Tests of the Pauli Exclusion Principle in bulk matter and their open problems: the case of the VIP experiment

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An experiment like VIP raises many fundamental questions like:

1. what is β ?
2. what is an anomalous X-ray?
3. how many scatterings are there?
4. what is a ”new electron”?
5. how many anomalous X-rays are there?

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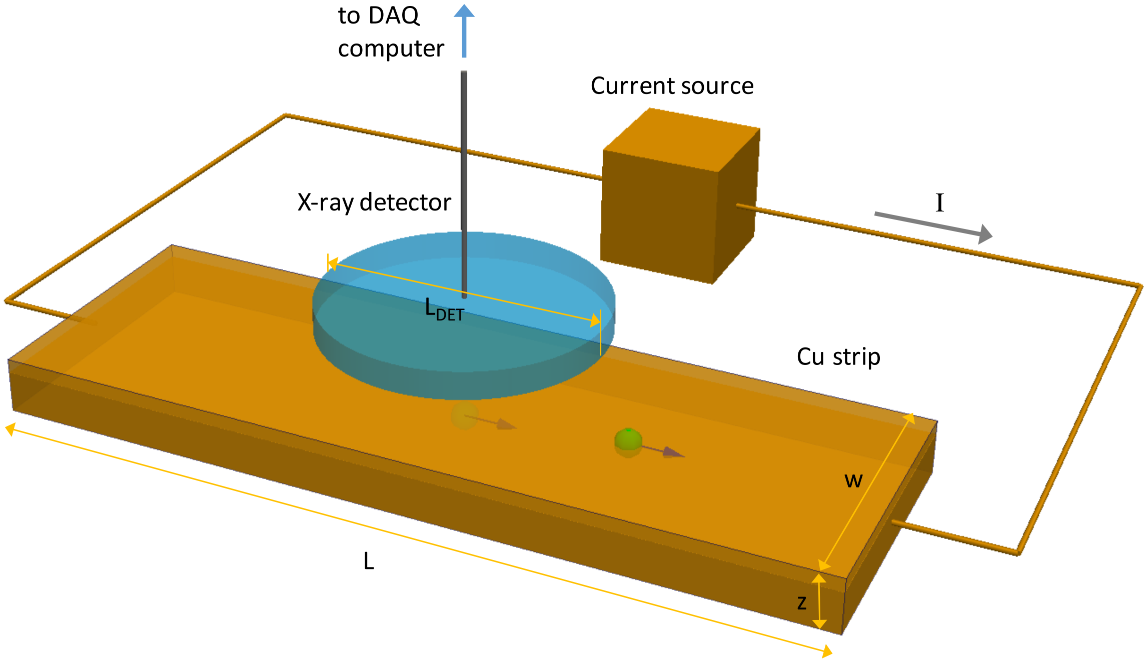
VIP belongs to that class of experiments that aims at detecting a very rare process by replicating it a huge number of times in bulk matter.

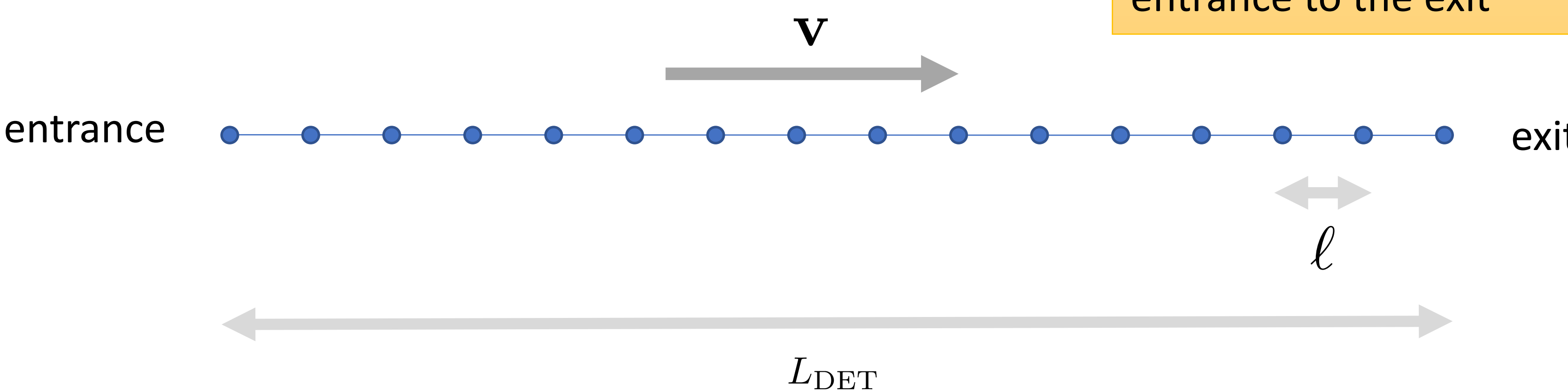
In this case the process is the formation of an anomalous non-Paulian atom with the capture of an electron that has a ”wrong symmetry” with respect to the electrons that belong to that atom.

In order to evaluate the probability of finding such non-Paulian configurations, we need a model for the capture of the “wrong symmetry” electrons.

Ramberg and Snow provided such a model in their original experimental proposal in Phys. Lett. 238 (1990) 438.



The RS model of electron transfer under the detector: a linear series of steps from the entrance to the exit



number of "fresh" electrons injected into copper strip

capture probability in the 2P state

probability of non-Paulian transition

number of electron-atom scatterings

$$N_X \geq \frac{1}{2}\beta^2 N_{\mathrm{new}} \times \frac{1}{10} N_{\mathrm{int}} \times (\text{geometric factor})$$

detector length

$$= \frac{\beta^2 \left(\sum I \Delta t\right) L_{\mathrm{DET}}}{e \ell \rho z \sigma} \times \frac{1}{8}\pi \times \frac{1}{20}$$

mean free path

density

strip thickness

X-ray cross-section

In the original RS paper, RS computed the number of scatterings as

$$L_{\mathrm{DET}}/\ell$$

detector linear size — mean free path of electrons in copper

RS implicitly assumed that electron drift due to the external power supply was much more important than random thermal motion.

But is it really so?

The VIP copper strips have length 9 cm, width 2 cm, thickness 50 µm. This means that the total volume of the strip is

$$V = 9 \times 10^{-8} \text{ m}^3$$

and the number of Cu atoms in the strip is about

$$N_{Cu} \approx 7.6 \times 10^{21}$$

We shall see that the mean free path in Cu is about 40 nm, this means that RS model corresponds to about

$$2 \times 10^6 \text{ steps}$$

which is a very small number compared with the total number of atoms in the strip.

There are several details that must be considered carefully

1. **electron density in the metal target**

it can be obtained from the density of states and from the Fermi-Dirac distribution

$$\frac{dn}{dE} = \rho(E) f_{\mathrm{FD}}(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\sqrt{E}\,\frac{1}{e^{(E-E_F)/k_B T}+1}$$

from which we find the electron density

$$n_{0\mathrm{K}} = \frac{8\sqrt{2}\pi m^{3/2}}{h^3}\left(\frac{2}{3}E_F^{3/2}\right)$$

The Fermi energy of copper is 7 eV, and therefore

$$n_{0\mathrm{K}}(\mathrm{Copper}) \approx 8.46 \times 10^{28}\,\mathrm{el/m^3}$$

2. electron speed in copper

The electron speed is, to a good approximation, determined by the Fermi energy

$$v_F = \sqrt{\frac{2E_F}{m}}$$

and therefore, in copper

$$v_F \approx 1.57 \times 10^6 \ \mathrm{m/s}$$

The corresponding electron wavelength is

$$\lambda = \frac{h}{m v_{\mathrm{F}}} \approx 4.6 \times 10^{-10} \ \mathrm{m}$$

3. *mean free path and mean time between scatterings*

The following equations relate mean free path, collision time and conductivity

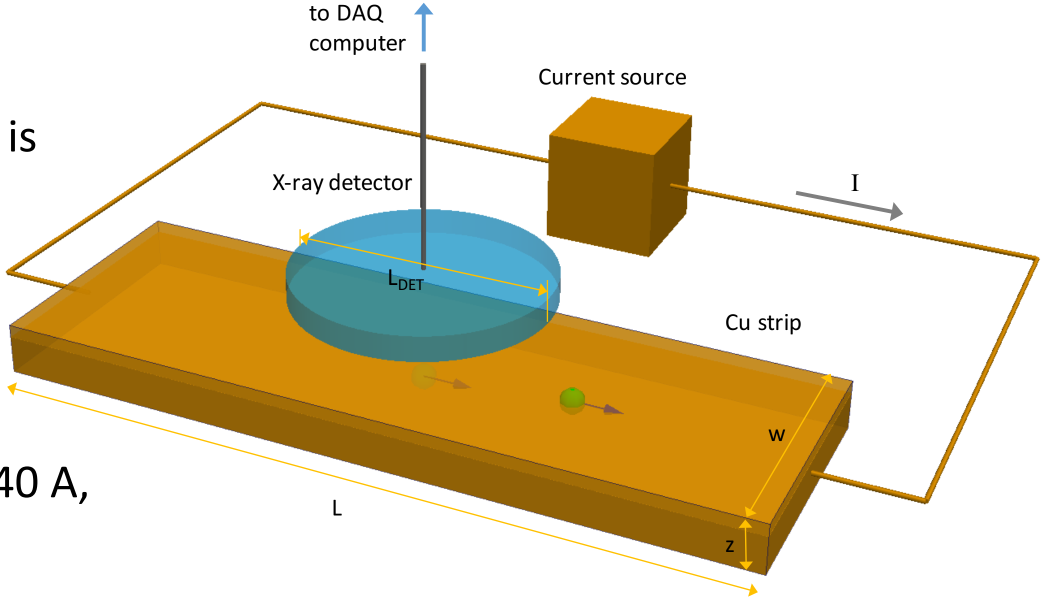
$$\tau = \frac{\ell}{v_F}; \quad \ell = \frac{m v_F}{n e^2} \sigma$$

Therefore, in copper we find

$$\ell \approx 39 \text{ nm}; \quad \tau \approx 2.5 \times 10^{-14} \text{ s}$$

Assuming – as RS did – that electrons follow a straight path, the total transit time under the VIP detector is just 56 ns, and the mean number of scatterings is 2.3 x 10^6.

4. *drift speed and drift time*



It is easy to see that the drift speed is

$$v_d = \frac{1}{nezw}\bar{I}$$

Therefore, in VIP, with a current of 40 A, we find

$$v_d \approx 0.21 \text{ mm/s}$$

This corresponds to a mean transit time of about 420 s.

Since the drift speed is so much smaller than the Fermi velocity, there must be important stochastic effects in electron transport.

5. effective number of scatterings

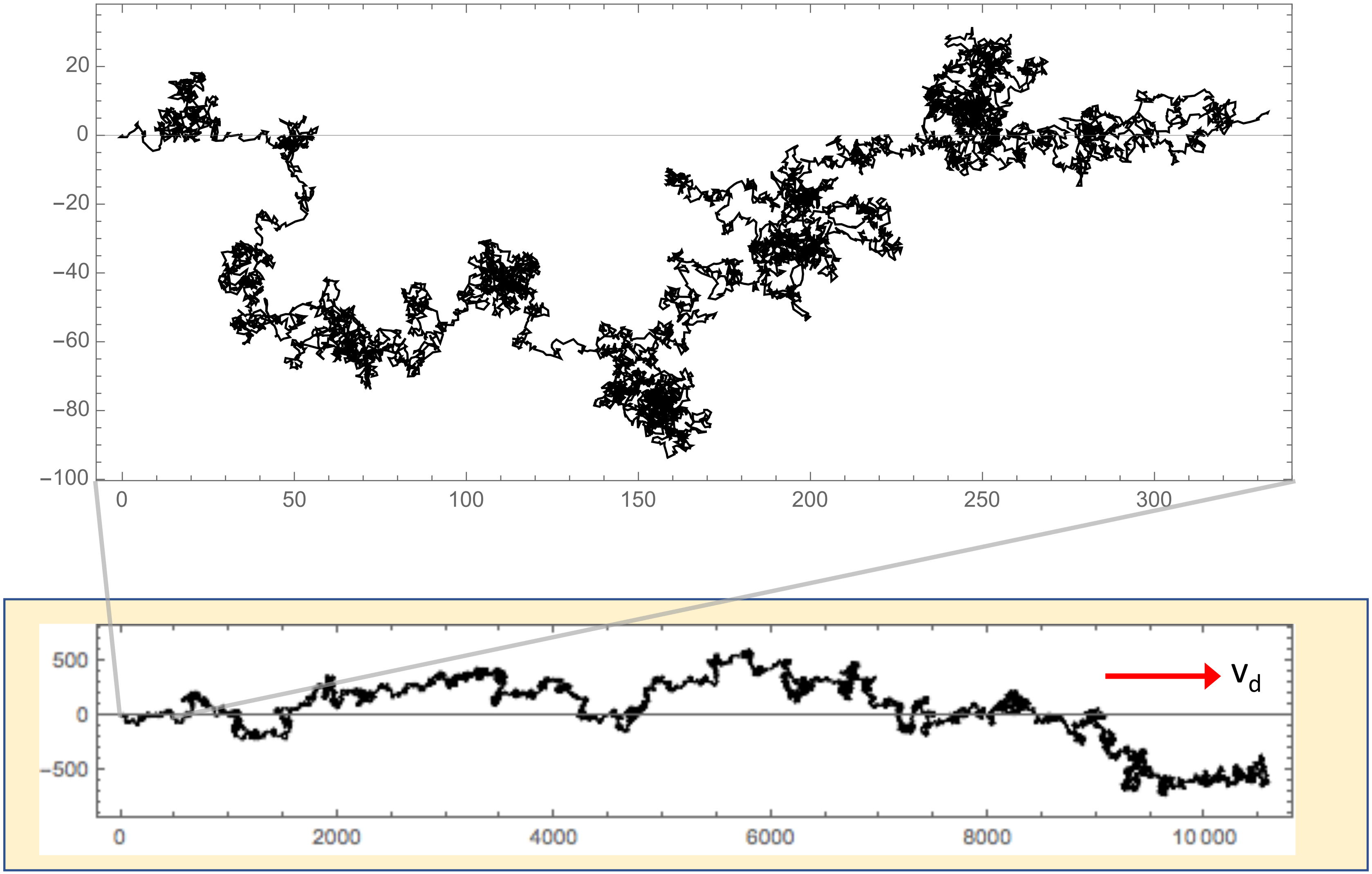
Using the drift speed that we just computed, we find that the transit time is about 420 s in VIP, and that the corresponding number of scatterings is

$$\frac{\Delta t}{\tau} = \frac{n^2 e^3}{m\sigma} \frac{zwL_{\mathrm{DET}}}{\bar{I}}$$

therefore the effective number of scatterings in VIP is about 1.8 x 10^{16}, 10 orders of magnitude above the RS estimate, and the total path length is about 5 x 10^{8} m.

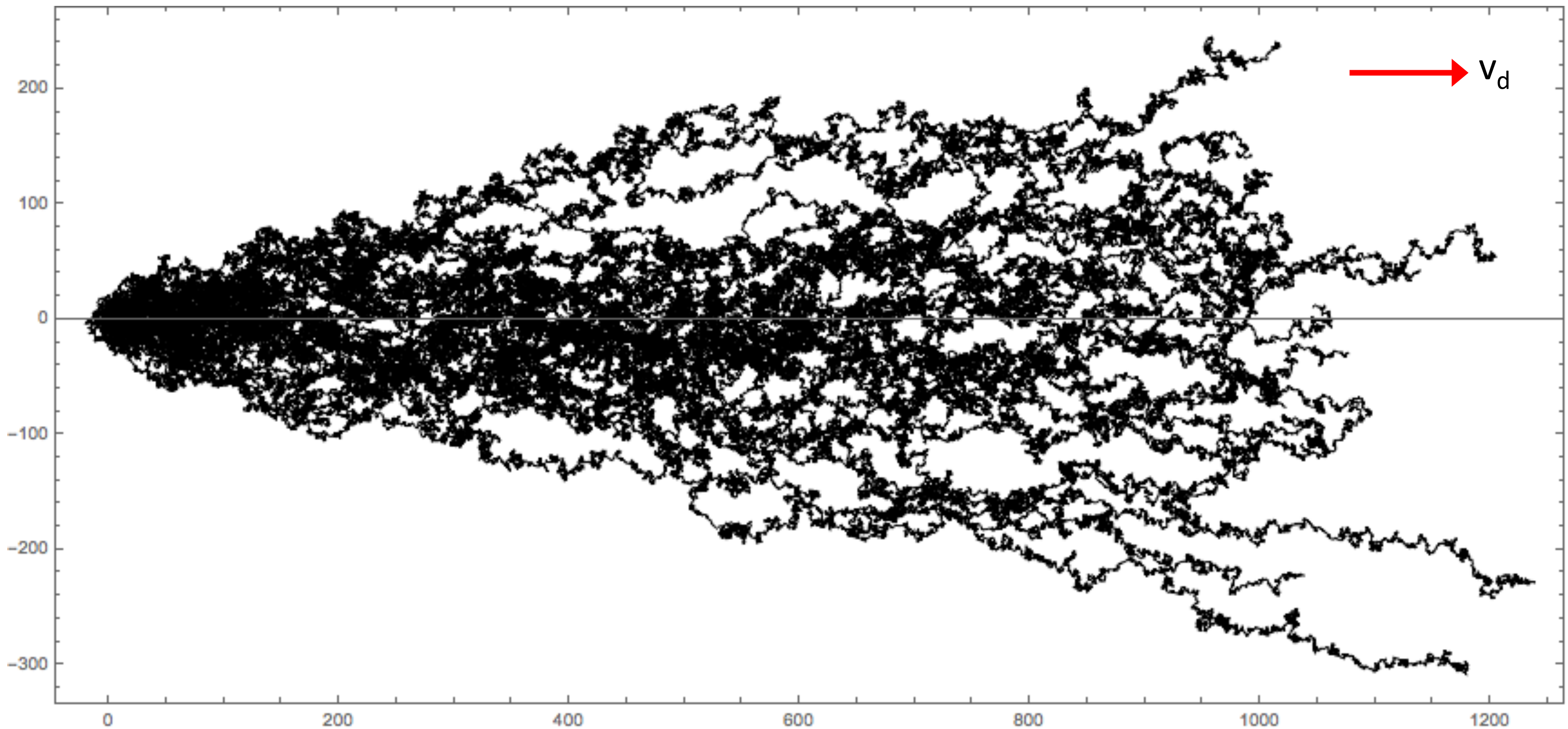
By lowering the current by 5 orders of magnitude, to the level of a few microamps, we increase the number of scatterings to a value close to the number of Cu atoms in a strip. This means that one electron on average scatters once on each atom. At the same time the transit time increases up to about 4 x 10^{7} s, which is of the order of 1 year.

These numbers are very much larger than the RS estimates. This shows that electrons are very good at exploring the copper target.



Path of a single electron along the copper target follows a biased random walk.

A set of drifting random walkers



The diffusion coefficient of electrons in Cu associated with this random motion can be computed from the Einstein formula

$$D = \frac{\mu k_B T}{q} = \frac{\tau k_B T}{m}$$

and in the case of copper we find

$$D_{\mathrm{Cu}} \approx 10^{-4}\ \mathrm{m}^2/\mathrm{s}$$

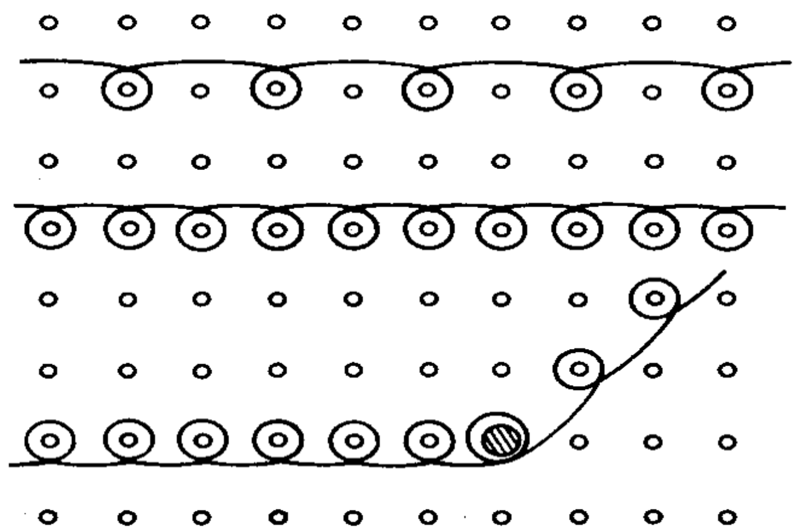
so that in the estimated crossing time of 420 s, the mean square radius covered by diffusion is about 20 cm.

This shows that eventually diffusion cannot be neglected.

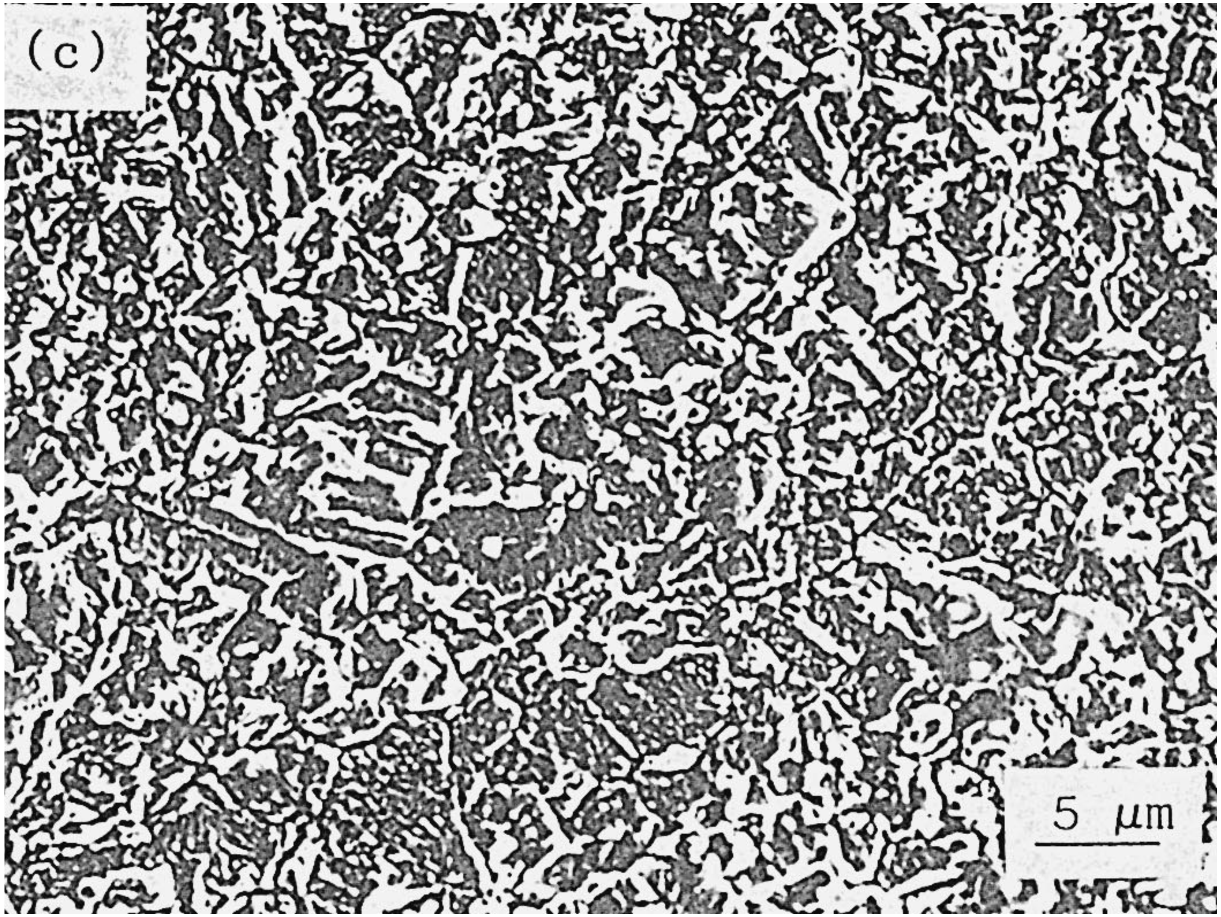
6. *is the computed number of scatterings the correct one?*

The scatterings have been computed in the approximation of the Lorentz-Drude-Sommerfeld model of conduction. If we turn to quantum mechanics, we find that a more refined description of the electrons in metals is provided by the Bloch wavefunctions. But what does a scattering mean in this context?

An intuitive view was provided by Seitz, as shown in the figure. The motion in periodic crystals is mapped onto periodic orbits. The orbits are broken by any imperfection.



So, in this view scatterings are only interruptions in the periodic motion. We should instead resort to close encounters to estimate the probability of capture.



The complex microstructure of annealed Cu, from Sadykov, Barykin and Aslanyan, Wear **225-229** (1999) 649.

7. the number of close encounters

The number of close encounters is obviously related to the number density of atoms in the sample. Taking the standard values of density and atomic weight of copper one finds

$$n_{\mathrm{Cu}} \approx 8.5 \times 10^{22}\ \mathrm{atoms/cm}^3$$

which corresponds to an average atom-atom distance

$$\delta \approx 2 \times 10^{-10}\ \mathrm{m}$$

This distance is traveled in a time

$$\delta / v_{\mathrm{F}} \approx 10^{-16}\ \mathrm{s}$$

and therefore the frequency of close encounters is about 10^{16} Hz.

The electron wavefunction has a significant overlap with the atom during a time which has the same order of magnitude

$$\Delta t \approx 2 \times 10^{-16}\ \mathrm{s}$$

and the radiative capture rate is of the order of the width of the K_α line, i.e. ,

$$\Gamma \approx 2.5\ \mathrm{eV} \rightarrow 6 \times 10^{14}\ \mathrm{Hz}$$

then **the radiative capture probability per close encounter is about 1/10.**

This means that the combined capture rate per electron is

$$\frac{1}{2}\beta^2 \times \left(6 \times 10^{14}\ \mathrm{Hz}\right)$$

8. an apparent paradox

Total traversal time: $T = L_{\mathrm{DET}}/v_d$

Drift velocity: $v_d = I/nqs$

Number of scatterings per electron: T/τ

Number of injected electrons per unit time: I/q

Total number of interactions per unit time:

$$\frac{I}{q} \times T = \frac{I}{q} \times \frac{nqsL_{\mathrm{DET}}}{I} = \frac{nqsL_{\mathrm{DET}}}{q}$$

which does not depend on current !

The solution of the paradox goes as follows:

The calculated number of interactions per unit time is attained only when the injected electrons start to exit the conduction channel, but we have seen that this takes time (420 s for a 40 A current in VIP). Lower currents require a larger amount of time.

This also means that large current can achieve a fast modulation of the potential X-ray signal. This could be an advantage in a possible modulated detection scheme that seeks for a given modulation frequency in the X-ray noise.

Conclusions

- The RS formula must be amended to take into account close encounters instead of scatterings, and the fact the path taken by electrons is an intricate random walk: the results presented here indicate that one should consider an enhancement of 10 (random walk) +2 (close encounters instead of scatterings) = 12 orders of magnitude

- As new electrons drift from the entrance to the exit, the interaction rate rises to a fixed, current-independent value; this value is attained faster for higher currents

- The dependence of the effect with current intensity could be exploited to search for a modulated signal in the large noise background and further increase the experimental sensitivity