

Testing Quantum Mechanics with Weak and Protective Measurements

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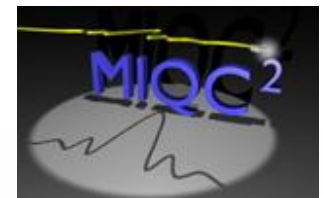
FP7: BRISQ2



Premiale
«Q-SecGroundSpace»



Seed «GeQuM»



Weak measurements

Weak measurements [Aharonov et al., PRL 60 (1988)]: little information is extracted from a single measurement, but the state does NOT collapse.

Weak value: $\langle \hat{A} \rangle_w = \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$

Pre-selected state: $|\psi_i\rangle$

Post-selected state: $|\psi_f\rangle$

Von Neumann coupling between an observable \hat{A} and a pointer observable \hat{P} :

$$\hat{U} = \exp(-ig\hat{A} \otimes \hat{P})$$

Projective measurement (post-selection on $|\psi_f\rangle$): $\hat{\Pi}_f = |\psi_f\rangle\langle\psi_f|$

\hat{X} and \hat{P}
canonically
conjugated

$$|\phi_{out}\rangle = \hat{\Pi}_f \hat{U} |\phi_{in}\rangle = \hat{\Pi}_f \hat{U} |\psi_i\rangle \otimes |f_i\rangle$$

In the weak interaction regime approximation:

$$\langle \hat{X} \rangle = \frac{\langle \phi_{out} | \hat{X} | \phi_{out} \rangle}{\langle \psi_i | \hat{\Pi}_f | \psi_i \rangle} = g \operatorname{Re}[\langle \hat{A} \rangle_w]$$



Weak measurements

$$\langle \hat{A} \rangle_w = \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

$$\hat{U} = \exp(-ig\hat{A} \otimes \hat{P})$$

$$\hat{\Pi}_f = |\psi_f\rangle\langle\psi_f|$$

Some interesting properties:

$\langle \hat{A} \rangle_w$ is a complex number

$\text{Re}[\langle \hat{A} \rangle_w]$ is unbounded!

□ Metrology:

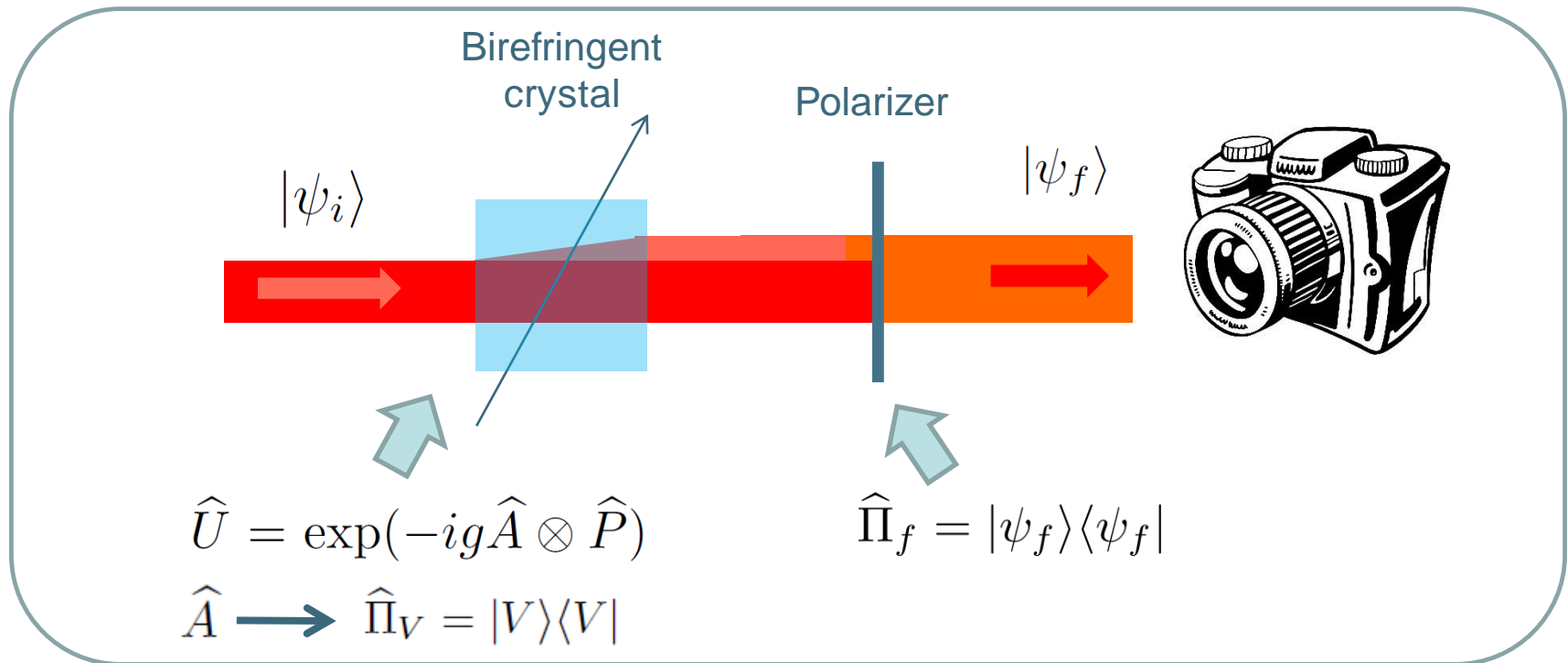
- Amplification of measurement of coupling strength, without amplifying unrelated noise [Boyd et al.]:
 - Light beam displacement [Kwiat et al.]
 - Angular deflection [Dixon et al.]

□ Foundations of Quantum Mechanics:

- Measurement of **incompatible observables on the same particle** [Mitchinson et al.]
- Tests of **Quantum Contextuality** [Pusey]
- Hints on Quantum Mechanics interpretations [TSVF, Aharonov et al., ...]



Weak measurement implementation



We measure the position observable \hat{X} ,
canonically conjugated to the pointer observable \hat{P}

$$\langle \hat{X} \rangle = g \text{Re}[\langle \hat{\Pi}_V \rangle_w]$$



Experiment Investigating the Connection between Weak Values and Contextuality

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(Received 5 February 2016; published 2 May 2016)

Weak value measurements have recently given rise to a great amount of interest in both the possibility of measurement amplification and the chance for further quantum mechanics foundations investigation. In particular, a question emerged about weak values being proof of the incompatibility between quantum mechanics and noncontextual hidden variables theories (NCHVTs). A test to provide a conclusive answer to this question was given by Pusey [Phys. Rev. Lett. 113, 200401 (2014)], where a theorem was derived showing the NCHVT incompatibility with the observation of anomalous weak values under specific conditions. In this Letter we realize this proposal, clearly pointing out the connection between weak values and the contextual nature of quantum mechanics.

DOI: 10.1103/PhysRevLett.116.180401



WMs and Quantum Contextuality

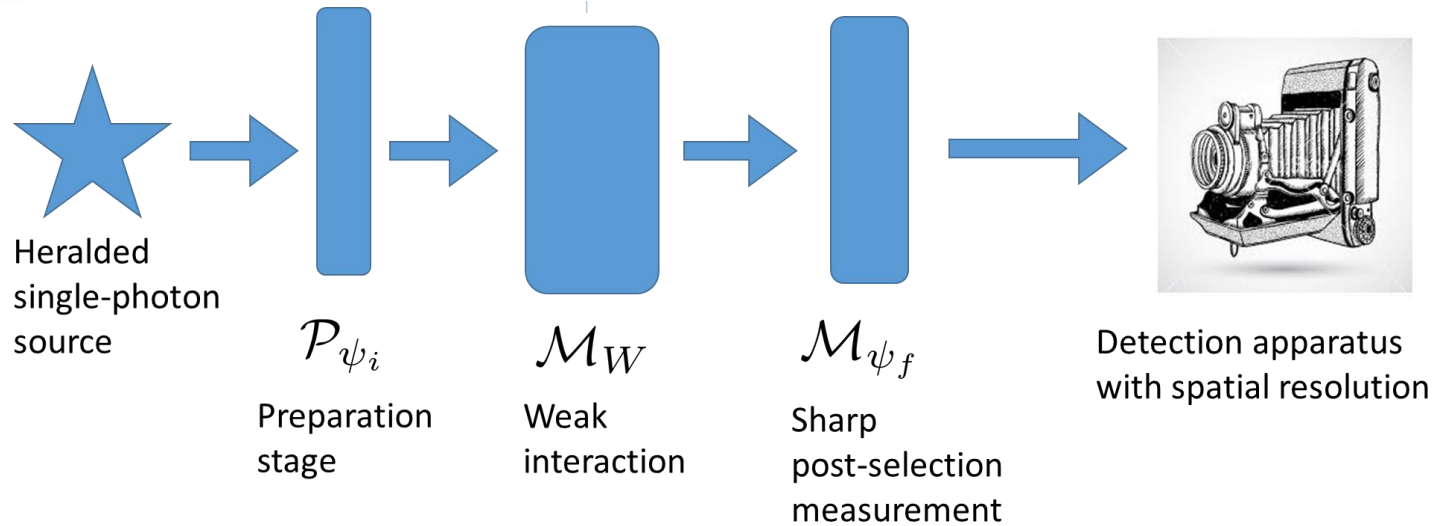
Non-Contextual Hidden Variable Theory: ontological model of an operational theory where, if two experimental procedures are operationally equivalent, then they have equivalent representations in such model [Spekkens, PRA 71 (2005)].

The measurement outcome depends only on the Hermitian operator associated with the measurement, not on the ones measured simultaneously with it: **each observable has a predetermined value, independent of the context.**

Question: Can Weak Values be a signature of Quantum Contextuality?

Answer: Yes! But only under specific conditions [Pusey, PRL 113 (2014)].

WMs and Quantum Contextuality



Initial and final states are non-orthogonal: $p_{\psi_f} := \mathbb{P}(\text{PASS} | \mathcal{P}_{\psi_i}, \mathcal{M}_{\psi_f}) > 0$

Without post-selection: $\mathbb{P}(x | \mathcal{P}, \mathcal{M}_W) = p_n(x-g)\mathbb{P}(1 | \mathcal{P}, \mathcal{M}_\Pi) + p_n(x)\mathbb{P}(0 | \mathcal{P}, \mathcal{M}_\Pi) \quad \forall \mathcal{P}$

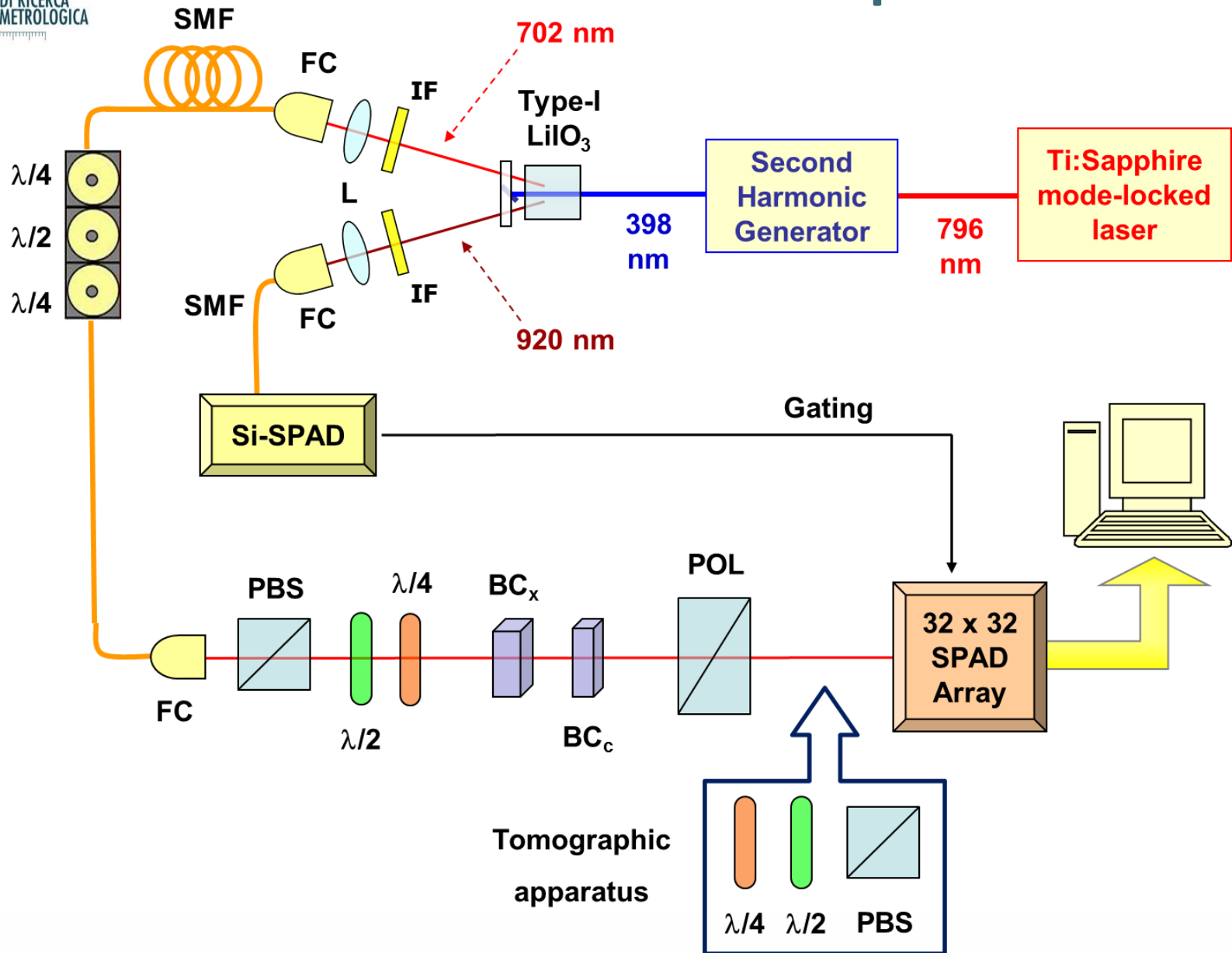
$\exists p_d : \mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_W, \mathcal{M}_{\psi_f}) = (1-p_d)\mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_{\psi_f}) + p_d\mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_d) \quad \forall \mathcal{P}$

$p_- := (p_{\psi_f})^{-1} \int_{-\infty}^0 \mathbb{P}(x, \text{PASS} | \mathcal{P}_{\psi_i}, \mathcal{M}_W, \mathcal{M}_{\psi_f}) dx : \quad \mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}} > 0$

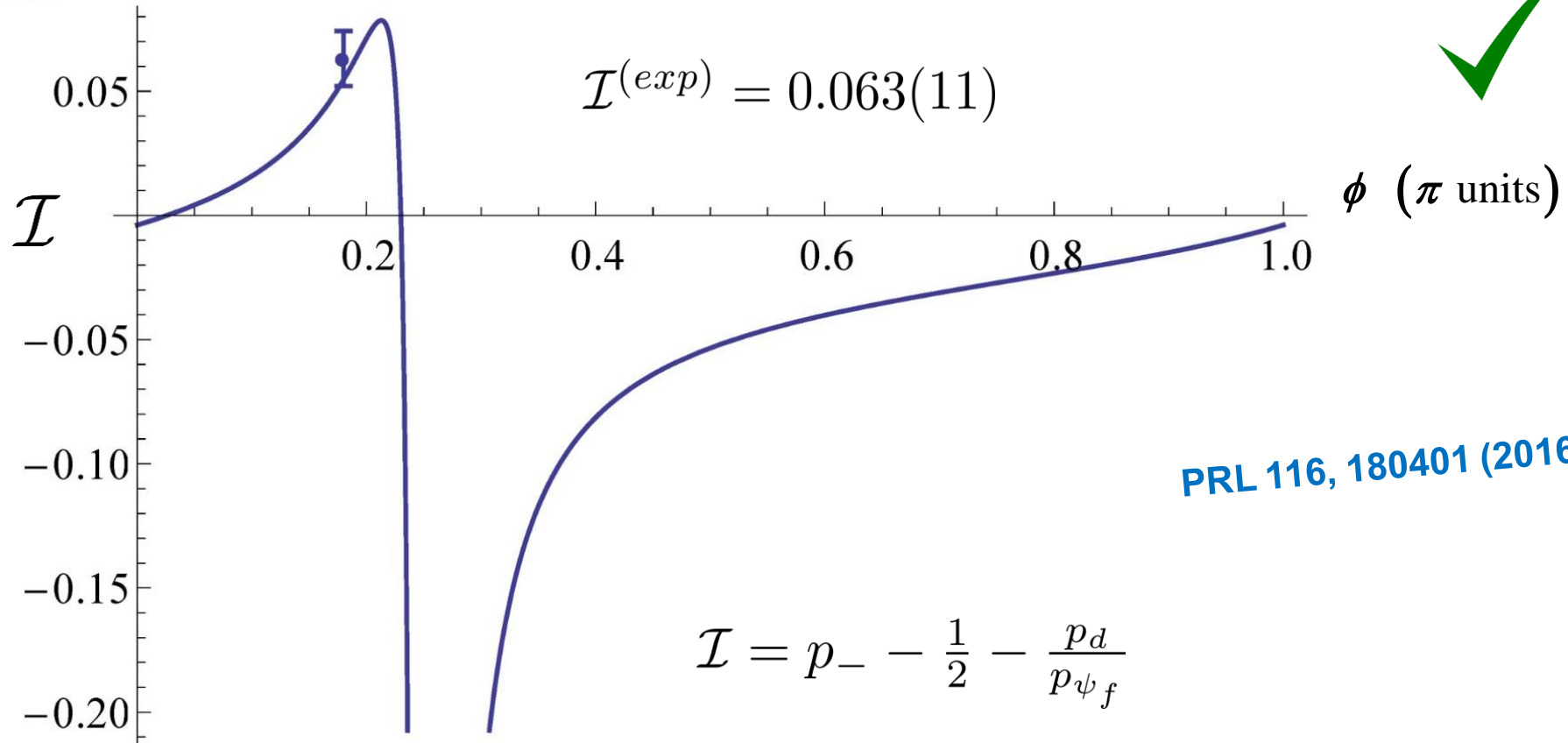


No non-contextual model satisfying outcome determinism for sharp measurements

Setup



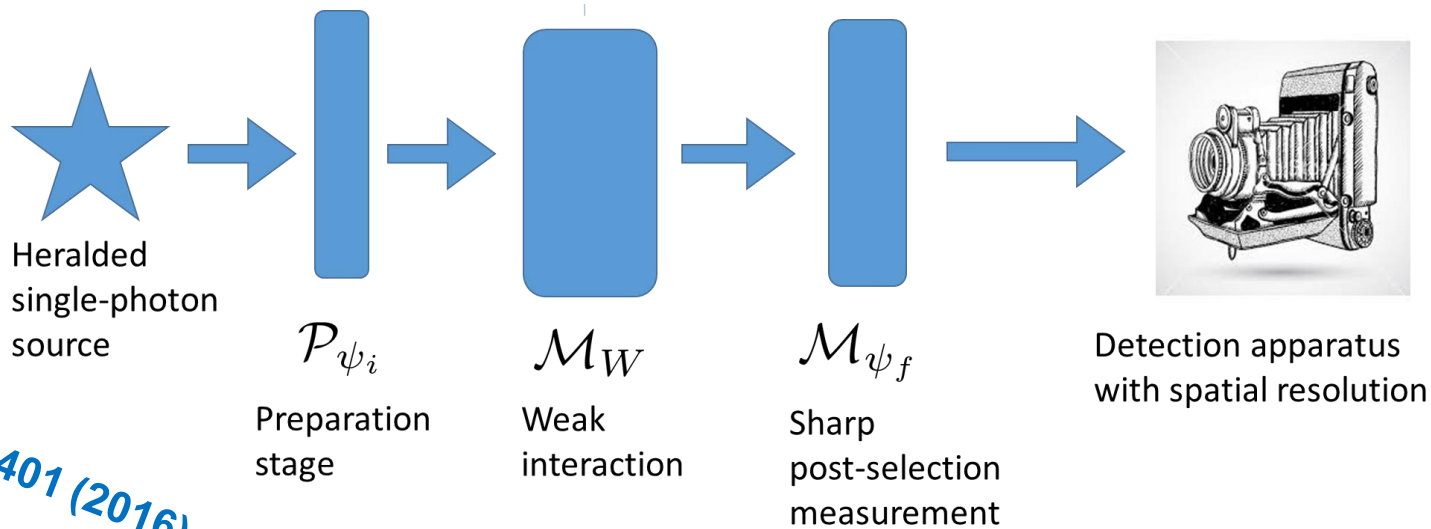
Condition 4: non-contextual bound violation



- input state: $|-\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$
- post-selection state: $|\psi_f\rangle = \cos \phi|H\rangle + \sin \phi|V\rangle$ ($\mathcal{I}^{(exp)}$: $\phi = 0.18\pi$)
- From experimental parameters: $p_d = 0.0019(2)$



WM and Contextuality: final check



PRL 116, 180401 (2016)

Initial and final states are non-orthogonal: $p_{\psi_f} := \mathbb{P}(\text{PASS} | \mathcal{P}_{\psi_i}, \mathcal{M}_{\psi_f}) > 0$ ✓

Without post-selection: $\mathbb{P}(x | \mathcal{P}, \mathcal{M}_W) = p_n(x-g)\mathbb{P}(1 | \mathcal{P}, \mathcal{M}_\Pi) + p_n(x)\mathbb{P}(0 | \mathcal{P}, \mathcal{M}_\Pi) \forall \mathcal{P}$ ✓

$\exists p_d : \mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_W, \mathcal{M}_{\psi_f}) = (1-p_d)\mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_{\psi_f}) + p_d\mathbb{P}(\text{PASS} | \mathcal{P}, \mathcal{M}_\alpha) \forall \mathcal{P}$ ✓

$p_- := (p_{\psi_f})^{-1} \int_{-\infty}^0 \mathbb{P}(x, \text{PASS} | \mathcal{P}_{\psi_i}, \mathcal{M}_W, \mathcal{M}_{\psi_f}) dx : \mathcal{I} = p_- - \frac{1}{2} - \frac{p_d}{p_{\psi_f}} \geq 0$ ✓

No non-contextual model allowed: weak measurements proved Quantum Contextuality



Measuring Incompatible Observables by Exploiting Sequential Weak Values

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(Received 4 May 2016; published 20 October 2016)

One of the most intriguing aspects of quantum mechanics is the impossibility of measuring at the same time observables corresponding to noncommuting operators, because of quantum uncertainty. This impossibility can be partially relaxed when considering joint or sequential weak value evaluation. Indeed, weak value measurements have been a real breakthrough in the quantum measurement framework that is of the utmost interest from both a fundamental and an applicative point of view. In this Letter, we show how we realized for the first time a sequential weak value evaluation of two incompatible observables using a genuine single-photon experiment. These (sometimes anomalous) sequential weak values revealed the single-operator weak values, as well as the local correlation between them.

DOI: 10.1103/PhysRevLett.117.170402



Joint and sequential weak measurements

Weak values «*challenge one of the canonical dicta of QM: that non commuting observables cannot be simultaneously measured*»

«*the fact that one hardly disturbs the systems in making WM means that one can in principle measure different variables in succession*» [Mitchison, Jozsa and Popescu, PRA 76 (2007)]

Joint weak measurement

Resch et al., PRL 92, 130402 (2004)

$$\hat{U} = \exp[-i(g_x \hat{A} \otimes \hat{P}_x + g_y \hat{B} \otimes \hat{P}_y)]$$

$$\langle \hat{X}\hat{Y} \rangle = \frac{1}{4} g_x g_y \text{Re} \left[\langle \hat{A}\hat{B} + \hat{B}\hat{A} \rangle_w + 2\langle \hat{A} \rangle_w^* \langle \hat{B} \rangle_w \right]$$

Sequential weak measurement

Mitchinson et al., PRA 76, 062105 (2007)

$$\hat{U}_y = \exp(-i g_y \hat{B} \otimes \hat{P}_y)$$



$$\hat{U}_x = \exp(-i g_x \hat{A} \otimes \hat{P}_x)$$

$$\langle \hat{X}\hat{Y} \rangle = \frac{1}{2} g_x g_y \text{Re} \left[\langle \hat{A}\hat{B} \rangle_w + \langle \hat{A} \rangle_w^* \langle \hat{B} \rangle_w \right]$$

Sequential weak measurement

$$\hat{A} \longrightarrow \hat{\Pi}_V = |V\rangle\langle V|$$

$$\hat{B} \longrightarrow \hat{\Pi}_\psi = |\psi\rangle\langle\psi|$$

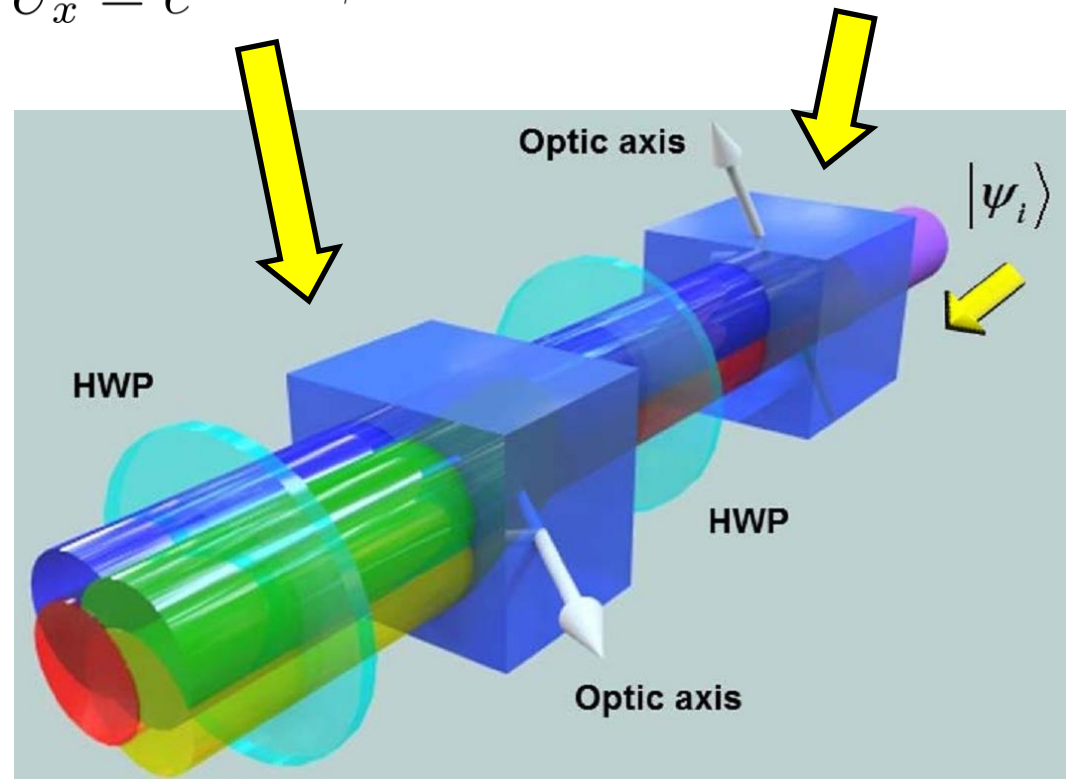
$$|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$$

Linearly polarized pre- and post-selection states $|\psi_i\rangle, |\psi_f\rangle$

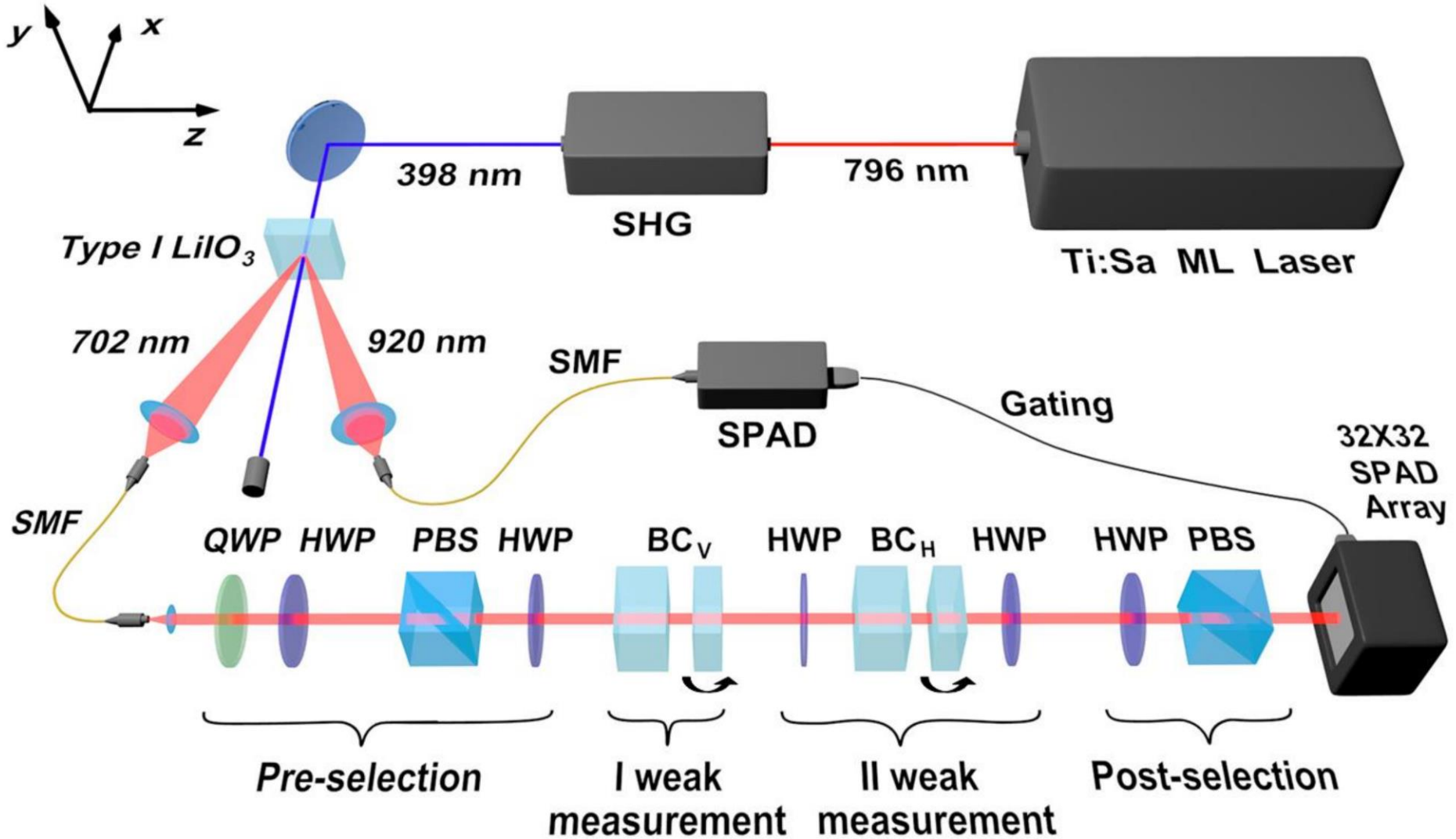
$$\left\{ \begin{array}{l} \langle \hat{X} \rangle = g_x \langle \hat{\Pi}_\psi \rangle_w \\ \langle \hat{Y} \rangle = g_y \langle \hat{\Pi}_V \rangle_w \\ \langle \hat{X}\hat{Y} \rangle = \frac{1}{2} g_x g_y \left(\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w + \langle \hat{\Pi}_\psi \rangle_w \langle \hat{\Pi}_V \rangle_w \right) \end{array} \right.$$

$$\hat{U}_x = e^{-ig_x \hat{\Pi}_\psi \otimes \hat{P}_x}$$

$$\hat{U}_y = e^{-ig_y \hat{\Pi}_V \otimes \hat{P}_y}$$



SETUP

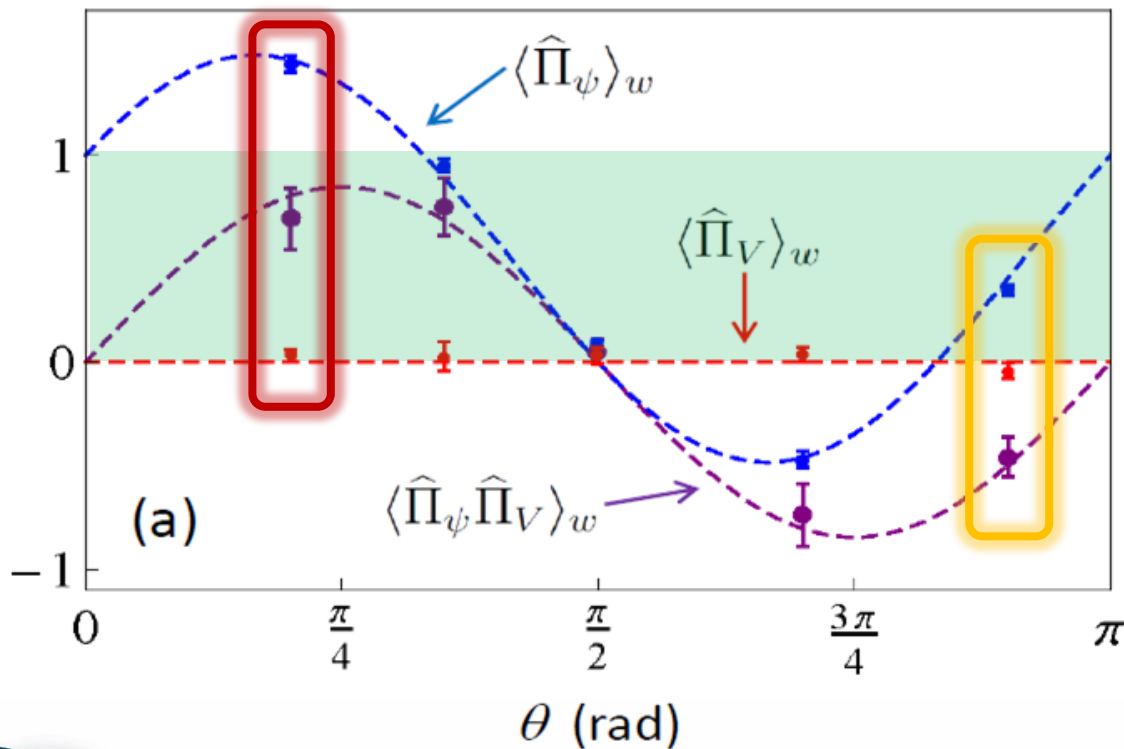


Results

Measured weak values (data points) compared with the theoretical predictions

$$\hat{\Pi}_V = |V\rangle\langle V| \quad \hat{\Pi}_\psi = |\psi\rangle\langle\psi| \quad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

$$|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle \quad |\psi_f\rangle = |H\rangle$$



$$\langle \hat{\Pi}_V \rangle_w = 0.03(3)$$

$$\langle \hat{\Pi}_\psi \rangle_w = 1.44(4)$$

$$\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w = 0.69(15)$$

$$\langle \hat{\Pi}_V \rangle_w = 0.04(3)$$

$$\langle \hat{\Pi}_\psi \rangle_w = 0.35(4)$$

$$\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w = -0.46(10)$$

PRL 117, 170402 (2016)



Results

Measured weak values (data points) compared with the theoretical predictions

$$\hat{\Pi}_V = |V\rangle\langle V| \quad \hat{\Pi}_\psi = |\psi\rangle\langle\psi| \quad (|\psi\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle)$$

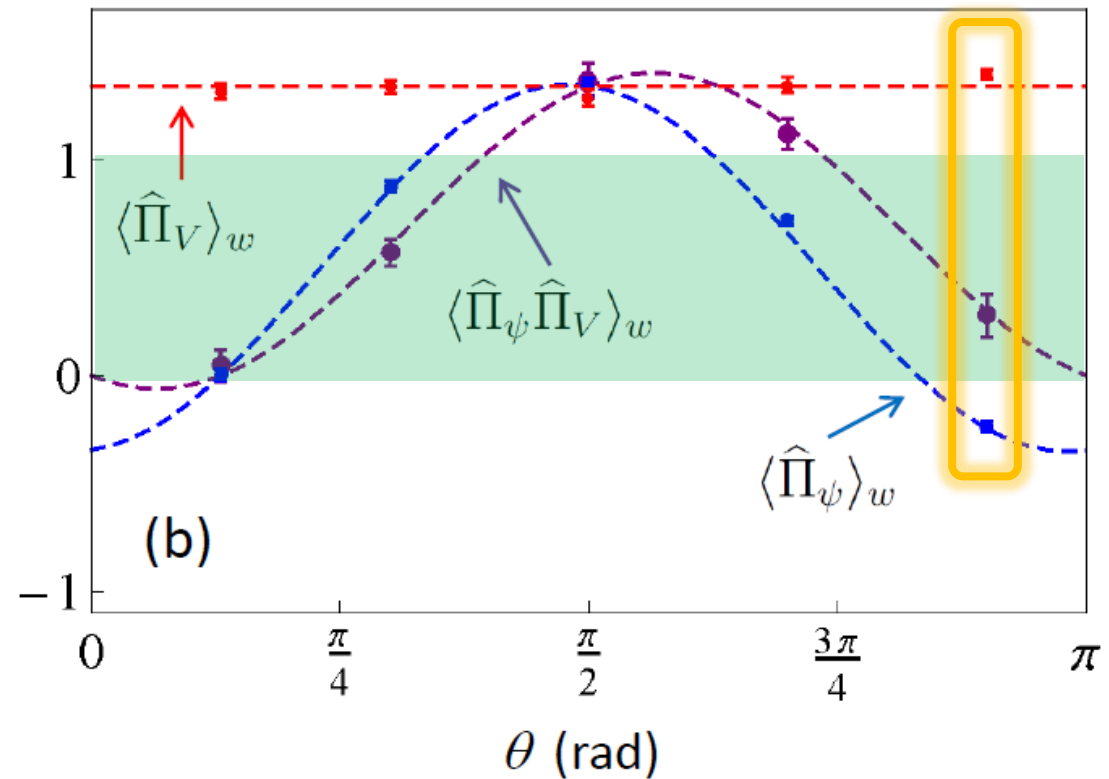
$$|\psi_i\rangle = 0.509|H\rangle + 0.861|V\rangle$$

$$|\psi_f\rangle = -0.397|H\rangle + 0.918|V\rangle$$

$$\langle\hat{\Pi}_V\rangle_w = 1.40(4)$$

$$\langle\hat{\Pi}_\psi\rangle_w = -0.24(3)$$

$$\langle\hat{\Pi}_\psi\hat{\Pi}_V\rangle_w = 0.28(10)$$



PRL 117, 170402 (2016)



Application: violation of a multiple-measurement Leggett-Garg inequality

Originally proposed to investigate macroscopic realism, Leggett-Garg inequalities can also be regarded as a tool for quantumness tests.

Sequential weak measurements allow extending the usual 3-measurement framework to a multiple-measurement one, e.g. violating the inequality:

$$|\mathcal{B}_4| = |\langle I_A I_B \rangle + \langle I_B I_C \rangle + \langle I_C I_D \rangle - \langle I_A I_D \rangle| \leq 2$$

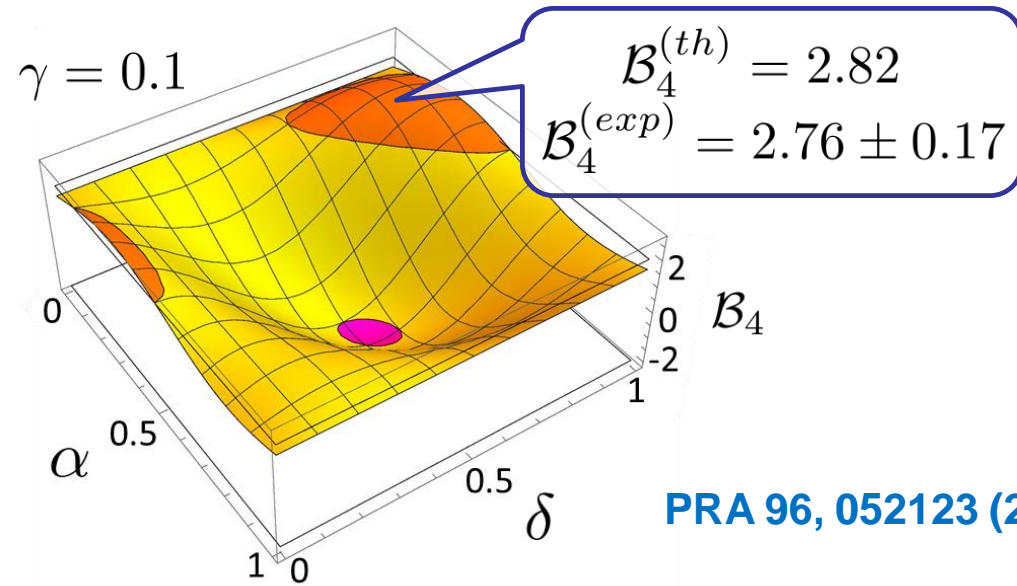
$$I_A = |\psi_A\rangle\langle\psi_A| - |\psi_A^\perp\rangle\langle\psi_A^\perp| \quad |\psi_A\rangle = \cos\alpha|H\rangle + \sin\alpha|V\rangle \quad (\text{preparation})$$

$$I_B = |\psi_\gamma\rangle\langle\psi_\gamma| - |\psi_\gamma^\perp\rangle\langle\psi_\gamma^\perp| \quad |\psi_\gamma\rangle = \cos\gamma|H\rangle + \sin\gamma|V\rangle$$

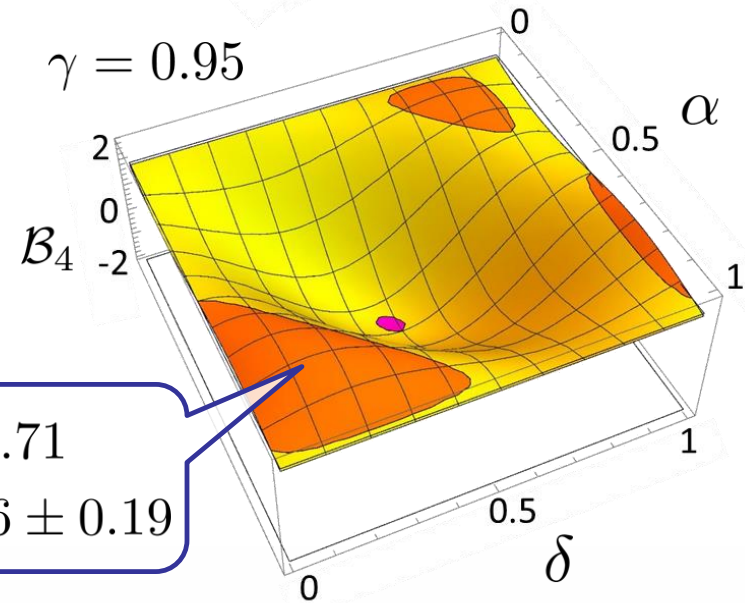
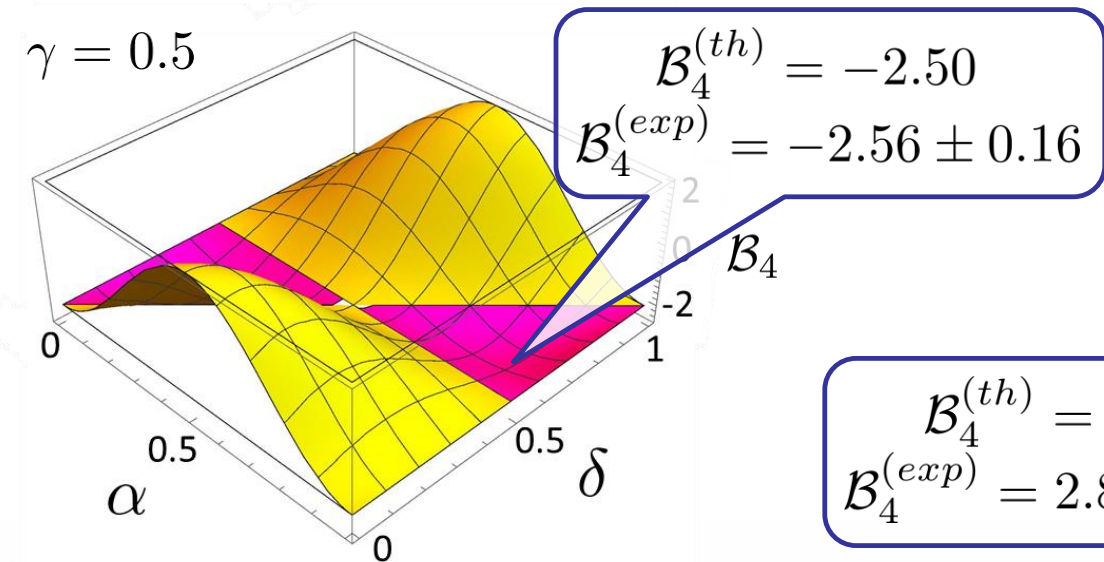
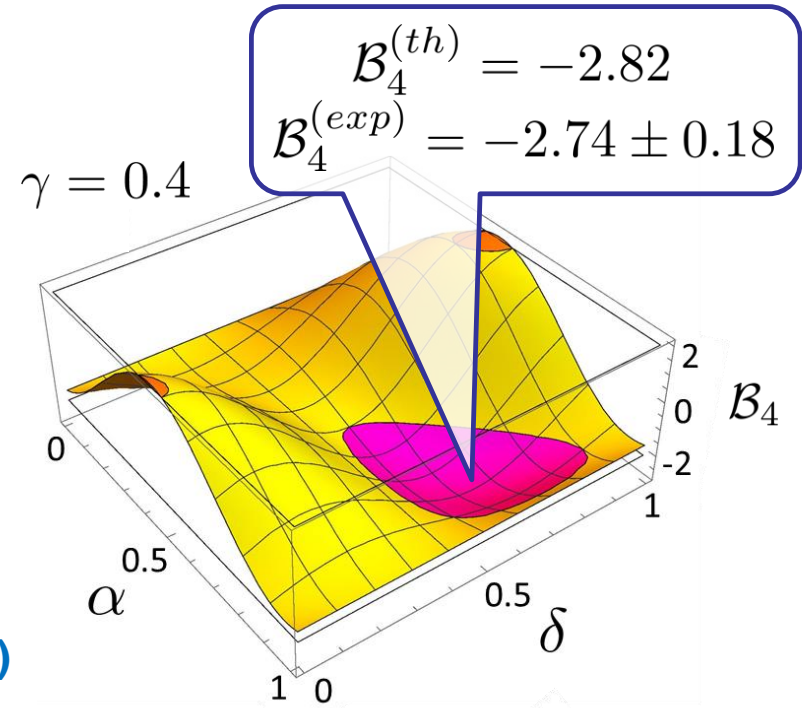
$$I_C = |H\rangle\langle H| - |V\rangle\langle V|$$

$$I_D = |\psi_D\rangle\langle\psi_D| - |\psi_D^\perp\rangle\langle\psi_D^\perp| \quad |\psi_D\rangle = \cos\delta|H\rangle + \sin\delta|V\rangle$$

Violations



PRA 96, 052123 (2017)



Determining the quantum expectation value by measuring a single photon

Fabrizio Piacentini^{1*}, Alessio Avella¹, Enrico Rebufello^{1,2}, Rudi Lussana³, Federica Villa³, Alberto Tosi³, Marco Gramegna¹, Giorgio Brida¹, Eliahu Cohen⁴, Lev Vaidman⁵, Ivo P. Degiovanni¹ and Marco Genovese¹

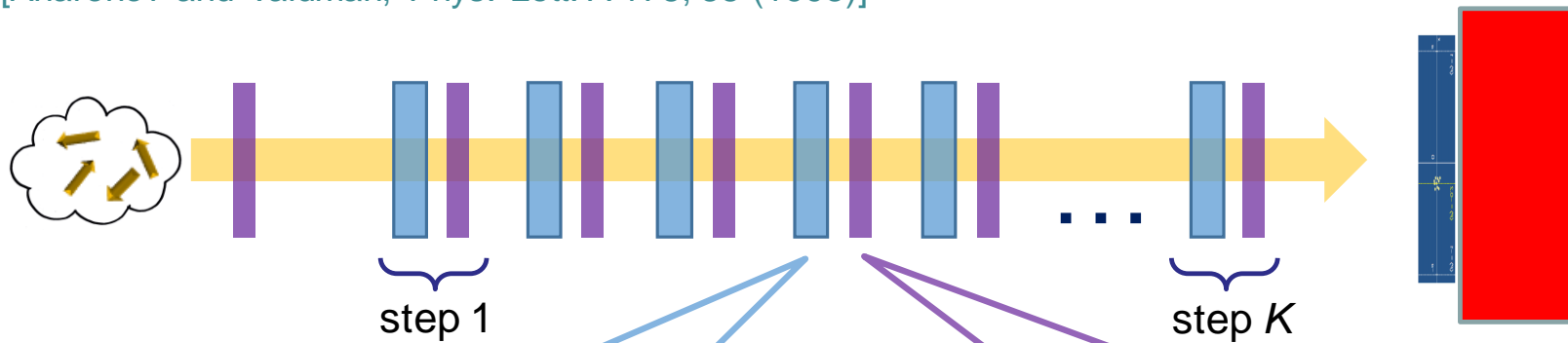
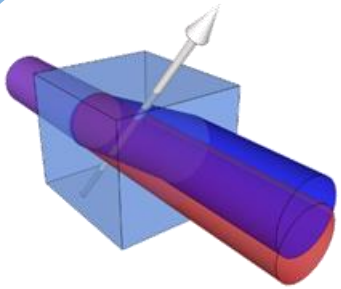
One of the most intriguing features of quantum mechanics is that variables might not have definite values. A complete quantum description provides only probabilities for obtaining various eigenvalues of a quantum variable. The eigenvalues and the corresponding probabilities specify the expectation value of a physical observable, which is known to be a statistical property of an ensemble of quantum systems. In contrast to this paradigm, here we demonstrate a method for measuring the expectation value of a physical variable on a single particle, namely, the polarization of a single protected photon. This realization of quantum protective measurements could find applications in the foundations of quantum mechanics and quantum-enhanced measurements.



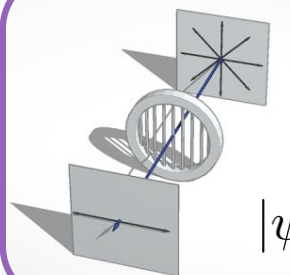
Can one extract the quantum expectation value by measuring a single particle?

Protective Measurement (PM): the interaction is smooth enough to leave the state unchanged. The pointer shift is proportional to the average value of the observable

[Aharonov and Vaidman, Phys. Lett. A 178, 38 (1993)]

$$\hat{U} = e^{-ig\hat{\Pi}_H \otimes \mathbf{P}}$$

$$\hat{\Pi}_H = |H\rangle\langle H|$$


$$\hat{\Pi}_\theta = |\psi_\theta\rangle\langle\psi_\theta|$$

$$|\psi_\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle$$

Ideal case: $K \rightarrow \infty, g \rightarrow 0$

No losses, photon survival probability $p_{sur}(K) = 1$

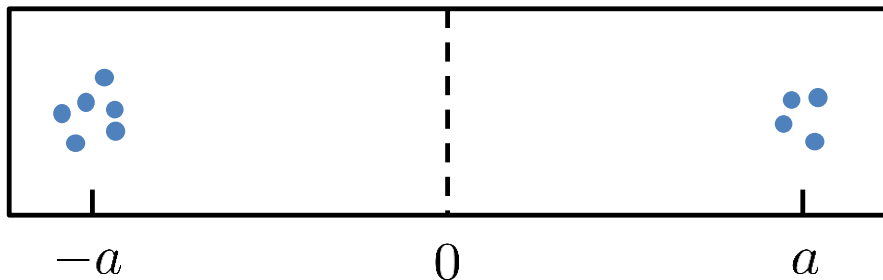


Can one extract the quantum expectation value by measuring a single particle?

Protective Measurement (PM): the interaction is smooth enough to leave the state unchanged. The pointer shift is proportional to the average value of the observable [Aharonov and Vaidman, Phys. Lett. A 178, 38 (1993)]

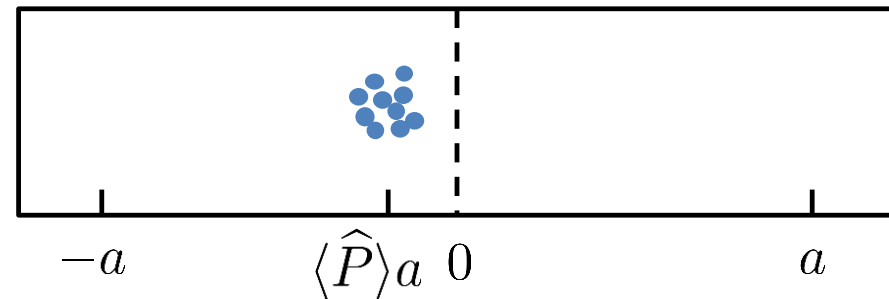
Measured observable: $\hat{P} = |H\rangle\langle H| - |V\rangle\langle V|$

Projective measurement



$$\langle \hat{P} \rangle = \frac{N_a - N_{-a}}{N_a + N_{-a}}$$

Protective measurement



Each photon carries the same information on $\langle \hat{P} \rangle$

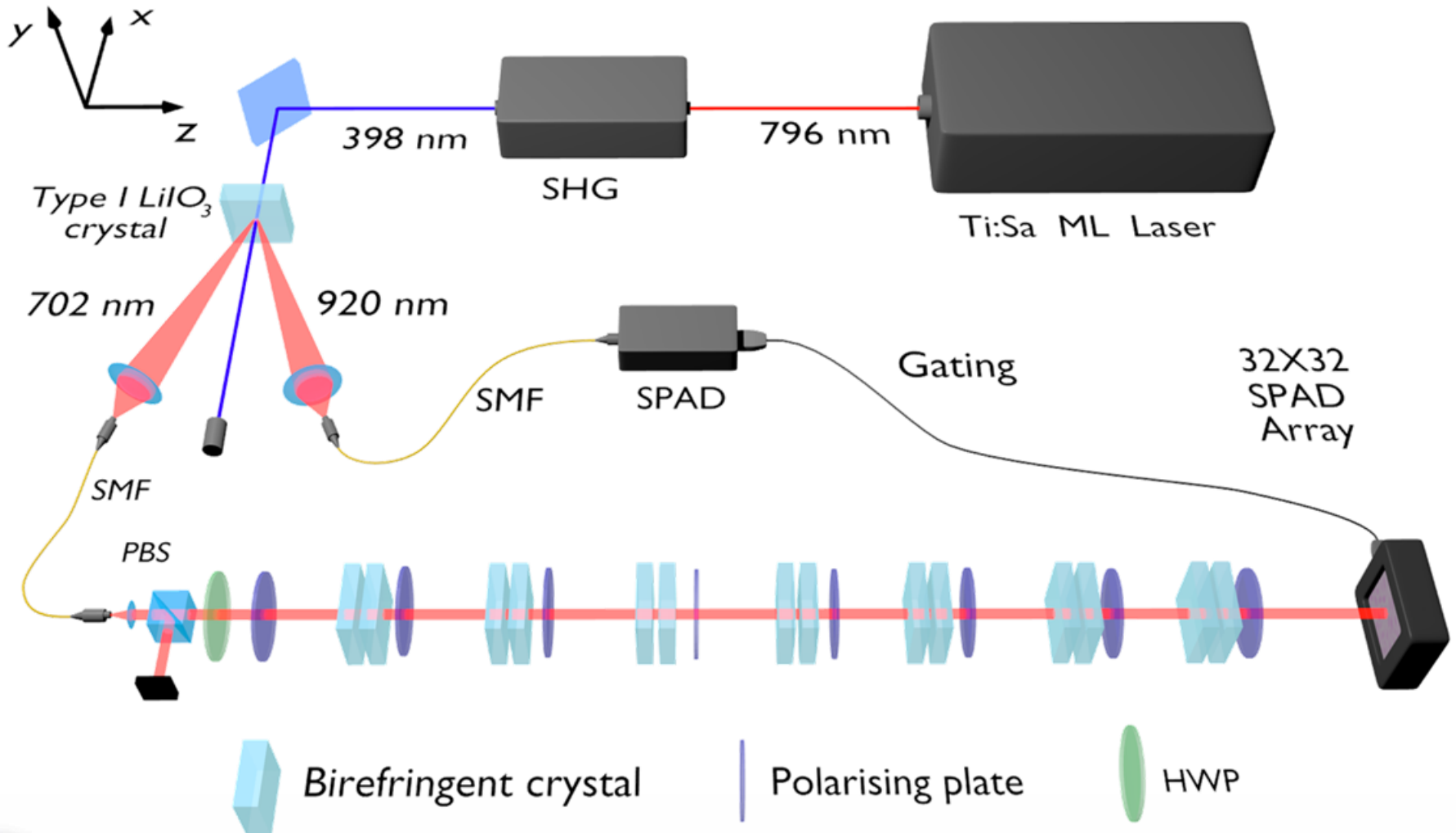


$$\langle \hat{P} \rangle = \frac{\langle \hat{X} \rangle}{a}$$



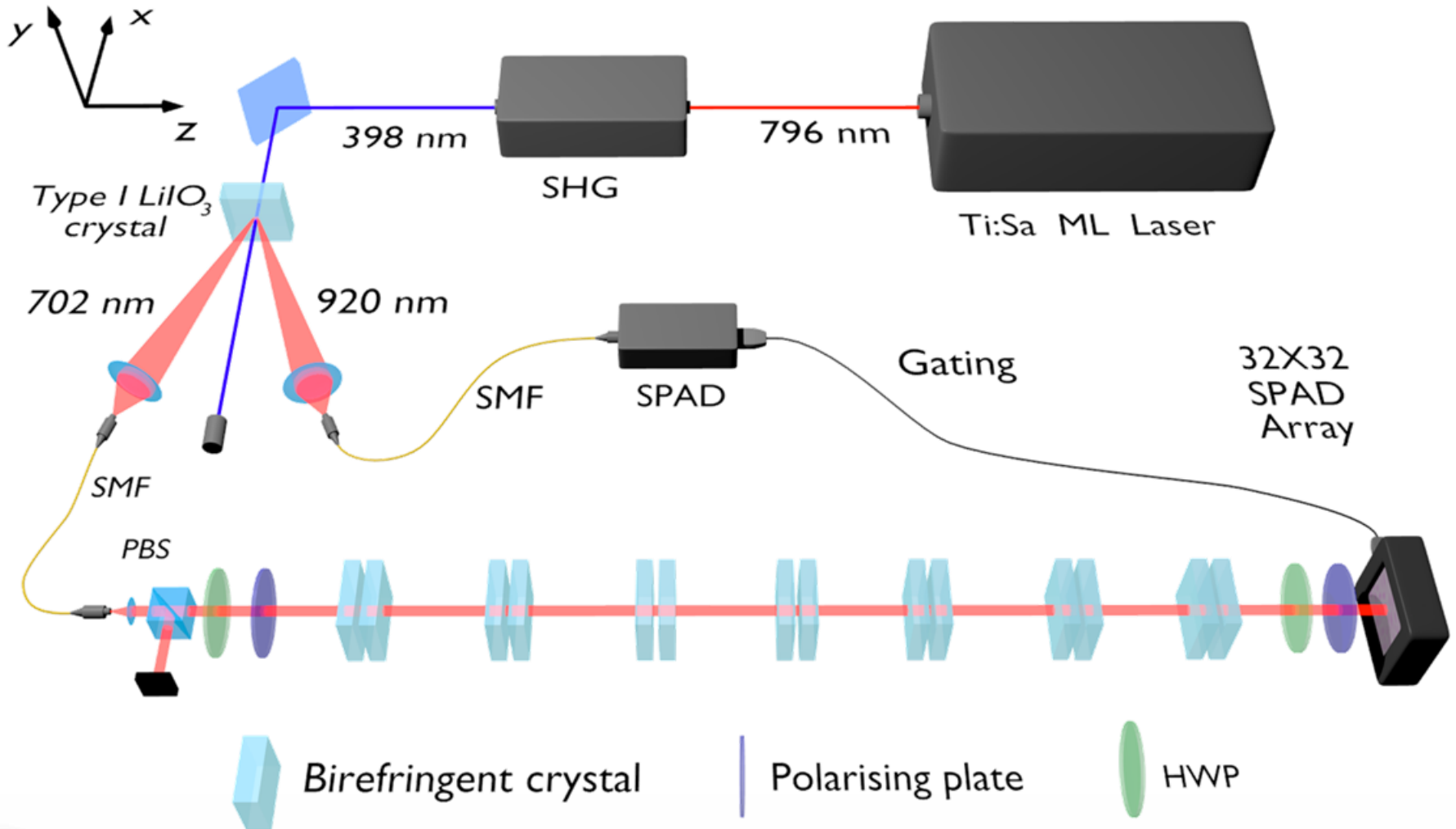
Protective measurements in the lab

$$K \rightarrow \infty, g \rightarrow 0, p_{sur}(K) = 1 \quad \longrightarrow \quad K = 7, g \ll 1, p_{sur}(K) \lesssim 1$$



Projective measurements in the lab

$K \rightarrow \infty, g \rightarrow 0, p_{sur}(K) = 1$ \longrightarrow $K = 7, g \ll 1, p_{sur}(K) \lesssim 1$

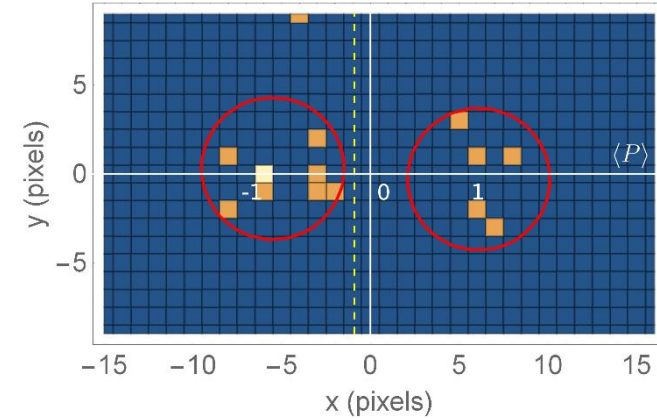
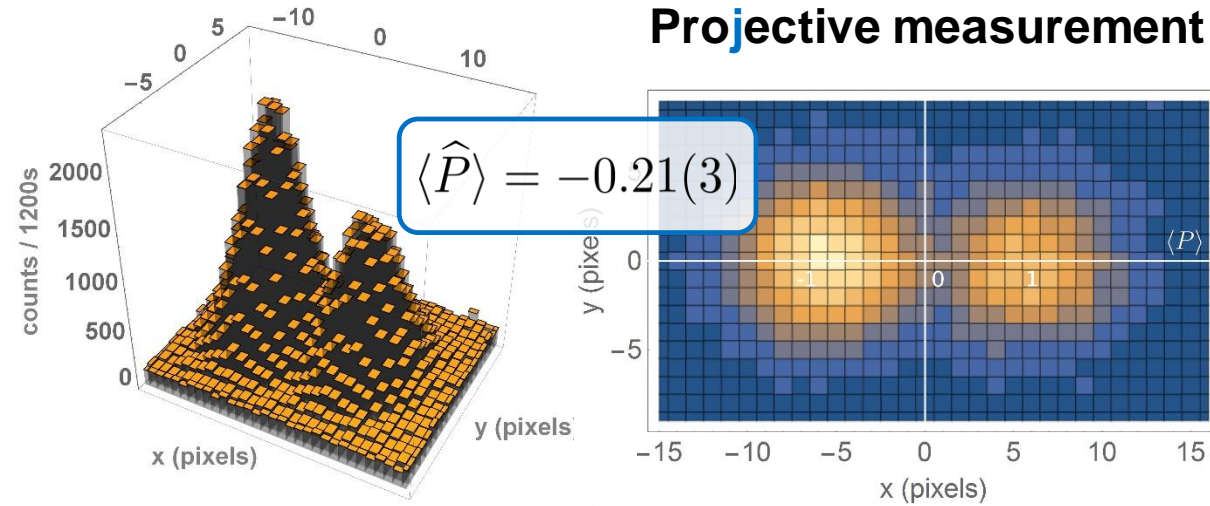


Experimental results

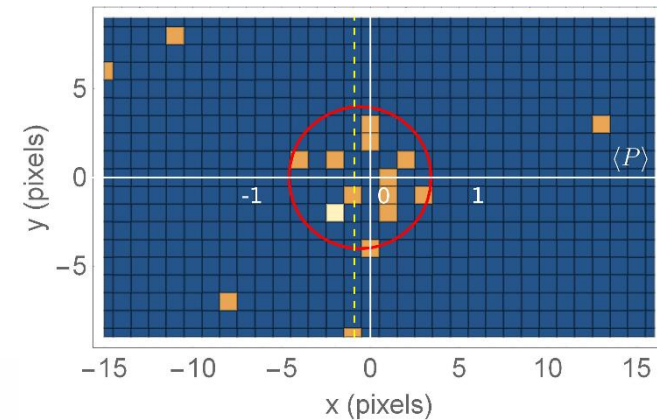
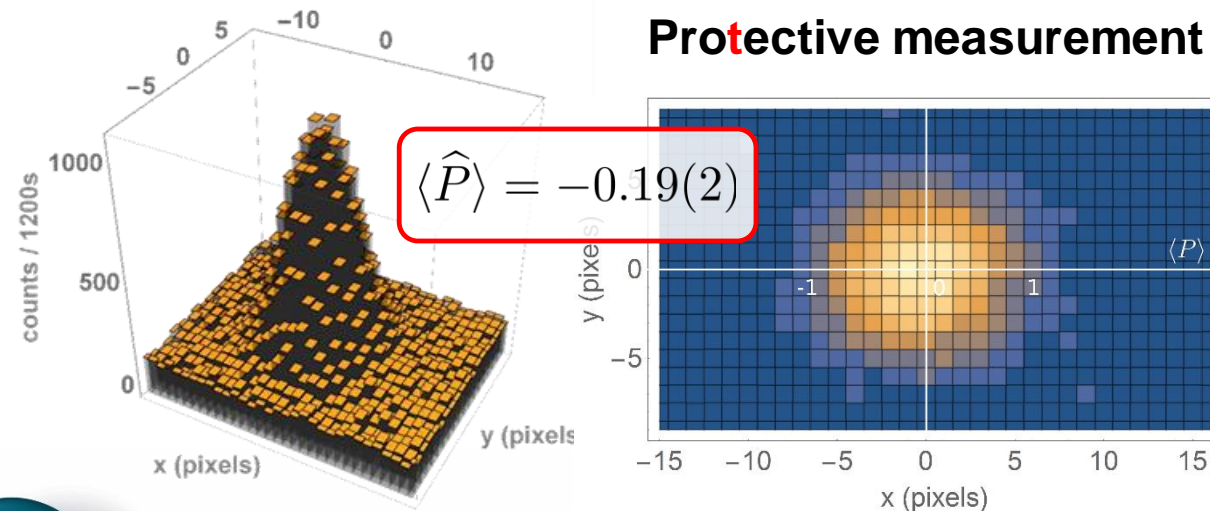
$$|\psi_{\frac{17\pi}{60}}\rangle = 0.629|H\rangle + 0.777|V\rangle \quad \langle \hat{P} \rangle^{th} = -0.208$$

Nat. Phys. 13, 1191 (2017)

Projective measurement

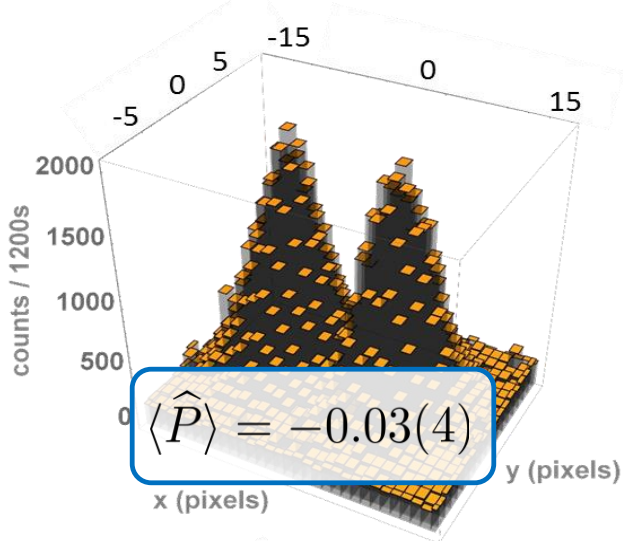


Protective measurement

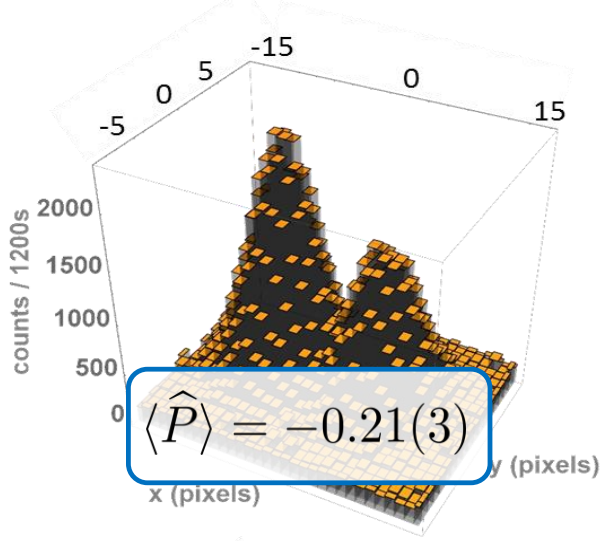


Experimental results

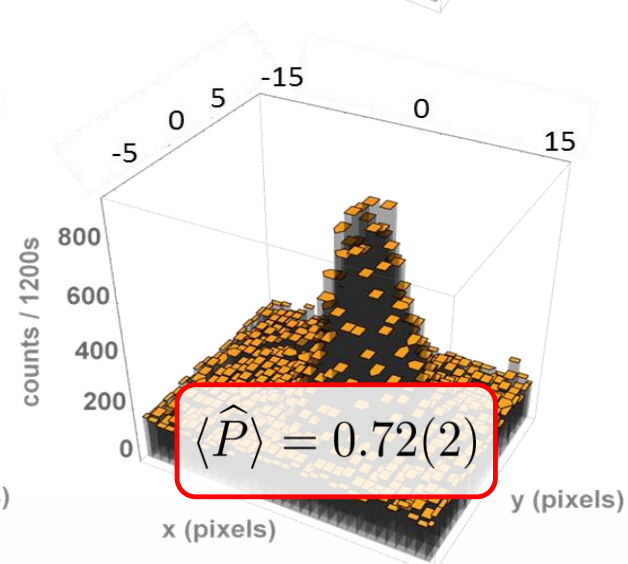
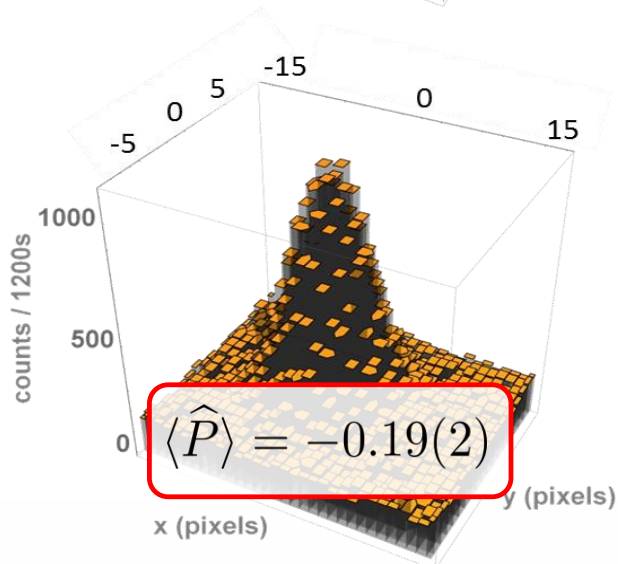
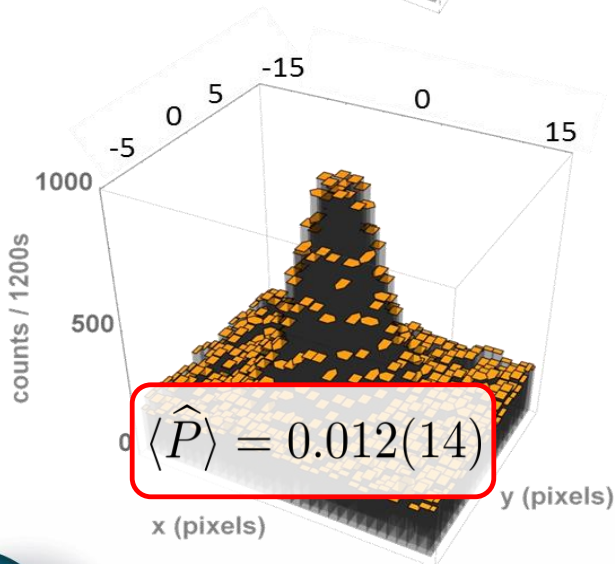
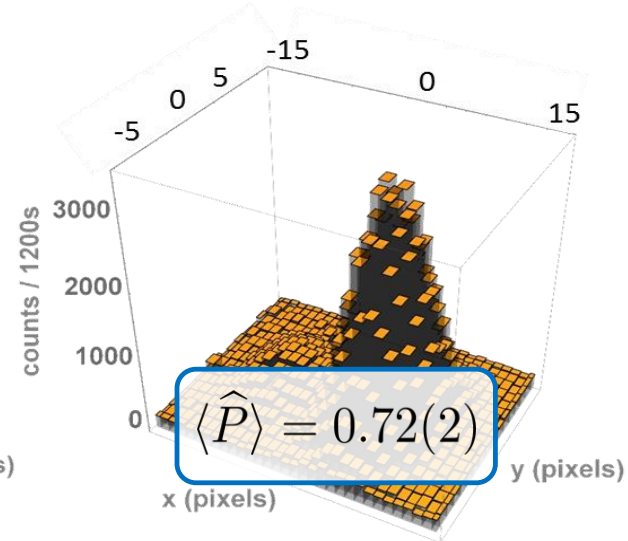
$$\langle \hat{P} \rangle^{th} = 0$$



$$\langle \hat{P} \rangle^{th} = -0.208$$

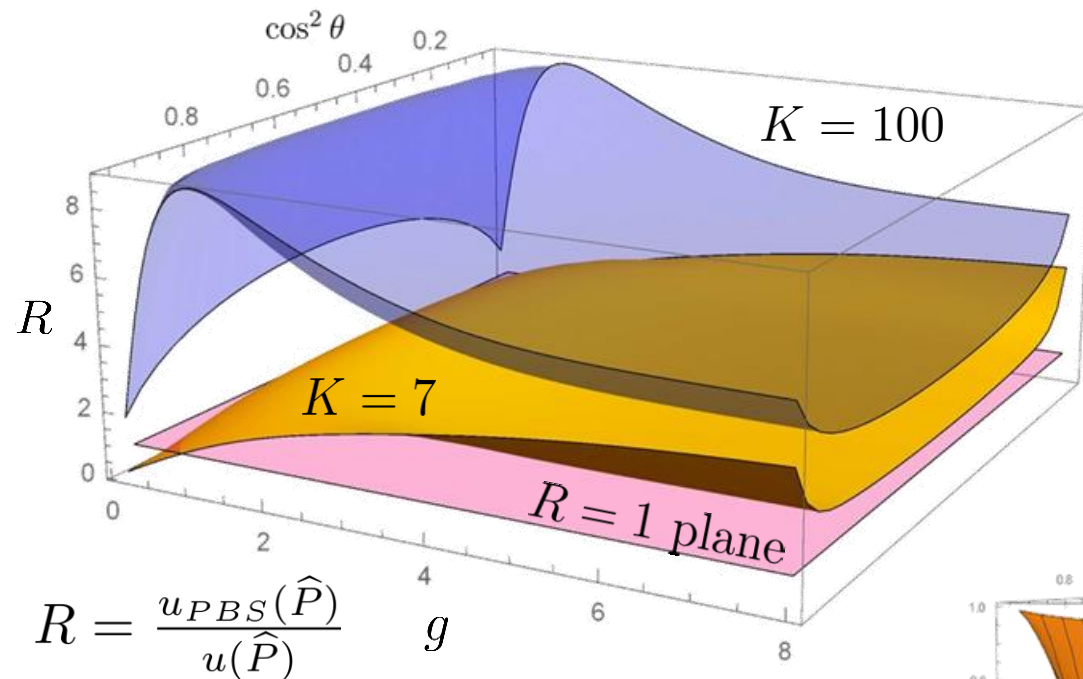


$$\langle \hat{P} \rangle^{th} = 0.707$$



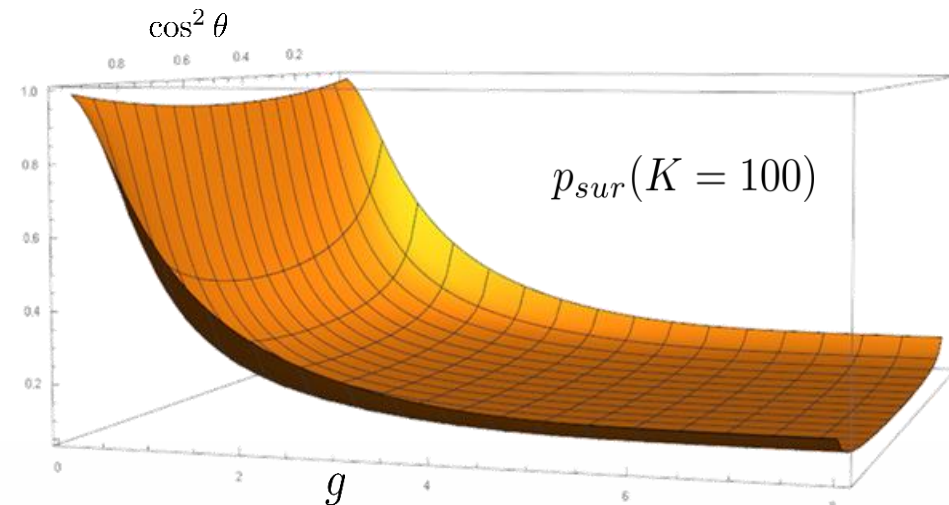
How accurate can PMs be?

Let us compare the uncertainty on \hat{P} given by a K -steps PM ($u(\hat{P})$) with the one obtained with a projective measurement ($u_{PBS}(\hat{P})$):



$u(\hat{P}) < u_{PBS}(\hat{P})$, even though in this framework projective measurements saturate the Quantum Cramér-Rao bound!

Nat. Phys. 13, 1191 (2017)



The advantage given by PMs is due to:

$$K \gg 1 \rightarrow p_{sur}(K) \gg (p_{sur}(1))^K$$



The weak crew

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Ivo P. Degiovanni

Marco Genovese



Eliahu Cohen



Lev Vaidman



Marco Barbieri



Rudi Lussana

Federica Villa

Alberto Tosi

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“Experiment Investigating the Connection between Weak Values and Contextuality”
Phys. Rev. Lett. 116, 180401 (2016)

“Measuring Incompatible Observables by Exploiting Sequential Weak Values”
Phys. Rev. Lett. 117, 170402 (2016)

“Anomalous weak values and the violation of a multiple-measurement Leggett-Garg inequality”
Phys. Rev. A 96, 052123 (2017)

“Determining the quantum expectation value by measuring a single photon”
Nat. Phys. 13, 1191 (2017)



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Thanks for
your
attention!!

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