mostame et al, New J. Phys. 14, 105013 (2012)

Main result

Tamascelli, Smirne, Huelga, Plenio. arXiv:1709.03509 (2017)

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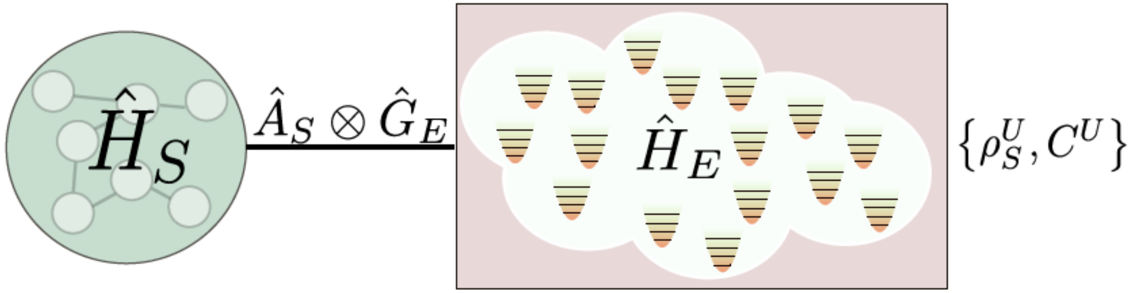

$$\hat{H}_{SE} = \hat{H}_S + \hat{H}_E + \sum_{j=1}^{\kappa} \hat{A}_{S,j} \otimes \hat{G}_{E,j}$$

$$\rho_S^U(t) = \mathrm{Tr}_E \left\{ e^{-i\hat{H}_{SE}t} \left(\rho_S(0) \otimes \rho_E(0)\right) e^{i\hat{H}_{SE}t} \right\}$$

$$G_{E,j}(t) = \mathrm{Tr}_E \left\{ \hat{G}_{E,j} e^{-i\hat{H}_E t} \rho_E(0) e^{i\hat{H}_E t} \right\}$$

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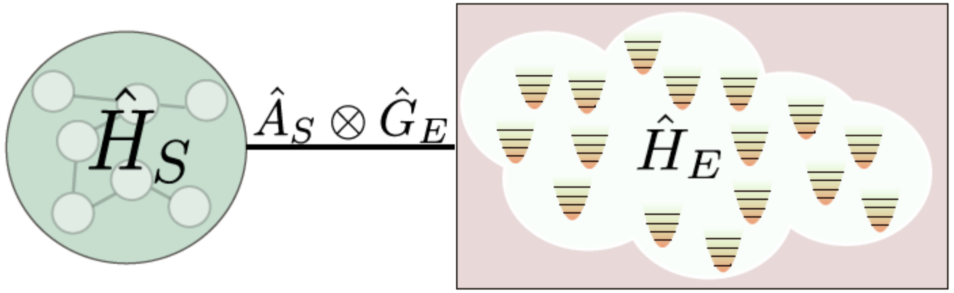
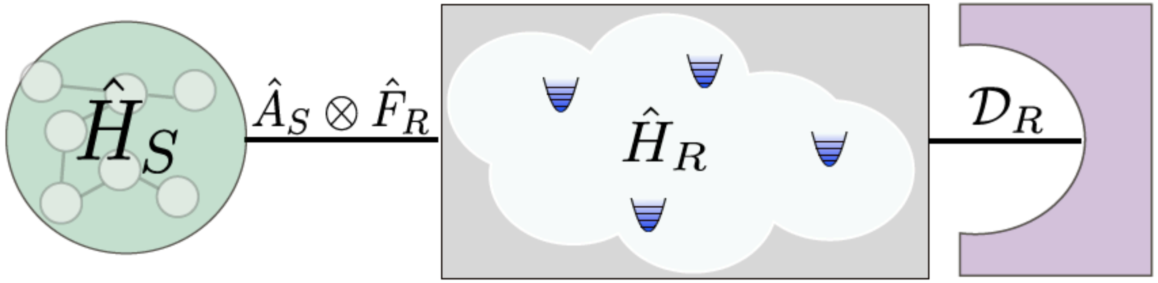


$\{\rho_S^U, C^U\}$



$\{\rho_S^L, C^L\}$

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$$\dot{\rho}_{SR}(t) = \mathcal{L}_{SR}\left[\rho_{SR}(t)\right] = -i\left[\hat{H}_{SR}, \rho_{SR}(t)\right] + \mathcal{D}_R\left[\rho_{SR}(t)\right]$$

$$\hat{H}_{SR} = \hat{H}_S + \hat{H}_R + \sum_{j=1}^{\kappa} \hat{A}_{S,j} \otimes \hat{F}_{R,j} \quad \mathcal{D}_R[\rho] = \sum_{j=1}^{\ell} \gamma_j \left(\hat{L}_{R,j} \rho \hat{L}^\dagger_{R,j} - \frac{1}{2}\left\{ \hat{L}^\dagger_{R,j}\hat{L}_{R,j}, \rho \right\} \right)$$

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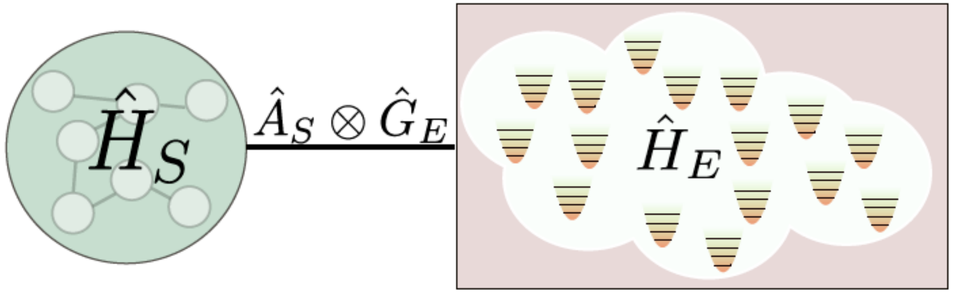
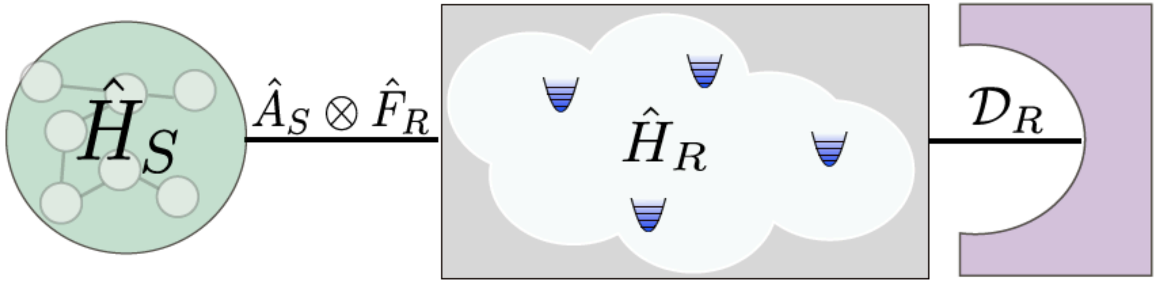
Main result

Tamascelli, Smirne, Huelga, Plenio. arXiv:1709.03509 (2017)



$\{\rho_S^U, C^U\}$



$\{\rho_S^L, C^L\}$

Same system S

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Free evolution of the environment E

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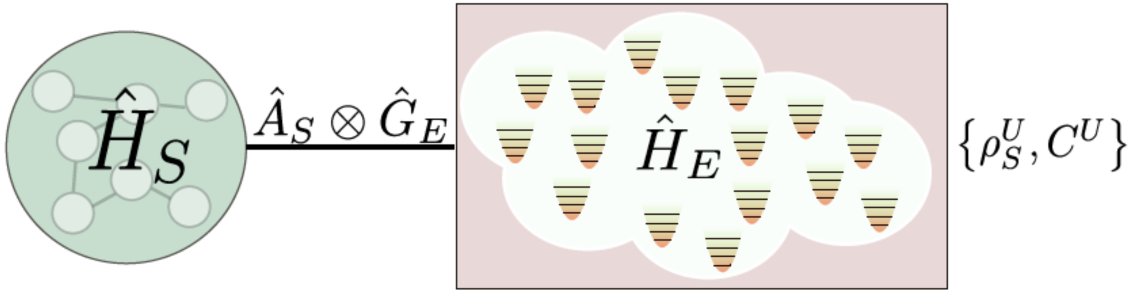
$$\mathcal{L}_R[\rho] = -i\left[\hat{H}_R, \rho\right] + \mathcal{D}_R[\rho]$$

“Free” evolution of the environment R

Main result

Tamascelli, Smirne, Huelga, Plenio. arXiv:1709.03509 (2017)



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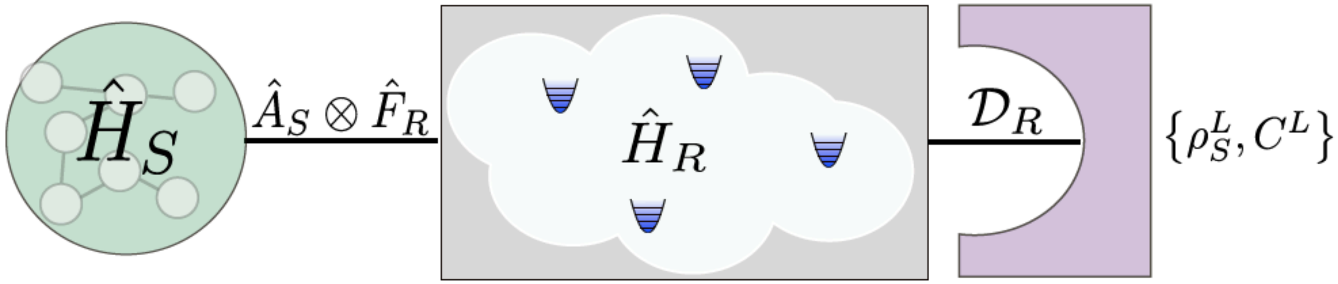

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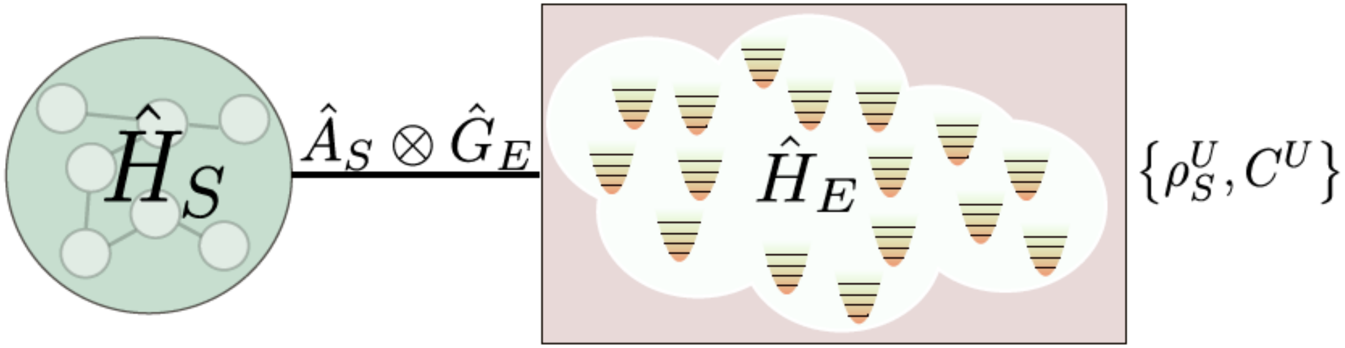
If E and R are bosonic baths, with $\rho_R(0)$ and $\rho_E(0)$ gaussian

$$\left.\begin{array}{rcl} F_{R,j}(t) & = & G_{E,j}(t) \\ C^L_{jj'}(t+s,s) & = & C^U_{jj'}(t+s,s) \end{array}\right\} \forall j, j', t, s \geq 0$$

$$\Longrightarrow \quad \rho_S^L(t) = \rho_S^U(t) \;\; \forall t.$$

Step 1: reduced dynamics from global unitaries



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- The reduced dynamics is fixed by

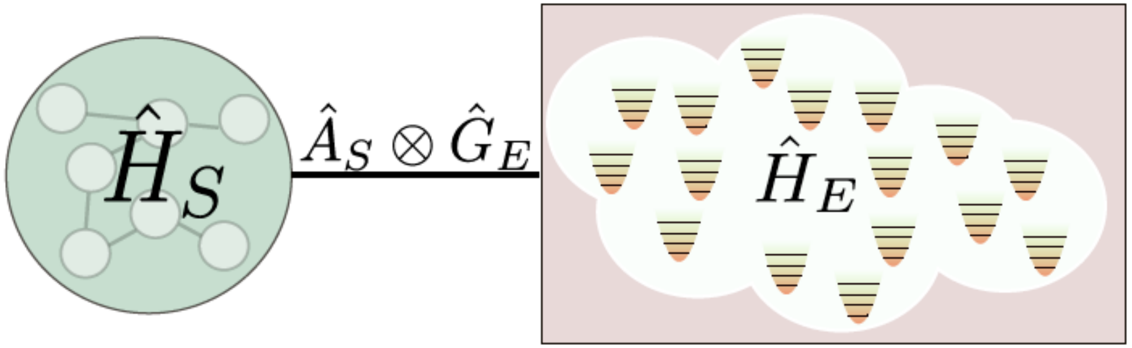
  Dyson expansion, TCL,…

I. The free system Hamiltonian $\hat{H}_S$

II. The system interaction terms $\hat{A}_{S,j}$

III. The environmental multi-time correlation functions $C_{\bar{E}}(t_1, \dots t_k) = \mathrm{Tr}_{\bar{E}} \left\{ \hat{G}_{\bar{E},j_1}(t_1) \dots \hat{G}_{\bar{E},j_k}(t_k) \rho_{\bar{E}}(0) \right\}$

Step 1: reduced dynamics from global unitaries



$\{\rho_S^U, C^U\}$

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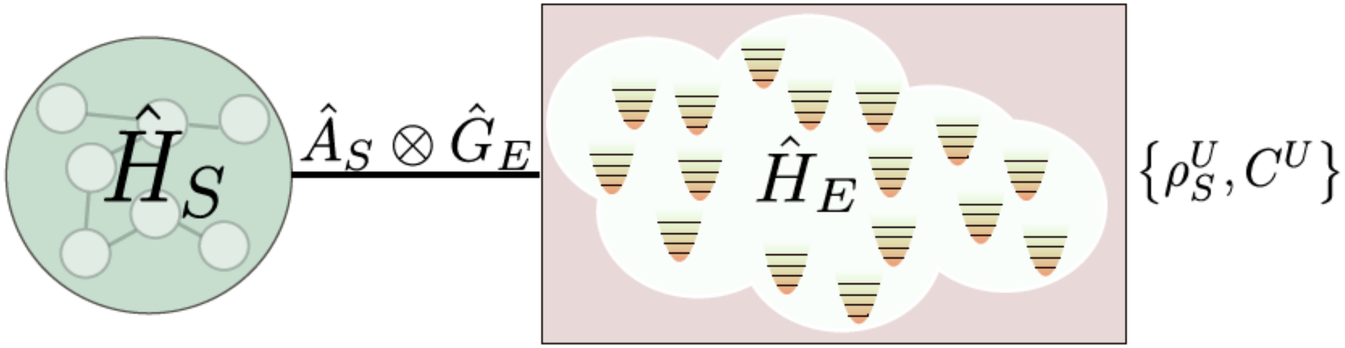
I. The free system Hamiltonian $\hat{H}_S$

II. The system interaction terms $\hat{A}_{S,j}$

III. The environmental multi-time correlation functions $C_{\bar{E}}(t_1, \dots t_k) = \mathrm{Tr}_{\bar{E}} \left\{ \hat{G}_{\bar{E},j_1}(t_1) \dots \hat{G}_{\bar{E},j_k}(t_k) \rho_{\bar{E}}(0) \right\}$

- For initial Gaussian states, the multi-time correlation functions are fixed by the one- and two-time correlation functions Breuer & Petruccione, *The theory of open quantum systems* (2002)

Step 1: reduced dynamics from global unitaries



$$\hat{H}_{SE} = \hat{H}_S + \hat{H}_E + \sum_{j=1}^{\kappa} \hat{A}_{S,j} \otimes \hat{G}_{E,j}$$

$$\rho_S^U(t) = \mathrm{Tr}_E \left\{ e^{-i\hat{H}_{SE}t} \left(\rho_S(0) \otimes \rho_E(0)\right) e^{i\hat{H}_{SE}t} \right\}$$

- The reduced dynamics is fixed by

  Dyson expansion, TCL,…

I. The free system Hamiltonian $\hat{H}_S$

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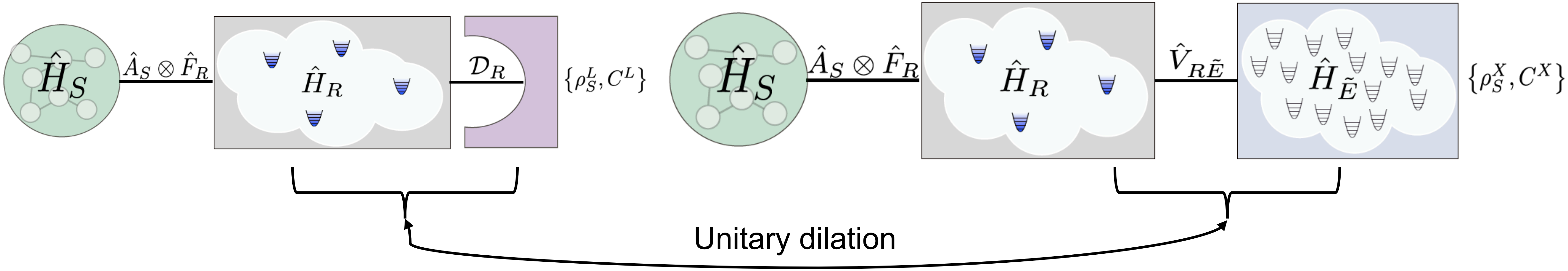
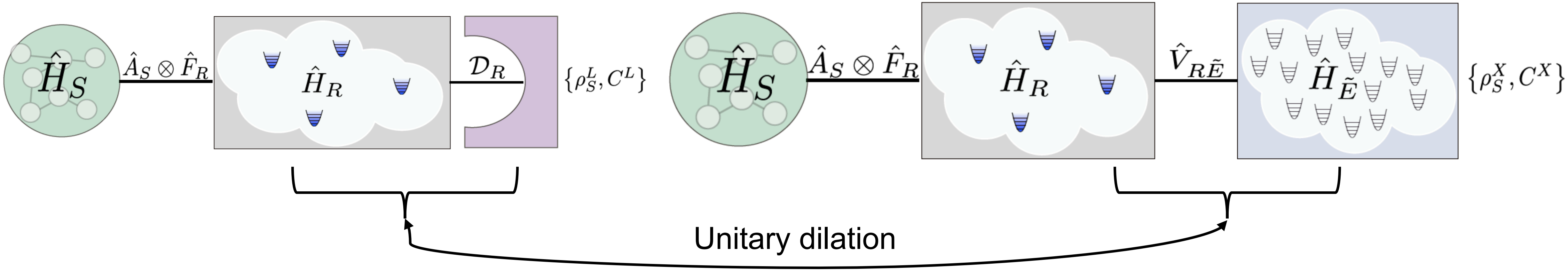
- For initial Gaussian states, the multi-time correlation functions are fixed by the one- and two-time correlation functions Breuer & Petruccione, *The theory of open quantum systems* (2002)

If we compare two unitaries with two initial Gaussian bath states, the reduced dynamics are the same if they have the same one- and two-time bath correlations

Step 2: unitary dilation of the Lindblad evolution



Step 2: unitary dilation of the Lindblad evolution



Bosonic modes on a proper Fock space

$$\left[\hat{b}_{\tilde{E}}(\omega, j), \hat{b}^\dagger_{\tilde{E}}(\omega', j')\right] = \delta_{jj'}\delta(\omega - \omega')$$

Initial vacuum state

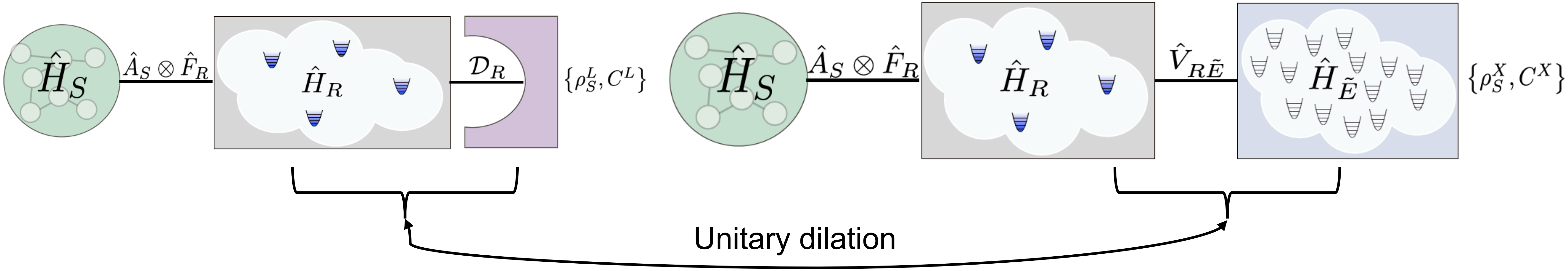
$$\rho_{SR\tilde{E}}(0) = \rho_{SR}(0) \otimes |0_{\tilde{E}}\rangle\langle 0_{\tilde{E}}|$$

$$\hat{H}_{SR\tilde{E}} = \hat{H}_{SR} + \hat{H}_{\tilde{E}} + \hat{V}_{R\tilde{E}},$$

$$\hat{H}_{\tilde{E}} = \sum_{j=1}^{\ell} \int_{-\infty}^{\infty} d\omega\, \omega\, \hat{b}^\dagger_{\tilde{E}}(\omega, j)\hat{b}_{\tilde{E}}(\omega, j),$$

$$\hat{V}_{R\tilde{E}} = \sum_{j=1}^{\ell} \sqrt{-\frac{\gamma_j}{2\pi}} \int_{-\infty}^{\infty} d\omega \hat{L}_{R,j}\hat{b}^\dagger_{\tilde{E}}(\omega, j) - \hat{L}^\dagger_{R,j}\hat{b}_{\tilde{E}}(\omega, j)$$

Step 2: unitary dilation of the Lindblad evolution



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$$\left[\hat{b}_{\tilde{E}}(\omega, j), \hat{b}^\dagger_{\tilde{E}}(\omega', j')\right] = \delta_{jj'}\delta(\omega - \omega')$$

Initial vacuum state

$$\rho_{SR\tilde{E}}(0) = \rho_{SR}(0) \otimes |0_{\tilde{E}}\rangle\langle 0_{\tilde{E}}|$$

Input field

$$\hat{b}_{in}(t, j) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \hat{b}_{\tilde{E}}(\omega, j)$$

$$\hat{H}_{SR\tilde{E}} = \hat{H}_{SR} + \hat{H}_{\tilde{E}} + \hat{V}_{R\tilde{E}},$$

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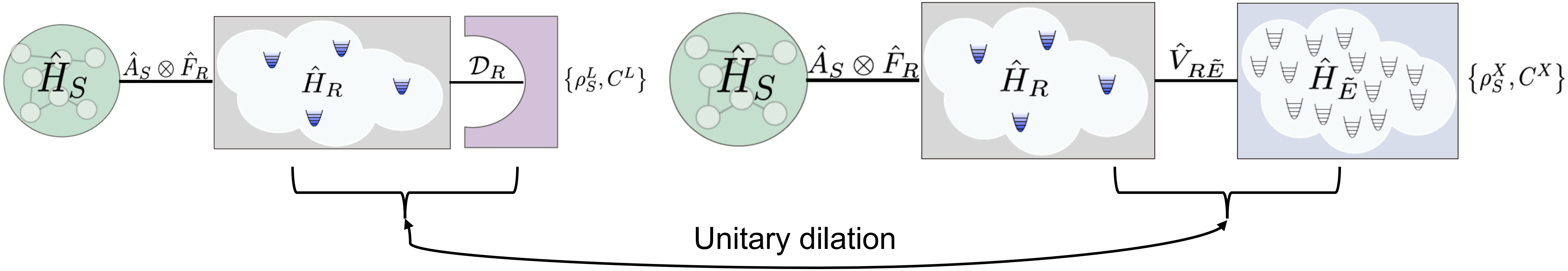
Gardiner & Zoller, *Quantum noise* (2004)

Step 2: unitary dilation of the Lindblad evolution



Bosonic modes on a proper Fock space

$$\left[\hat{b}_{\tilde{E}}(\omega, j), \hat{b}^\dagger_{\tilde{E}}(\omega', j')\right] = \delta_{jj'}\delta(\omega - \omega')$$

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$$\hat{b}_{in}(t, j) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \hat{b}_{\tilde{E}}(\omega, j)$$

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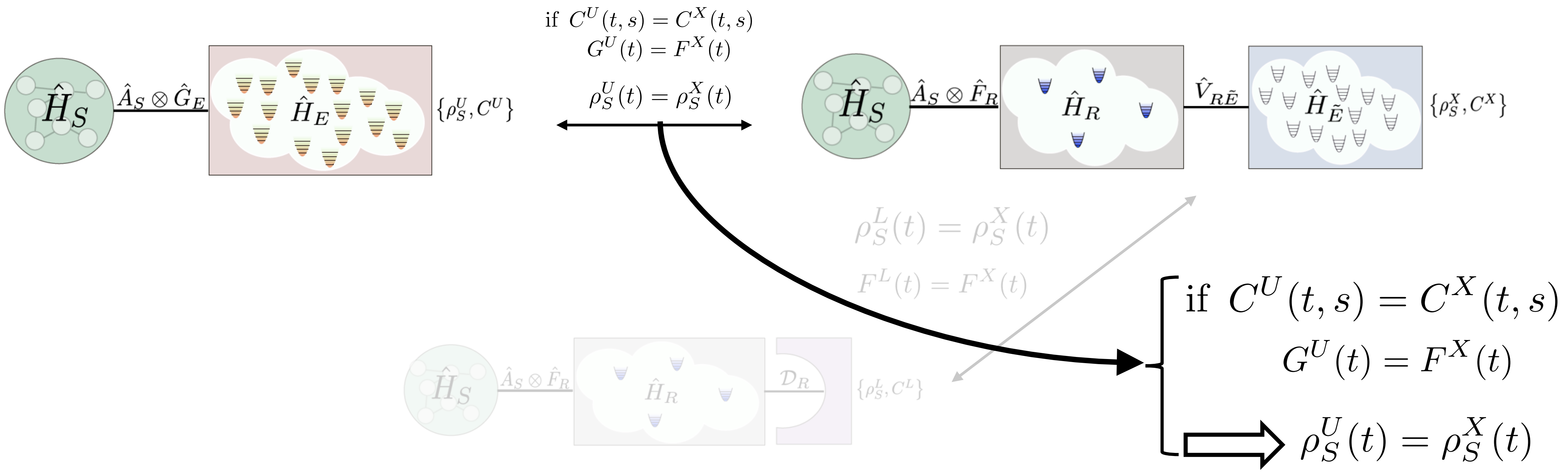
$$\hat{V}_{R\tilde{E}} = \sum_{i=1}^{\ell} \sqrt{-\frac{\gamma_j}{2\pi}} \int_{-\infty}^{\infty} d\omega \hat{L}_{R,j} \hat{b}^\dagger_{\tilde{E}}(\omega, j) - \hat{L}^\dagger_{R,j} \hat{b}_{\tilde{E}}(\omega, j)$$

$$\rho^L_{SR}(t) = e^{\mathcal{L}_{SR} t}[\rho_{SR}(0)] = \mathbf{Tr}_{\tilde{E}} \left\{ e^{-i\hat{H}_{SR\tilde{E}} t} \left(\rho_{SR}(0) \otimes |0_{\tilde{E}}\rangle\langle 0_{\tilde{E}}|\right) e^{i\hat{H}_{SR\tilde{E}} t} \right\}$$

Lindblad EXACT reduced dynamics

Step 1 plus 2



if $C^U(t,s) = C^X(t,s)$
$G^U(t) = F^X(t)$
$\rho_S^U(t) = \rho_S^X(t)$
$\hat{H}_S$
$\hat{A}_S \otimes \hat{G}_E$
$\hat{H}_E$
$\{\rho_S^U, C^U\}$
$\hat{A}_S \otimes \hat{F}_R$
$\hat{H}_R$
$\hat{V}_{R\tilde{E}}$
$\hat{H}_{\tilde{E}}$
$\{\rho_S^X, C^X\}$
$\rho_S^L(t) = \rho_S^X(t)$
$F^L(t) = F^X(t)$
$\mathcal{D}_R$
$\{\rho_S^L, C^L\}$
if $C^U(t,s) = C^X(t,s)$
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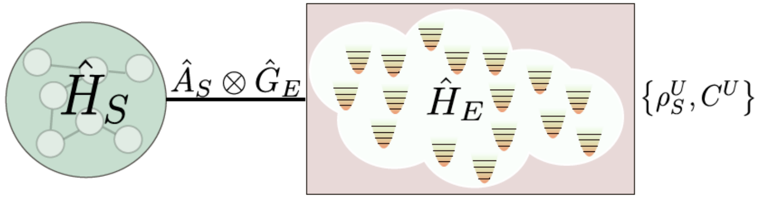
Step 1 plus 2



if $C^U(t,s) = C^X(t,s)$
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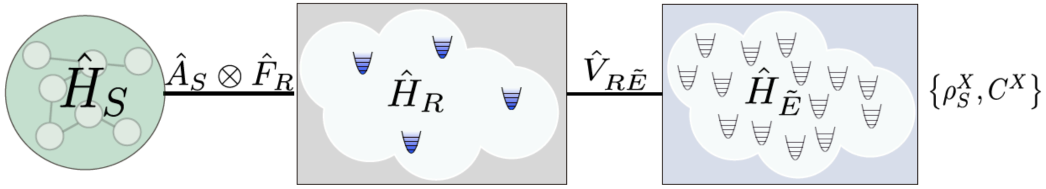
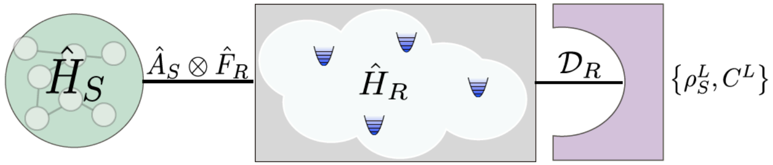
$\rho_S^U(t) = \rho_S^X(t)$



$\rho_S^L(t) = \rho_S^X(t)$

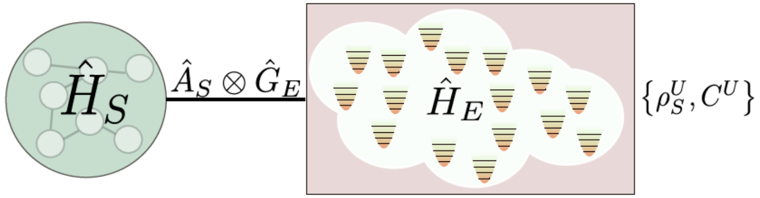
$F^L(t) = F^X(t)$



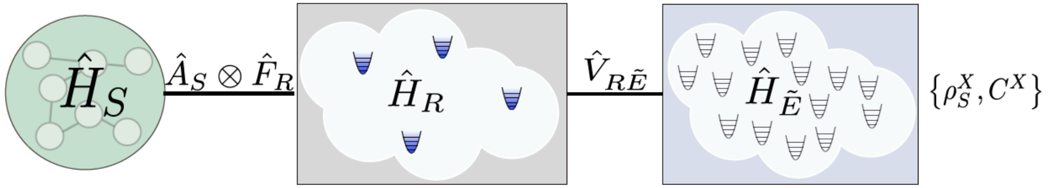
if $C^U(t,s) = C^X(t,s)$
$G^U(t) = F^L(t)$

$\Rightarrow \rho_S^U(t) = \rho_S^L(t)$

Step 3: quantum regression theorem



$$\text{if } C^U(t,s) = C^X(t,s)$$
$$G^U(t) = F^X(t)$$
$$\rho_S^U(t) = \rho_S^X(t)$$

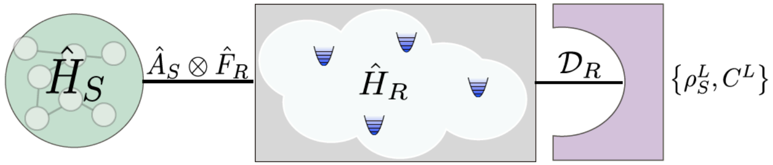


$$\rho_S^L(t) = \rho_S^X(t)$$
$$F^L(t) = F^X(t)$$

To establish the theorem

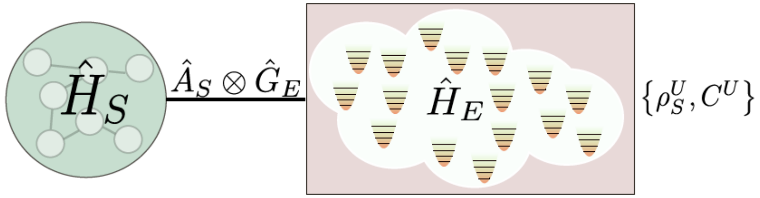
we need to replace

C^X with C^L



$$\left\{ \begin{array}{l} \text{if } C^U(t,s) = C^X(t,s) \\ G^U(t) = F^L(t) \end{array} \right. \Rightarrow \rho_S^U(t) = \rho_S^L(t)$$

Step 3: quantum regression theorem



if $C^U(t,s) = C^X(t,s)$
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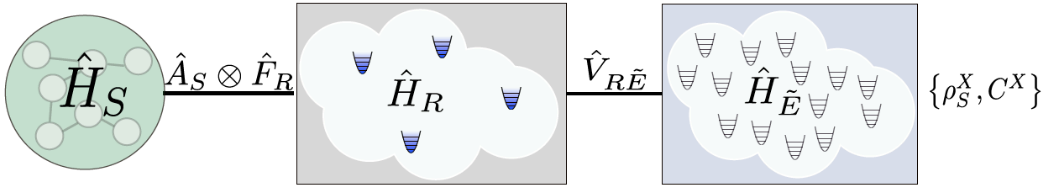
$\rho_S^U(t) = \rho_S^X(t)$



$\rho_S^L(t) = \rho_S^X(t)$

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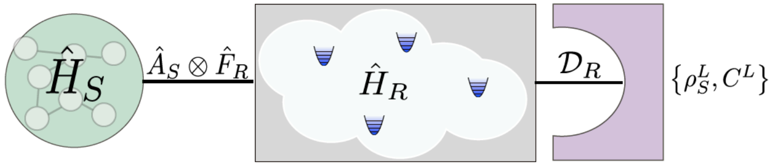
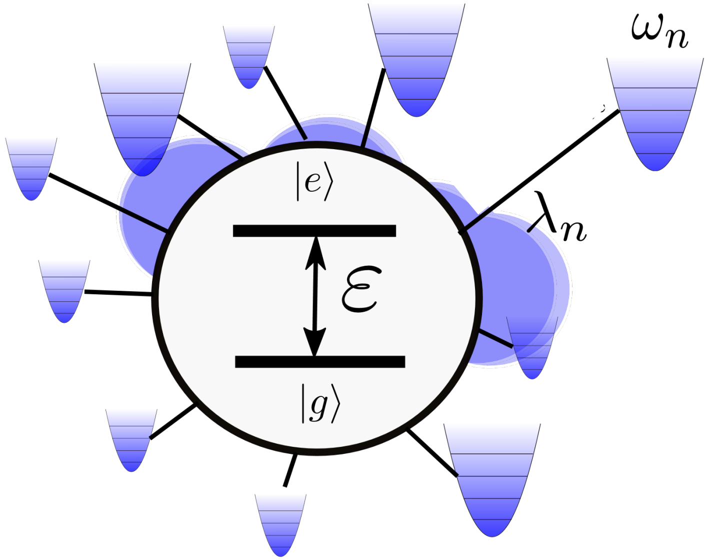


QUANTUM REGRESSION THEOREM

$$C^L_{jj'}(t+s,s) = C^X_{jj'}(t+s,s)$$
$$= \mathrm{Tr}_{RE}\left\{e^{i\hat{H}_{R\tilde{E}}(t+s)}\hat{F}_{R,j}e^{-i\hat{H}_{R\tilde{E}}(t+s)}\, e^{i\hat{H}_{R\tilde{E}}s}\hat{F}_{R,j'}e^{-i\hat{H}_{R\tilde{E}}s}(\rho_R(0)\otimes\rho_{\tilde{E}}(0))\right\}$$

The multitime correlations of $\hat{O}_R \otimes \hat{1}_{\tilde{E}}$ on R+$\tilde{E}$ can be directly evaluated by means of the reduced dynamics on R, i.e. with the Lindblad generator $\mathcal{L}_R$

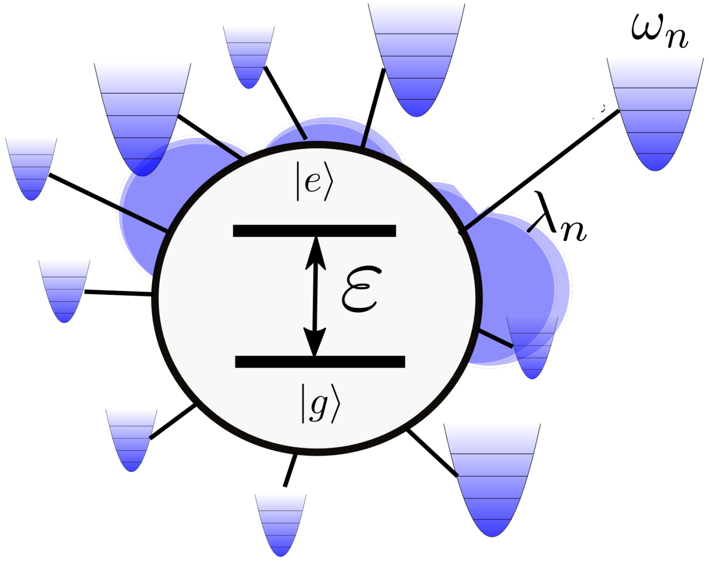
Spin-boson model



Archetypal model of OQS and common testbed for numerical approaches

$$\underbrace{\omega\sigma_z}_{H_S} + \underbrace{\sigma_x \otimes \int_{-\infty}^{\infty} d\omega (g(\omega)\hat{a}_\omega + g^*(\omega)\hat{a}^\dagger_\omega)}_{H_I} + \underbrace{\int_{-\infty}^{+\infty} d\omega\, \omega\, \hat{a}^\dagger_\omega \hat{a}_\omega}_{H_E}$$

Spin-boson model



Archetypal model of OQS and common testbed for numerical approaches

H_S | H_I | H_E

$$\omega\sigma_z + \sigma_x \otimes \int_{-\infty}^{\infty} d\omega (g(\omega)\hat{a}_\omega + g^*(\omega)\hat{a}_\omega^\dagger) + \int_{-\infty}^{+\infty} d\omega\, \omega\, \hat{a}_\omega^\dagger \hat{a}_\omega$$

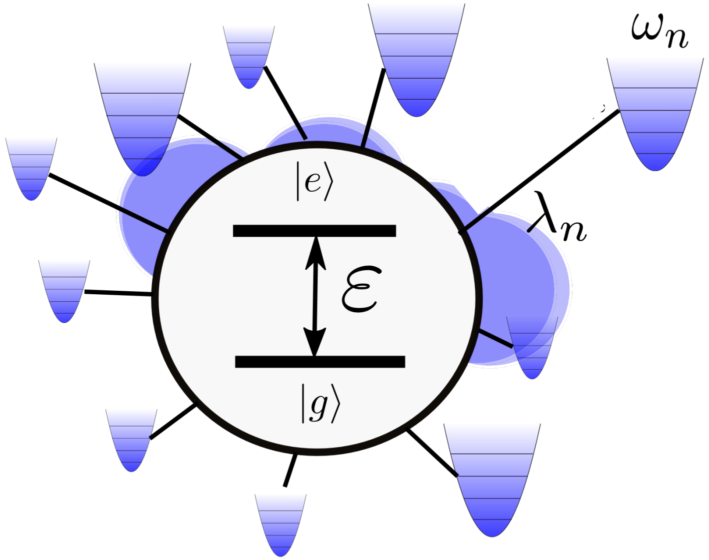
- If RWA

H_I

$$\sigma_+ \otimes \int_{-\infty}^{\infty} d\omega g(\omega)\hat{a}_\omega + \sigma_- \otimes \int_{-\infty}^{\infty} d\omega g^*(\omega)\hat{a}_\omega^\dagger$$

The number of excitations is conserved

Spin-boson model



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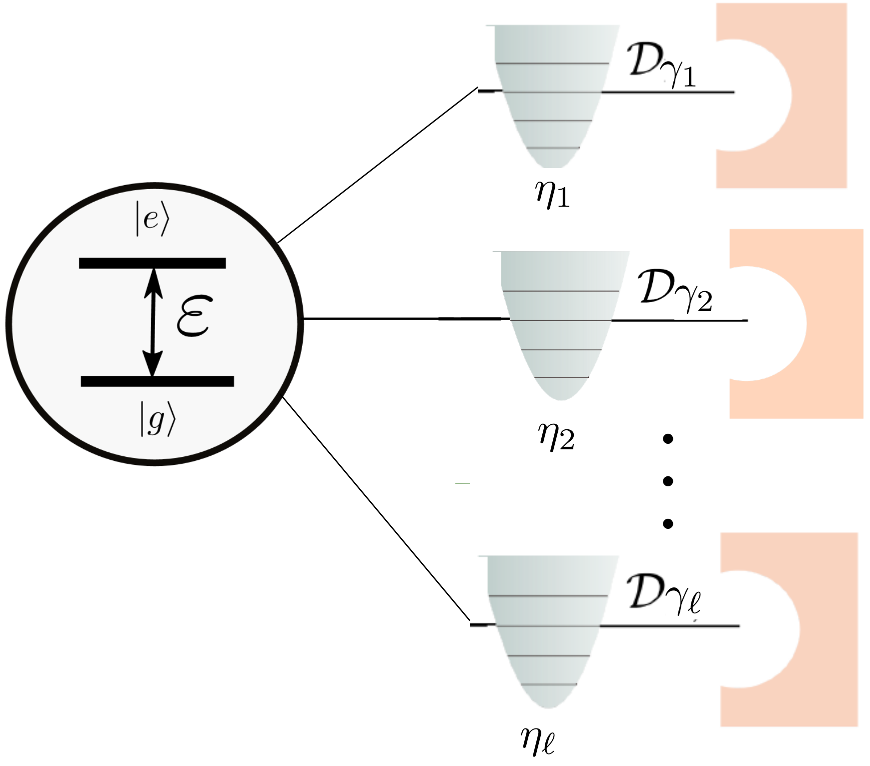
The number of excitations is conserved

$$\rho_E(0) = |0\rangle\langle 0|$$

In both cases expectation values are 0 and there is only one 2-time correlation (fixed by the correlation spectrum $S(\omega) = |g(\omega)|^2$)

$$C^U_{SB}(t) = \int_{-\infty}^{\infty} d\omega |g(\omega)|^2 e^{-i\omega t}$$

The pseudomodes



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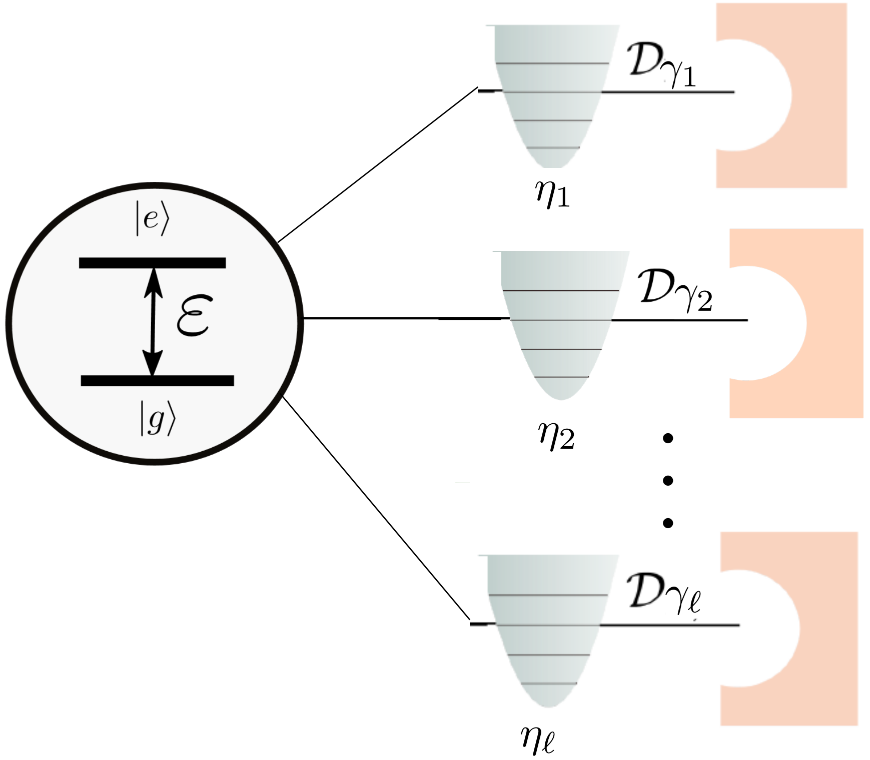


Finite (small) number of auxiliary modes

$$[\hat{c}_j, \hat{c}_l^\dagger] = \delta_{jl}$$

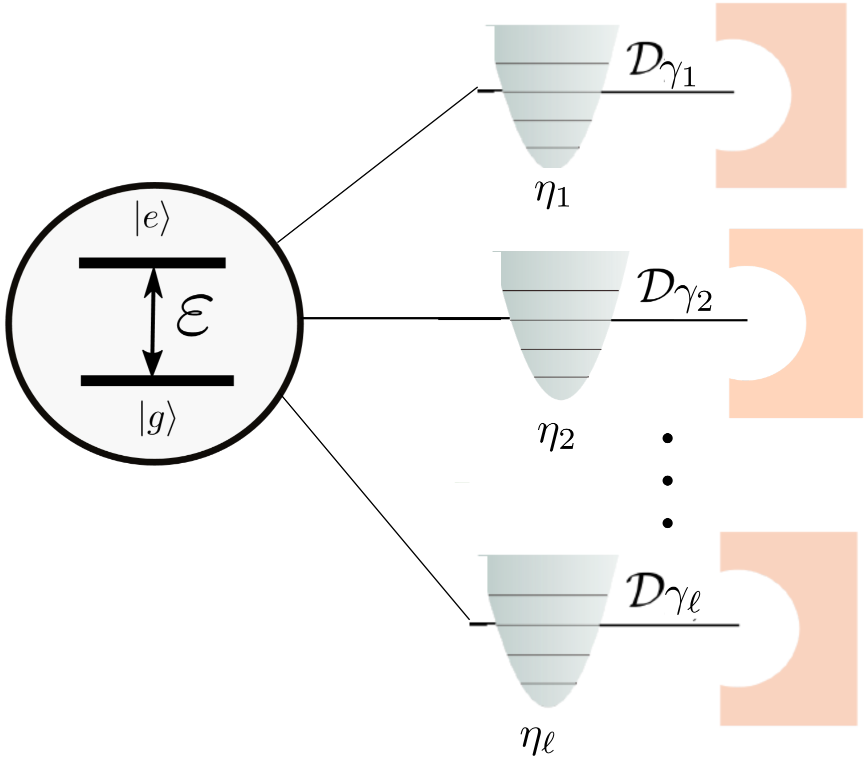
$$\hat{H}_R = \sum_{j=1}^{\ell} \eta_j \hat{c}_j^\dagger \hat{c}_j \qquad \hat{H}_{SR} = \hat{H}_S + \hat{H}_R + \sigma_x \otimes \sum_j (\lambda \hat{c}_j + \lambda^* \hat{c}_j^\dagger)$$

After RWA $\sigma_+ \otimes \sum_j \lambda \hat{c}_j + \sigma_- \otimes \lambda^* \hat{c}_j^\dagger$

$$\mathcal{D}_R[\rho] = \sum_{j=1}^{\ell} \gamma_j \left(\hat{c}_j \rho \hat{c}_j^\dagger - \frac{1}{2} \left\{ \hat{c}_j^\dagger \hat{c}_j, \rho \right\} \right)$$

Lindblad spontaneous emission

The pseudomodes



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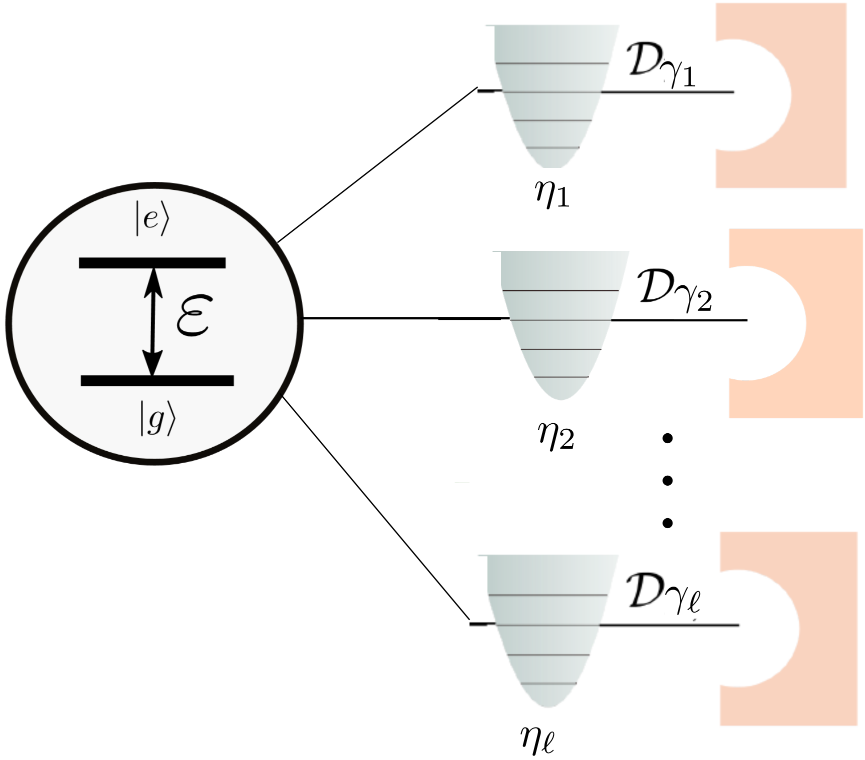
Lindblad spontaneous emission

$$\rho_R(0) = |0\rangle\langle 0|$$

$\Downarrow$

$$C_{SB}^L(t) = |\lambda|^2 \sum_{j=1}^{\ell} e^{(i\eta_j - \gamma_j/2)t}$$

The pseudomodes



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$$\Downarrow$$

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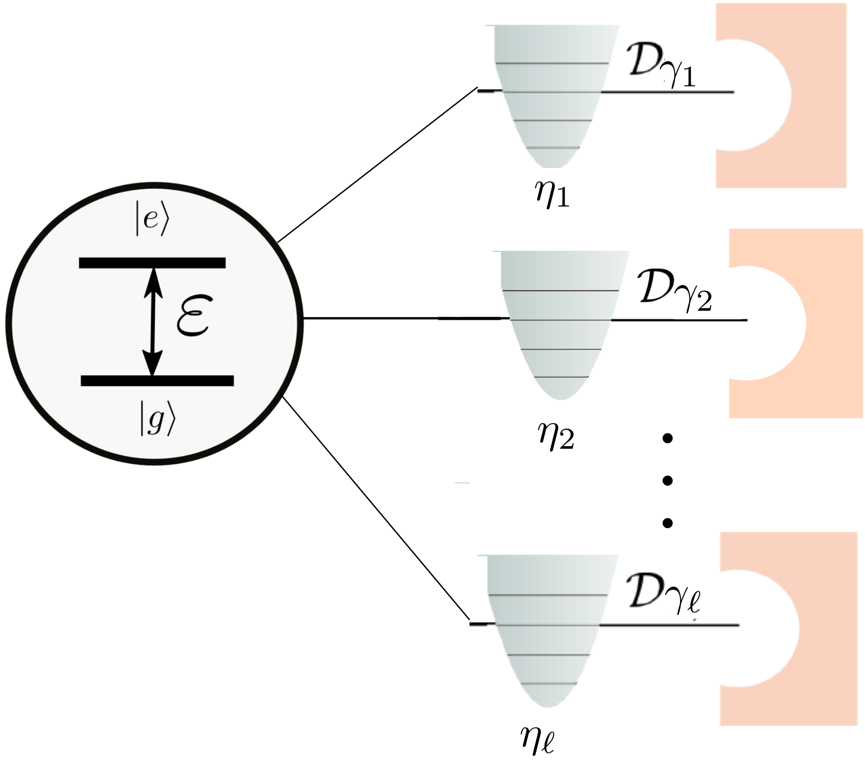
$$C_{SB}^U(t) = \int_{-\infty}^{\infty} d\omega |g(\omega)|^2 e^{-i\omega t}$$

$\Rightarrow$

ℓ pseudomodes for a $C_{SB}^U(t)$ which is the sum of ℓ exponentials

The pseudomodes



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$$\mathcal{D}_R[\rho] = \sum_{j=1}^{\ell} \gamma_j \left(\hat{c}_j \rho \hat{c}_j^\dagger - \frac{1}{2} \left\{ \hat{c}_j^\dagger \hat{c}_j, \rho \right\} \right)$$

Lindblad spontaneous emission

$$\rho_R(0) = |0\rangle\langle 0|$$

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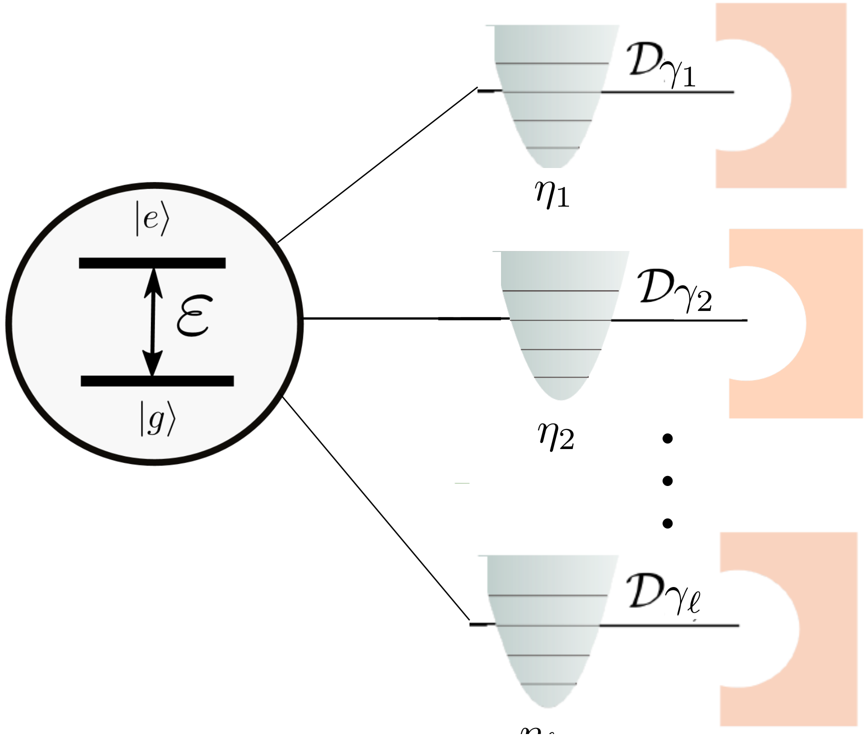
$$C_{SB}^U(t) = \int_{-\infty}^{\infty} d\omega |g(\omega)|^2 e^{-i\omega t}$$

$\Rightarrow$ ℓ pseudomodes for a $C_{SB}^U(t)$ which is the sum of ℓ exponentials

- If RWA Garraway, PRA 55,2290 (1997)
- Generalized to non-excitation-conserving coupling

The pseudomodes



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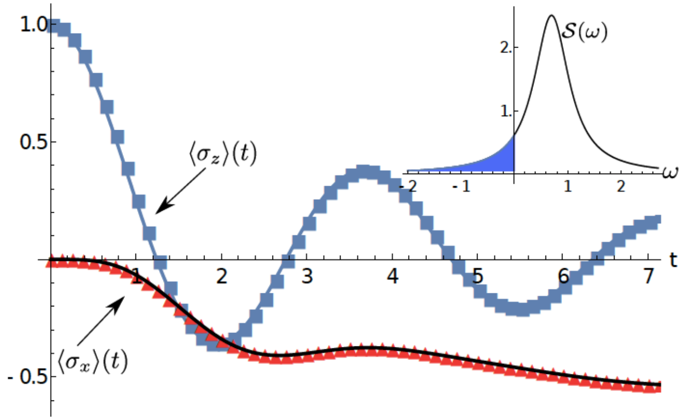
Lindblad spontaneous emission



Lorentzian spectrum

Strong coupling, non-excitation-conserving: beyond

Garraway, PRA 55,2290 (1997)

Imamoglu, PRA 50, 3650 (1994)
[Approximated analysis]

Approach for an approximated equivalence

What if one needs a very high number of pseudomodes, or, even, there is no Lindblad such that $C^L = C^U$*?*

$$C^U(t) \approx C^L(t) \Longrightarrow C^X(t) \approx C^U(t) \Longrightarrow \rho_S^U(t) \approx \rho_S^L(t)$$

Mascherpa, Smirne, Huelga &Plenio, PRL 118, 100401 (2017) to bound the error!!

Step 2 and 3 still work, without adding any further approximation!!

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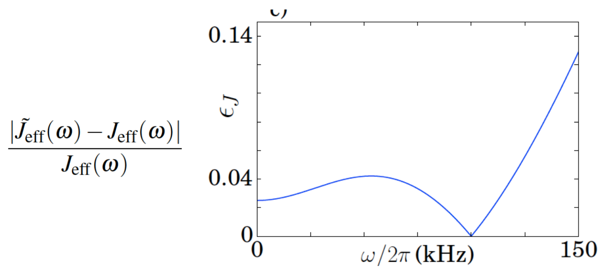
Mascherpa, Smirne, Huelga &Plenio, PRL 118, 100401 (2017) to <u>bound</u> the error!!

<u>Step 2 and 3 still work, without adding any further approximation!!</u>

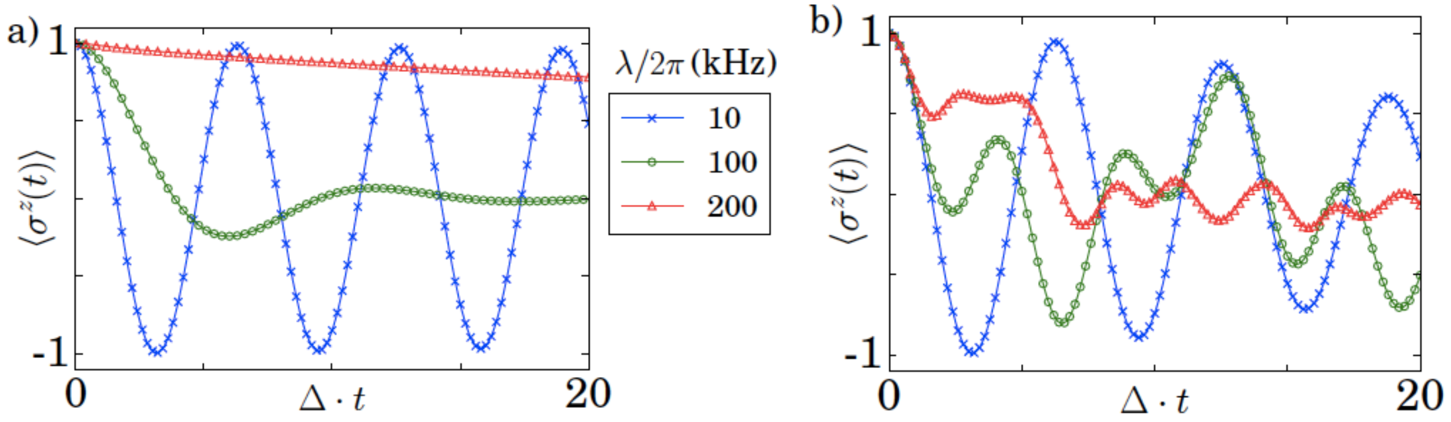
- EXAMPLE: Simulation of an antisymmetrized Lorentzian spectrum, finite T with ion traps

$$\mathscr{D}_{\kappa,\bar{n}}\rho = \kappa(\bar{n}+1)[a\rho a^\dagger - a^\dagger a\rho] + \kappa\bar{n}[a^\dagger\rho a - aa^\dagger\rho] + \text{H.c.}$$



Solid lines: numerically exact (TEDOPA)
Markers: Lindblad-type dynamics

Lemmer, Cormick, Tamascelli, Schaetz, Huelga & Plenio, arXiv: 1704.00629 (2017)

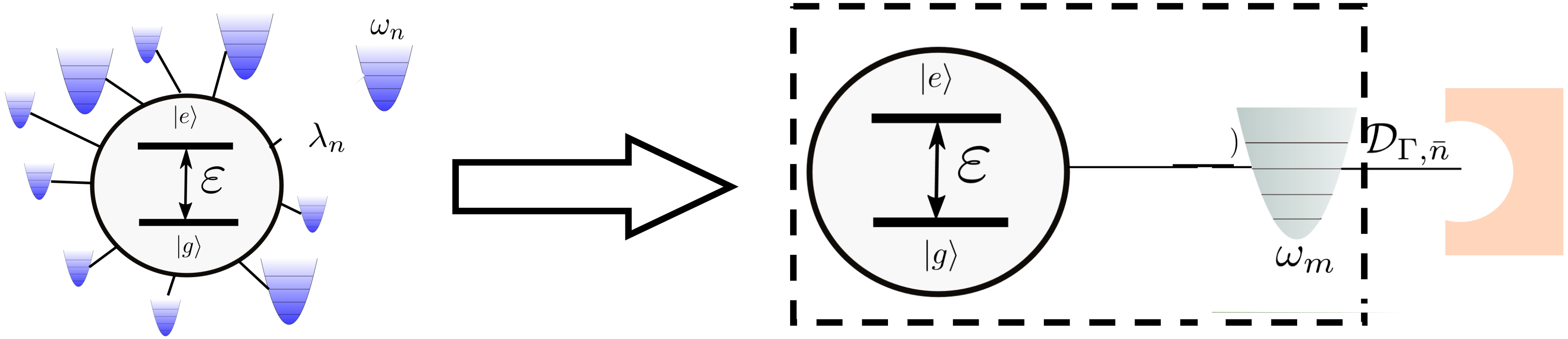


Non-resonant regime

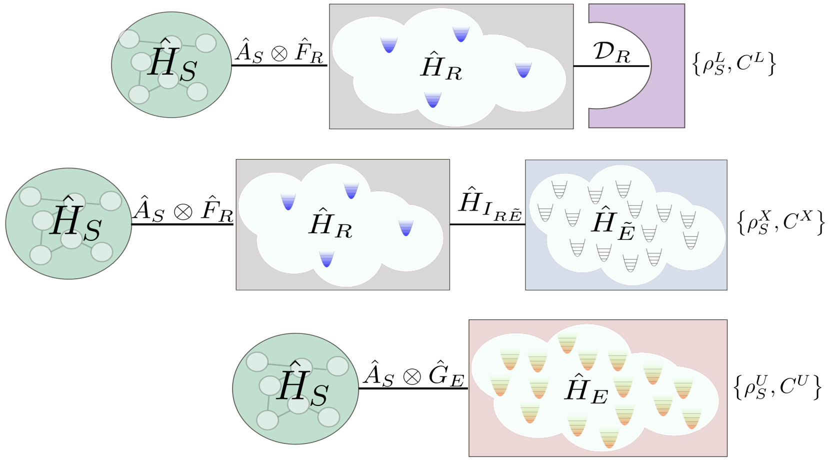
Resonant regime

Summary

Physical idea:
non-Markovian core



Proof of a sufficient condition guarantying the equivalence



$$\left.\begin{array}{rcl} F_{R,j}(t) & = & G_{E,j}(t) \\ C^L_{jj'}(t+s,s) & = & C^U_{jj'}(t+s,s) \end{array}\right\} \forall j,j',t,s \geq 0$$

$$\implies \quad \rho_S^L(t) = \rho_S^U(t) \ \ \forall t.$$

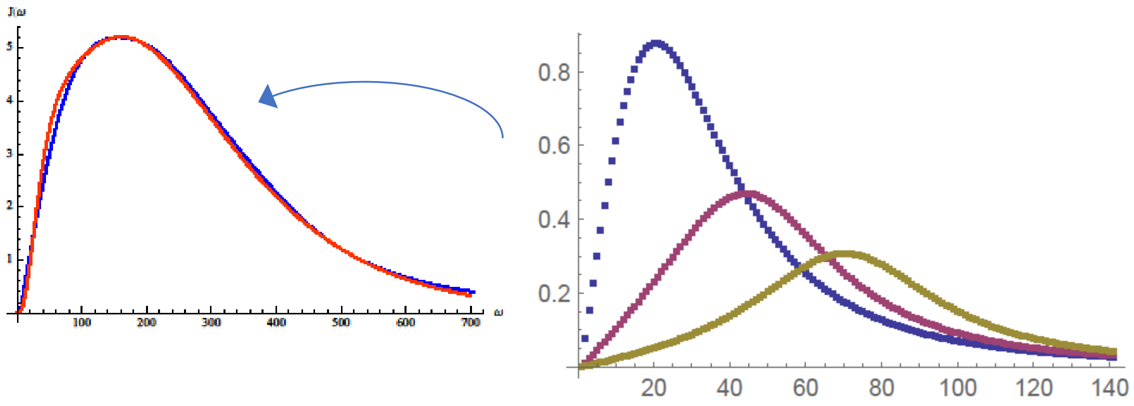
Direct applications

- Generalization of the pseudomodes approach to different couplings
- Simulation of the spin-boson model with ion traps

Outlook

Formulating a ”dictionary” of correlation functions obtained from some (simple) reference Lindblad structures

$$J(\omega) \approx \sum_i w_i \, J_{AL}^{\omega_m^i, \Gamma^i}(\omega)$$

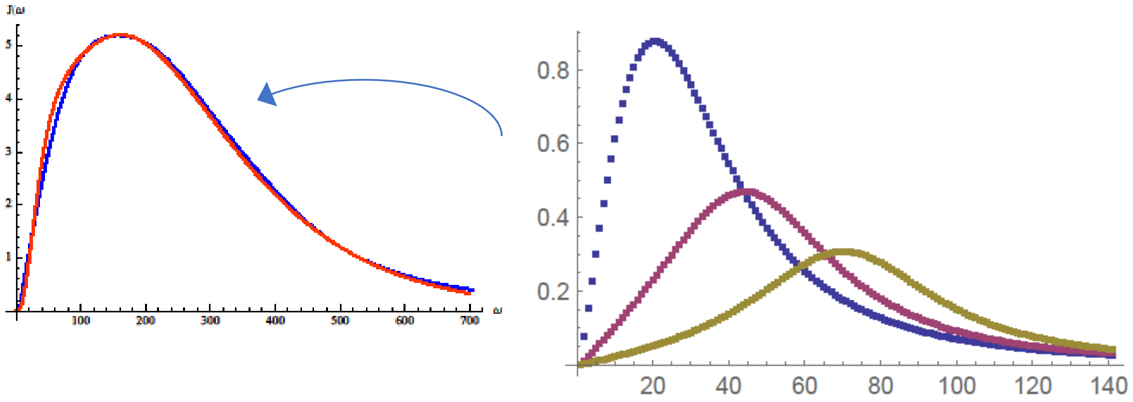


Optimization of the decomposition of the environmental correlation function

Outlook

Formulating a ”dictionary” of correlation functions obtained from some (simple) reference Lindblad structures

$$J(\omega) \approx \sum_i w_i \, J_{AL}^{\omega_m^i,\Gamma^i}(\omega)$$



Optimization of the decomposition of the environmental correlation function

Extending the approach (fermionic baths, system multitime…)

Outlook

Formulating a ”dictionary” of correlation functions obtained from some (simple) reference Lindblad structures

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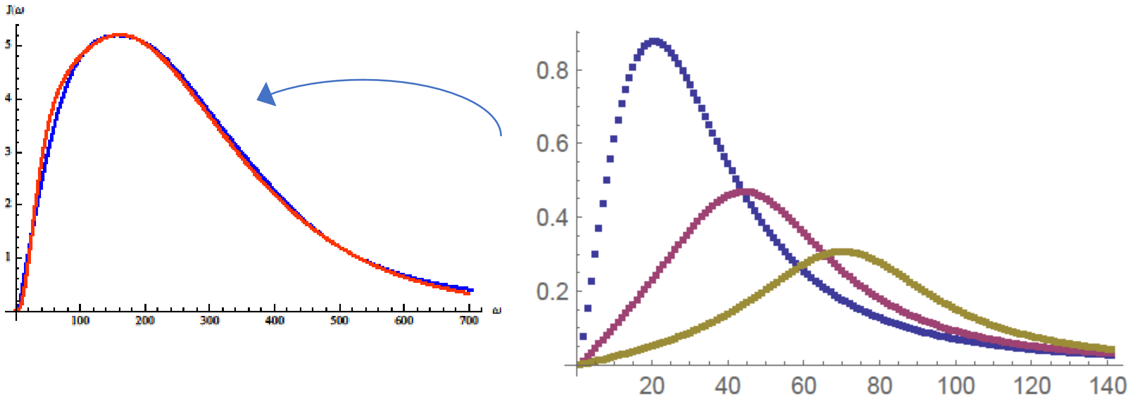
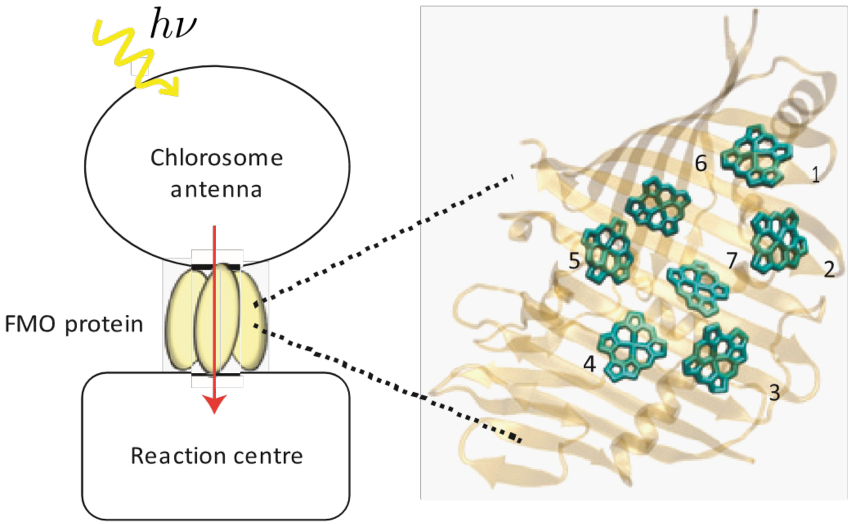


Optimization of the decomposition of the environmental correlation function

Extending the approach (fermionic baths, system multitime…)



…one step toward full/extended simulation of complex open-system dynamics

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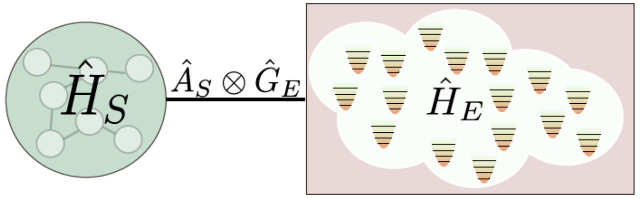
PAPETS

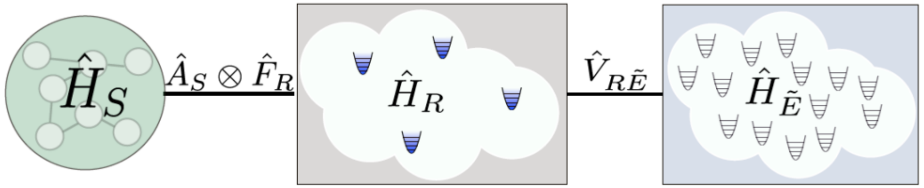
Institut für Theoretische Physik

Summarizing the proof



$\rho_S^U(t) = \rho_S^X(t)$

if $C^U(t,s) = C^X(t,s)$
$G^U(t) = F^X(t)$



$\rho^L(t) = \rho^X(t)$

$F^L(t) = F^X(t)$

if $C^U(t,s) = C^X(t,s)$
$G^U(t) = F^X(t)$

$\Rightarrow \rho_S^U(t) = \rho_S^X(t)$

QUANTUM REGRESSION THEOREM

$$C^L_{jj'}(t+s,s) = C^X(t+s,s)$$
$$= \mathrm{Tr}_{RE}\left\{e^{i\hat{H}_{R\tilde{E}}(t+s)}\hat{F}_{R,j}e^{-i\hat{H}_{R\tilde{E}}(t+s)}\, e^{i\hat{H}_{R\tilde{E}}s}\hat{F}_{R,j'}e^{-i\hat{H}_{R\tilde{E}}s}(\rho_R(0)\otimes\rho_{\tilde{E}}(0))\right\}$$

Summarizing the proof

$\rho_S^U(t) = \rho_S^X(t)$

if $C^U(t,s) = C^X(t,s)$

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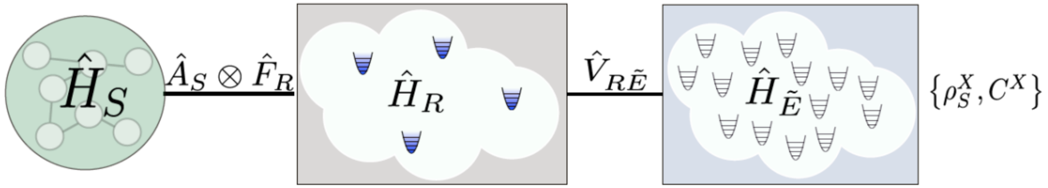


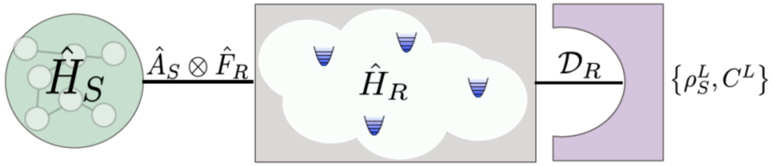
$\rho^L(t) = \rho^X(t)$

$F^L(t) = F^X(t)$

if $C^U(t,s) = C^X(t,s)$

$G^U(t) = F^L(t)$

$\rho_S^U(t) = \rho_S^L(t)$



QUANTUM REGRESSION THEOREM

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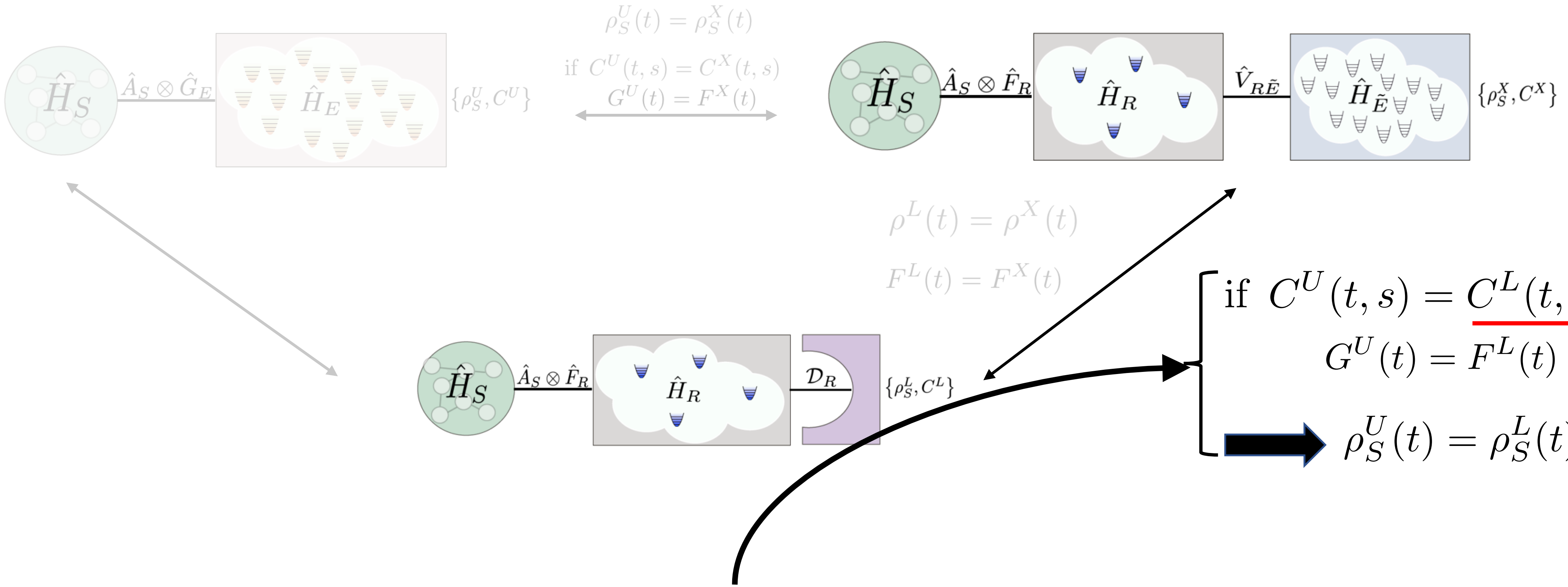
Summarizing the proof



if $C^U(t,s) = C^L(t,s)$

$G^U(t) = F^L(t)$

$\Rightarrow \rho_S^U(t) = \rho_S^L(t)$

QUANTUM REGRESSION THEOREM

$$C^L_{jj'}(t+s,s) = C^X(t+s,s)$$
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