Generating hierarchies with Anderson localisation (in theory space)

based on 1710.01354 with N. Craig, which is based on 1211.7149 by I. Rothstein

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The Anderson tight binding model

$$\hat{H} = \sum_i \epsilon_i \hat{a}_i^\dagger \hat{a}_i - (\sum_i t \hat{a}_i^\dagger \hat{a}_{i+1} + \text{h.c.})$$

The ϵ_i are sampled from a uniform distribution over $[-\frac{W}{2}, \frac{W}{2}]$.

$$H_{ij} = \begin{pmatrix} \epsilon_1 & -t & 0 & \cdots & 0 \\ -t & \epsilon_2 & -t & \cdots & 0 \\ 0 & -t & \epsilon_3 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -t \\ 0 & 0 & 0 & -t & \epsilon_N \end{pmatrix}$$

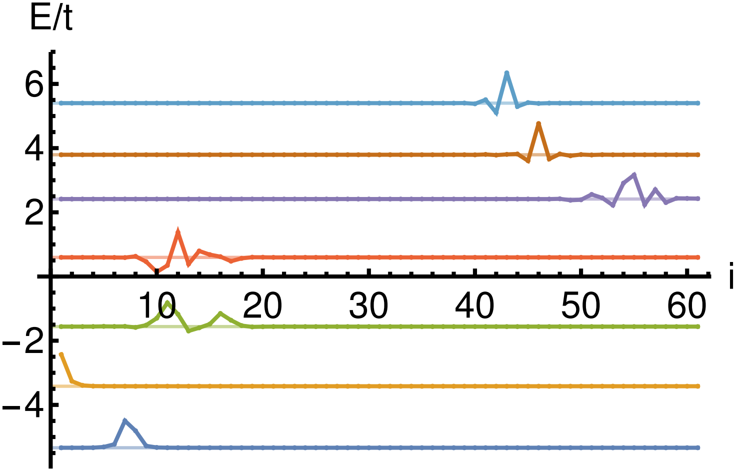
All eigenstates are localised. If $H_{ij} v_j^n = \lambda^n v_i^n$

$$|v_i^n| \sim \exp\left(-\frac{|i - i_0^n|}{L_n}\right)$$

$W \gg t$: the strong localisation limit

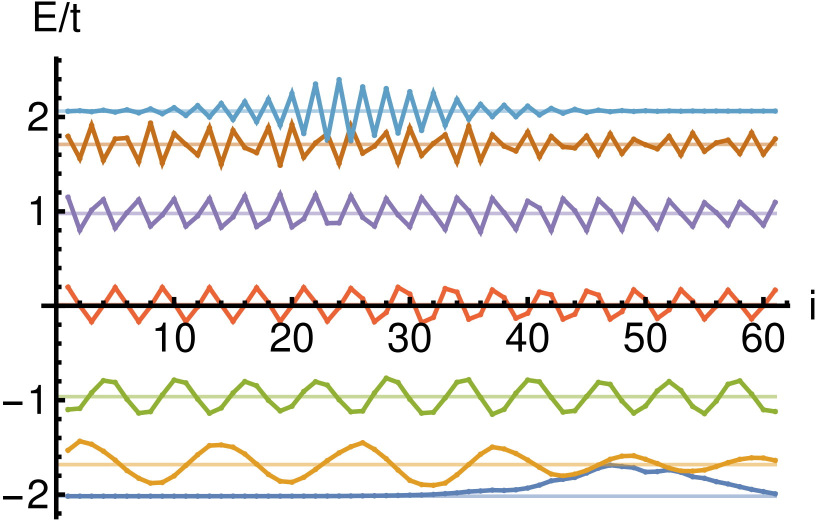
$(N = 61,\ W = 10,\ t = 1)$

$$|\psi\rangle = |i\rangle + \sum_{j \neq i} \frac{\langle j|\, V\, |i\rangle}{E_i - E_j} |j\rangle + \ldots \sim |i\rangle + \frac{t}{W} |i \pm 1\rangle + \frac{t^2}{W^2} |i \pm 2\rangle + \ldots$$



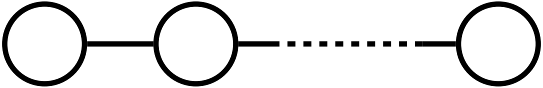
$W \ll t$: the weak localisation limit

$(N = 61,\, W = 0.4,\, t = 1)$



(Many other matrices exhibit this behaviour too!)

Example: Clockwork has a localised zero mode



(A corresponding UV lagrangian comprises fields $\Phi_i = f \exp(\frac{i\pi_i}{f\sqrt{2}})$,

$$\mathcal{L} = \sum_{i=1}^{N} |\partial\Phi_i|^2 - (\sum_{i=1}^{N-1} f^{q-2} m^2 \Phi_i^\dagger \Phi_{i+1}^q + \text{h.c.}) - \lambda \sum_{i=1}^{N} (|\Phi_i|^2 - f^2)^2 \text{ .)}$$

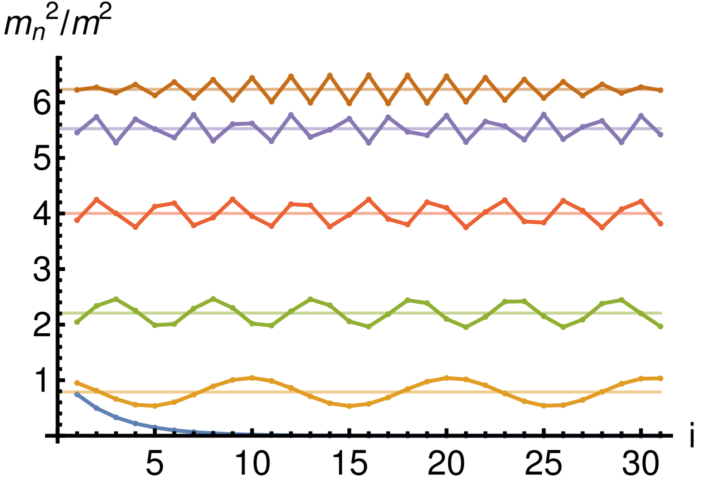
$$\mathcal{L}_{\text{CW}} = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 - \frac{1}{2}\sum_{i=1}^{N-1} m^2(\pi_i - q\pi_{i+1})^2 + \text{O}(\pi^4) + \frac{1}{f}\pi_N G_N \tilde{G}_N$$

$$m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & 0 \\ -q & 1+q^2 & -q & \cdots & 0 \\ 0 & -q & 1+q^2 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & -q \\ 0 & 0 & 0 & -q & 1+q^2 \end{pmatrix} \begin{pmatrix} q^{N-1} \\ q^{N-2} \\ q^{N-3} \\ \vdots \\ 1 \end{pmatrix} = \vec{0}$$

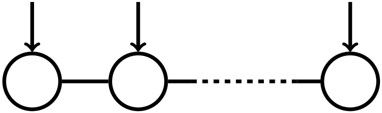
. . . and a band of 'extended' massive states

$(N = 31, q = 1.5)$

$$\mathcal{L}_\pi = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 - \frac{1}{2}\sum_{i=1}^{N-1} m^2(\pi_i - q\pi_{i+1})^2.$$



A toy scalar model



(Coset rep lagrangian $\Phi_i = f\exp(\frac{i\pi_i}{f\sqrt{2}})$:

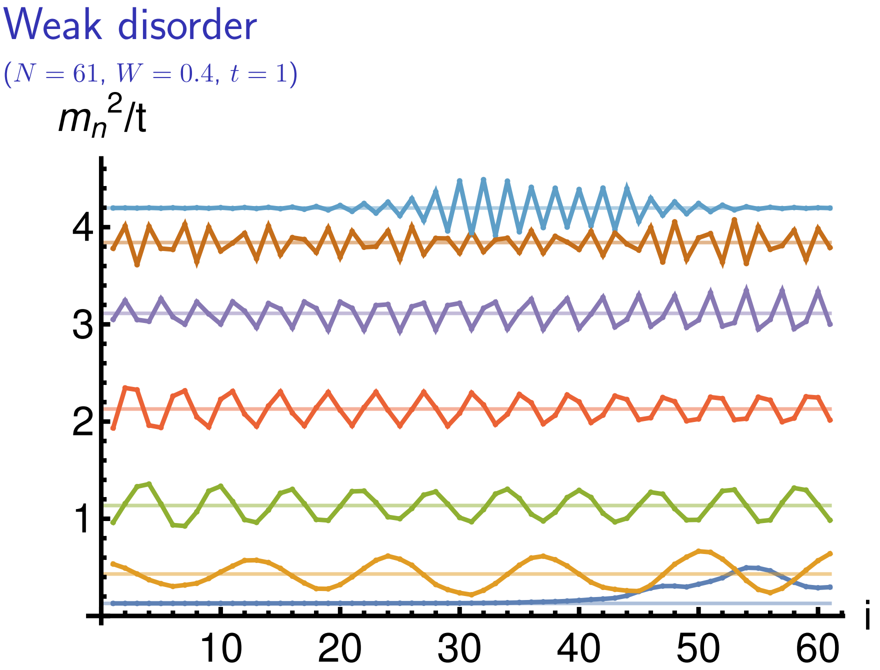
$$\mathcal{L} = \sum \partial\Phi_i^\dagger \partial\Phi_i - (\sum \frac{1}{4}\epsilon_i \Phi_i \Phi_i + \sum t\Phi_i^\dagger \Phi_{i+1} + \text{h.c.}) - V(\Phi_i)\ .)$$

$$\mathcal{L}_\pi = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 - \frac{1}{2}\sum_{i=1}^{N}\epsilon_i\pi_i^2 - \frac{1}{2}\sum_{i=1}^{N-1} t(\pi_i - \pi_{i+1})^2 + \frac{1}{f}\pi_N G_N \tilde{G}_N$$

Sample ϵ_i uniformly from $[0, W]$. If $\epsilon_i > 0, \forall i$, all mass squareds are positive.

Save for edge effects, the mass matrix is a constant shift away from the Anderson tight binding model.

$$\begin{pmatrix} t+\epsilon_1 & -t & 0 & \cdots \\ -t & 2t+\epsilon_2 & -t & \cdots \\ 0 & -t & 2t+\epsilon_3 & \ddots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix}$$

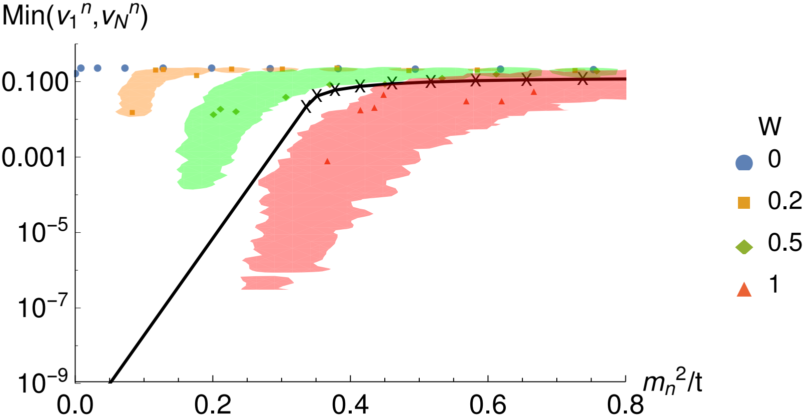


Weak disorder
$(N = 61,\ W = 0.4,\ t = 1)$
m_n^2/t
4
3
2
1
10
20
30
40
50
60
i

End site coupling

$N = 35,\ t = 1$

$$\mathcal{L}_\pi = \frac{1}{2}\sum_{i=1}^{N}(\partial\pi_i)^2 - \frac{1}{2}\sum_{i=1}^{N}\epsilon_i\pi_i^2 - \frac{1}{2}\sum_{i=1}^{N-1}t(\pi_i - \pi_{i+1})^2.$$



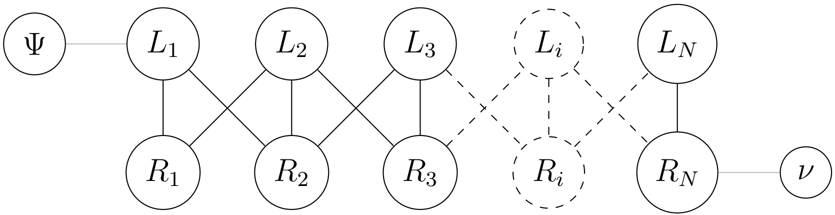
(Black line shows clockwork mass matrix, $q = 2$, $m^2 = t/3$.)

An ‘Anderson see-saw’ mechanism

It’s similar to the ‘clockwork WIMP’ of Hambye, Teresi, & Tytgat 1612.06411

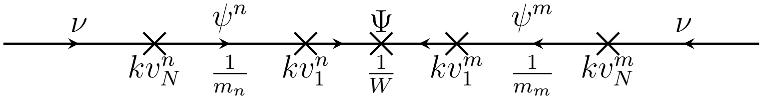
Comprises an SM neutrino ν, N left- and right-handed Weyl fermions L_i and R_i with Dirac mass matrix, and a right-handed Majorana fermion Ψ

$$\begin{aligned}\mathcal{L}_{\text{mass}} &= -k\bar{L_1}\Psi - \bar{L_i}H_{ij}R_j - k\bar{\nu}R_N - W\Psi\Psi + \text{h.c.}\\ &= -\sum_n \left(kv_1^n\bar{\psi_L^n}\Psi - m_n\bar{\psi_L^n}\psi_R^n - kv_N^n\bar{\nu}\psi_R^n\right) - W\Psi\Psi + \text{h.c.}\,.\end{aligned}$$



The lattice ‘insulates’ ν from Ψ

$$
\begin{aligned}
\mathcal{L}_{\text{mass}} &= -k\bar{L_1}\Psi - \bar{L}_i H_{ij} R_j - k\bar{\nu} R_N - W\Psi\Psi + \text{h.c.} \\
&= -\sum_n \left(k v_1^n \bar{\psi_L^n}\Psi - m_n \bar{\psi_L^n}\psi_R^n - k v_N^n \bar{\nu}\psi_R^n \right) - W\Psi\Psi + \text{h.c.}\,.
\end{aligned}
$$

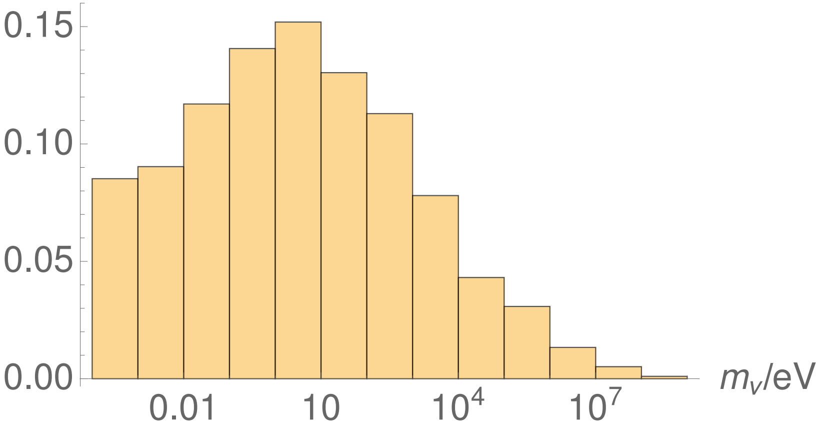


$$
m_\nu \sim \left(\sum_{n=1}^{N} \frac{k v_N^n k v_1^n}{m_n} \right)^2 \frac{1}{W} \sim \left(\sum_{n=1}^{N} \frac{k^2}{m_n} e^{-\frac{N}{L_n}} \right)^2 \frac{1}{W},
$$

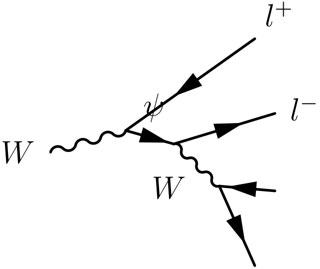
where L_n is the localisation length of the nth mode.

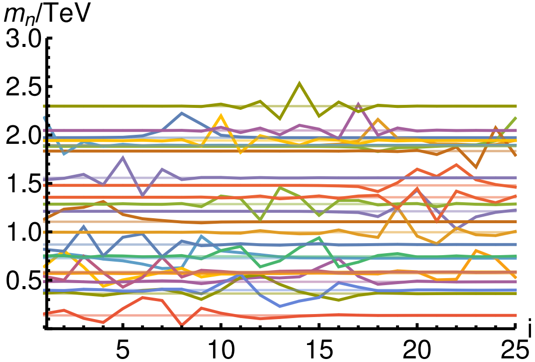
O(TeV) fundamental parameters $\to$ O(eV) ν mass

Set $N = 25$, $W = 2$ TeV, $t = \frac{3}{10}W$, $k = \frac{1}{10}W$ and draw ϵ_i uniformly from $[-W, W]$.



L conserving signatures from TeV mass states





Summary

Band matrices with enough of their entries drawn at random from a smooth distribution will typically have localised eigenvectors.

Shoehorned into an N-site model, this can be used quite generically to generate small numbers/couplings, and may have BSM applications.

As a proof of concept, I described a neutrino mass model. See `1710.01354` a model of quark masses.