From Quantum Cellular Automata to Doubly Special Relativity

Alessandro Bisio

Workshop Quantum Foundations
New frontiers in testing quantum mechanics from underground to the space

November 30th 2017
Laboratori Nazionali di Frascati

in collaboration with:

Giacomo Mauro D'Ariano Paolo Perinotti

Alessandro Tosini Nicola Mosco Marco Erba

supported by

JOHN TEMPLETON FOUNDATION

SUPPORTING SCIENCE~INVESTING IN THE BIG QUESTIONS

Quantum Theory

Von Neumann, 1932

Each physical system is associated with a Hilbert space

Unit vectors are associated with states of the system

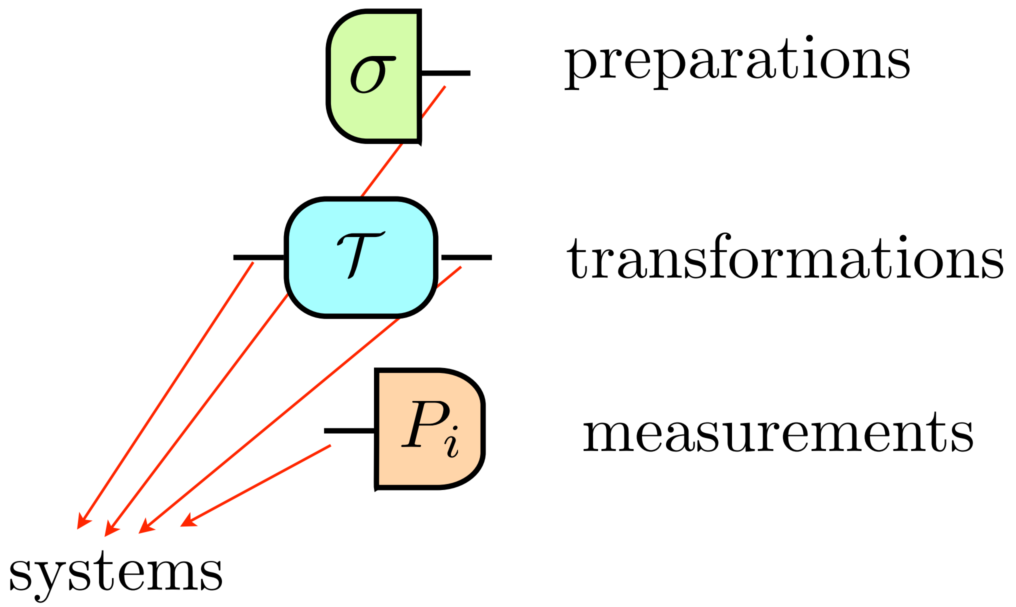
Physical observables are represented by self adjoint operators

The Hilbert space of a composite system is the tensor product of the state spaces associated with the component systems

The probabilities of the outcomes are given by the Born rule

Reconstruction of Quantum Theory

Operational Probabilistic Theory



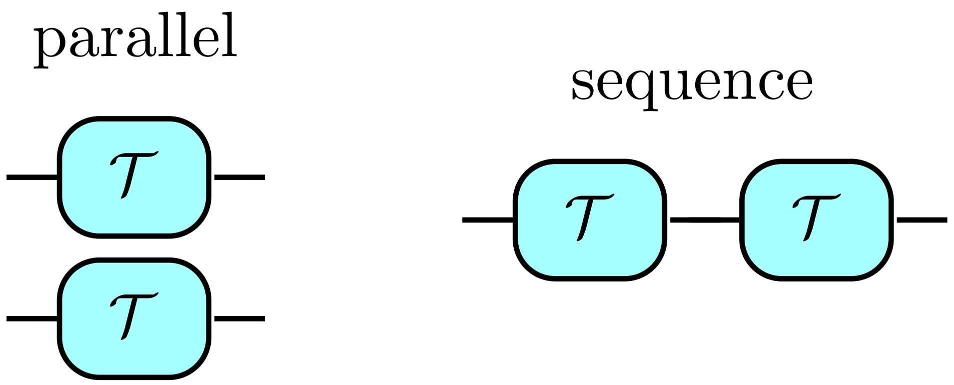
preparations

transformations

measurements

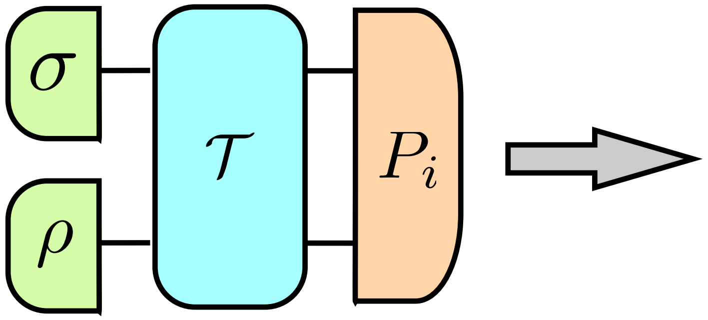
systems

composition



parallel

sequence



outcomes

probability

theory of information

G. Ludwig, *Foundations of Quantum Mechanics* (Springer, New York, 1985).

L. Hardy, e-print arXiv:quant-ph/0101012.

G. Chiribella, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 84, 012311 (2011)

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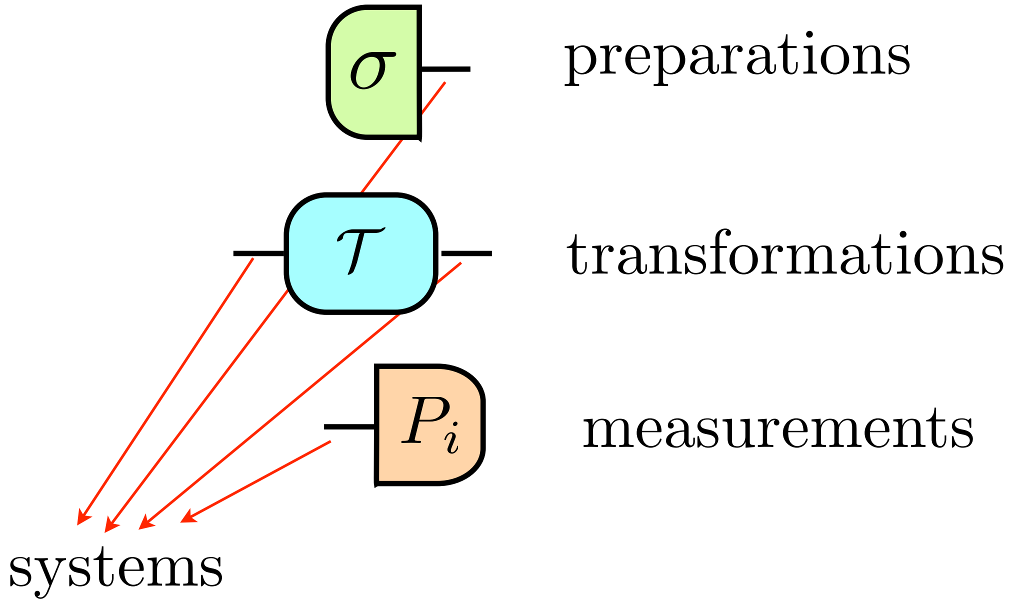

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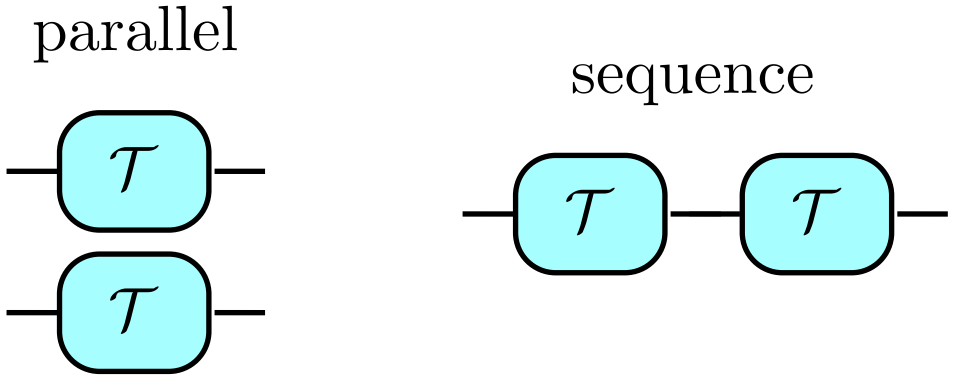

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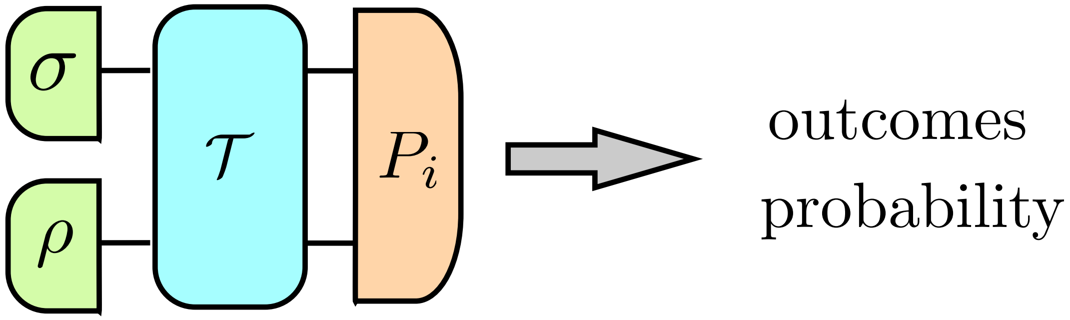

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Space? Dynamics?

Energy? Time?

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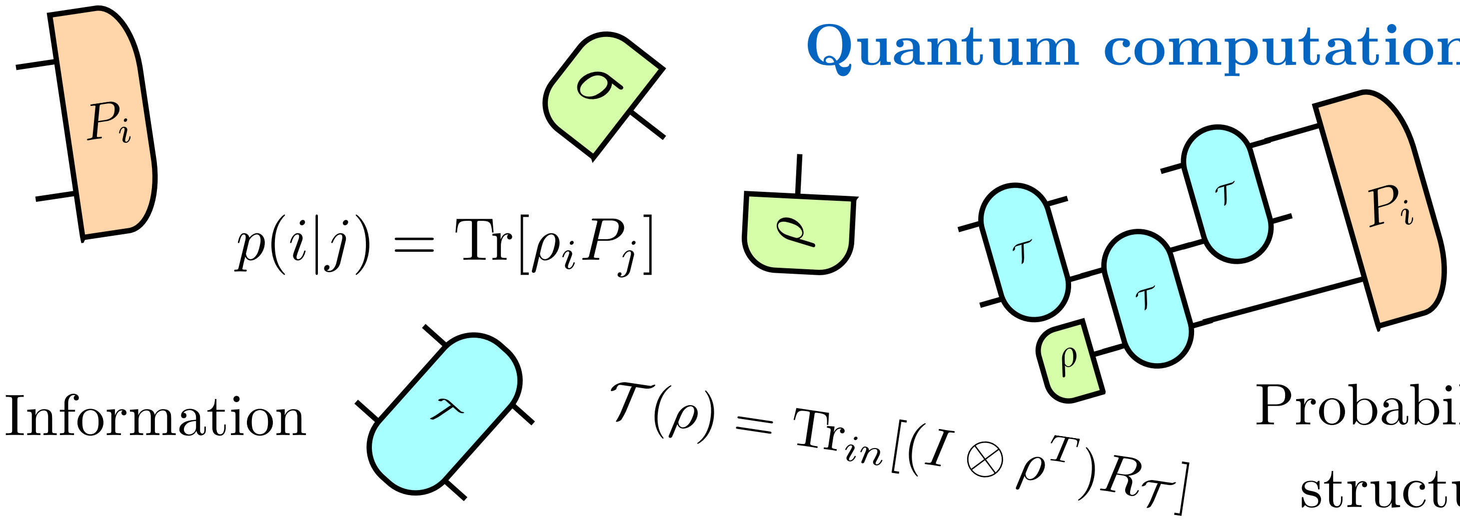
Reconstruction of Quantum Field Theory

Quantum Theory

Quantum computation



$p(i|j) = \mathrm{Tr}[\rho_i P_j]$

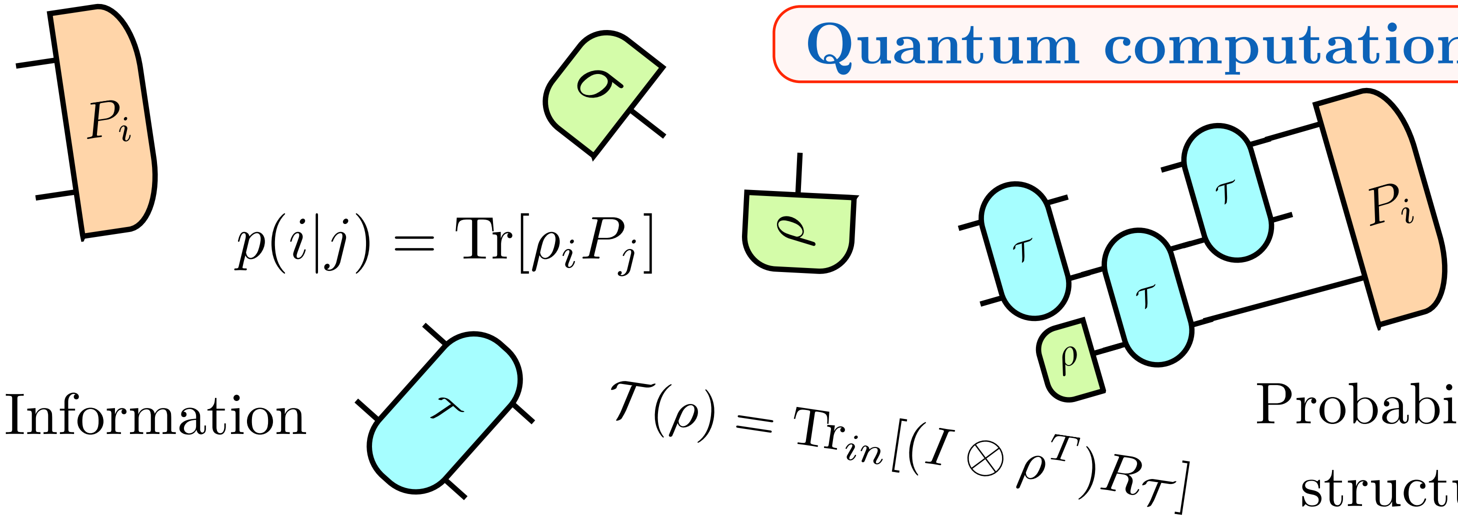
Information

$\mathcal{T}(\rho) = \mathrm{Tr}_{in}[(I \otimes \rho^T) R_{\mathcal{T}}]$

Probabilistic structure

Reconstruction of Quantum Field Theory

Quantum Theory



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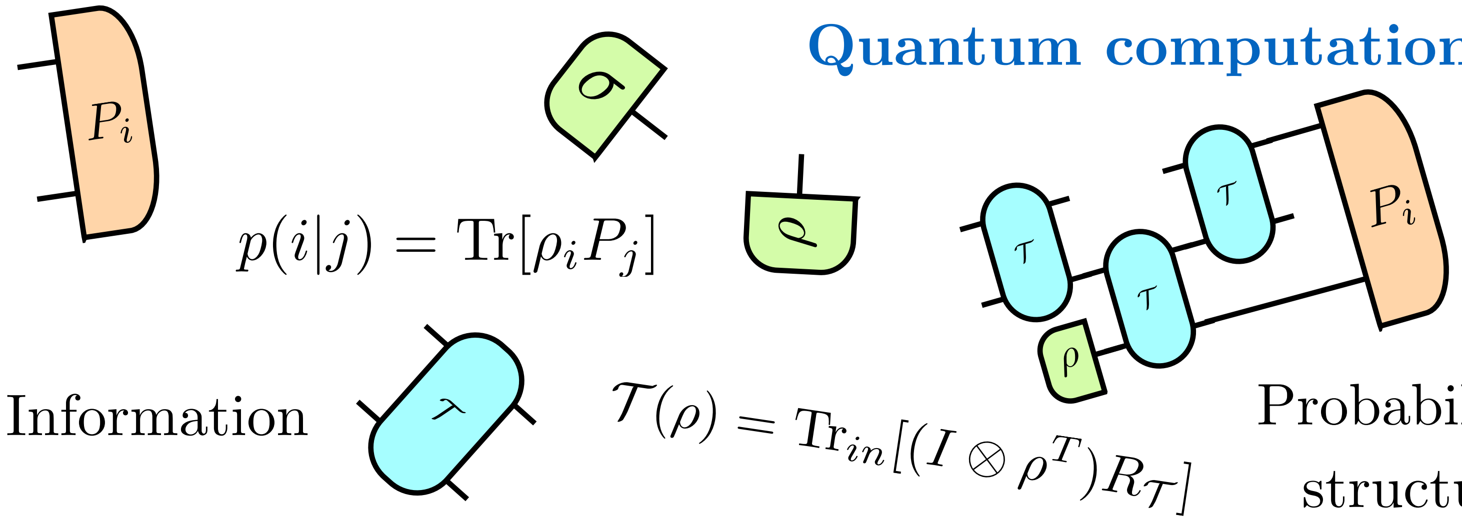
Simulating Physics with Computers

R. P. Feynman, Int. J. Theo. Phys. 21, 467 (1982)

Can a Quantum Computer **exactly** simulate physical systems?

Replace physical laws with quantum algorithms

Reconstruction of Quantum Field Theory



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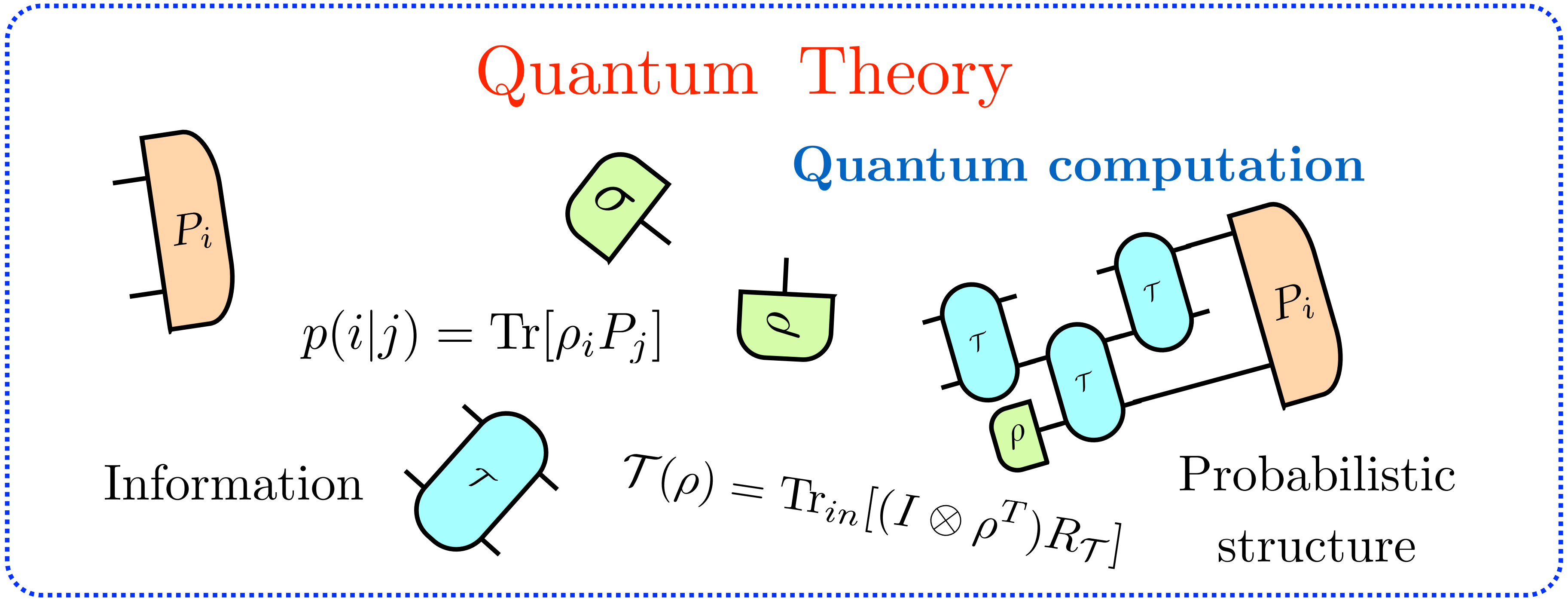

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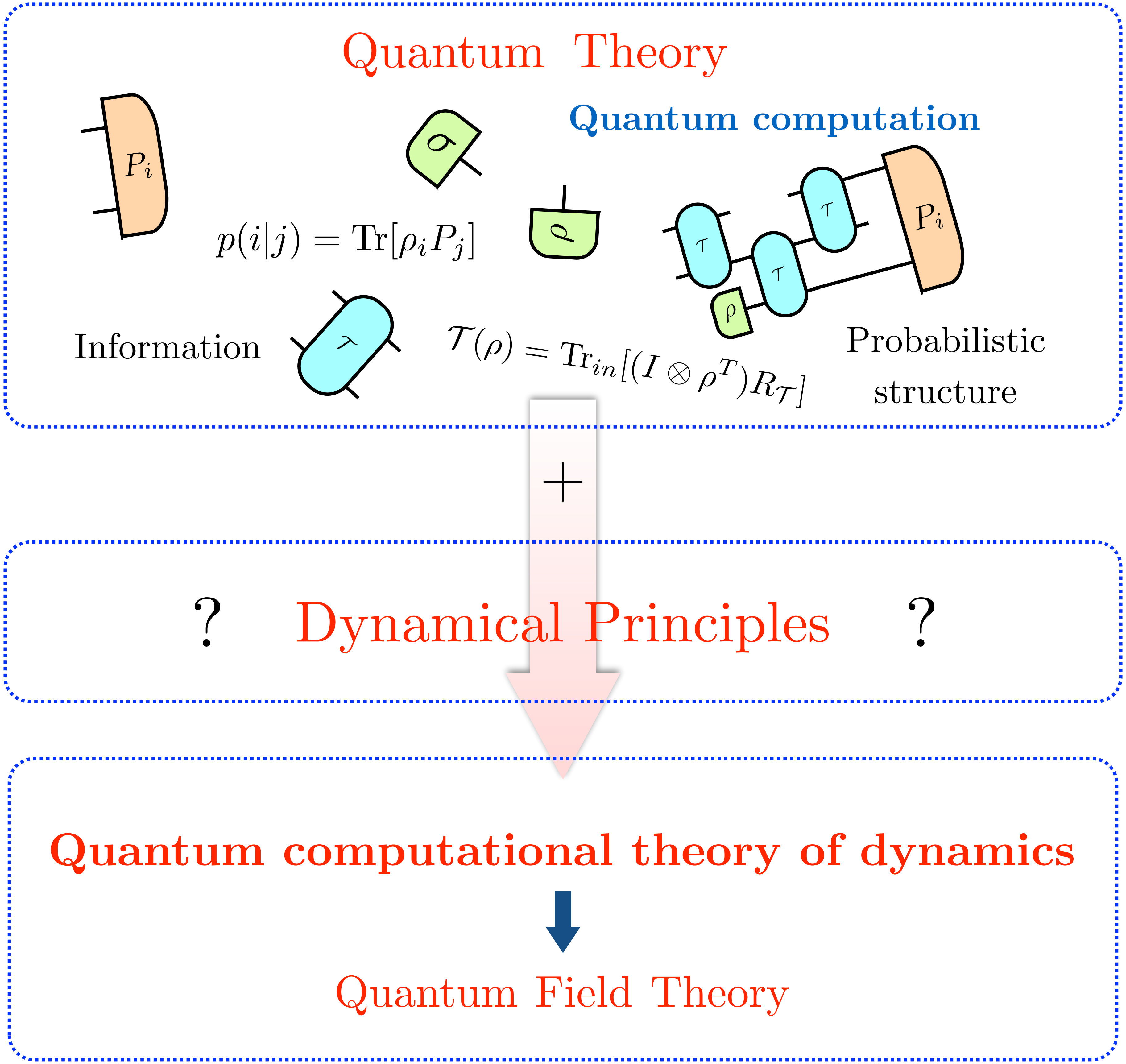

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Probabilistic structure

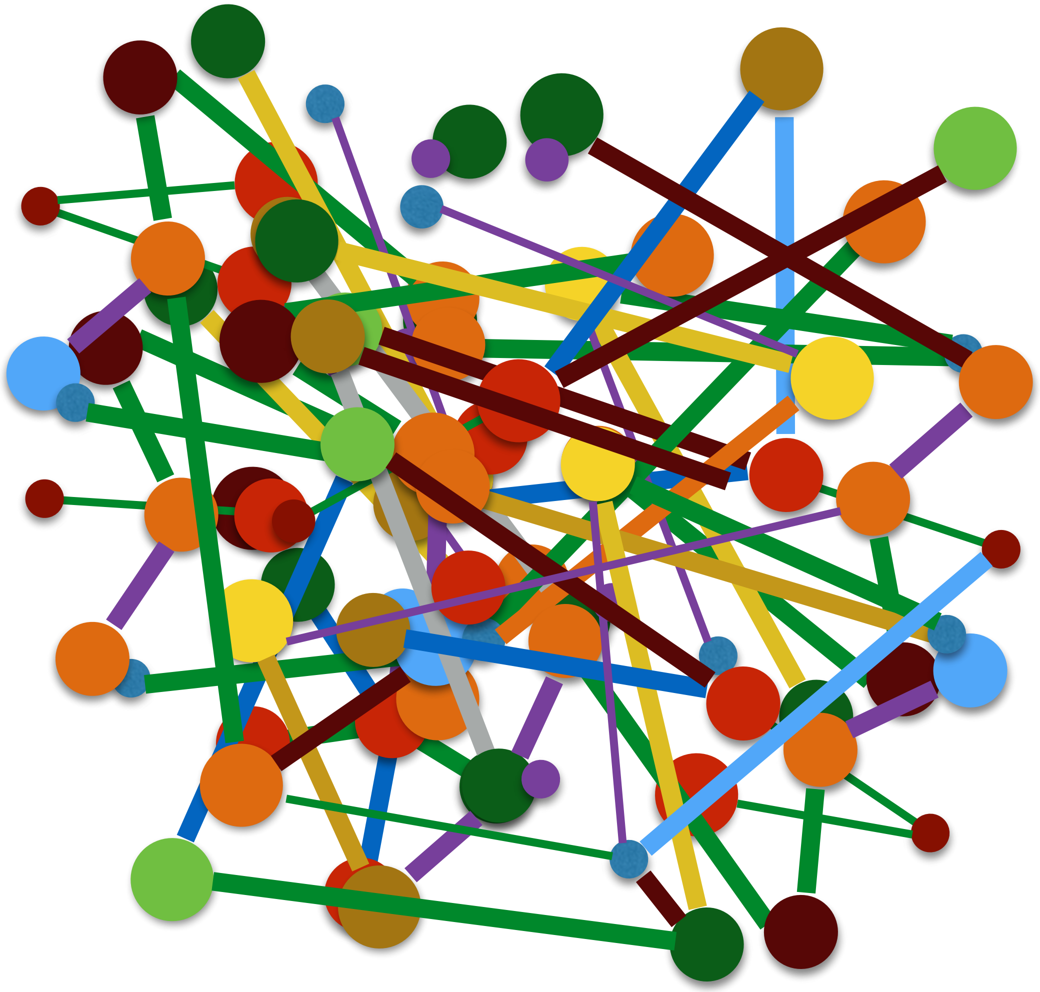
? Dynamical Principles ?

Quantum computational theory of dynamics

Quantum Field Theory

What kind of computer?

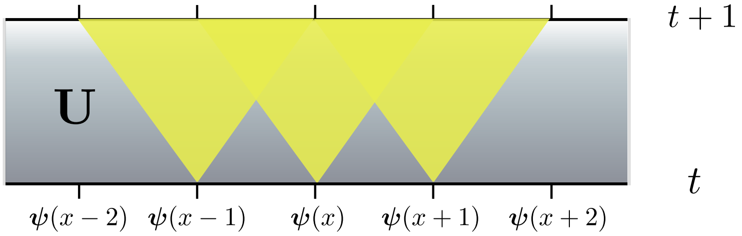
Quantum Circuit

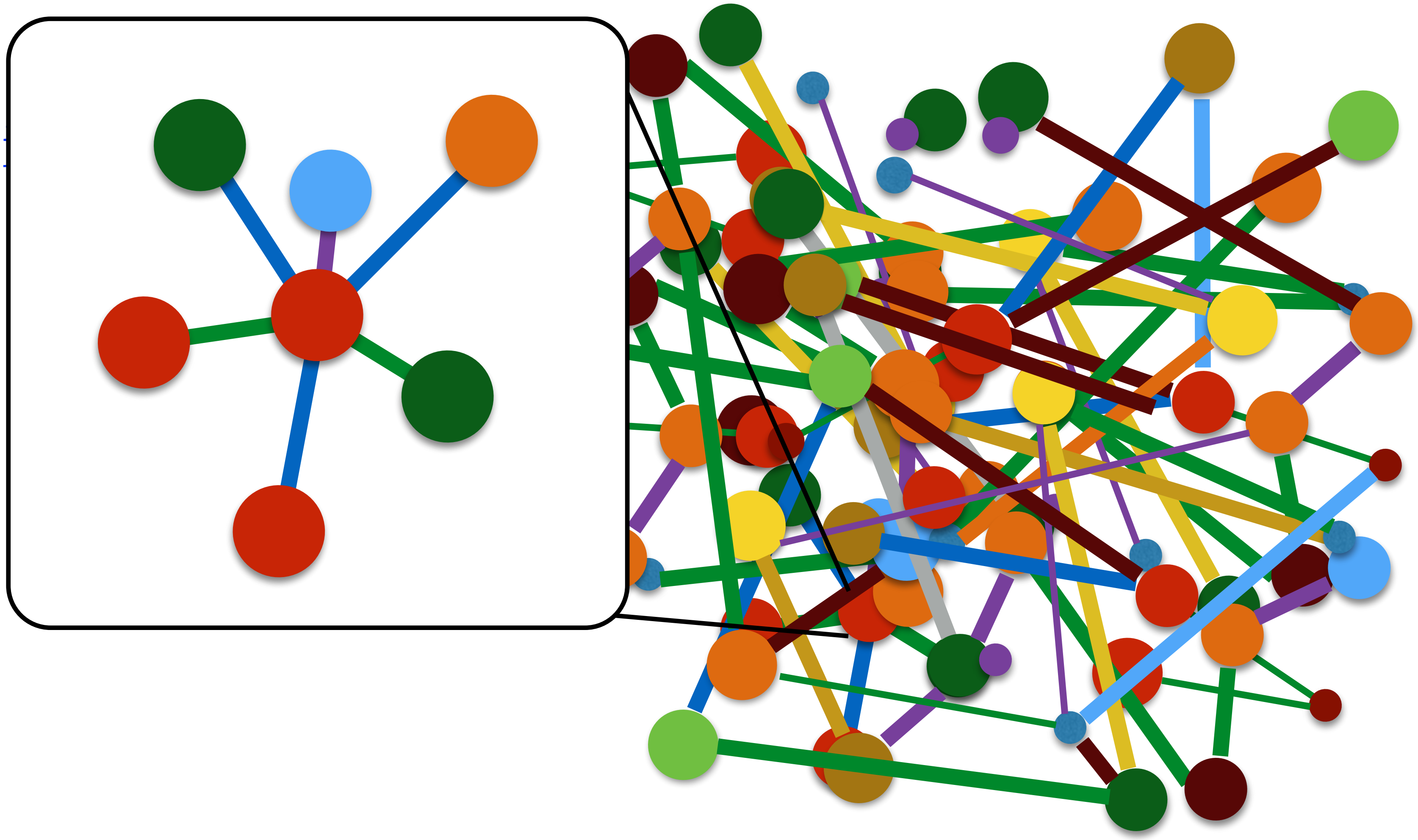


Rules of the game $\Longleftrightarrow$ axioms

What kind of computer?

Quantum Circu





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"[...] everything that happens in a finite volume of space and time would have to be exactly analyzable with a finite numbers of logical operations" R. Feynman

Each system interacts with a finite number of neighbors: **locality**

Reversible Quantum Computation: **unitary evolution**

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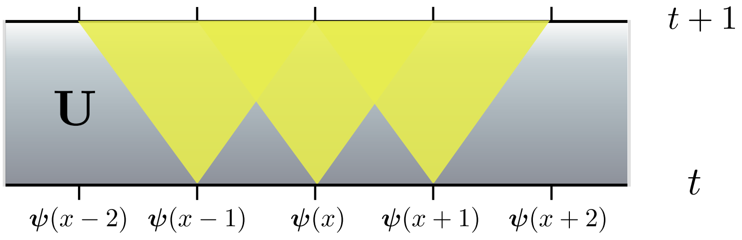

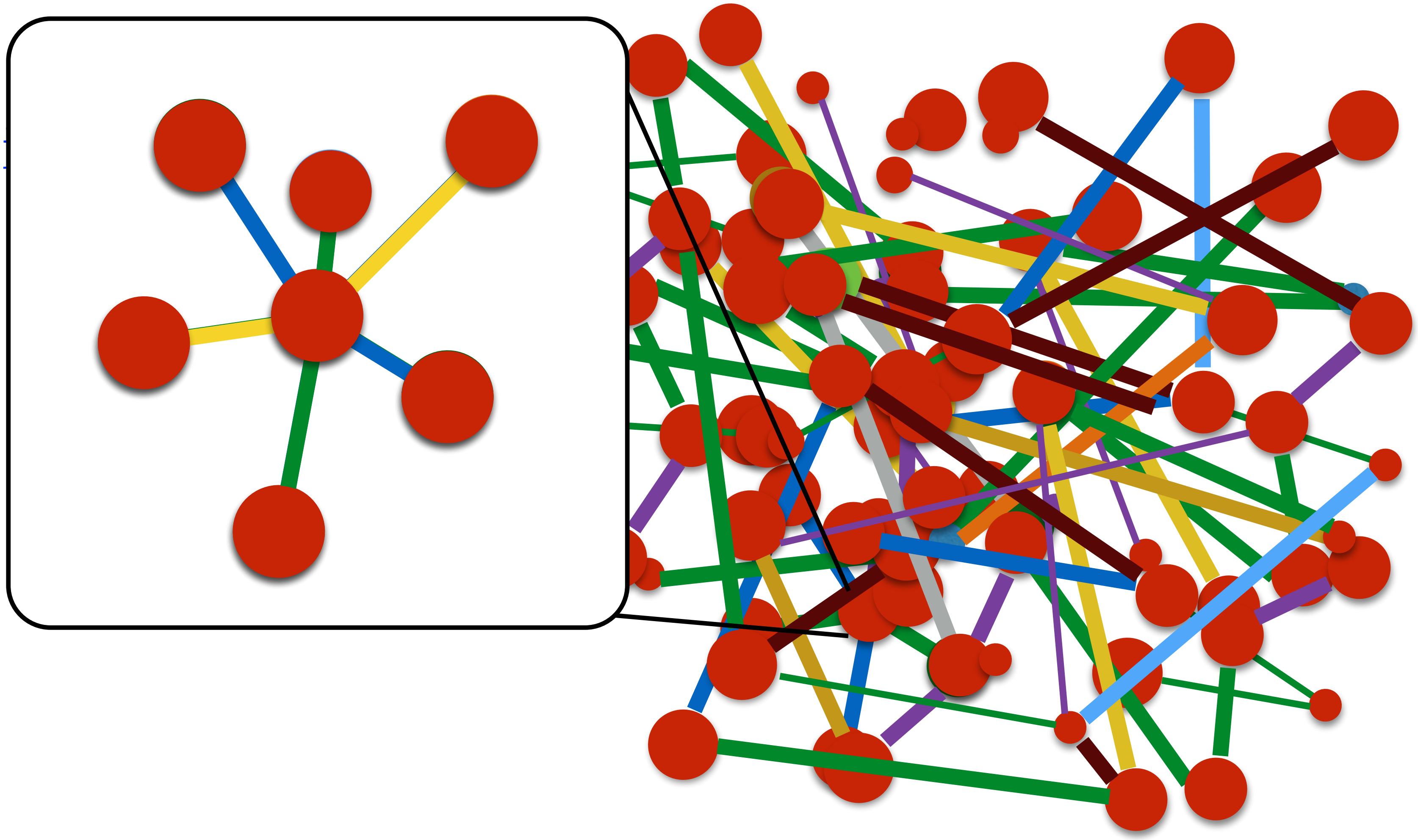

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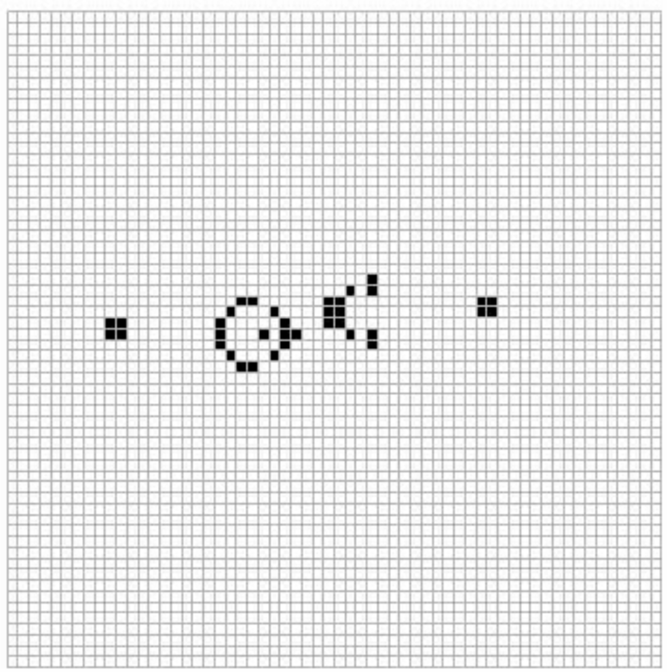
$\Downarrow$

Quantum Cellular Automaton

B. Schumacher, R.F. Werner
e-print arXiv:0405174.

like Conway's game of life...



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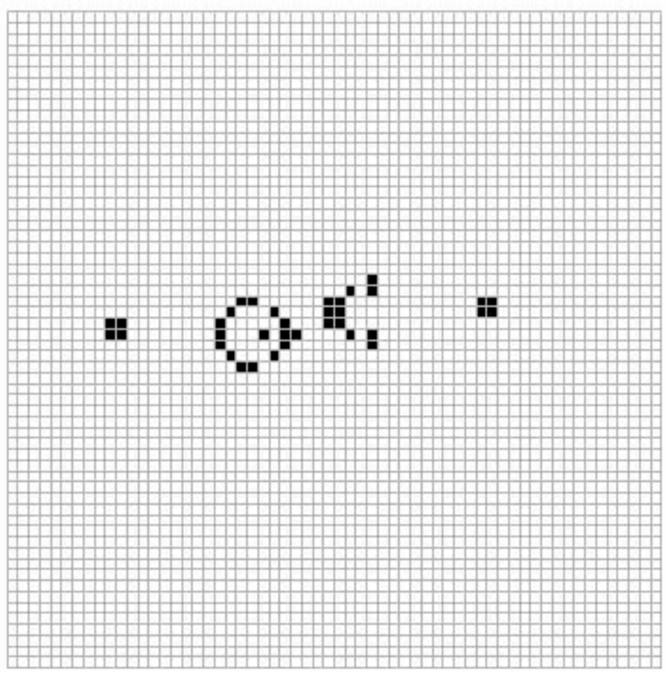

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Cayley Graph

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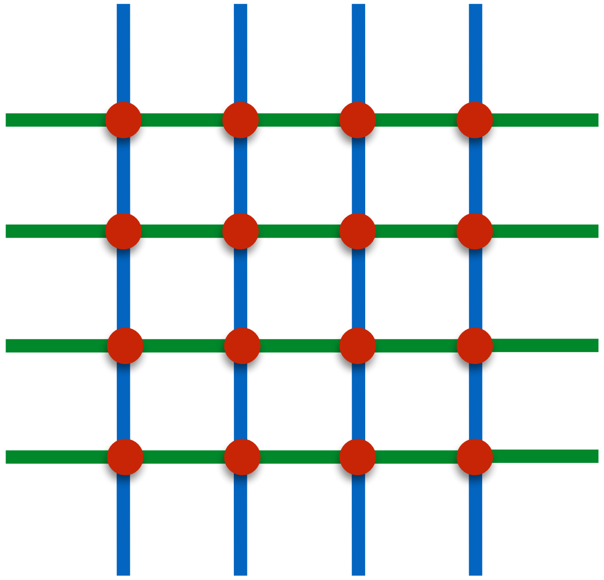
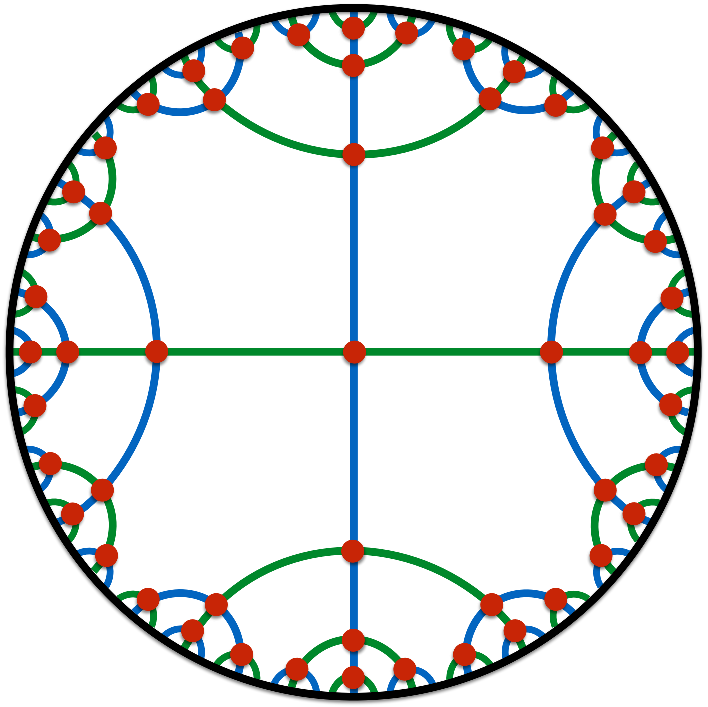
flat



curved (e.g. hyperbolic)



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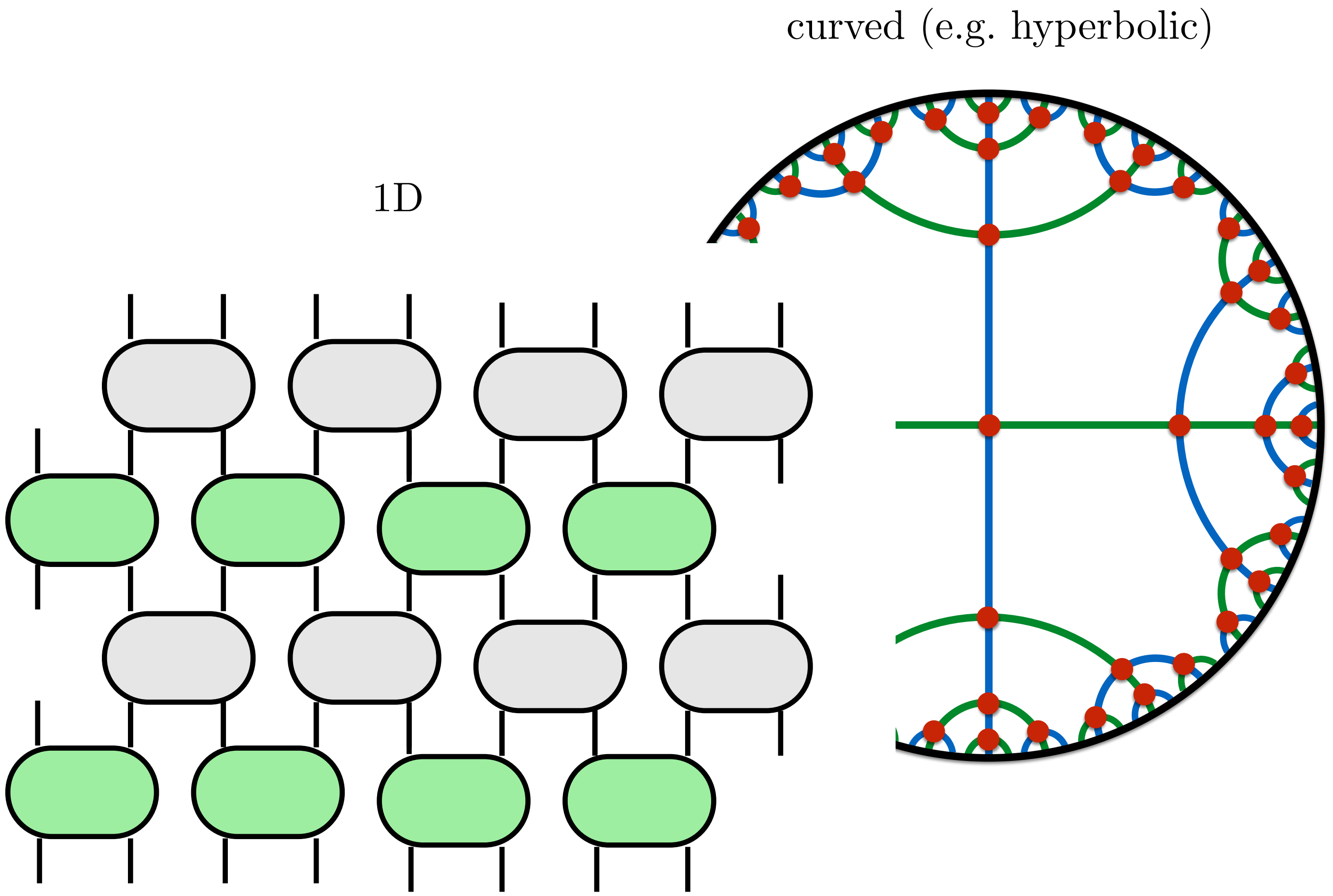

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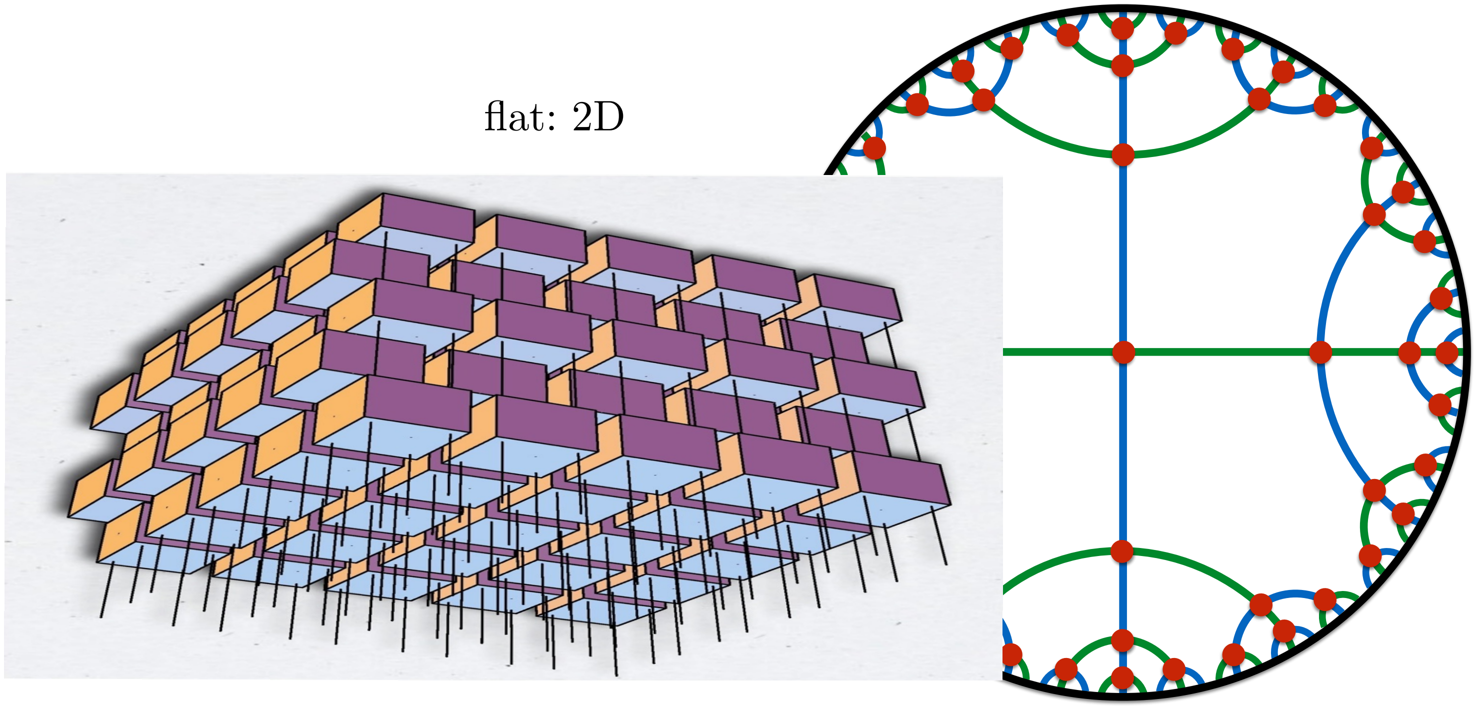

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Towards free Quantum Field Theory

Cayley graph quasi-isometrically embeddable in flat space

⇕

The group G must be virtually abelian ⇒ We restrict to the abelian case $\mathbb{Z}^3$

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Linear evolution $U\psi_i^s U^\dagger = \mathbf{U}_{i,j}^{s,r}\psi_j^r$ ⇔ Quantum Walk

field operators

site label

internal degree of freedom (e.g. spinorial index)

unitary matrix

Towards free Quantum Field Theory

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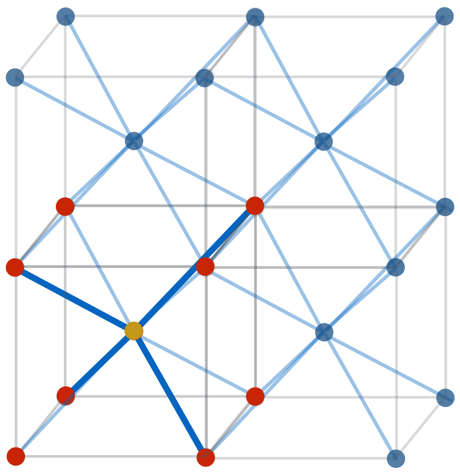
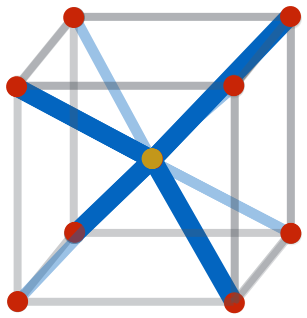
Weyl Quantum Walk





BCC lattice

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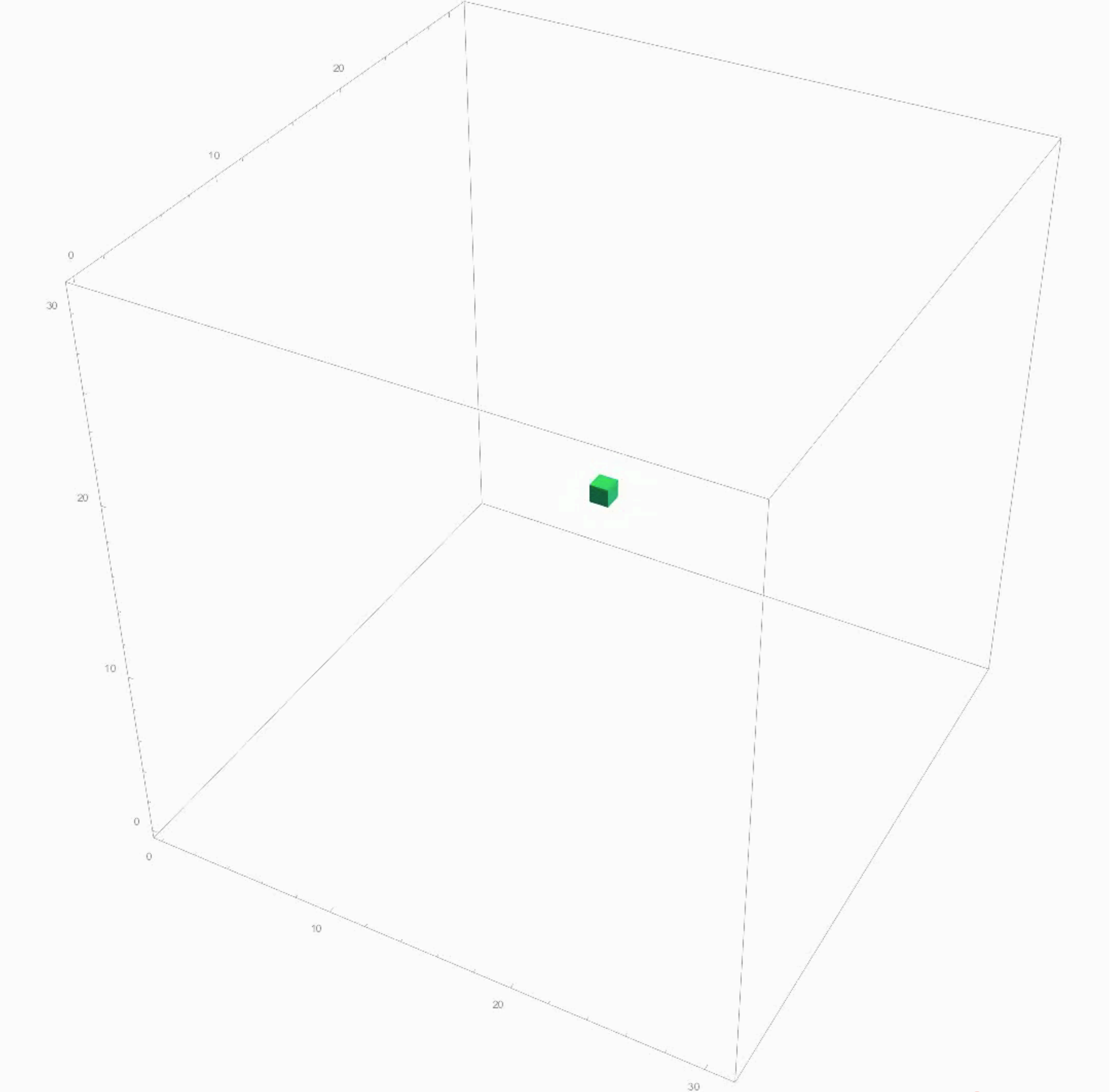

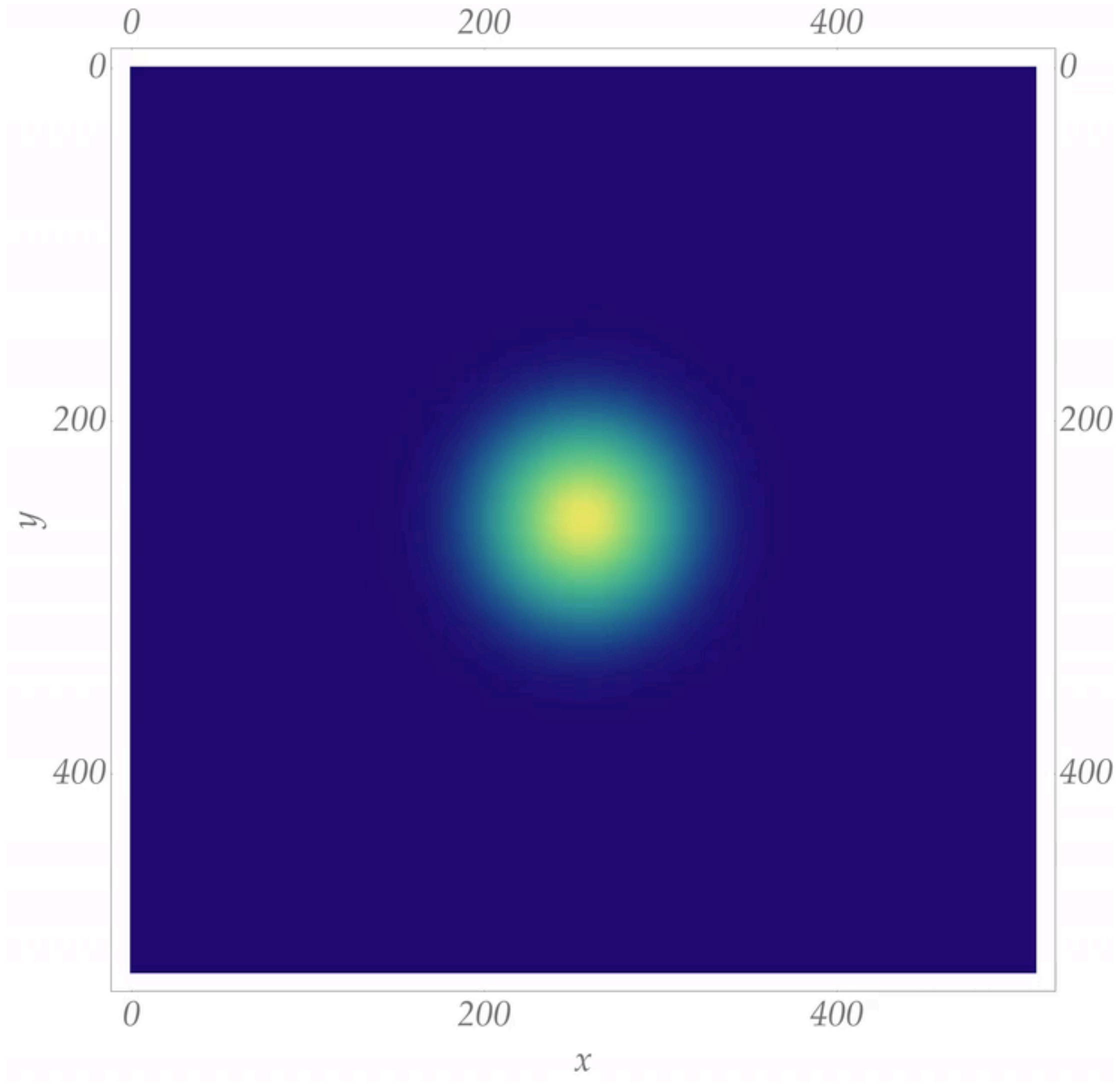

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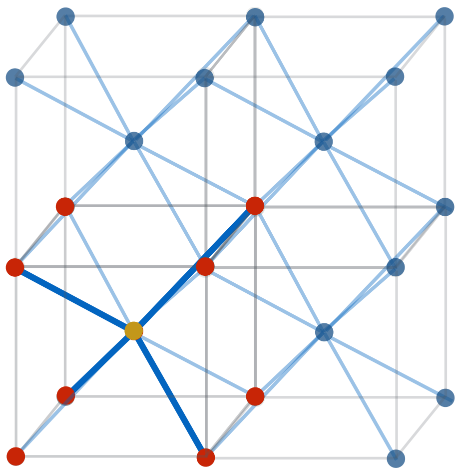
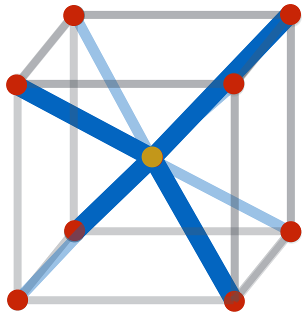
G. M. D’Ariano, P. Perinotti, N. Mosco, A. Tosini Entropy 18, 228 (2016).





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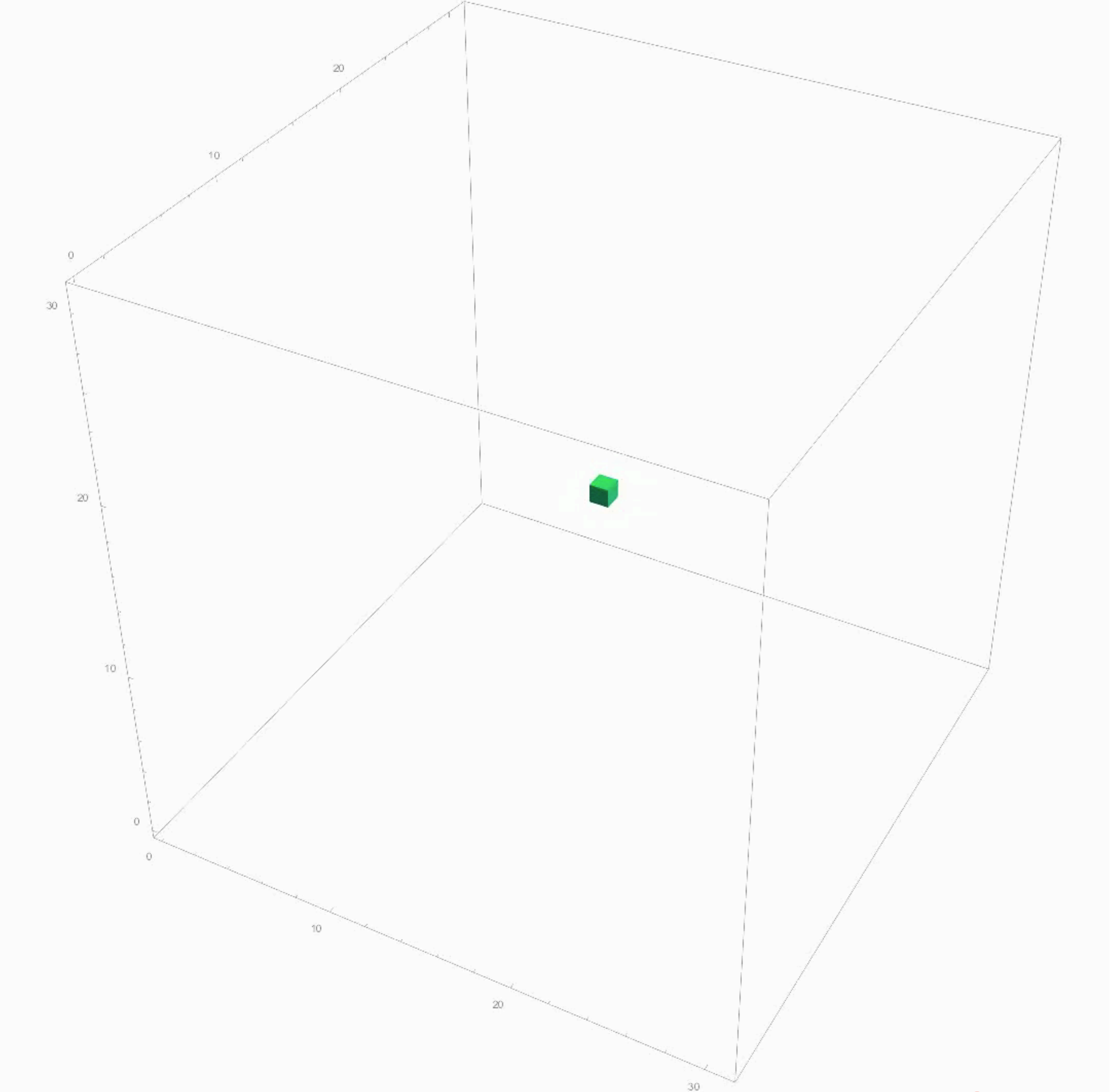
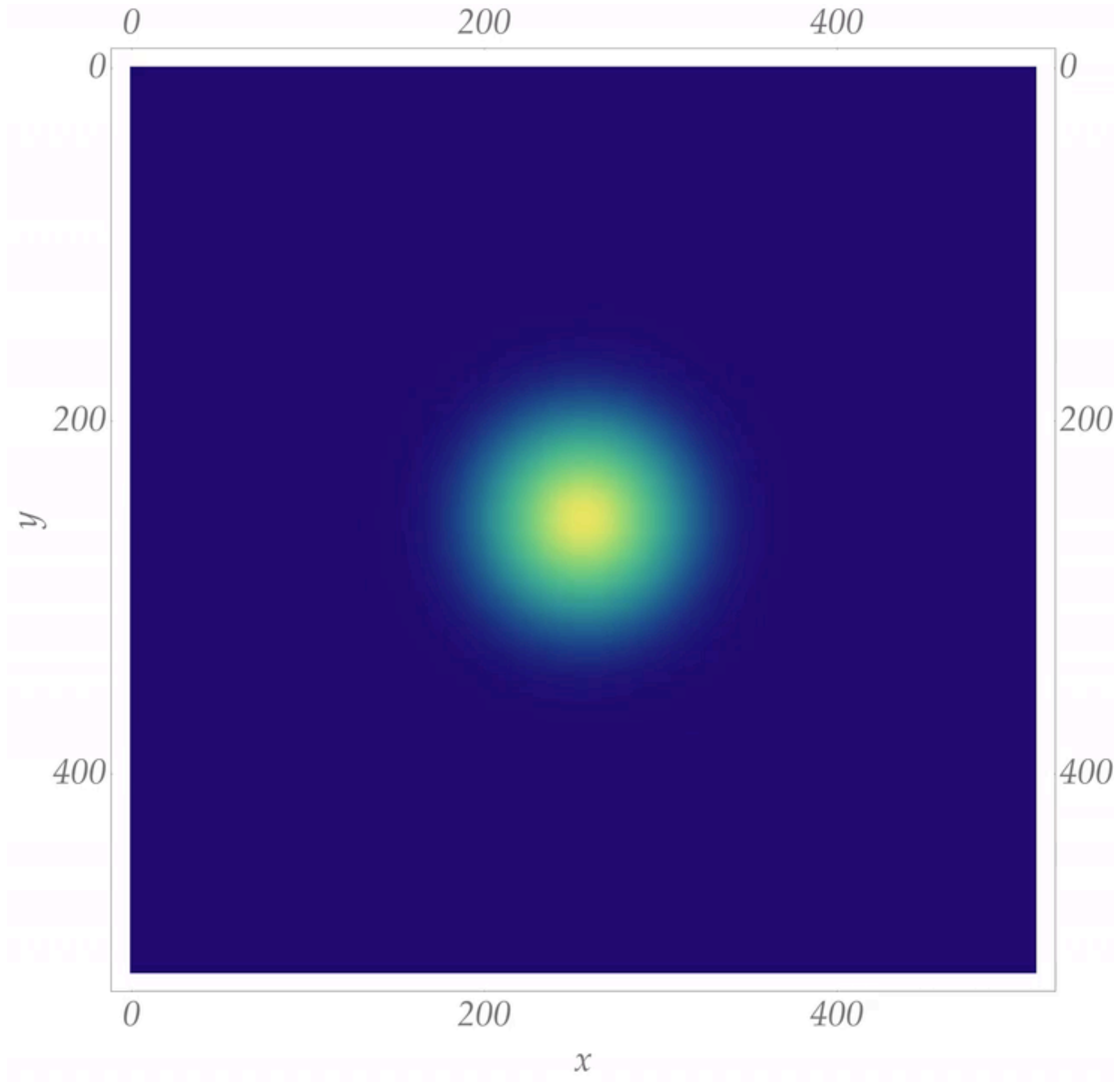
Towards free Quantum Field Theory





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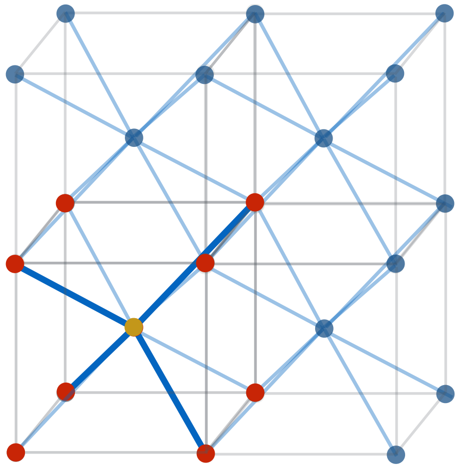

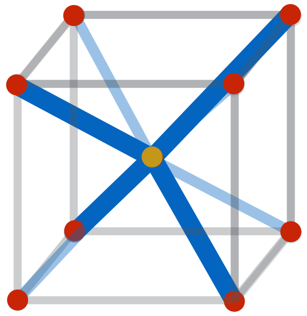

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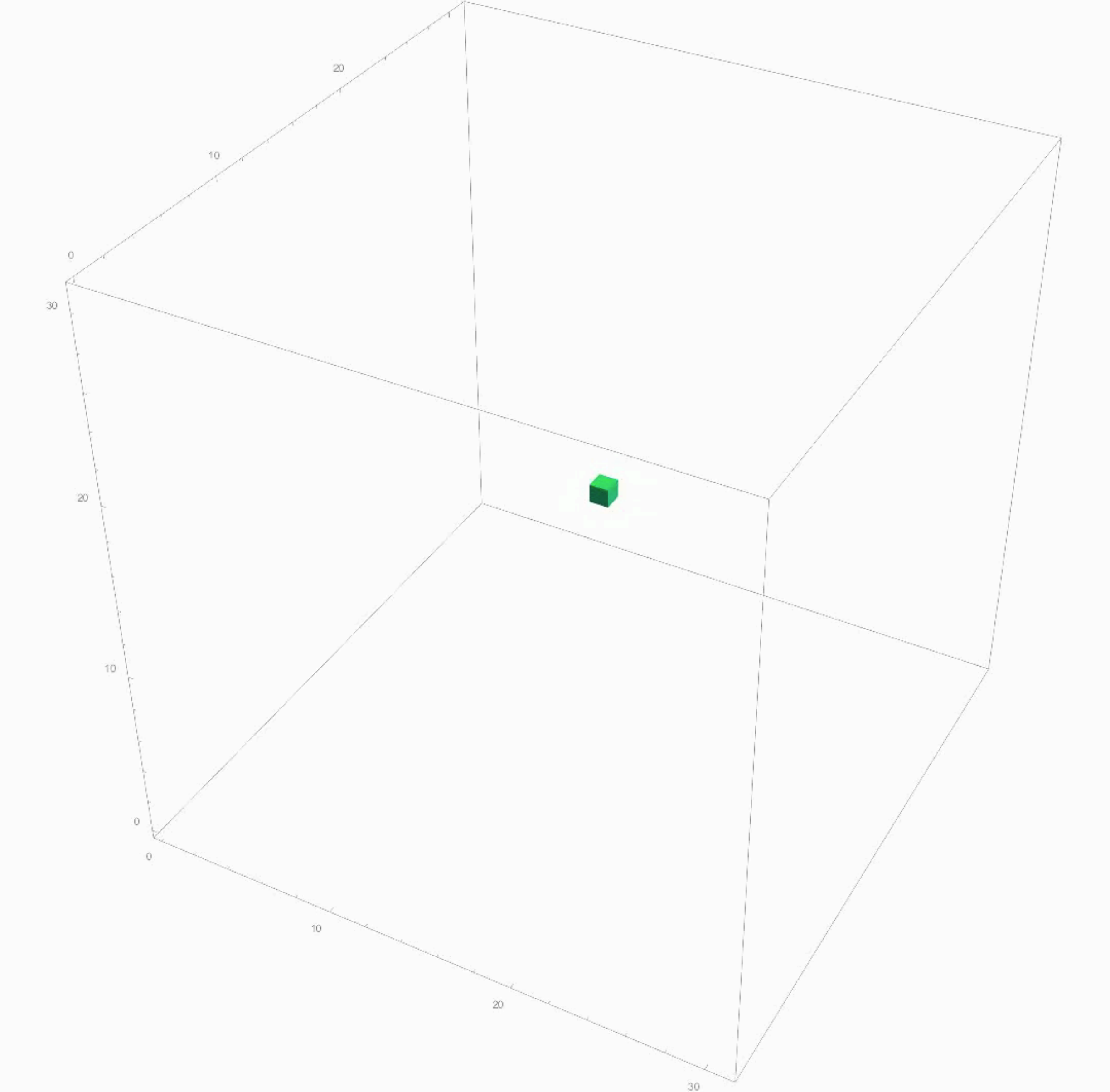
generators

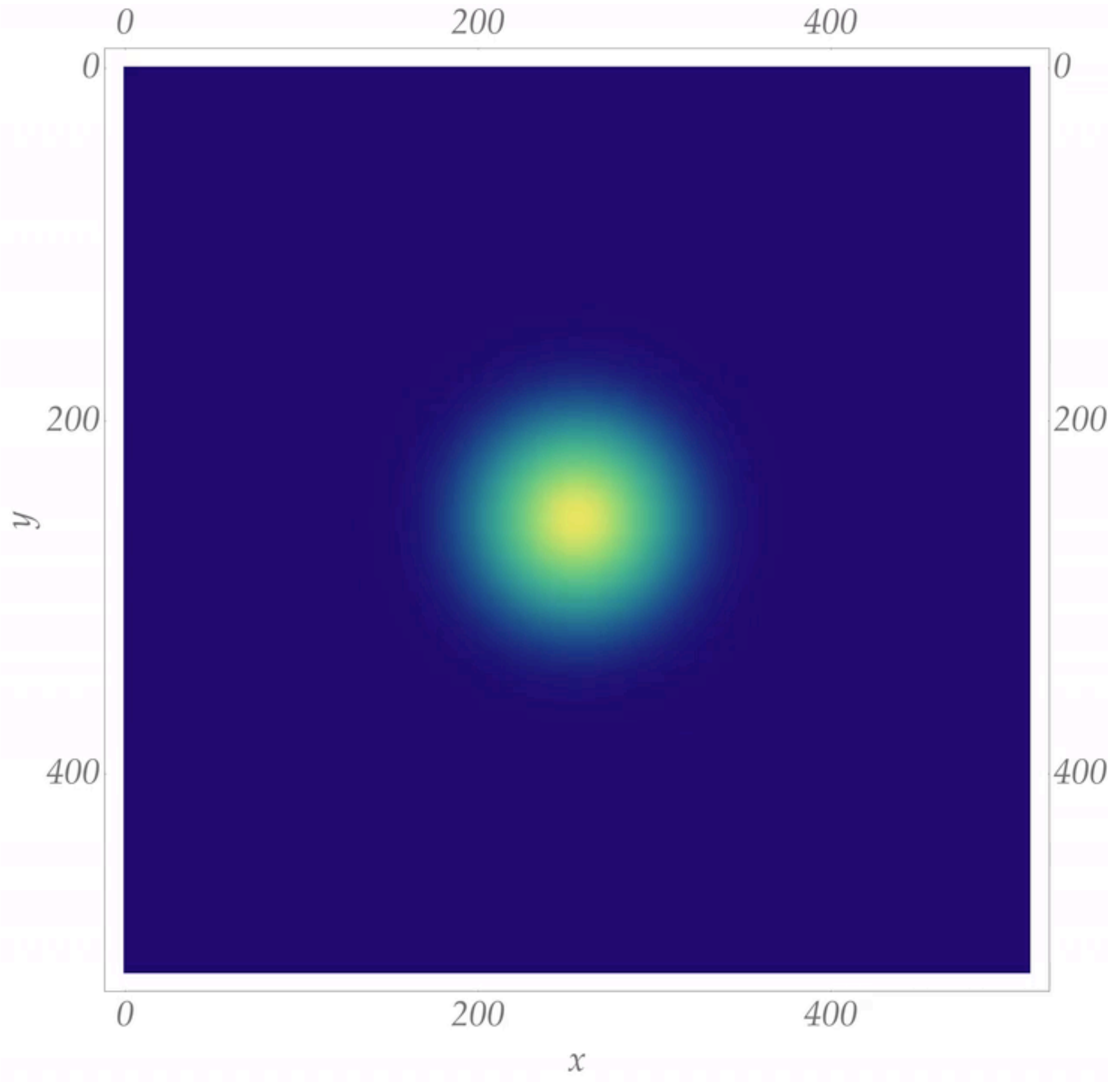
translations

Brillouin zone

Momentum cutoff

Towards free Quantum Field Theory





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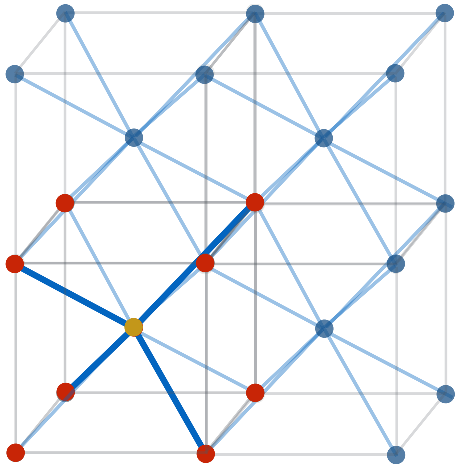

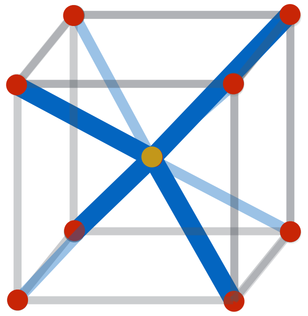

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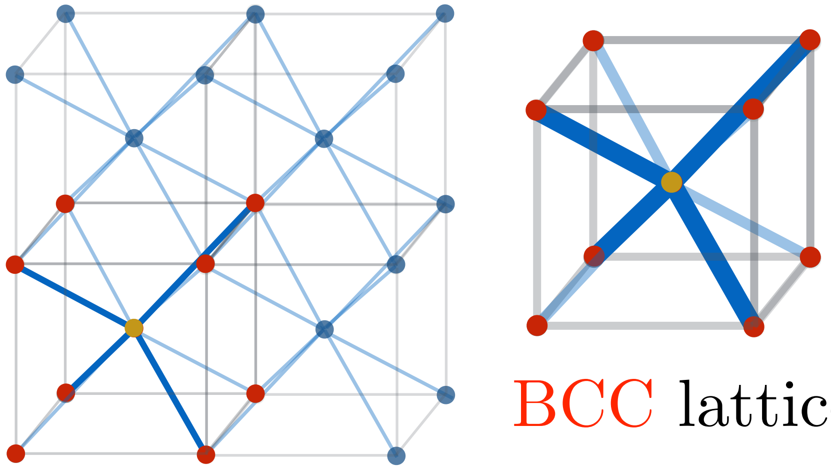
$$\xrightarrow{k \ll 1} \sigma^\mu k_\mu \psi = 0$$

Weyl equation

Towards free Quantum Field Theory

Weyl Quantum Walk



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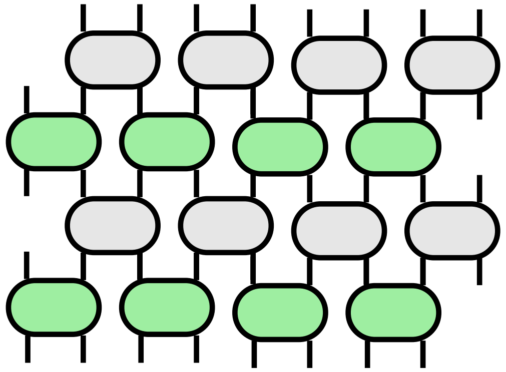
(1+1)-dimensional Dirac Quantum Walk



$$\mathbf{U} = \begin{pmatrix} nS & -im \\ -im & nS^\dagger \end{pmatrix} \qquad \begin{aligned} &S\psi(x) = \psi(x+1) \\ &n^2 + m^2 = 1, \quad \boxed{0 \leqslant m \leqslant 1} \end{aligned}$$

bounded rest mass

$$\xrightarrow{m,k\to 0} \quad i\partial_t \psi(k,t) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix} \psi(k,t)$$

Dirac equation

$$\boxed{\cos^2(\omega_A) = (1-m^2)\cos^2(k) \xrightarrow{m,k\to 0} \omega_A^2 - k^2 = m^2}$$

Relativistic dispersion relation

Futher generalization to 3+1 dimensions and to free Maxwell's equations is possible

AB, G. M. D'Ariano, A. Tosini, Annals of Physics 354, 244 (2015).

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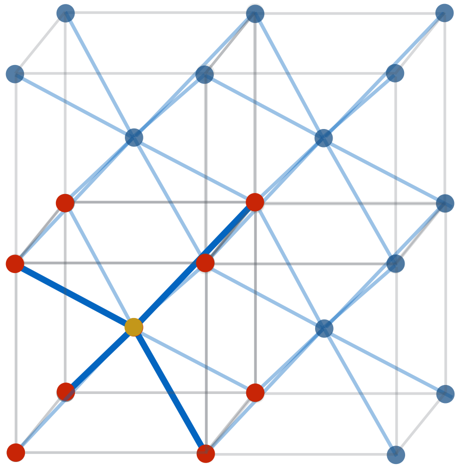
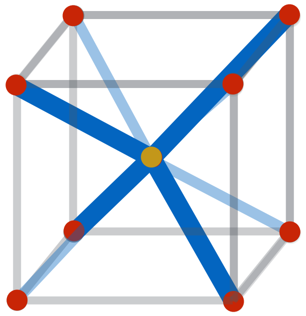
Towards free Quantum Field Theory

Weyl Quantum Walk





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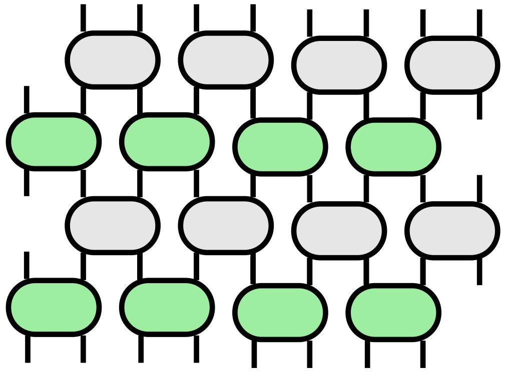

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$n^2 + m^2 = 1,\quad 0 \leqslant m \leqslant 1$

bounded rest mass

$$\xrightarrow{m,k \to 0} i\partial_t \psi(k,t) = \begin{pmatrix} -k & m \\ m & k \end{pmatrix} \psi(k,t)$$

Dirac equation

$$\boxed{\cos^2(\omega_A) = (1-m^2)\cos^2(k) \xrightarrow{m,k \to 0} \omega_A^2 - k^2 = m^2}$$

Relativistic dispersion relation

Futher generalization to 3+1 dimensions and to free Maxwell's equations is possible

AB, G. M. D'Ariano, A. Tosini, Annals of Physics 354, 244 (2015).

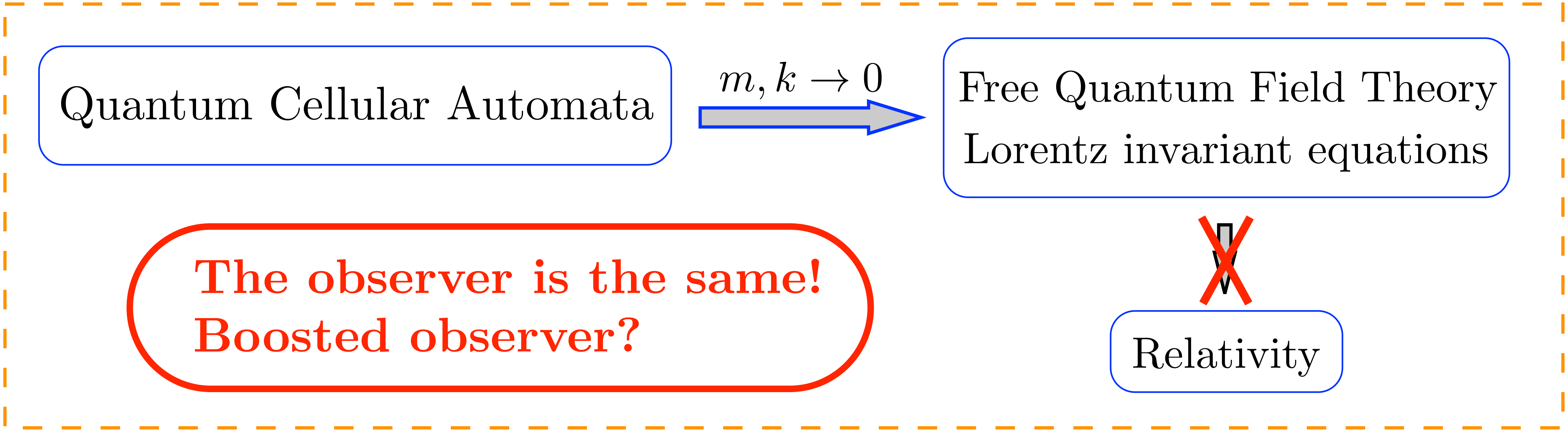
AB, G. M. D'Ariano, P. Perinotti, Annals of Physics 368, 177 (2016).

QCA and Lorentz transformations

Quantum Cellular Automata $\xrightarrow{m, k \to 0}$ Free Quantum Field Theory

Lorentz invariant equations

QCA and Lorentz transformations



Quantum Cellular Automata

$m, k \to 0$

Free Quantum Field Theory

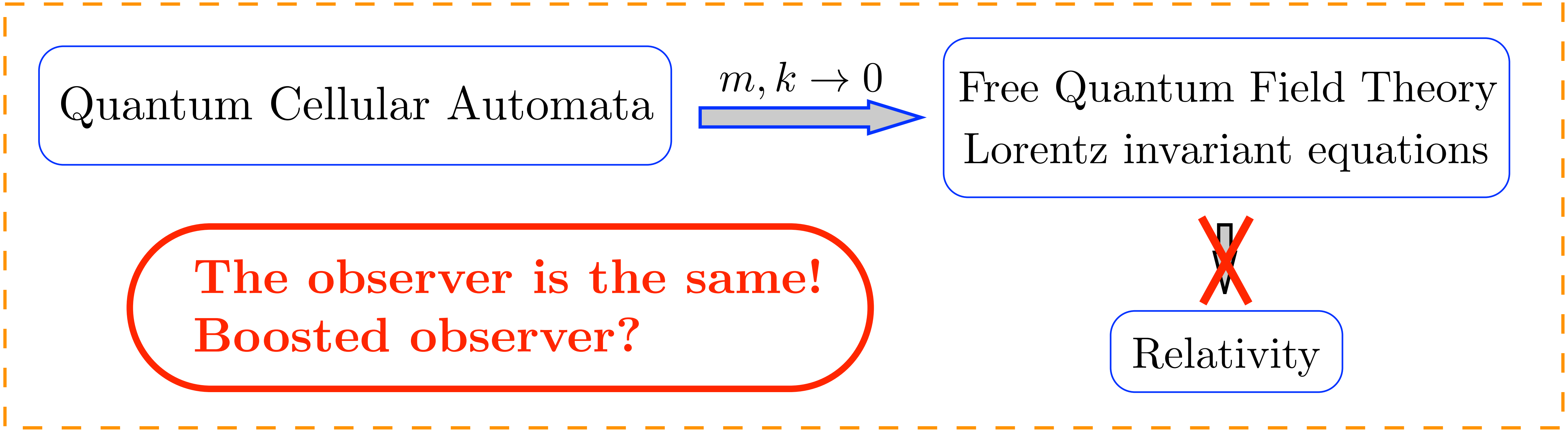
Lorentz invariant equations

Relativity

The observer is the same!

Boosted observer?

QCA and Lorentz transformations



Quantum Cellular Automata

$m, k \to 0$

Free Quantum Field Theory
Lorentz invariant equations

Relativity

The observer is the same!
Boosted observer?

1D Dirac QW dispersion relation

$$\cos^2(\omega) = (1 - m^2)\cos^2(k)$$

classical kinematics
emergent from the automaton

non Lorentz invariant

Lorentz transformation

$$\begin{pmatrix} \omega' \\ k' \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix} \begin{pmatrix} \omega \\ k \end{pmatrix}$$

$$\gamma := \frac{1}{\sqrt{1-\beta^2}}$$

Violations of Lorentz invariance
at ultra-relativistic scales

different
transformation

or

privileged
reference frame

Deformed relativity

A simple speculation from Quantum Gravity → In whose reference frame is the Planck energy the threshold for new phenomena?

Preserve relativity principle
Lorentz group

AND

invariant energy scale

G. Amelino-Camelia, Physics Letters B 510, 255 (2001).

J. Magueijo, L. Smolin, Phys. Rev. Lett. 88, 190403 (2002).

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⇓

Modify the action of Lorentz group

non-linear action in momentum space

$$L_\beta^D := \mathcal{D}^{-1} \circ L_\beta \circ \mathcal{D},$$

$$L_\beta = \gamma \begin{pmatrix} 1 & -\beta \\ -\beta & 1 \end{pmatrix}$$

momentum space
is more fundamental

$\mathcal{D}$ is a non-linear map

- $J_{\mathcal{D}}(0,0) = I$
- singular point ⇔ invariant energy
- invertible

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which $\mathcal{D}$?

G. Amelino-Camelia, Physics Letters B 510, 255 (2001).

J. Magueijo, L. Smolin, Phys. Rev. Lett. 88, 190403 (2002).

Deformed relativity and QW

Dirac QW dispersion relation

$$\cos^2(\omega) = (1 - m^2)\cos^2(k) \Rightarrow \frac{\sin^2(\omega)}{\cos^2(k)} - \tan^2(k) = m^2$$

$$\tilde{\omega}^2 - \tilde{k}^2 = m^2$$

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

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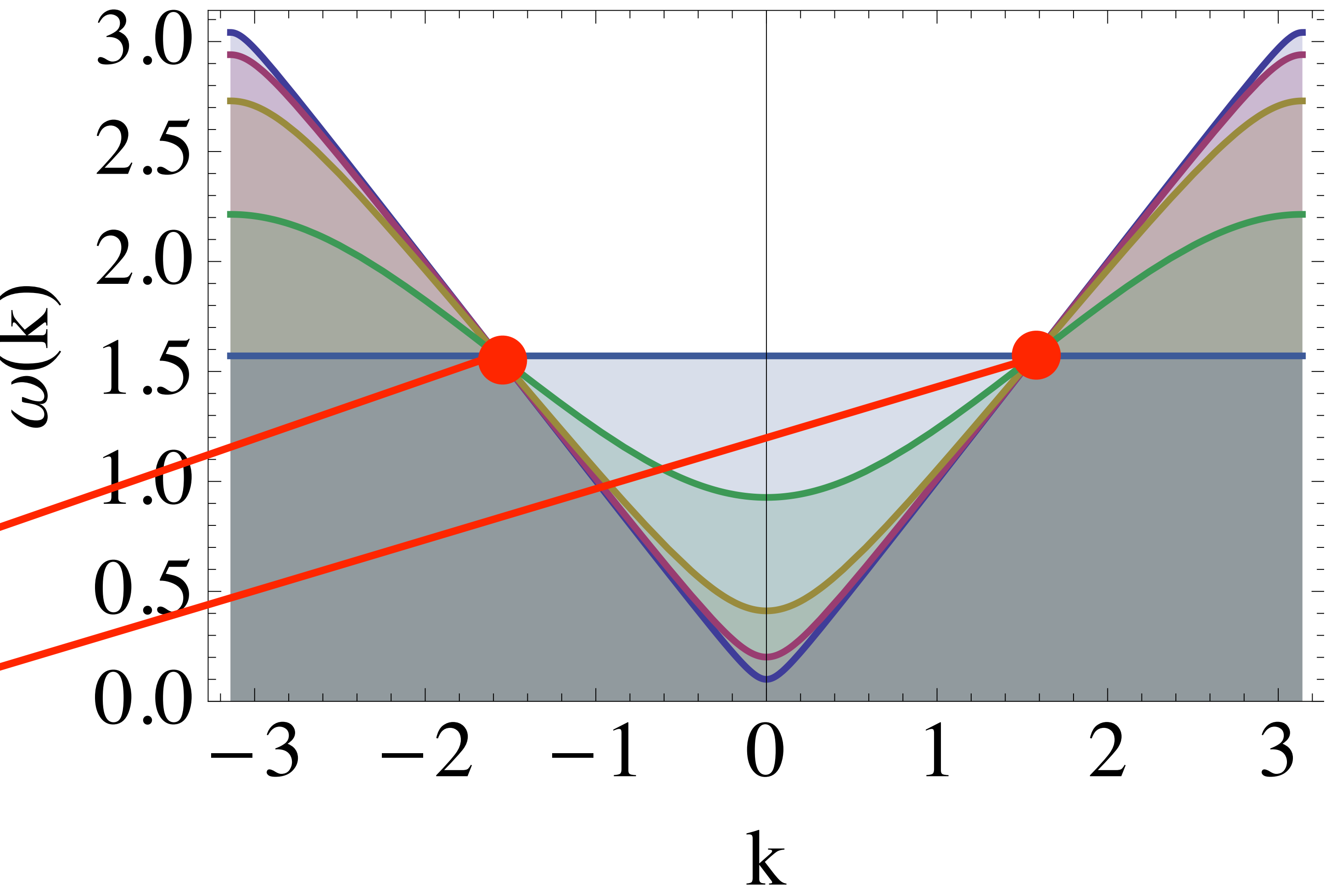
$$\mathcal{D}\begin{pmatrix}\omega\\ k\end{pmatrix} = \begin{pmatrix}\frac{\sin(\omega)}{\cos(k)}\\ \tan(k)\end{pmatrix}$$

$$-\frac{\pi}{2} \leqslant k \leqslant \frac{\pi}{2}$$

$$\omega_{inv} = \frac{\pi}{2}$$



A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

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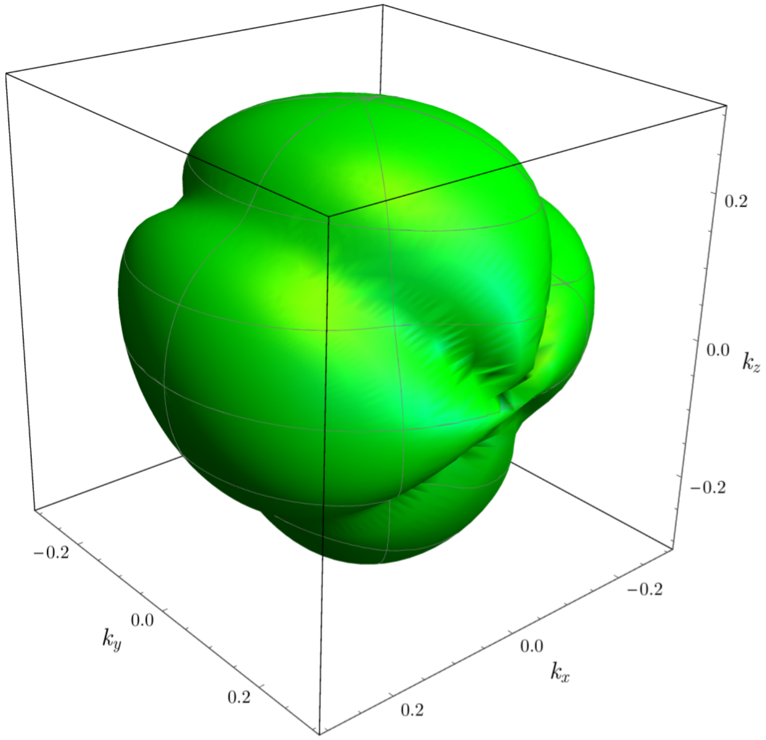
AB, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 94, 042120 (2016).

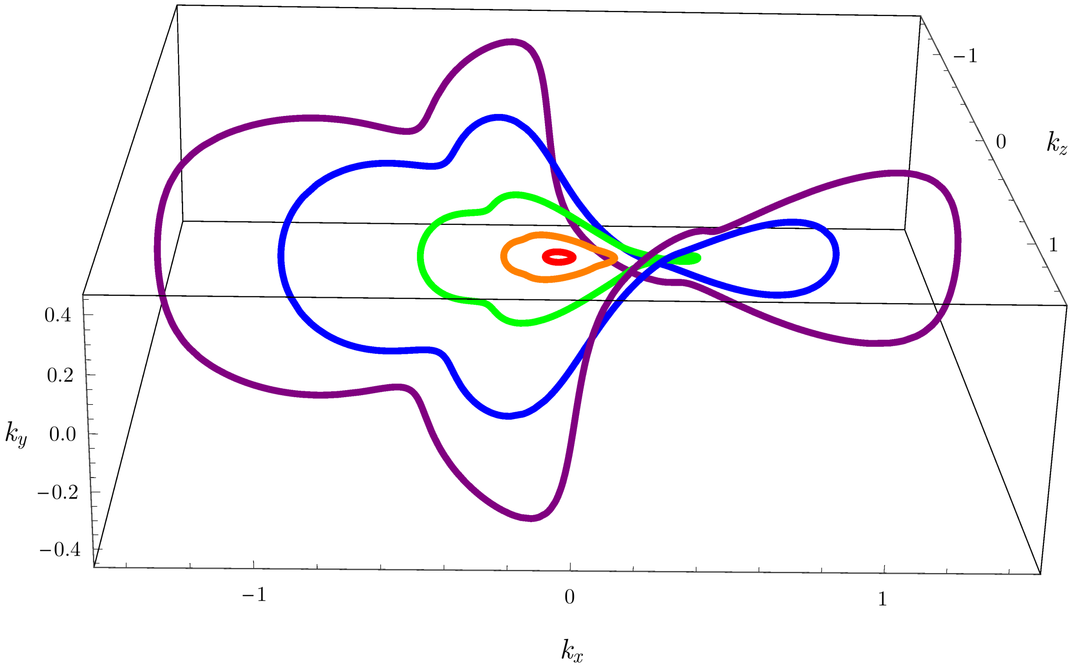
Deformed relativity and QW: 3 spatial dimensions

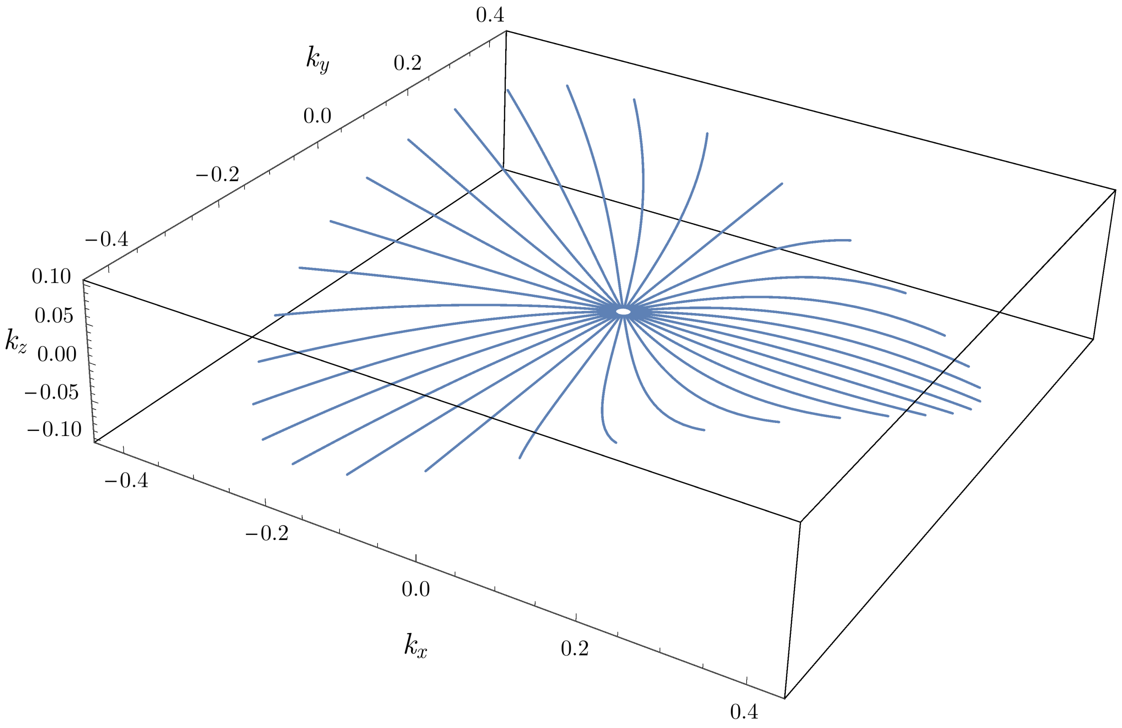
Weyl QW dispersion relation

$$n_\mu(k)\sigma^\mu\psi(k)=0$$

$$n(k):=\begin{pmatrix}\sin\omega\\ s_xc_yc_z+c_xs_ys_z\\ c_xs_yc_z-s_xc_ys_z\\ c_xc_ys_z+s_xs_yc_z\end{pmatrix}\qquad c_i=\cos\left(\frac{k_i}{\sqrt{3}}\right)\qquad s_i=\sin\left(\frac{k_i}{\sqrt{3}}\right)$$







AB, G. M. D'Ariano, P. Perinotti, Foundations of Physics, 47, 8,1065(2017).

AB, G. M. D'Ariano, P. Perinotti, Phil. Trans. R. Soc. A 374 20150232 (2016).

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Deformed relativity in position space

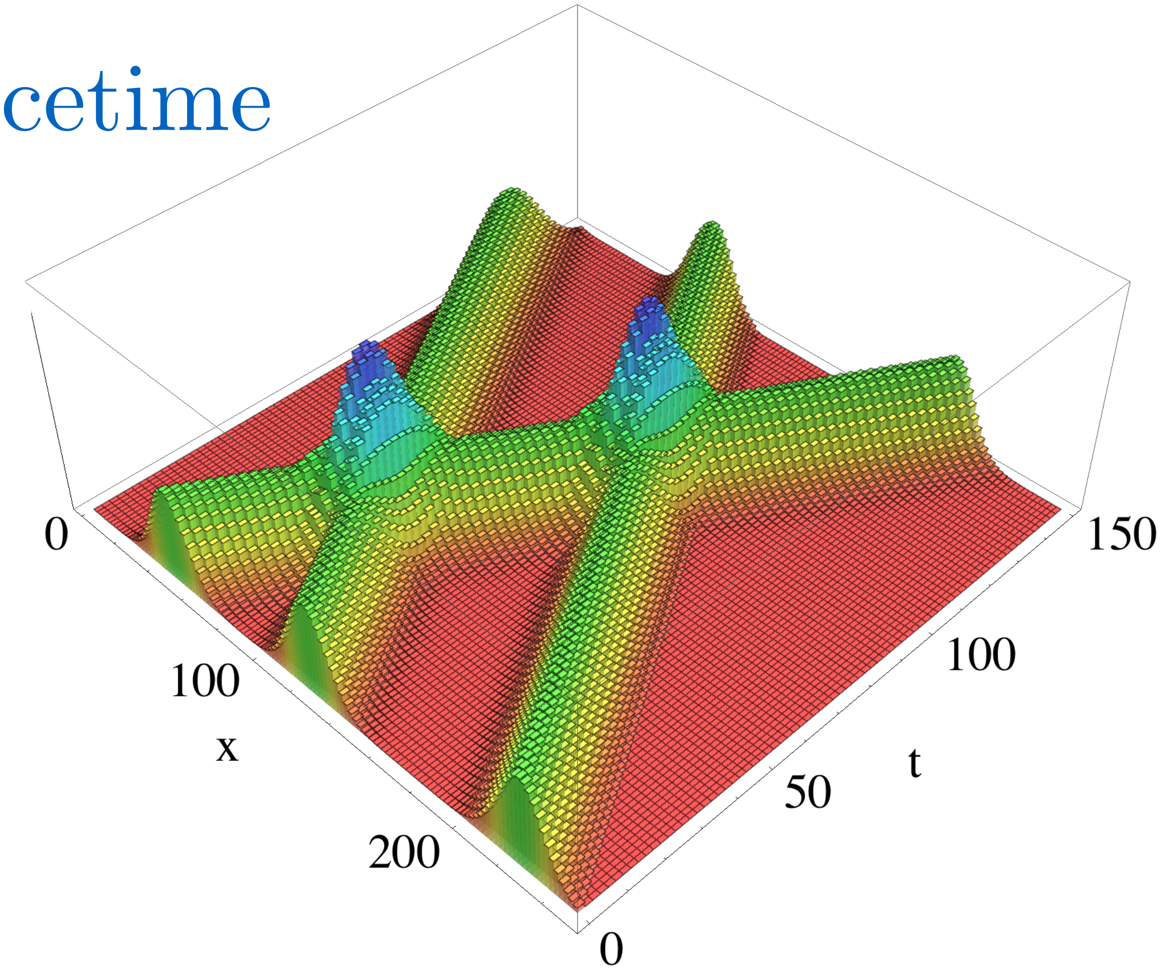
The model is defined in the momentum space → ? → Transformations in the position space?

Operational toy model of spacetime

coincidence of wavepackets

⇕

point in spacetime



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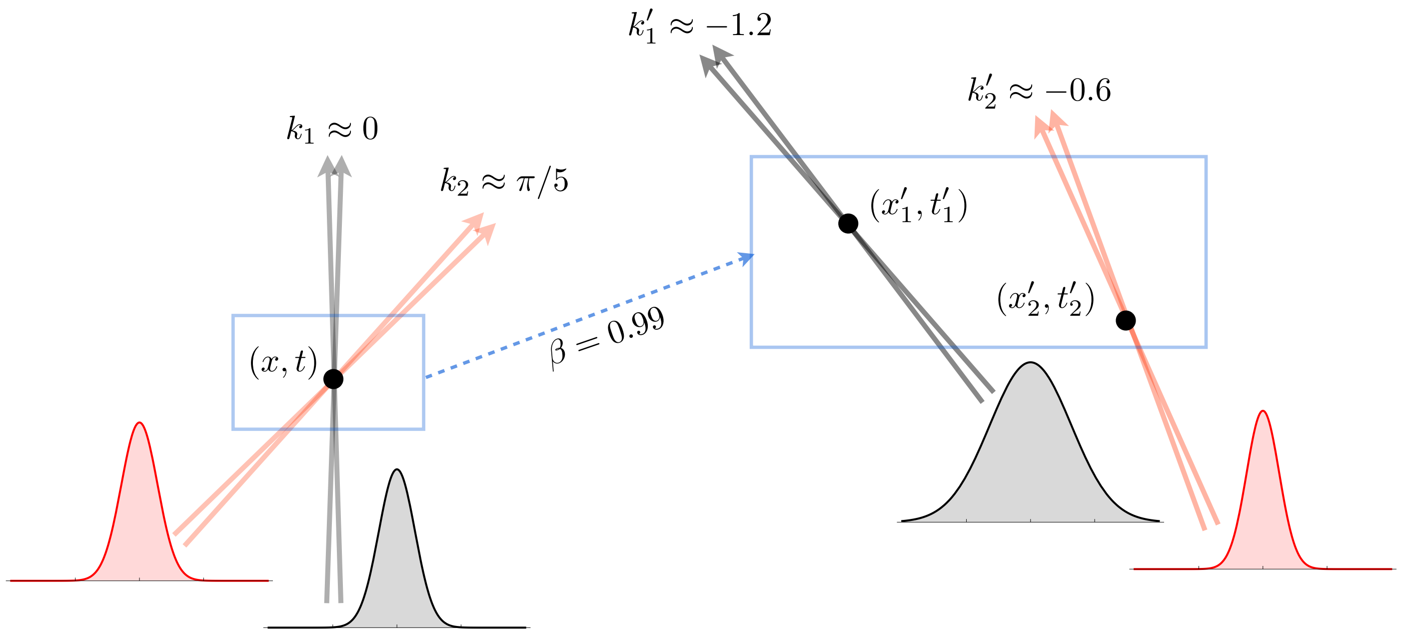

transformation of the coincidence points:

$$\begin{pmatrix} t' \\ x' \end{pmatrix} \approx \begin{pmatrix} -\partial_{\omega'} k & \partial_{k'} k \\ \partial_{\omega'} \omega & -\partial_{k'} \omega \end{pmatrix}_{\boxed{k'=k'_0}} \begin{pmatrix} t \\ x \end{pmatrix}$$

momentum-dependent spacetime

Relative Locality

A. Bibeau-Delisle, AB, G. M. D'Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

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AB, G. M. D'Ariano, P. Perinotti, Phys. Rev. A 94, 042120 (2016).

Deformed relativity in position space

Relative locality ⟹ Observer-dependent spacetime

“before”

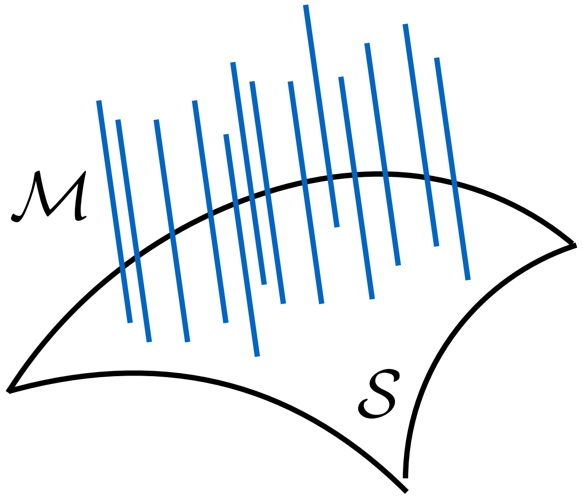
spacetime $\mathcal{S}$

“objective arena”

flat momentum space $\mathcal{M}$

phase space

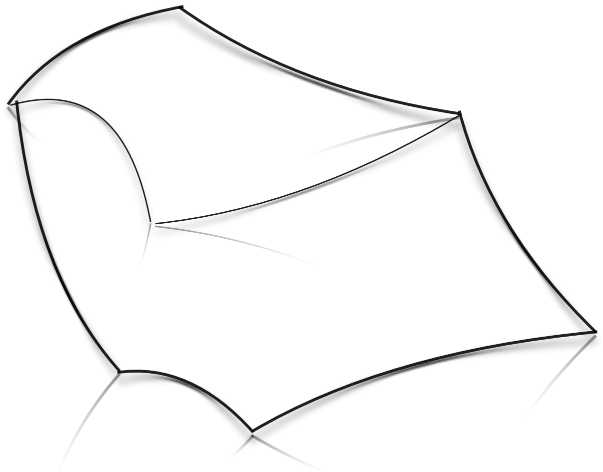
$\mathcal{P} = \mathcal{T}^*\mathcal{S}$



“after”

phase space

$\mathcal{P} \neq \mathcal{T}^*\mathcal{S}$



no canonical projection that gives a description of processes in spacetime

R. Schutzhold, W. G. Unruh, JETP Lett. 78, 431 (2003).

G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman, L. Smolin, Phys. Rev. D 84, 084010 (2011).

A. Bibeau-Delisle, AB, G. M. D’Ariano, P. Perinotti, A. Tosini, EPL 109, 50003 (2015).

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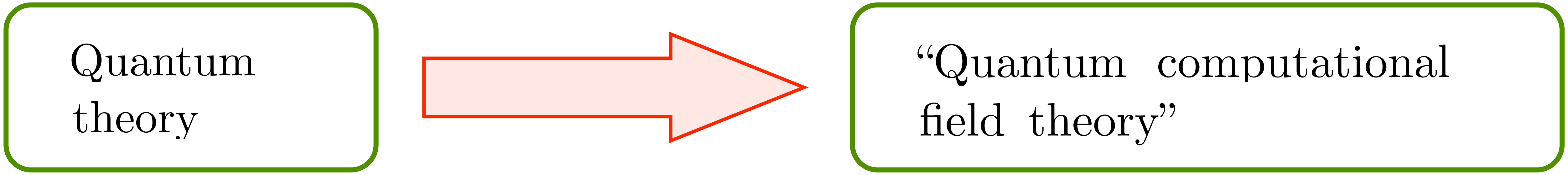
A final overlook

Main idea

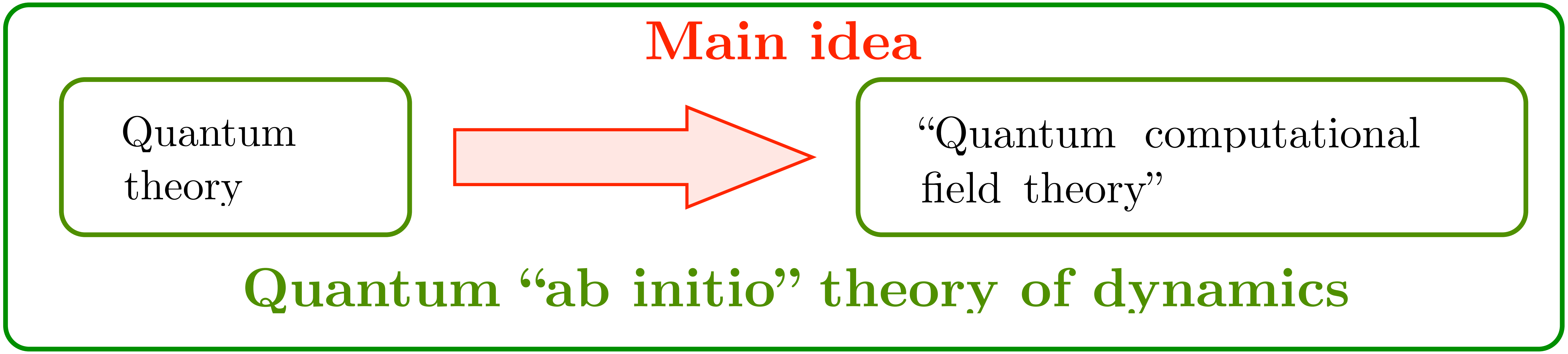


Quantum theory

"Quantum computational field theory"

Quantum "ab initio" theory of dynamics

A final overlook



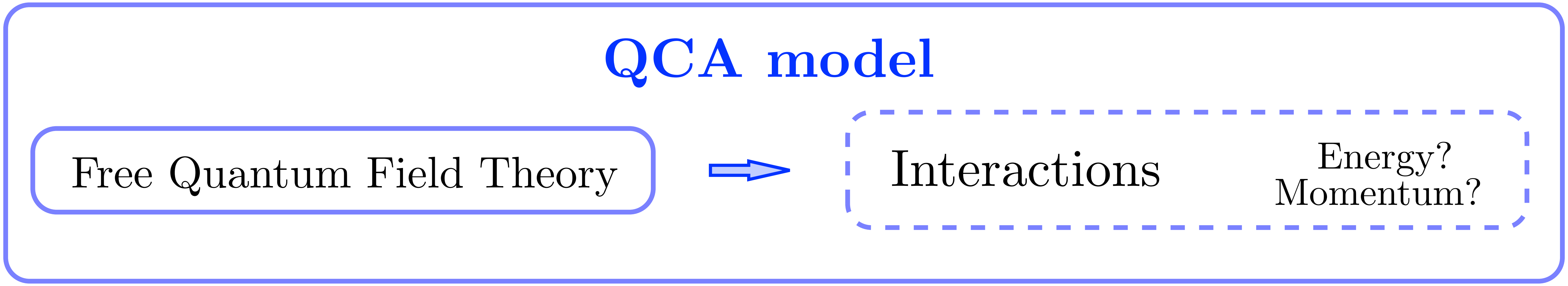
Main idea

Quantum theory

→

"Quantum computational field theory"

Quantum "ab initio" theory of dynamics



QCA model

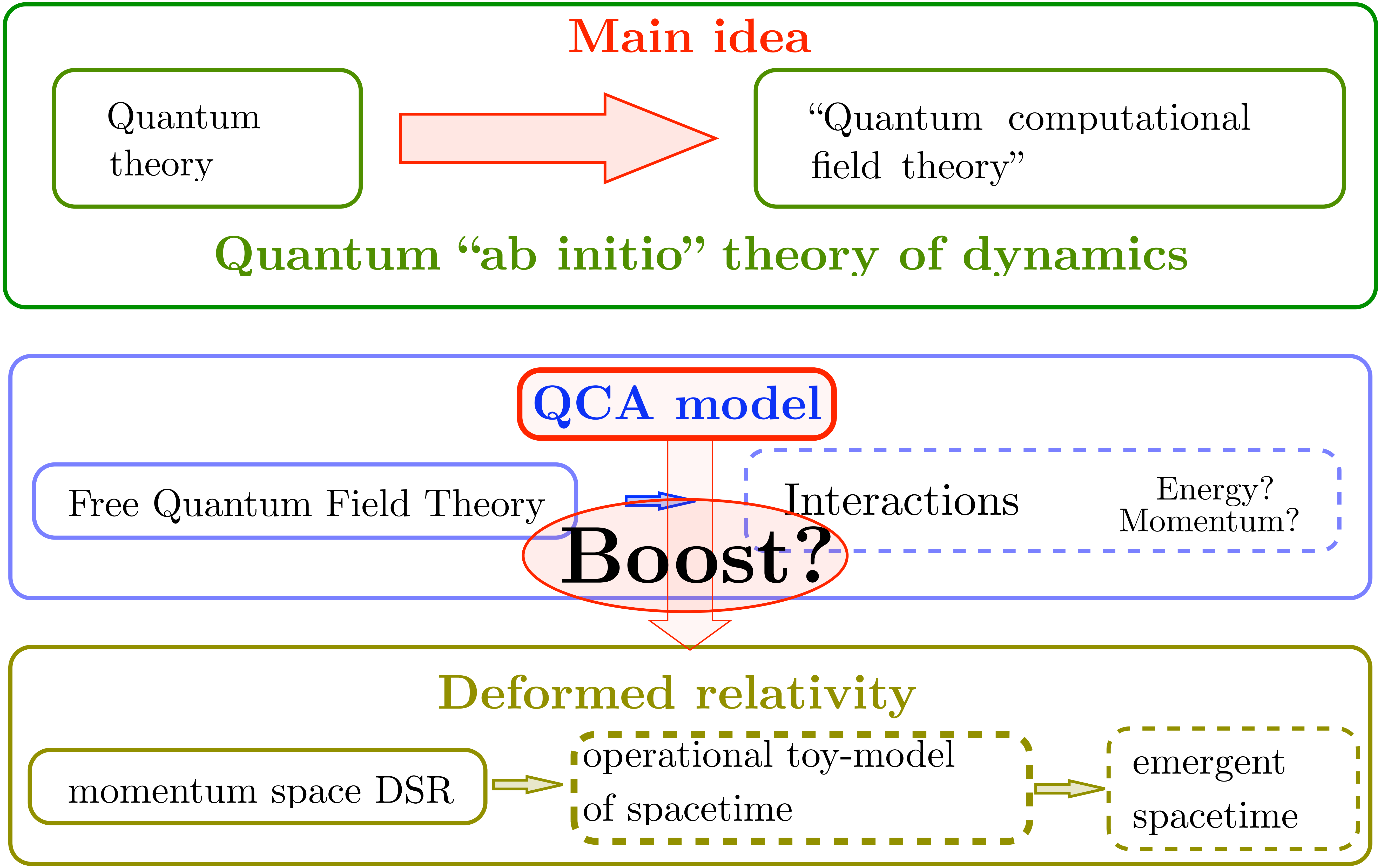
Free Quantum Field Theory

⇒

Interactions

Energy?
Momentum?

A final overlook



Main idea

Quantum theory → "Quantum computational field theory"

Quantum "ab initio" theory of dynamics

QCA model

Free Quantum Field Theory → Interactions

Energy?
Momentum?

Boost?

Deformed relativity

momentum space DSR → operational toy-model of spacetime → emergent spacetime