Non-equilibrium quantum bounds to Landauer's principle

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in collaboration with: Giacomo Guarnieri, Mauro Paternostro, John Goold, Simon Pigeon, and Bassano Vacchini

New J. Phys. **19** 103038 (2017)

Phys. Rev. A **96**, 042109 (2017)

Landauer’s Principle

…holds that "any **logically irreversible** manipulation of information, such as the **erasure** of a bit or the merging of two computation paths, must be accompanied by a corresponding entropy increase in non-information-bearing degrees of freedom of the information-processing apparatus or its environment".

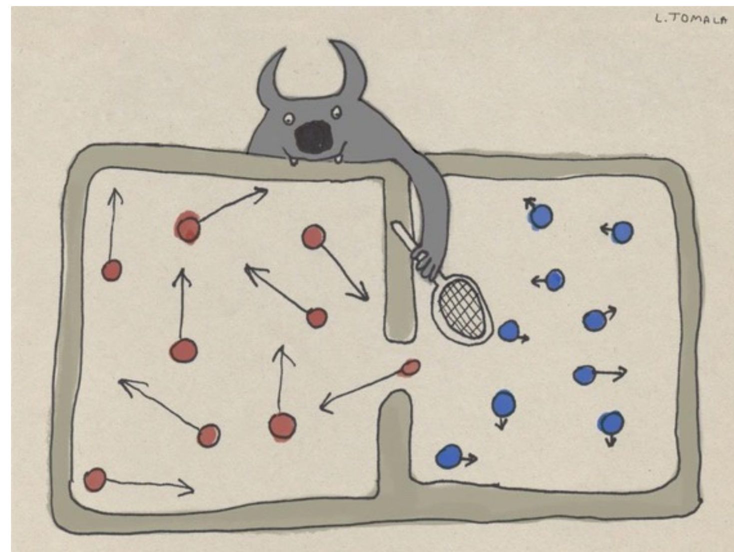
Endows information with strong evidence that it is a physical resource and not just some abstract entity*

(*BUT an important caveat: Landauer only pertains to logically irreversible processes - information can be processed in perfectly reversible ways also)

C. H. Bennet, “Notes on Landauer’s principle, reversible computation, and Maxwell’s Demon”, Studies in History and Philosophy of Modern Physics **34**, 501–510 (2003)

Landauer's Principle

One of the most remarkable outcomes is the exorcism of Maxwell's Demon



It asserts that energy cost to the Demon of erasing the one bit of knowledge (where the atom is) comes with an energy cost of

$$E = k_B T \ln 2$$

This sets the lower bound on the dissipated heat from the system into its environment

$$\langle Q \rangle = \mathrm{Tr}\left[\mathcal{H}_E \left(\varrho_E(t) - \varrho_E(0)\right)\right]$$

Interestingly, Landauer's principle holds for both classical and quantum systems and has been experimentally verified

Landauer's Principle - The Experiments

Classical

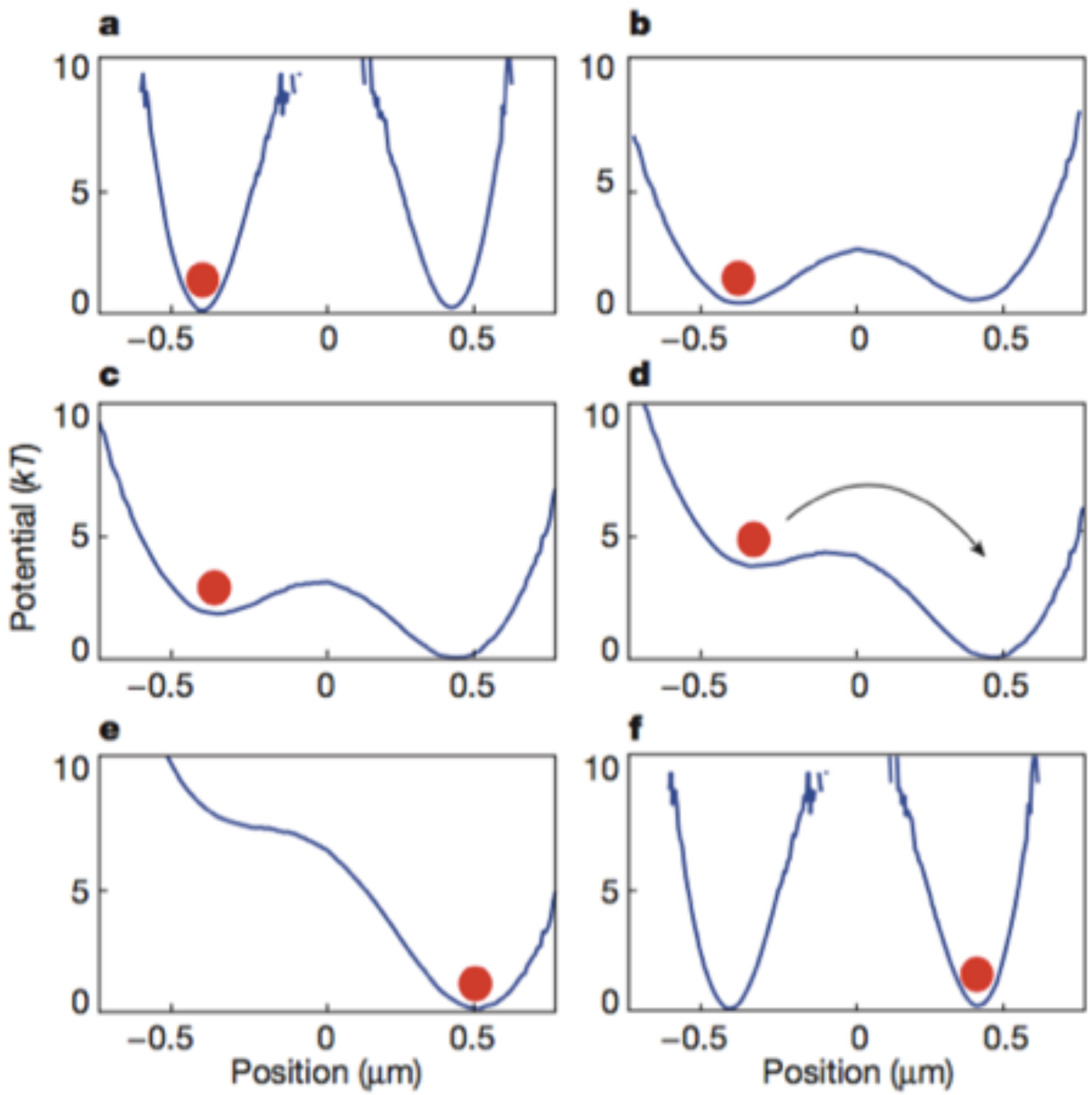


Figure 1 | The erasure protocol used in the experiment. One bit of information stored in a bistable potential is erased by first lowering the central barrier and then applying a tilting force. In the figures, we represent the transition from the initial state, 0 (left-hand well), to the final state, 1 (right-hand well). We do not show the obvious 1 → 1 transition. Indeed the procedure is such that irrespective of the initial state, the final state of the particle is always 1. The potential curves shown are those measured in our experiment (Methods).

A. Berut et al, "Experimental verification of Landauer's principle linking information and thermodynamics" Nature 483, 187-189 (2012)

Quantum

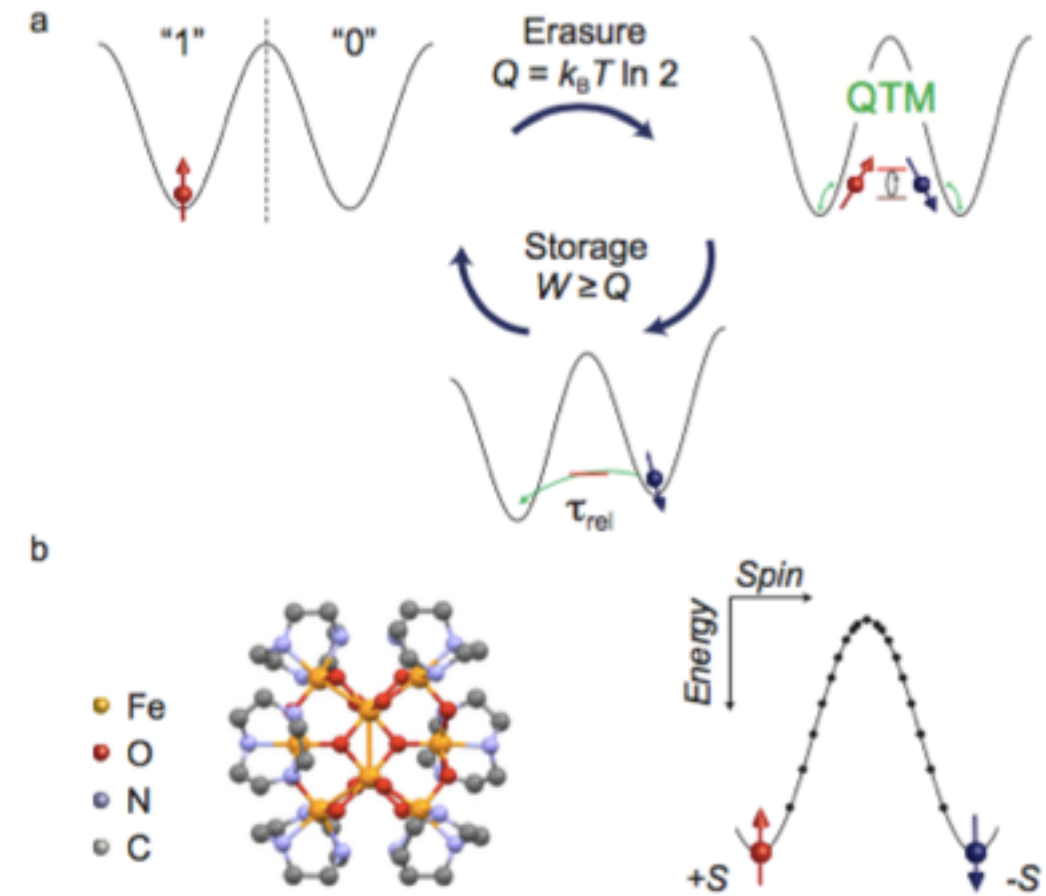
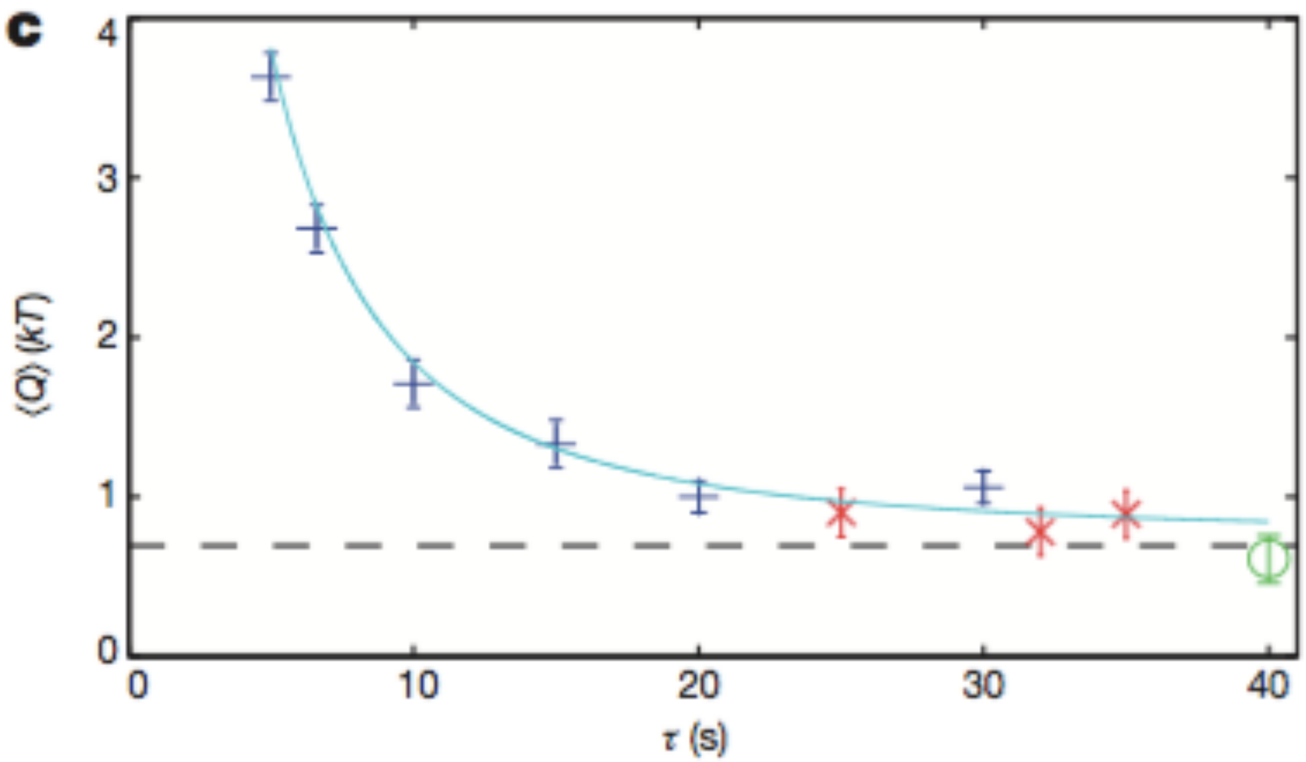


FIG. 1. **Quantum-enhanced Landauer erasure and storage of a molecular bit.** (a) Schematics of the Landauer erasure process employing a quantum nanomagnet. In order to erase the spin bit, the effective barrier separating the two binary states is lowered by inducing quantum tunneling of magnetization (QTM). A small bias magnetic field is then used to initialise the spin in the desired state within a time $\tau_{\rm rel}$ and store the new information. The Landauer principle fixes the minimal dissipated heat Q and work W involved in the cycle. (b) Sketch of the Fe_8 easy-axis molecular magnet. In the absence of magnetic field, the double-well potential favors the two $S_z = \pm 10$ easy-axis spin eigenstates.

R. Gaudenzi et al, "Quantum-enhanced Landauer erasure and storage" arXiv:1703.04607

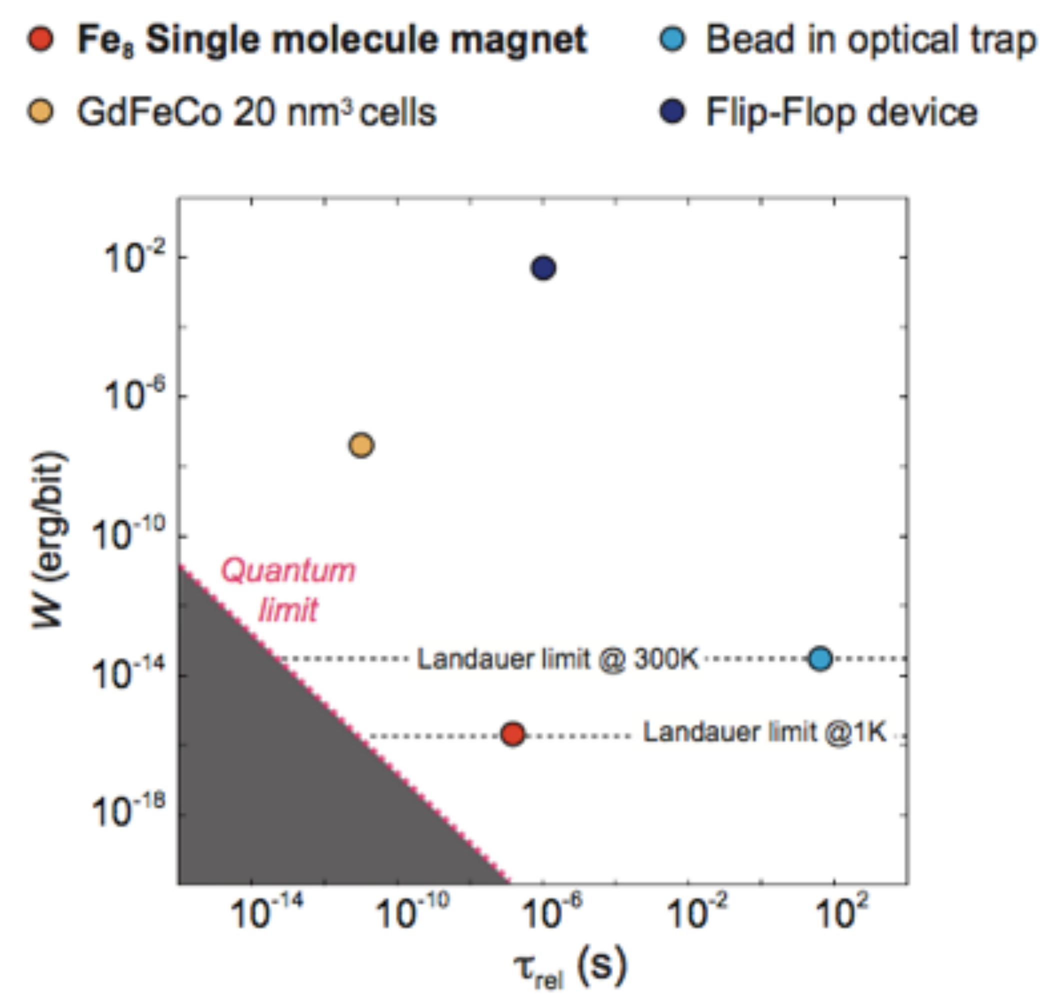
Landauer's Principle - The Experiments

Classical



A. Berut et al, "Experimental verification of Landauer's principle linking information and thermodynamics" Nature 483, 187-189 (2012)

Quantum



R. Gaudenzi et al, "Quantum-enhanced Landauer erasure and storage" arXiv:1703.04607

Landauer's Principle - The Details

"Entropic" Formulation

Recall the heat dissipated by the system into its environment is given by

$$\langle Q \rangle = \mathrm{Tr}\left[\mathcal{H}_E \left(\varrho_E(t) - \varrho_E(0)\right)\right]$$

Landauer and Bennetts formulations relate this quantity to the associate change in entropy of the system and provide us with the "entropic" bound that will be our focus

$$\beta\langle Q \rangle \geq \Delta S = S(\varrho_S(0)) - S(\varrho_S(t))$$

Where the entropy is given by the von Neumann entropy

D. Reeb and M. M. Wolf, "An improved landauer principle with finite-size corrections," New J. Phys. 16, 103011 (2014)

Landauer's Principle

"Thermodynamic" Formulation

However, while the entropic bound arises due to Landauer's considerations, we can seek other paths to find alternative bounds on the dissipated heat

$$\langle Q \rangle = \mathrm{Tr}\left[\mathcal{H}_E\left(\varrho_E(t) - \varrho_E(0)\right)\right]$$

By considering the details of the dynamical map governing the evolution of the system a "Landauer-like" bound can be derived that involves only the Kraus operators and initial system state

$$\beta\langle Q \rangle \geq \mathcal{B} = -\ln\left(\mathrm{Tr}\left[\sum_i K_i^\dagger \varrho_S(0) K_i\right]\right)$$

Kraus operators s.t.:

$$\sum_i K_i^\dagger K_i = \mathbb{1} \quad \text{and} \quad \varrho_S(t) = \sum_i K_i \varrho_S(0) K_i^\dagger$$

J. Goold, M. Paternostro, and K. Modi, "Nonequilibrium quantum landauer principle," Phys. Rev. Lett. 114, 060602 (2015)

G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B. Vacchini, and M. Paternostro, "Full counting statistics approach to the quantum non-equilibrium landauer bound," New J. Phys. **19** 103038 (2017)

Landauer's Principle

Full Counting Statistics Formulation

Using a two time measurement approach we can establish the conditional probability to record a given amount of heat being transferred from the system to its environment

$$P_t\left[E_m, E_n\right] = \mathrm{Tr}[\Pi_m U(t) \Pi_n \rho_S(0) \otimes \rho_\beta \Pi_n U^\dagger(t) \Pi_m]$$

Thus the corresponding probability distribution is given

$$p_t(Q) = \sum_{E_n, E_m} \delta(Q - (E_m - E_n)) P_t\left[E_m, E_n\right]$$

The cumulant generating function, with counting parameter η, is then

$$\Theta(\eta, \beta, t) \equiv \ln\langle e^{-\eta Q}\rangle_t = \ln \int p_t(Q) e^{-\eta Q} dQ$$

P. Talkner, E. Lutz, and P. Hanggi, "Fluctuation theorems: Work is not an observable" Phys. Rev. E **75**, 050102 (2007)

G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B. Vacchini, and M. Paternostro, "Full counting statistics approach to the quantum non-equilibrium landauer bound," New J. Phys. **19** 103038 (2017)

Landauer's Principle

Full Counting Statistics Formulation

The cumulant of n-th order is simply obtained by differentiation

$$\langle Q^n \rangle_t = (-1)^n \frac{\partial^n}{\partial \eta^n} \Theta(\eta, \beta, t)|_{\eta=0}$$

Hence n=1 corresponds to the average heat. The convexity of Θ requires

$$\Theta(\eta, \beta, t) \geq \eta \frac{\partial}{\partial \eta} \Theta(\eta, \beta, t)\Big|_{\eta=0}$$

Thus combining these expressions we arrive at a new Landauer bound on the dissipated heat valid for arbitrary non-equilibrium processes

$$\beta \langle Q \rangle_t \geq -\frac{\beta}{\eta} \Theta(\eta, \beta, t) \equiv \mathcal{B}_{\mathcal{Q}}^{\eta}(t) \quad (\eta > 0)$$

Remarkably for $\eta \to \beta$ we recover the previous bound exactly.

G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B. Vacchini, and M. Paternostro, "Full counting statistics approach to the quantum non-equilibrium landauer bound," arXiv:1704.01078 in press New J. Phys.

Landauer's Principle

Therefore we have three separate bounds on the dissipated heat emerging from fundamentally different viewpoints

$$\beta\langle Q\rangle \geq \Delta S = S(\varrho_S(0)) - S(\varrho_S(t))$$

$$\beta\langle Q\rangle \geq \mathcal{B} = -\ln\left(\mathrm{Tr}\left[\sum_i K_i^\dagger \varrho_S(0) K_i\right]\right) \qquad \beta\langle Q\rangle_t \geq -\frac{\beta}{\eta}\Theta(\eta,\beta,t) \equiv \mathcal{B}_{\mathcal{Q}}^{\eta}(t) \quad (\eta > 0)$$

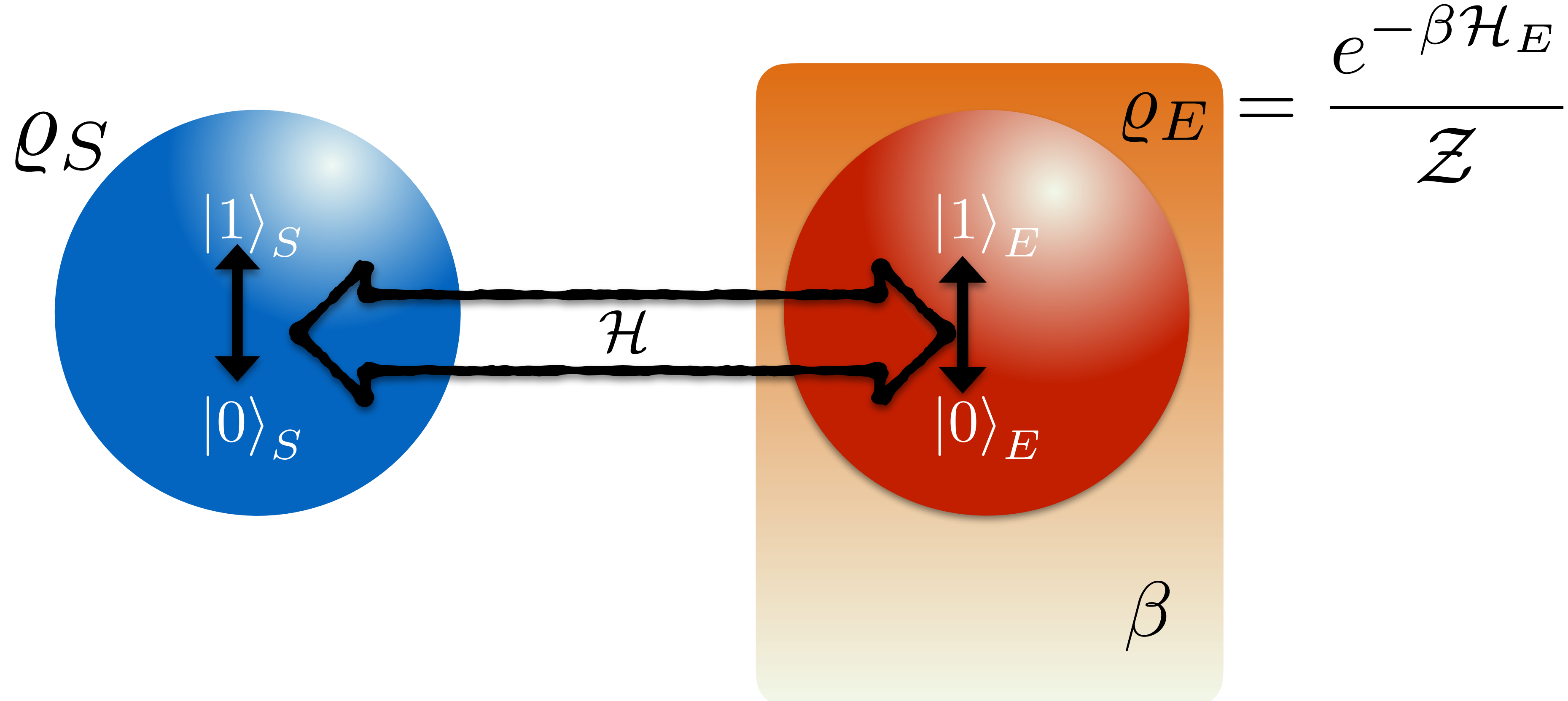
Each with its own advantages and insights that can be drawn.

It may then be natural to ask how these bounds relate to one another and their "relative" performance.

More importantly we are interested in assessing a point often overlooked when discussing Landauer:

does the presence of quantumness change things?

The model



We consider the excitation preserving *XX* model

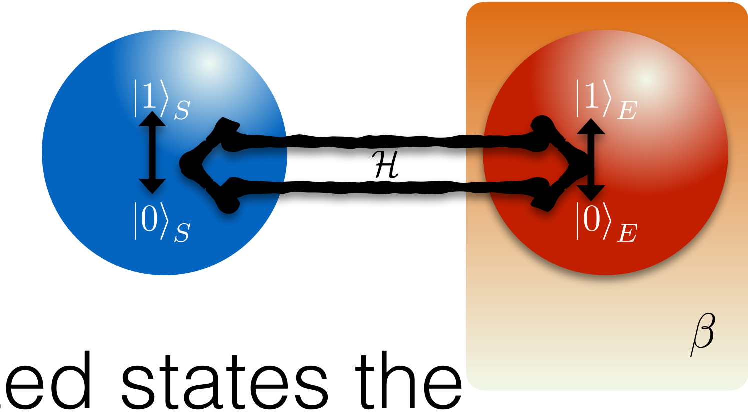
$$\mathcal{H} = J(\sigma_S^x \otimes \sigma_E^x + \sigma_S^y \otimes \sigma_E^y)$$

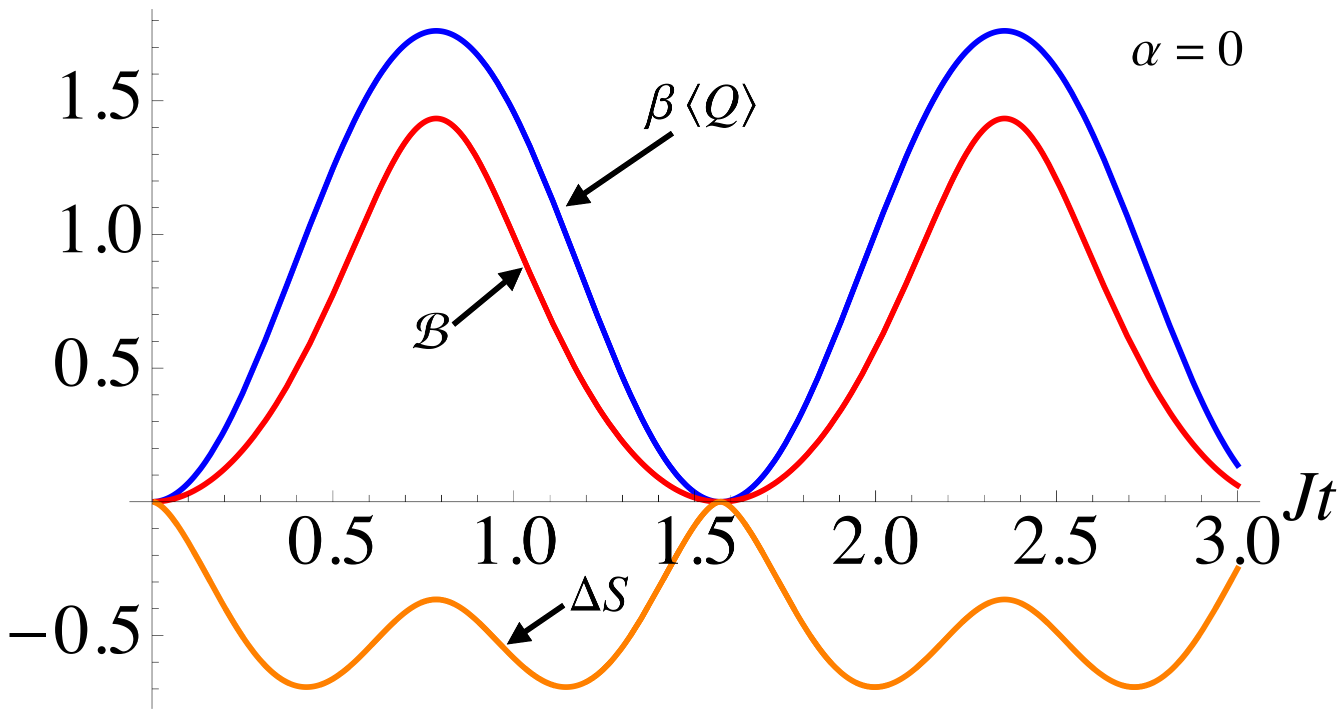
We know that the dissipated heat will necessarily be affected by different initial states - but what role does the quantum nature play?

$$\varrho_S(0) = \begin{pmatrix} 1-\alpha^2 & \delta \\ \delta & \alpha^2 \end{pmatrix} \qquad \delta = w\left(\alpha\sqrt{1-\alpha^2}\right)$$

$$\varrho_S(0) = \begin{pmatrix} 1-\alpha^2 & \delta \\ \delta & \alpha^2 \end{pmatrix} \quad \delta = w\left(\alpha\sqrt{1-\alpha^2}\right)$$

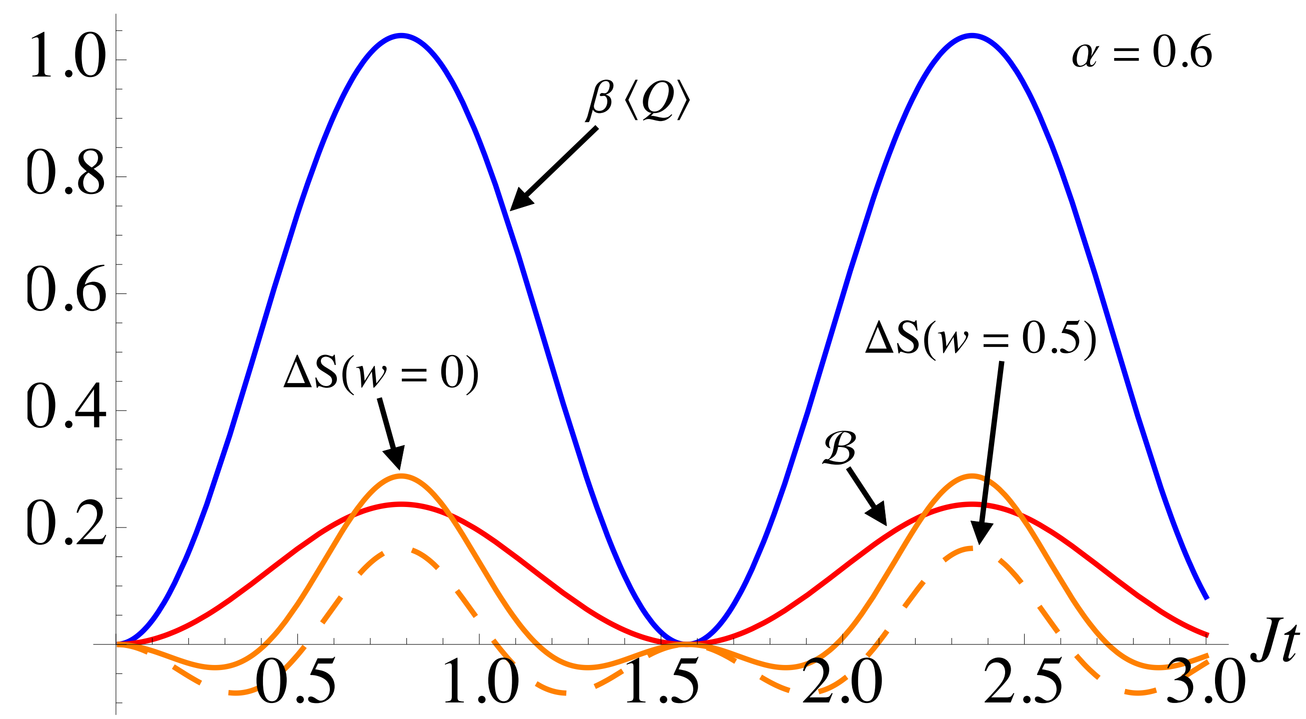
Dynamical Comparison





For pure excited states the thermodynamic bound closely tracks the dissipated heat while the entropic bound appears largely useless

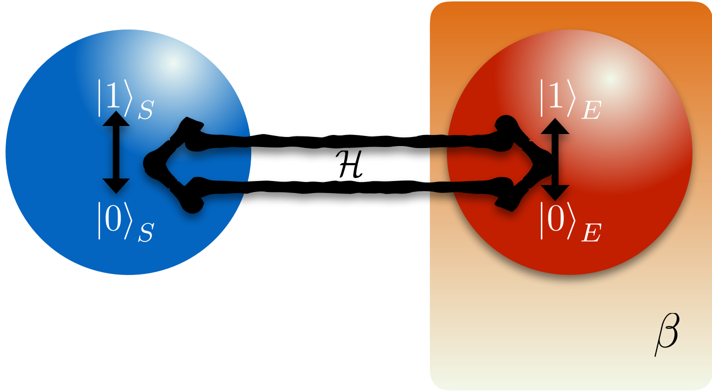
Changing the initial state we see that very different behaviours can be observed and that the entropic bound can be tighter

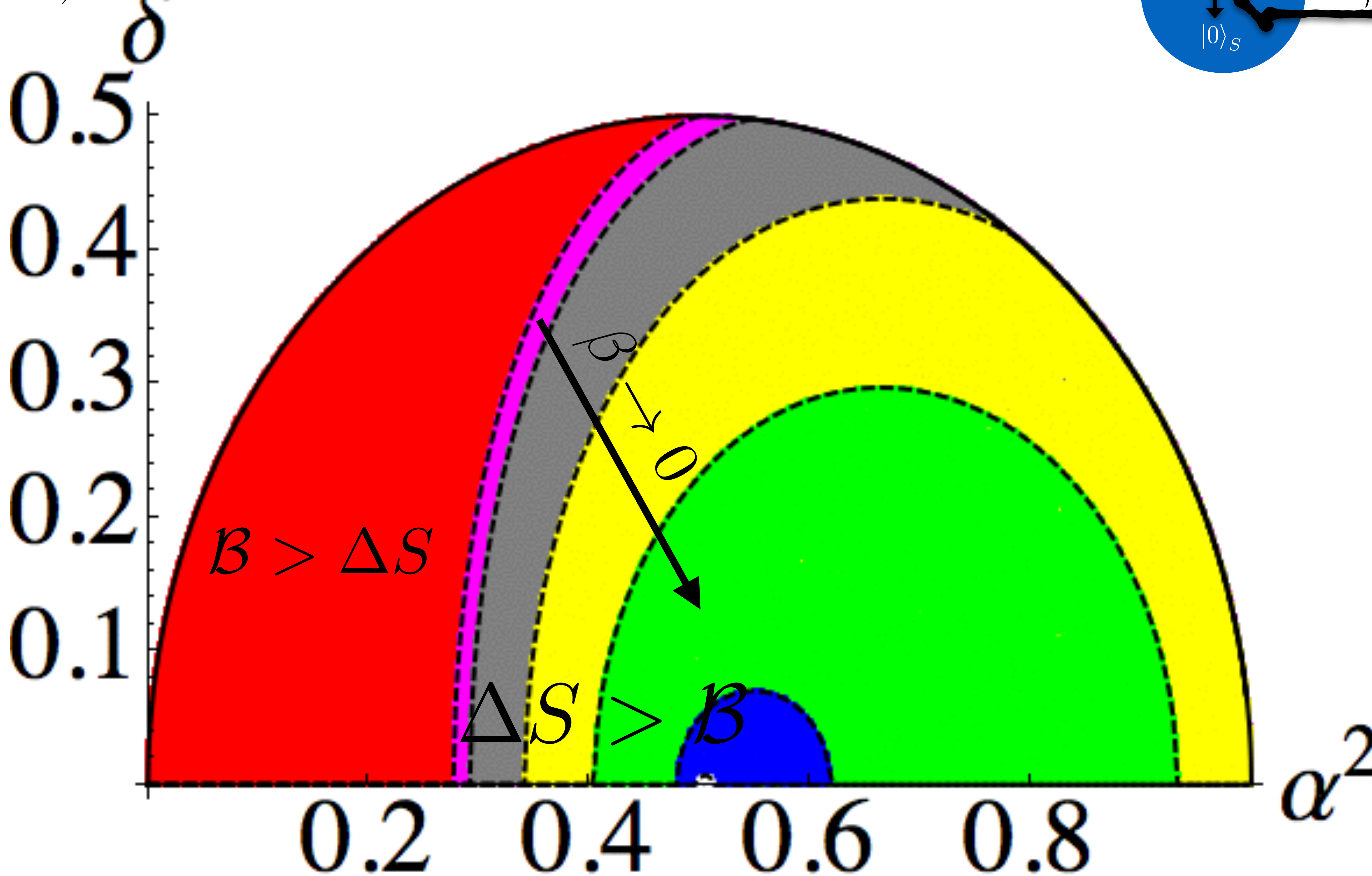


Clearly the initial state plays a crucial role in dictating the performance of these bounds, on the actual dissipated heat, and therefore in information erasure

$\varrho_S(0) = \begin{pmatrix} 1-\alpha^2 & \delta \\ \delta & \alpha^2 \end{pmatrix}$ $\delta = w\left(\alpha\sqrt{1-\alpha^2}\right)$

Which bound is better?



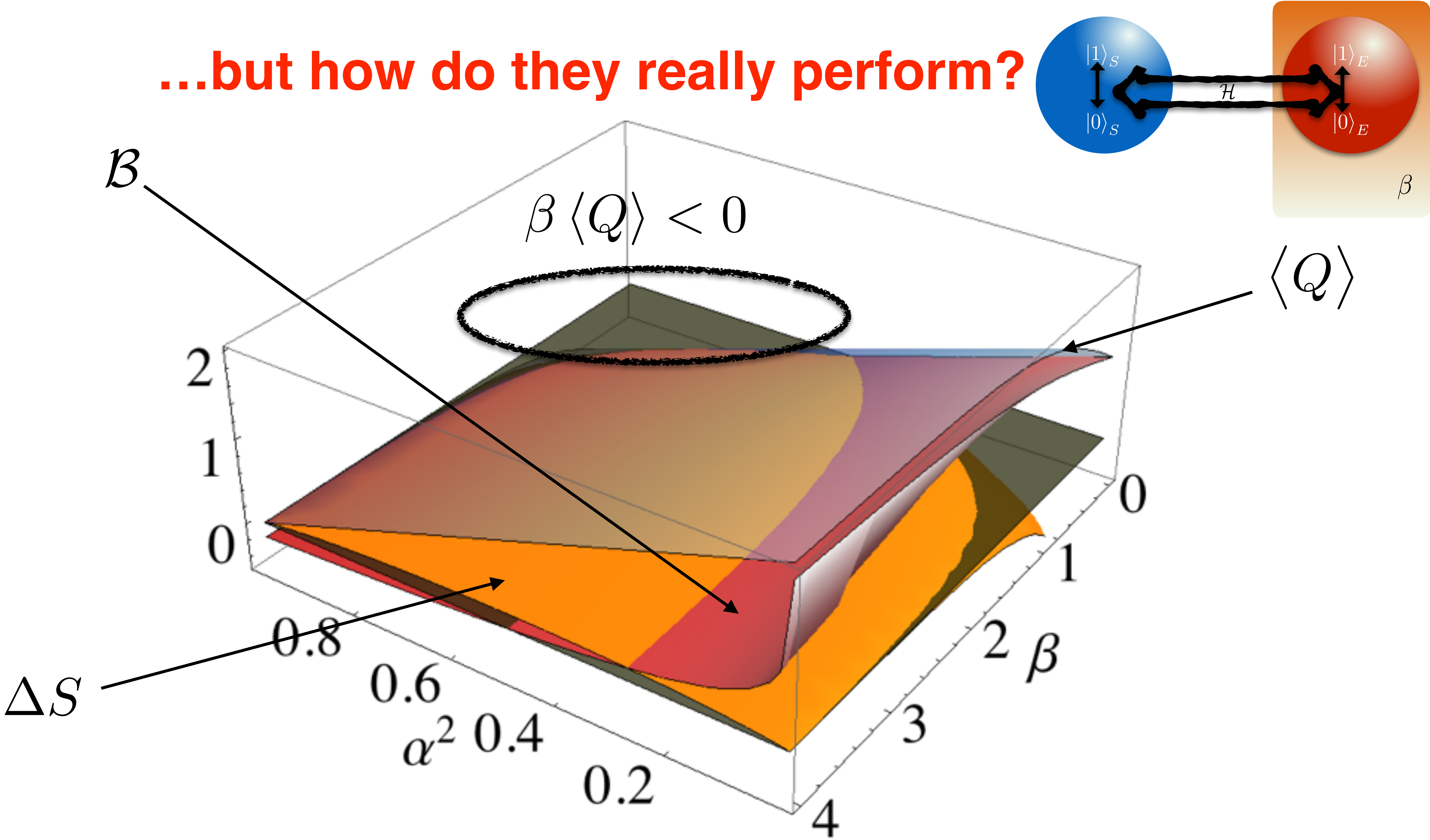


Choosing the point when <*Q*> is maximised we can determine which bound is tighter for the whole initial state parameter space

Higher temperatures tend to favour the thermodynamic bound

We see sharp boundaries between the relative performance of the bounds….but

…but how do they really perform?

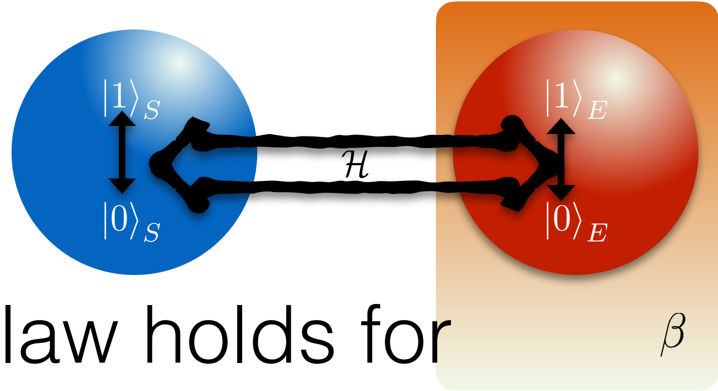


It is important to caveat the previous plots with how tight either bound actually is: Remarkably, both bounds can be negative despite a positive dissipated heat

We also see that the dissipated heat can be negative

$$\varrho_S(0) = \begin{pmatrix} 1-\alpha^2 & \delta \\ \delta & \alpha^2 \end{pmatrix} \quad \delta = w\left(\alpha\sqrt{1-\alpha^2}\right)$$

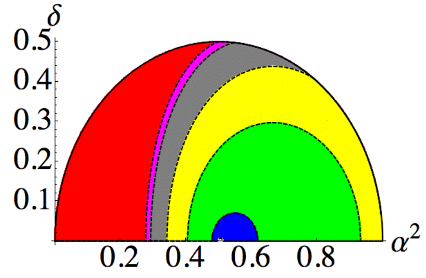
Clausius' Law

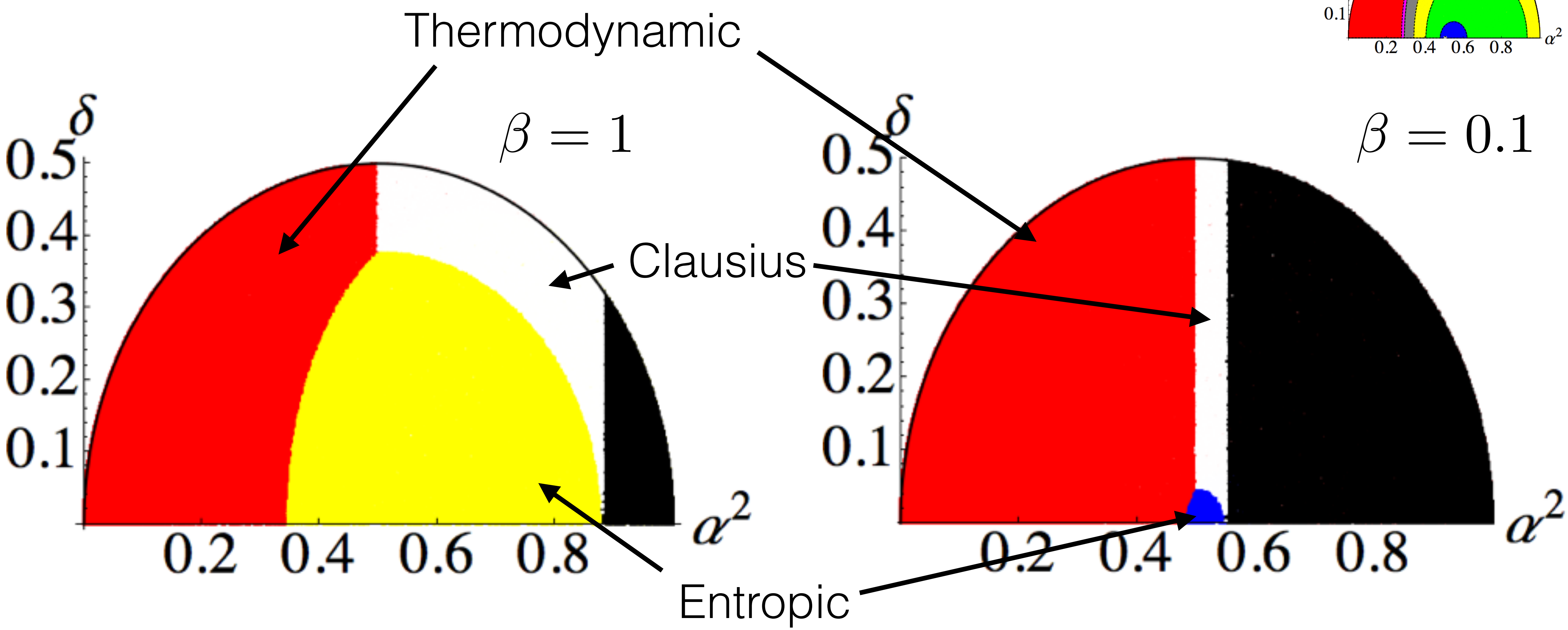


It is easy to find that Clausius' statement of the second law holds for

$$\beta\langle Q\rangle \geq 0 \qquad \alpha^2 \leq \tfrac{1}{2}\left[1+\tanh(\beta)\right]$$

We know both bounds can be negative and therefore fail to accurately capture the behaviour of the heat



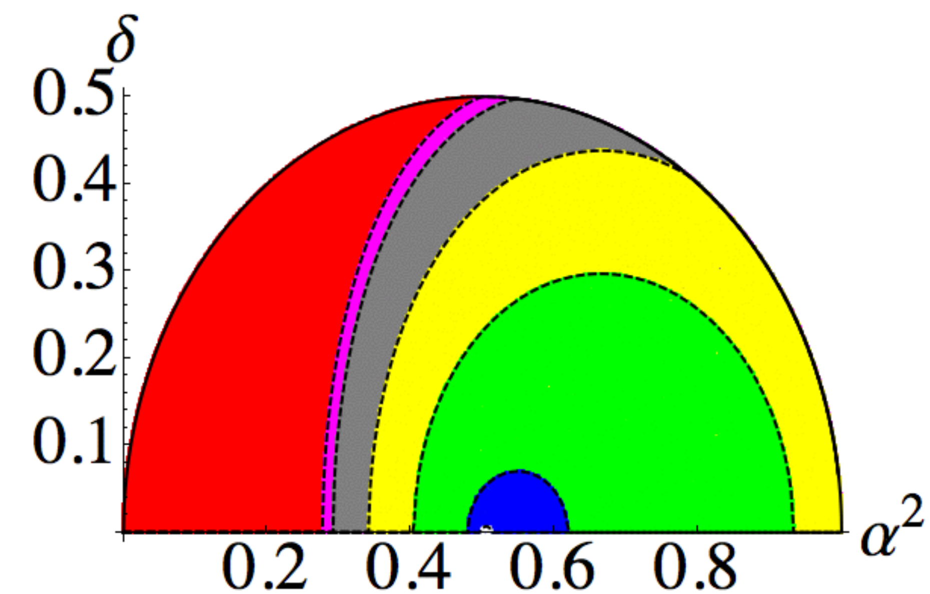


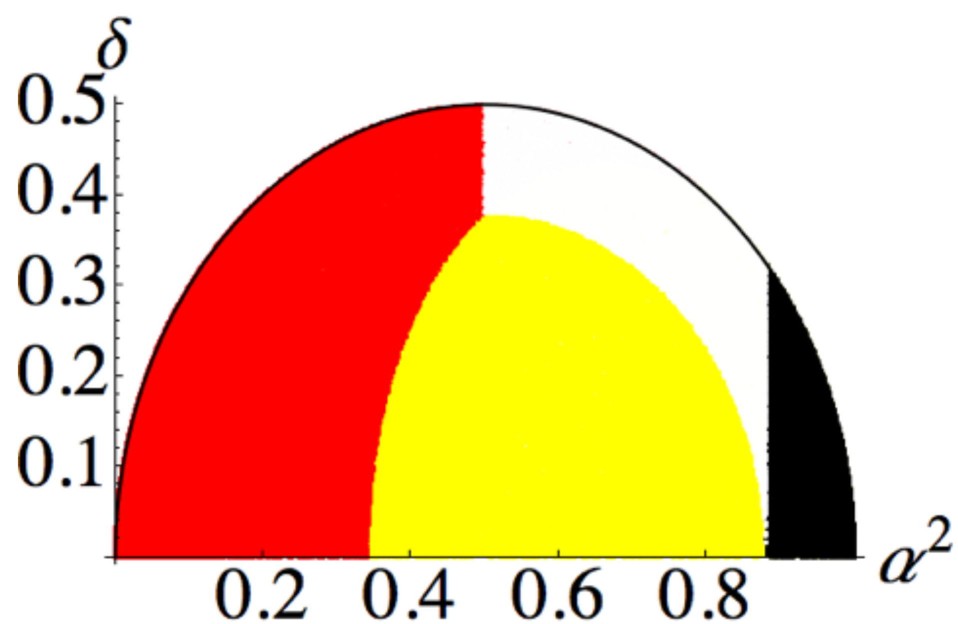
…why do we care again?

We have different formulations of non-equilibrium Landauer type bounds

$$\beta\langle Q\rangle \geq \Delta S = S(\varrho_S(0)) - S(\varrho_S(t)) \qquad \beta\langle Q\rangle \geq \mathcal{B} = -\ln\left(\mathrm{Tr}\left[\sum_i K_i^\dagger \varrho_S(0) K_i\right]\right) \qquad \beta\langle Q\rangle_t \geq -\frac{\beta}{\eta}\Theta(\eta,\beta,t) \equiv \mathcal{B}_Q^\eta(t) \quad (\eta > 0)$$

Sharp crossovers between the bounds appear, and more interestingly quantum coherence appears to play an almost negligible role





Furthermore, the performance of these bounds sometimes fail to provide any meaningful information

In the context of information erasure the role that quantum coherence plays is not fully understood yet. Our results indicate it is not particularly relevant

Many Thanks!

Steve Campbell, Giacomo Guarnieri, Mauro Paternostro, and Bassano Vacchini,
"Non-equilibrium quantum bounds to Landauer's principle: Tightness and effectiveness"
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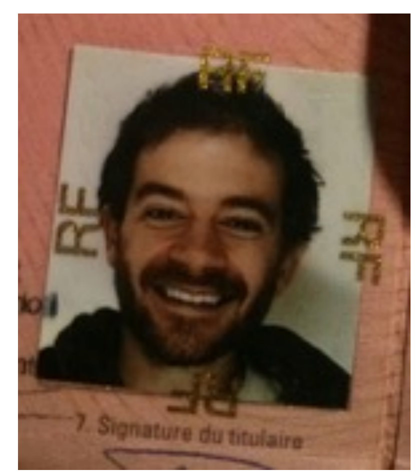

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