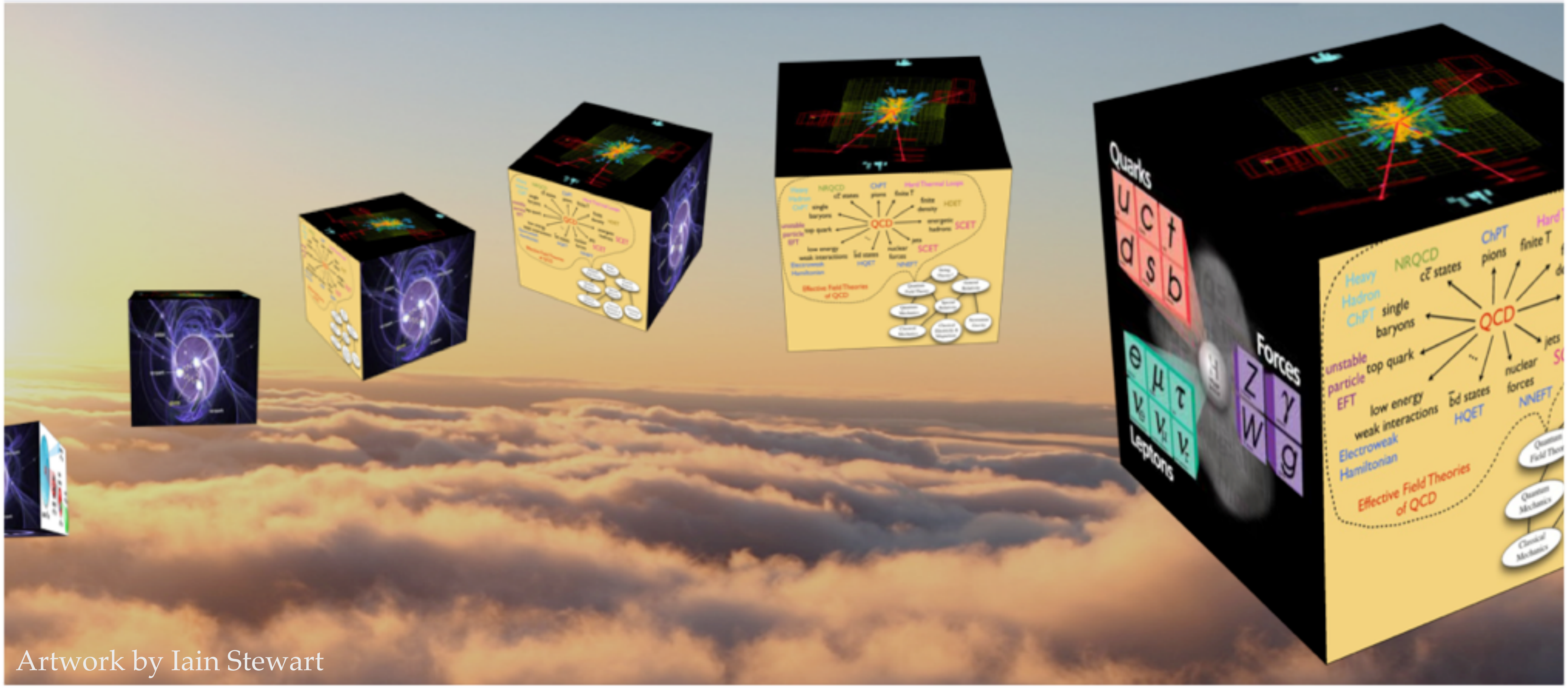

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Effective Field Theory after a New Physics Discovery

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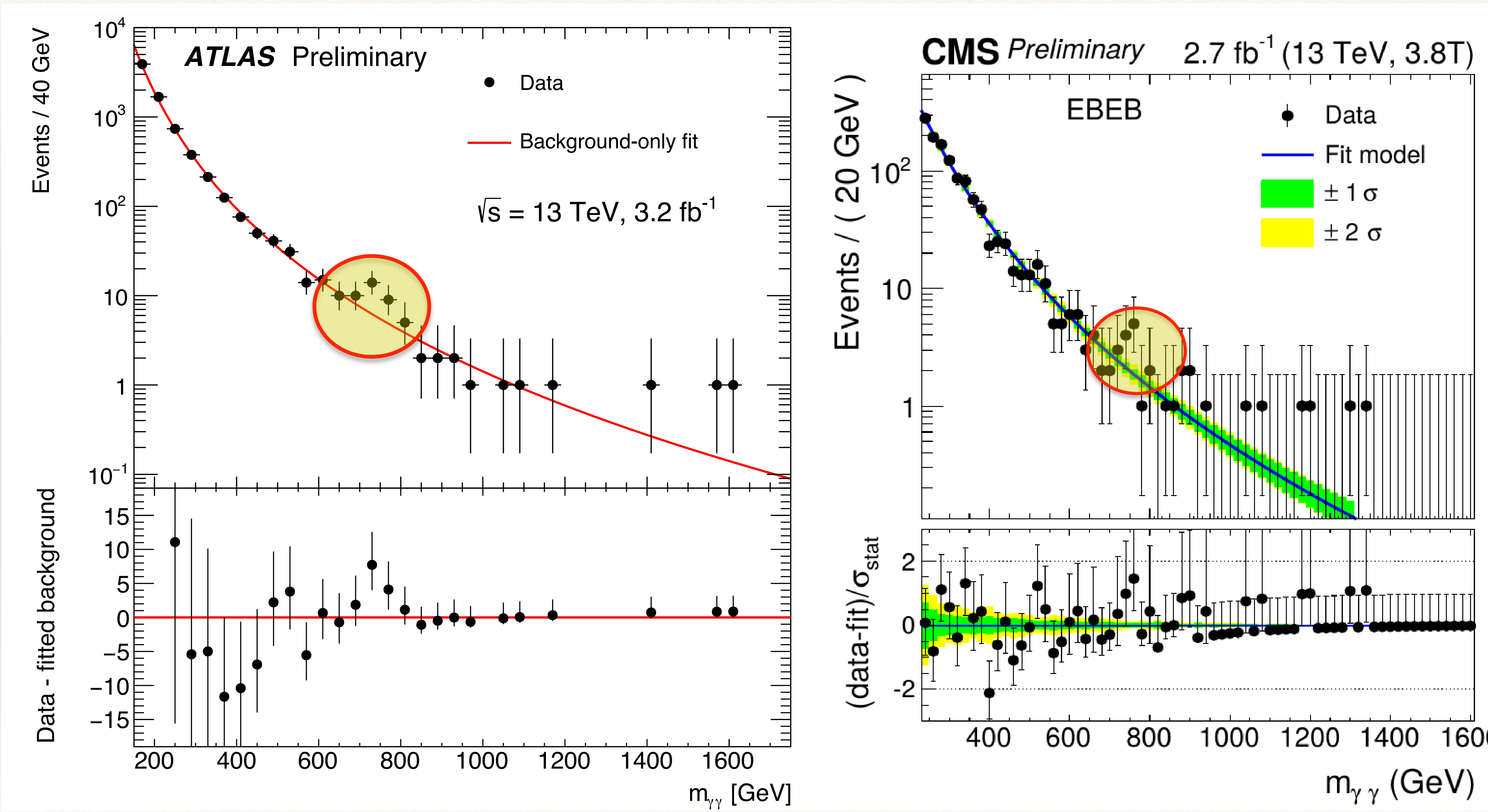
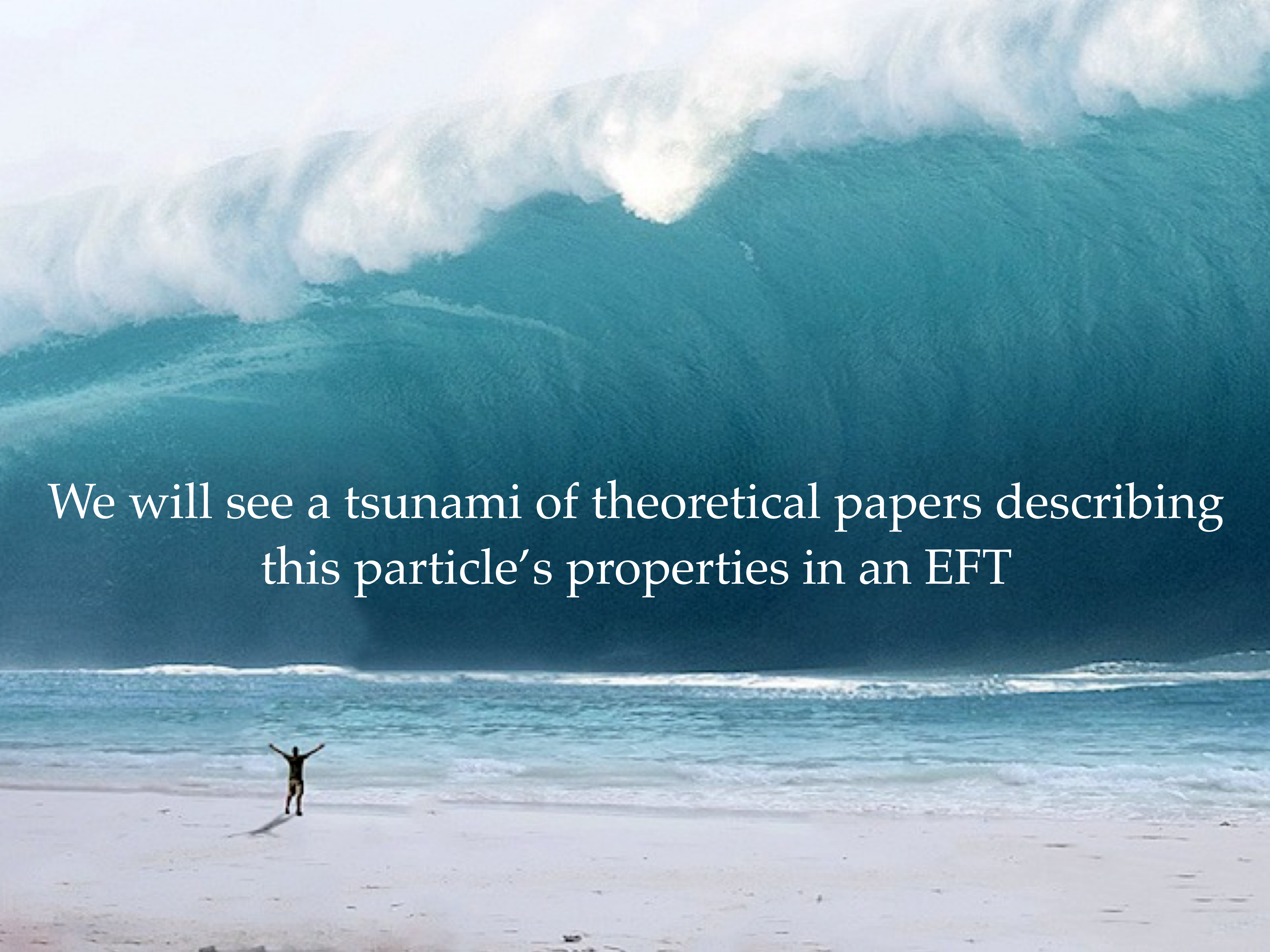
(based on 1806.01278 with Stefan Alte and Matthias König)

Imagine we feel like in paradise …

… after the theory discovery of a new particle



What will happen?


We will see a tsunami of theoretical papers describing
this particle’s properties in an EFT

For the example of a spin-0 singlet S

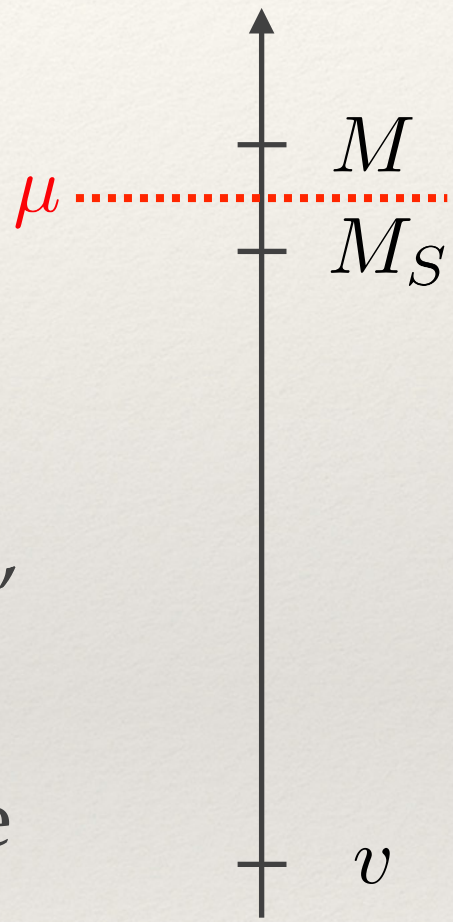
- Most general effective Lagrangian at D=5:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = &-M\lambda_1\, S\,\phi^\dagger\phi - \frac{\lambda_2}{2}\, S^2\,\phi^\dagger\phi - \frac{\lambda_3}{6M}\, S^3\,\phi^\dagger\phi - \frac{\lambda_4}{M}\, S\left(\phi^\dagger\phi\right)^2 \\ &+ \frac{c_{GG}}{M}\,\frac{\alpha_s}{4\pi}\, S\, G^a_{\mu\nu}G^{\mu\nu,a} + \frac{c_{WW}}{M}\,\frac{\alpha}{4\pi s_w^2}\, S\, W^a_{\mu\nu}W^{\mu\nu,a} + \frac{c_{BB}}{M}\,\frac{\alpha}{4\pi c_w^2}\, S B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{\tilde{c}_{GG}}{M}\,\frac{\alpha_s}{4\pi}\, S\, G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + \frac{\tilde{c}_{WW}}{M}\,\frac{\alpha}{4\pi s_w^2}\, S\, W^a_{\mu\nu}\tilde{W}^{\mu\nu,a} + \frac{\tilde{c}_{BB}}{M}\,\frac{\alpha}{4\pi c_w^2}\, S B_{\mu\nu}\tilde{B}^{\mu\nu} \\ &- \frac{1}{M}\left(S\,\bar{Q}_L\,\hat{\mathbf{Y}}_u\,\tilde{\phi}\,u_R + S\,\bar{Q}_L\,\hat{\mathbf{Y}}_d\,\phi\,d_R + S\,\bar{L}_L\,\hat{\mathbf{Y}}_e\,\phi\,e_R + \text{h.c.}\right) + \mathcal{O}\left(\frac{1}{M^2}\right)\end{aligned}$$

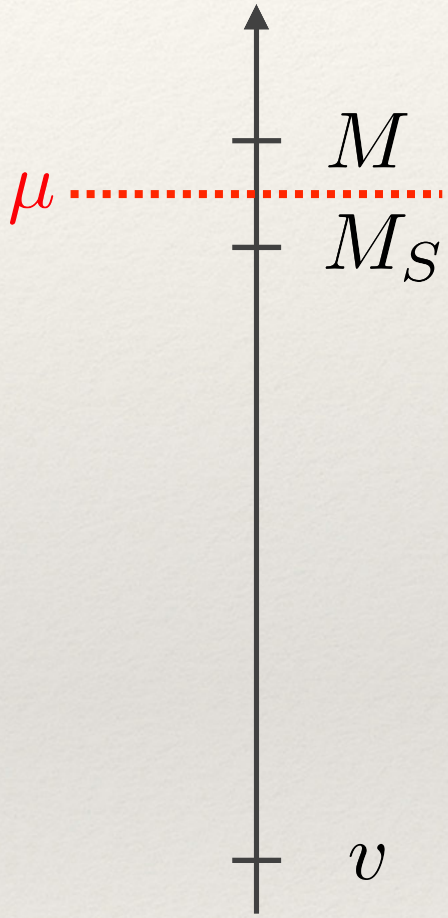
- Can describe the production and decay rates of S in terms of a hand full of parameters

What's wrong about it?

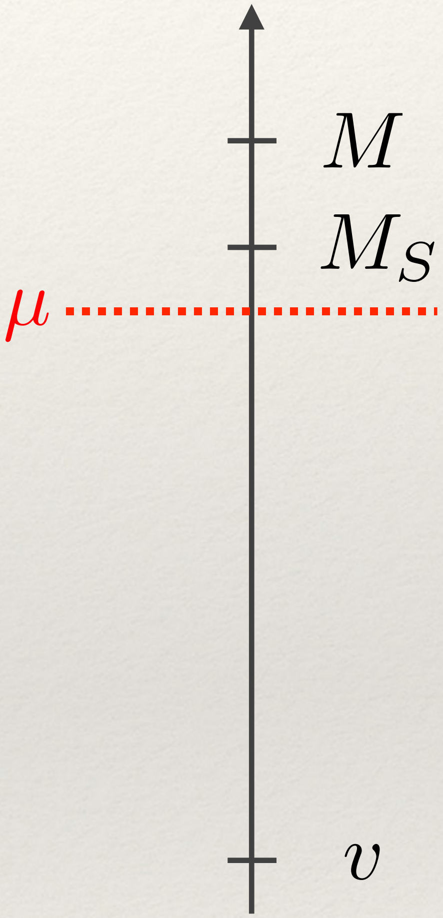
- An EFT is used to separate the physics on different (length or mass/energy) scales
- Goal: separate new-physics scale *M* from the electroweak scale v
- But the EFT **contains the heavy resonance** ***S*** **as a field**, so it's scales are v (masses of SM particles) and M_S
- Widely different scales $M_S \gg v$ are not separated by the EFT (no control over large logs)
- For $M \sim M_S$, an **infinite tower** of higher-dimensional operators gives rise to unsuppressed contributions



Is there a way out?



Is there a way out?



Soft-collinear effective theory (SCET)

- SCET is the proper effective field theory to describe the properties of highly energetic light particles produced in the decay of a heavy particle:

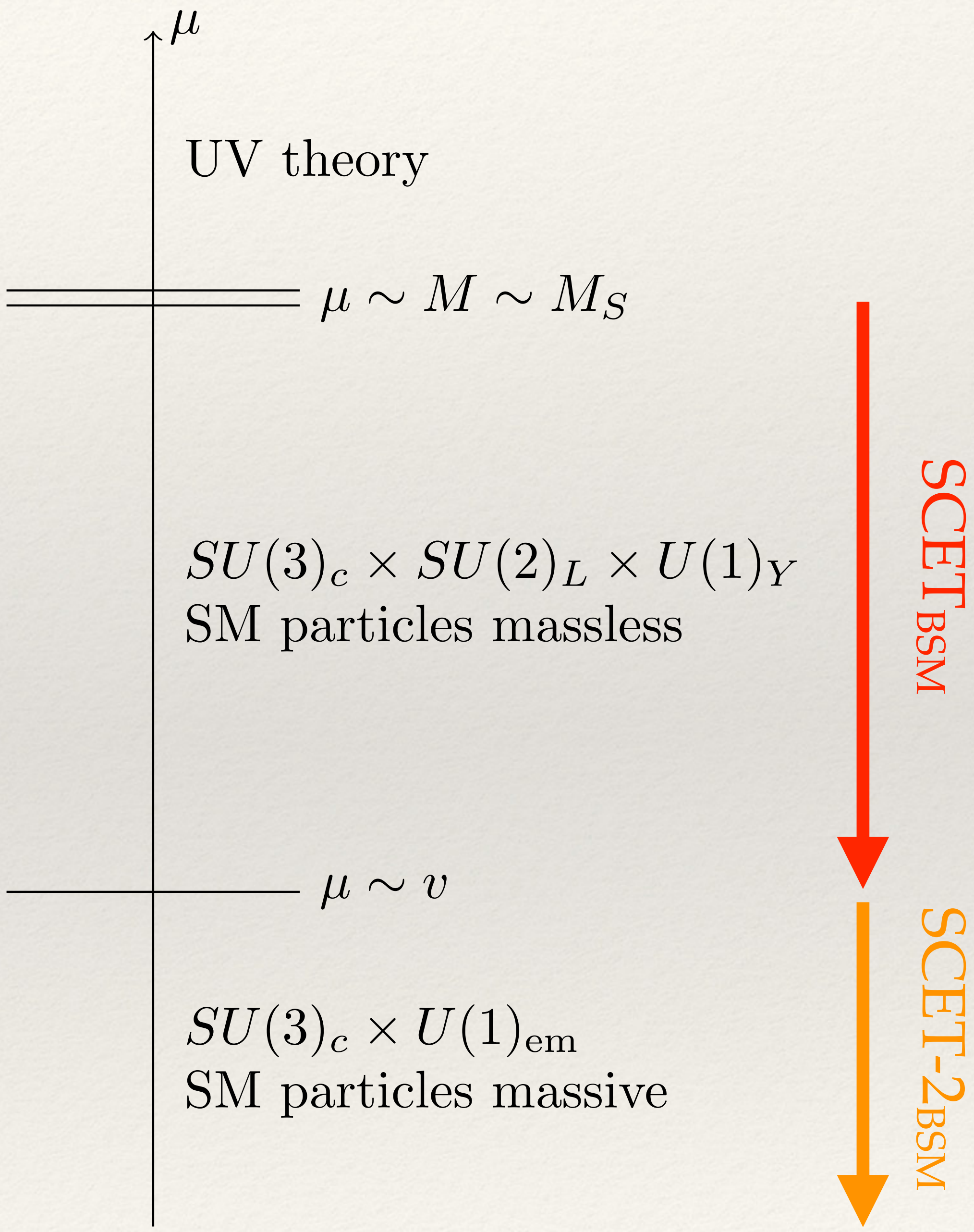
  Bauer, Fleming, Pirjol, Stewart 2001;
  Bauer, Pirjol, Stewart 2002
  Beneke, Chapovsky, Diehl, Feldmann 2002

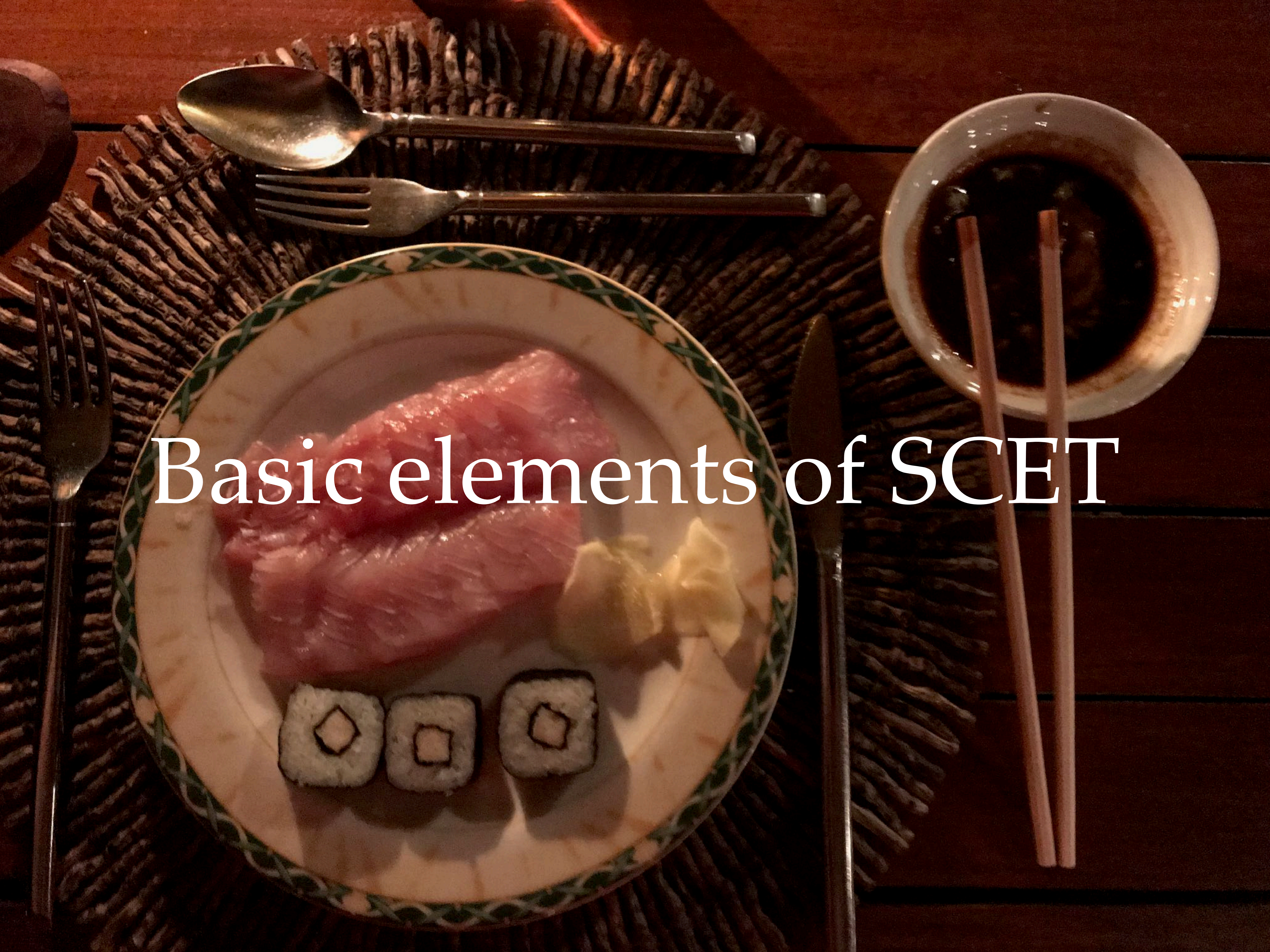
  - systematic scale separation between M_S and v, including resummation of large logarithms
  - case where $M \sim M_S$ can be dealt with naturally
- Effective Lagrangian is process dependent: will consider 2-body decays of a heavy, spin-0 resonance S which is a singlet under the SM

Outline of the construction

- At the new-physics scale $M \sim M_S$ the full theory is matched onto SCET$_{\text{BSM}}$, giving rise to Wilson coefficient functions
- The SCET operators are evolved from M_S down to the scale v using RGEs and anomalous dimensions derived in the effective theory
- At the scale v a matching onto a theory with massive SM fields is performed
- If desired, the operators can be evolved down further when very light SM particles are involved

see also: Chiu, Golf, Kelley, Manohar 2007




Basic elements of SCET

Basic elements of SCET

- Intrinsic complication consists of fact that large mass M_S enters low-energy theory as a parameter characterizing the large energies $E_i \sim M_S$ of the light final-state particles

- Gives rise to **non-local operators** in the effective Lagrangian, with fields separated along the light-like directions in which these particles travel

Bauer, Fleming, Pirjol, Stewart 2001; Bauer, Pirjol, Stewart 2002
Beneke, Chapovsky, Diehl, Feldmann 2002

- After Fourier transformation, this introduces a dependence of the Wilson coefficients on M_S

Basic elements of SCET

- In a given decay process of S, final state contains jets defining directions $\{\boldsymbol{n}_1, \dots, \boldsymbol{n}_k\}$ of large energy flow
- Each jet consists of one or more $\boldsymbol{n_i}$**-collinear particles**, which carry energies much larger than their rest mass
- Define light-like reference vectors $n_i^\mu = (1, \boldsymbol{n}_i)$ and $\bar{n}_i^\mu = (1, -\boldsymbol{n}_i)$ with $n_i \cdot \bar{n}_i = 2$; then:

$$p^\mu = \bar{n}_i \cdot p \, \frac{n_i^\mu}{2} + n_i \cdot p \, \frac{\bar{n}_i^\mu}{2} + p_\perp^\mu$$

$$(n_i \cdot p, \bar{n}_i \cdot p, p_\perp) \sim M_S \, (\lambda^2, 1, \lambda)$$

$$\lambda = \frac{v}{M_S}$$

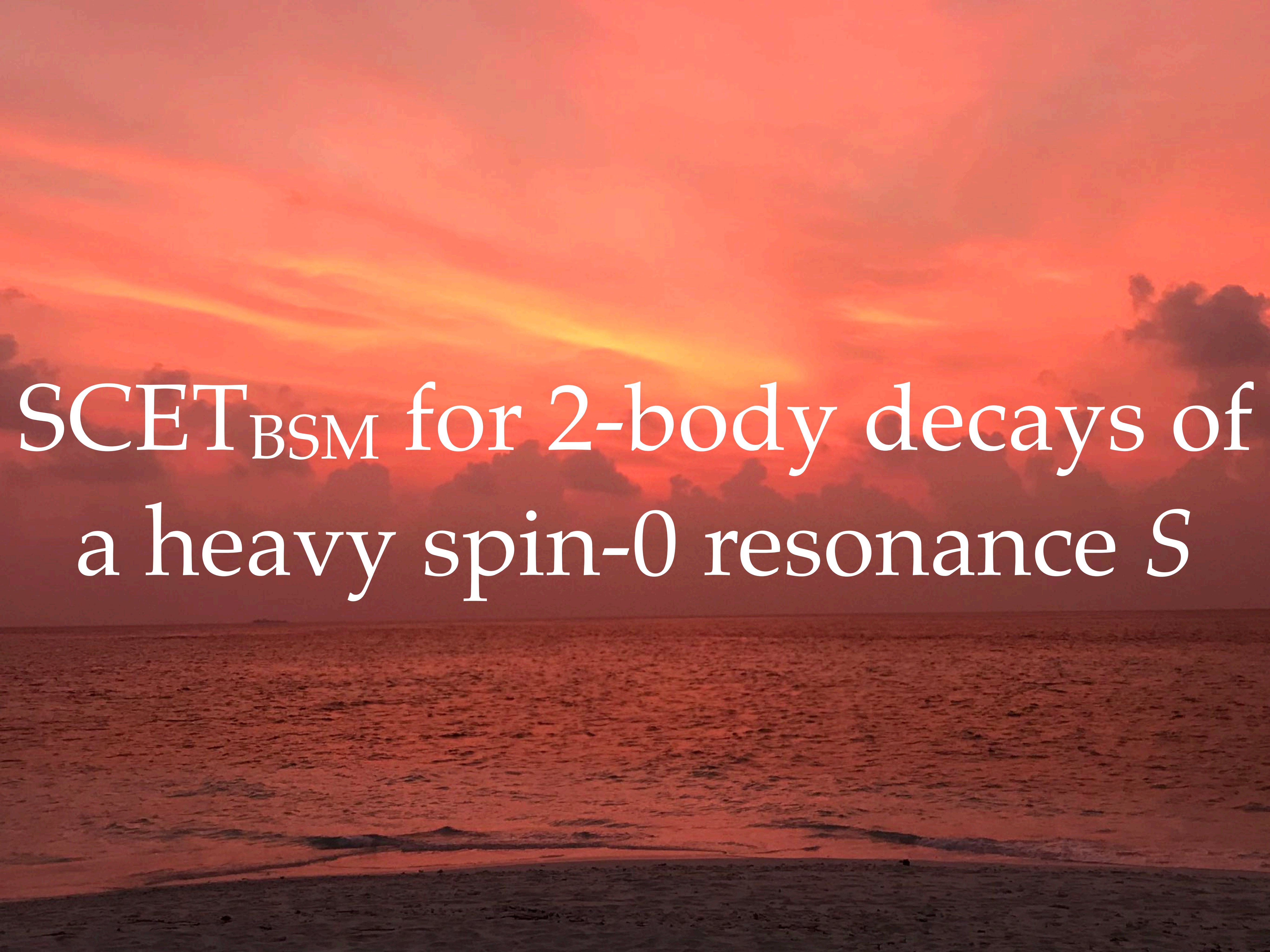
expansion parameter

Basic elements of SCET

- Particles inside a jet can interact with each other, but only soft particles can mediate between different jets
- In SCET, particles are described by gauge-invariant collinear building blocks defined using Wilson lines:
  - scalar doublet: $\Phi_{n_i}(x) = W_{n_i}^\dagger(x)\,\phi(x)$
  - fermions: $\mathcal{X}_{n_i}(x) = \frac{\not{n}_i \bar{\not{n}}_i}{4}\,W_{n_i}^\dagger(x)\,\psi(x)$
  - gauge bosons: $\mathcal{A}_{n_i}^\mu(x) = W_{n_i}^{(A)\dagger}(x)\left[iD_{n_i}^\mu\,W_{n_i}^{(A)}(x)\right]$

$$= g_A \int_{-\infty}^0 ds\,\bar{n}_{i\alpha}\left[W_{n_i}^{(A)\dagger} A_{n_i}^{\alpha\mu}\,W_{n_i}^{(A)}\right](x+s\bar{n}_i)$$

Bauer, Pirjol, Stewart 2002
Hill, MN 2002


SCET$_{\mathrm{BSM}}$ for 2-body decays of
a heavy spin-0 resonance S

Basic elements of SCET

- Power counting rules:

$$\Phi_{n_i} \sim \lambda\,, \qquad \mathcal{X}_{n_i} \sim \lambda\,, \qquad \mathcal{A}^{\mu}_{n_i\perp} \sim \lambda\,, \qquad n_i \cdot \mathcal{A}_{n_i} \sim \lambda^2$$

  imply that adding fields gives rise to power suppression

- Because of EWSB, the effective theory also contains scalar fields carrying no 4-momentum:

$$\Phi_0 \stackrel{\text{EWSB}}{\to} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \sim \lambda$$

- For 2-body decays, operators need to contain collinear fields in two opposite directions

- Obtain Lagrangian by constructing gauge-invariant operators built out of these fields, starting at $O(\lambda^2)$

Effective Lagrangian at O(λ^2)

- Most general expression:

$$\mathcal{L}_{\rm eff}^{(2)} = M\, C_{\phi\phi}(M_S, M, \mu)\, O_{\phi\phi}(\mu) + M \sum_{A=G,W,B} \Big[C_{AA}(M_S, M, \mu)\, O_{AA}(\mu) + \widetilde{C}_{AA}(M_S, M, \mu)\, \widetilde{O}_{AA}(\mu) \Big]$$

with:

$$O_{\phi\phi} = S \left(\Phi_{n_1}^\dagger \Phi_{n_2} + \Phi_{n_2}^\dagger \Phi_{n_1} \right)$$

$$O_{AA} = S\, g_{\mu\nu}^{\perp}\, \mathcal{A}_{n_1}^{\mu,a}\, \mathcal{A}_{n_2}^{\nu,a}$$

$$\widetilde{O}_{AA} = S\, \epsilon_{\mu\nu}^{\perp}\, \mathcal{A}_{n_1}^{\mu,a} \mathcal{A}_{n_2}^{\nu,a}$$

- Model-independent predictions for all diboson decay rates in terms of 7 Wilson coefficient functions!

Effective Lagrangian at O(λ²)

- After EWSB, Φ_{n_i} contains both the Higgs boson and the longitudinal modes of the electroweak gauge bosons:

$$\Phi_{n_i}(0) = \frac{1}{\sqrt{2}} W_{n_i}^\dagger(0) \begin{pmatrix} 0 \\ v + h_{n_i}(0) \end{pmatrix}$$

with:

$$W_{n_i}(0) = P \exp\left[\frac{ig}{2}\int_{-\infty}^0 ds \begin{pmatrix} \frac{c_w^2 - s_w^2}{c_w}\,\bar{n}_i \cdot Z_{n_i} + 2s_w\,\bar{n}_i \cdot A_{n_i} & \sqrt{2}\,\bar{n}_i \cdot W_{n_i}^+ \\ \sqrt{2}\,\bar{n}_i \cdot W_{n_i}^- & -\frac{1}{c_w}\,\bar{n}_i \cdot Z_{n_i} \end{pmatrix}(s\bar{n}_i)\right]$$

- Hence:

$$O_{\phi\phi} = S(0)\,h_{n_1}(0)\,h_{n_2}(0) + m_Z^2 \int_{-\infty}^0 ds \int_{-\infty}^0 dt\, S(0)\,\bar{n}_1 \cdot Z_{n_1}(s\bar{n}_1)\,\bar{n}_2 \cdot Z_{n_2}(t\bar{n}_2)$$

$$+\, m_W^2 \int_{-\infty}^0 ds \int_{-\infty}^0 dt\, S(0)\left[\bar{n}_1 \cdot W_{n_1}^-(s\bar{n}_1)\,\bar{n}_2 \cdot W_{n_2}^+(t\bar{n}_2) + (+ \leftrightarrow -)\right] + \ldots$$

Diboson decay rates

- For the $S \to hh$ decay mode we obtain:

$$\mathcal{M}(S \to hh) = M\, C_{\phi\phi}$$

- For the $S \to V_1 V_2$ decay modes we define the form factor decomposition:

$$\mathcal{M}(S \to V_1 V_2) = M \left[F_\perp^{V_1 V_2}\, \varepsilon_{1\perp}^* \cdot \varepsilon_{2\perp}^* + \widetilde{F}_\perp^{V_1 V_2}\, \epsilon_{\mu\nu}^\perp\, \varepsilon_{1\perp}^{*\mu}\, \varepsilon_{2\perp}^{*\nu} + F_\parallel^{V_1 V_2}\, \frac{m_1 m_2}{k_1 \cdot k_2}\, \varepsilon_{1\parallel}^* \cdot \varepsilon_{2\parallel}^* \right]$$

- Both amplitudes scale like λ^0 and hence are of leading order in power counting

Diboson decay rates

- For the form factors we obtain:

$$F_\perp^{gg} = g_s^2\, C_{GG}\,, \qquad \widetilde{F}_\perp^{gg} = g_s^2\, \widetilde{C}_{GG}$$

$$F_\perp^{\gamma\gamma} = e^2\,(C_{WW} + C_{BB})\,, \qquad \widetilde{F}_\perp^{\gamma\gamma} = e^2\,(\widetilde{C}_{WW} + \widetilde{C}_{BB})$$

$$F_\perp^{\gamma Z} = e^2 \left(\frac{c_w}{s_w}\, C_{WW} - \frac{s_w}{c_w}\, C_{BB} \right), \qquad \widetilde{F}_\perp^{\gamma Z} = e^2 \left(\frac{c_w}{s_w}\, \widetilde{C}_{WW} - \frac{s_w}{c_w}\, \widetilde{C}_{BB} \right)$$

$$F_\perp^{ZZ} = e^2 \left(\frac{c_w^2}{s_w^2}\, C_{WW} + \frac{s_w^2}{c_w^2}\, C_{BB} \right), \qquad \widetilde{F}_\perp^{ZZ} = e^2 \left(\frac{c_w^2}{s_w^2}\, \widetilde{C}_{WW} + \frac{s_w^2}{c_w^2}\, \widetilde{C}_{BB} \right)$$

$$F_\perp^{WW} = \frac{e^2}{s_w^2}\, C_{WW}, \qquad \widetilde{F}_\perp^{WW} = \frac{e^2}{s_w^2}\, \widetilde{C}_{WW}\,,$$

and:

$$F_\parallel^{ZZ} = -C_{\phi\phi}\,, \qquad F_\parallel^{WW} = -C_{\phi\phi}$$

→ Goldstone boson equivalence theorem!

Example of a UV completion

- Consider for illustration a model containing a doublet of heavy, vector-like quarks $\psi = (T\ B)^T$ transforming as $(\mathbf{3}, \mathbf{2}, 1/6)$, with Lagrangian $\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_S + \mathcal{L}_\psi$ and:

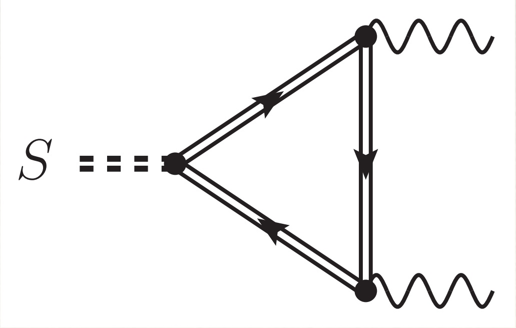
$$\mathcal{L}_S = \frac{1}{2}(\partial_\mu S)(\partial^\mu S) - \frac{M_S^2}{2} S^2 - c_1 S\, \bar\psi\psi - c_2\, S\, (\bar{Q}_L \psi + \bar\psi\, Q_L)$$

$$\mathcal{L}_\psi = \bar\psi\,(i\not{D} - M)\,\psi - \Big(g_t\,\bar\psi\,\tilde\phi\, t_R + g_b\,\bar\psi\,\phi\, b_R + \text{h.c.}\Big)$$

- We then find the non-trivial matching conditions:

$$C_{GG} = -\frac{1}{4\pi^2}\,\arcsin^2\!\Big(\frac{M_S}{2M}\Big)$$

$$C_{WW} + C_{BB} = -\frac{5}{6\pi^2}\,\arcsin^2\!\Big(\frac{M_S}{2M}\Big)$$



Effective Lagrangian at O(λ^3)

- Most general result (*i,j* are generation indices):

$$\mathcal{L}_{\rm eff}^{(3)} = \frac{1}{M}\left[C^{ij}_{F_L\bar{f}_R}(M_S,M,\mu)\,O^{ij}_{F_L\bar{f}_R}(\mu) + \sum_{k=1,2}\int_0^1 du\,C^{(k)\,ij}_{F_L\bar{f}_R\,\phi}(u,M_S,M,\mu)\,O^{(k)\,ij}_{F_L\bar{f}_R\,\phi}(u,\mu) + \text{h.c.}\right]$$
$$+\frac{1}{M}\sum_{A=G,W,B}\left[\int_0^1 du\,C^{ij}_{F_L\bar{F}_L A}(u,M_S,M,\mu)\,O^{ij}_{F_L\bar{F}_L A}(u,\mu) + (F_L\to f_R) + \text{h.c.}\right]$$

with:

$$O^{ij}_{F_L\bar{f}_R}(\mu) = S\,\bar{\mathcal{X}}^i_{L,n_1}\,\Phi_0\,\mathcal{X}^j_{R,n_2} + (n_1\leftrightarrow n_2)$$
$$O^{(1)\,ij}_{F_L\bar{f}_R\,\phi}(u,\mu) = S\,\bar{\mathcal{X}}^i_{L,n_1}\,\Phi^{(u)}_{n_1}\,\mathcal{X}^j_{R,n_2} + (n_1\leftrightarrow n_2)$$
$$O^{(2)\,ij}_{F_L\bar{f}_R\,\phi}(u,\mu) = S\,\bar{\mathcal{X}}^i_{L,n_1}\,\Phi^{(u)}_{n_2}\,\mathcal{X}^j_{R,n_2} + (n_1\leftrightarrow n_2)$$
$$O^{ij}_{F_L\bar{F}_L A}(u,\mu) = S\,\bar{\mathcal{X}}^i_{L,n_1}\,\not{\mathcal{A}}^{\perp(u)}_{n_1}\,\mathcal{X}^j_{L,n_2} + (n_1\leftrightarrow n_2)$$
$$O^{ij}_{f_R\bar{f}_R A}(u,\mu) = S\,\bar{\mathcal{X}}^i_{R,n_1}\,\not{\mathcal{A}}^{\perp(u)}_{n_1}\,\mathcal{X}^j_{R,n_2} + (n_1\leftrightarrow n_2)$$

Decay rates into fermions

- For the $S \to \psi_1 \bar{\psi}_2$ decay modes, we find:

$$\mathcal{M}(S \to f_{iL}\, \bar{f}_{jR}) = \frac{v}{\sqrt{2}} \frac{M_S}{M}\, \mathrm{C}^{ij}_{f_L \bar{f}_R}\, e^{i\varphi_{ij}}\,,$$

$$\mathcal{M}(S \to f_{iR}\, \bar{f}_{jL}) = \frac{v}{\sqrt{2}} \frac{M_S}{M}\, \mathrm{C}^{ji\,*}_{f_L \bar{f}_R}\, e^{-i\varphi_{ji}}$$

where after rotation to the mass basis:

$$\boldsymbol{C}_{F_L \bar{f}_R} \to \boldsymbol{U}^\dagger_{f_L} \boldsymbol{C}_{F_L \bar{f}_R} \boldsymbol{W}_{f_R} \equiv \mathbf{C}_{f_L \bar{f}_R}$$

- Both amplitudes scale like λ and hence are of subleading order in power counting

Effective Lagrangian at O(λ^4)

- The only decay mode not yet accounted for is $S \to Zh$, and at tree level a single operator contributes

- We find:

$$\mathcal{L}_{\rm eff}^{(4)} \ni \frac{\widetilde{C}_{\phi\phi\phi\phi}(M_S, M, \mu)}{M} \Big[iS \Big(\Phi_{n_1}^\dagger \Phi_0 - \Phi_0^\dagger \Phi_{n_1} \Big) \Big(\Phi_{n_2}^\dagger \Phi_0 + \Phi_0^\dagger \Phi_{n_2} \Big) + (n_1 \leftrightarrow n_2) \Big]$$

and:

$$\mathcal{M}(S \to Z_\parallel h) = -iC_{\phi\phi\phi\phi} \frac{v^2}{M}$$

→ resolves a puzzle!

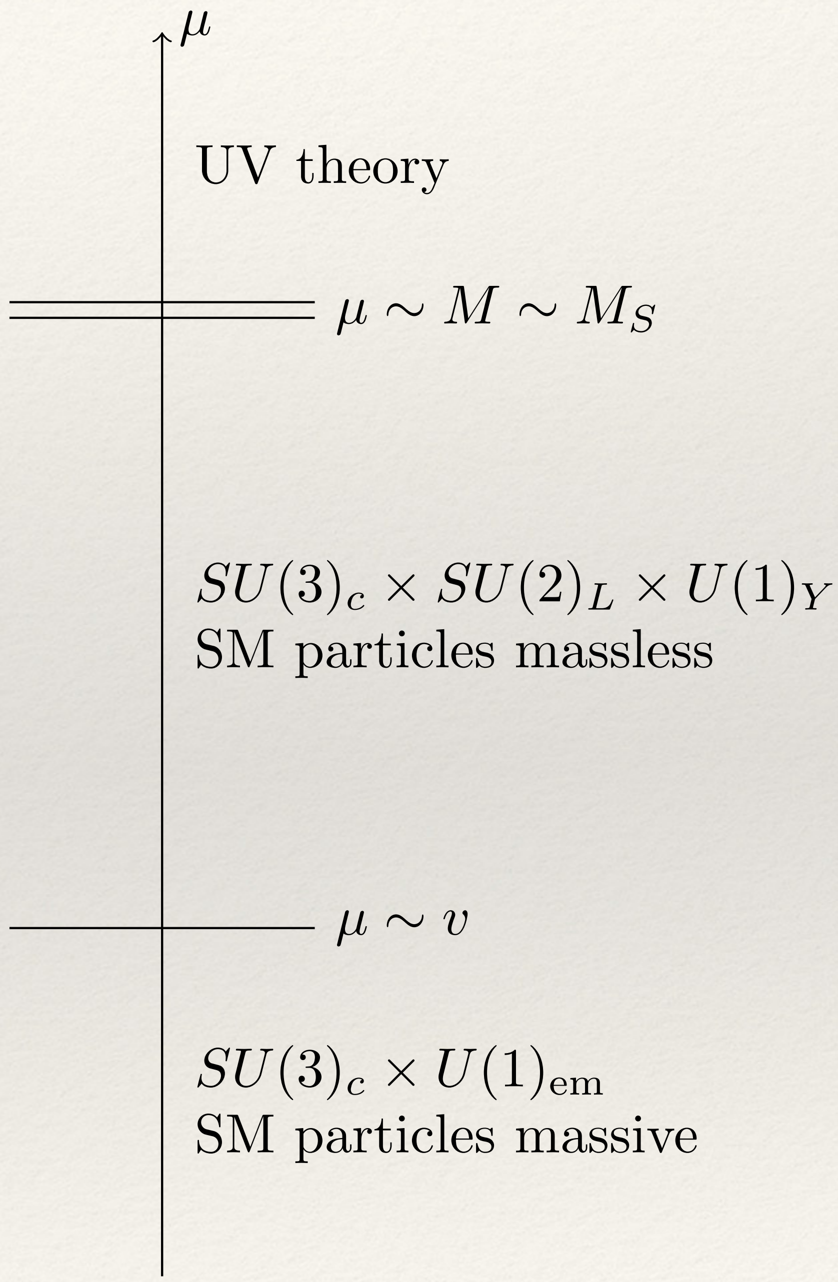
Bauer, MN, Thamm 2016

- Amplitude scales like λ^2 and hence is of subsubleading order in power counting


Resummation of large logs

Resumming logarithms of M_S/v

- At the new-physics scale $M \sim M_S$ the full theory is matched onto SCET_{BSM}, giving rise to Wilson coefficient functions
- The SCET operators are **evolved from M_S down to the scale v using RGEs and anomalous dimensions** derived in the effective theory
- At the scale v a matching onto a theory with massive SM fields is performed



Resumming logarithms of M_S/v

- Anomalous dimensions in SCET contain terms enhanced by a logarithm $\ln(M_S^2/\mu^2)$ times the cusp anomalous dimension
- These are responsible for resuming Sudakov double logarithms $\sim [\alpha_s \ln^2(M_S^2/v^2)]^n$
- Some of the RGEs also contain convolutions over the momentum-fraction variable u (à la DGLAP)
- We find that a non-trivial operator mixing occurs starting at $O(\lambda^3)$, where also other subtleties arise

QCD evolution at O(λ³)

- We find, for example:

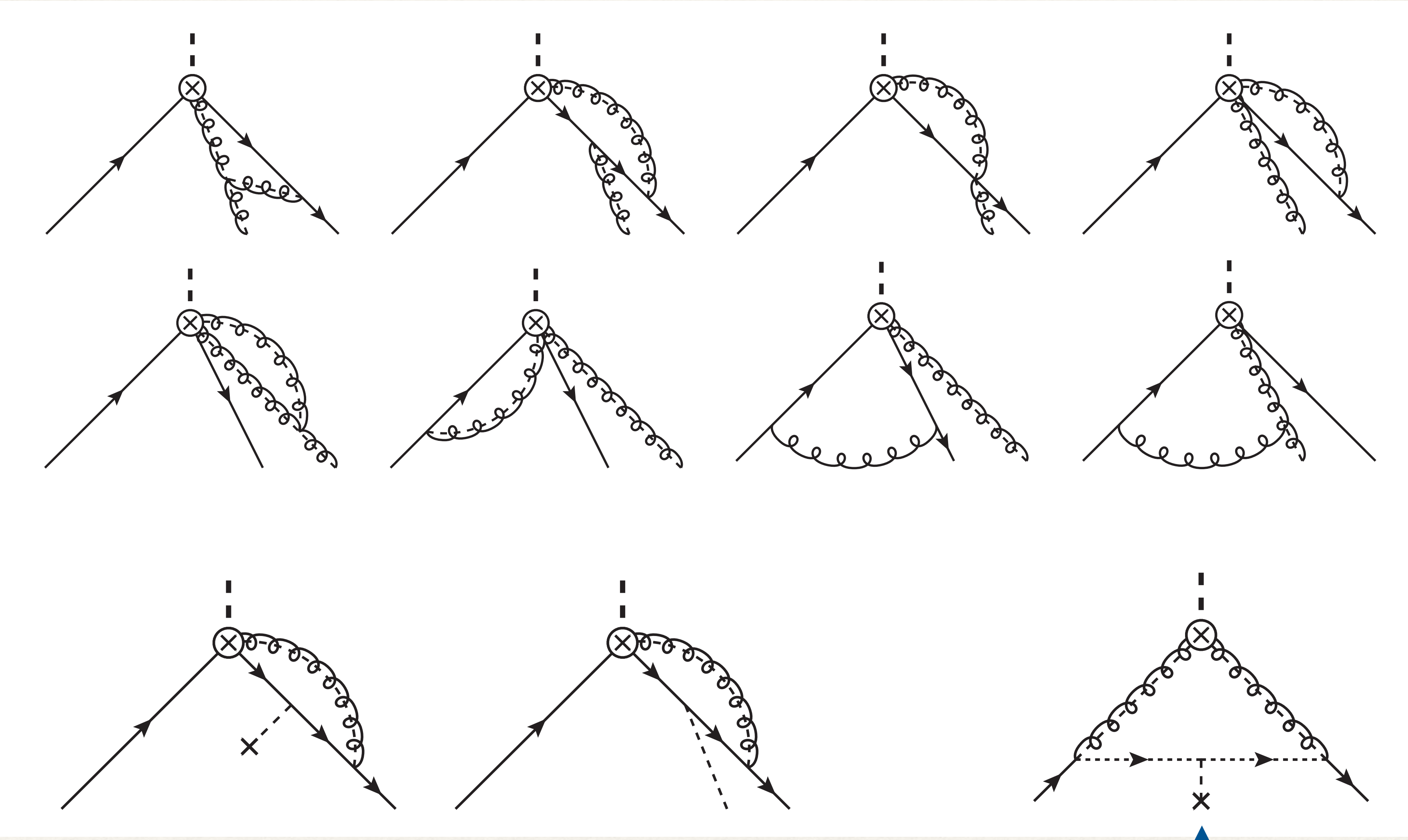
mixing of $O(\lambda^2)$ operators into $O(\lambda^3)$ operators via subleading interactions in the SCET Lagrangian

$$\mu \frac{d}{d\mu} \boldsymbol{C}_{Q_L\bar{q}_R}(\mu) = \Gamma_{Q_L\bar{q}_R}(\mu)\, \boldsymbol{C}_{Q_L\bar{q}_R}(\mu) + \frac{M^2}{M_S^2} \left[\gamma_{\text{cusp}}^{q\bar{q}} \left(\ln \frac{M_S^2}{\mu^2} - i\pi \right) + \tilde{\gamma}_{q\bar{q}} \right] g_s^2(\mu) \left(C_{GG}(\mu) + i\widetilde{C}_{GG}(\mu) \right) \boldsymbol{Y}_q(\mu) + \int_0^1 dw\, \Gamma_{\text{mix}}(0,w,\mu) \left[\boldsymbol{Y}_q(\mu)\, \bar{\boldsymbol{C}}_{q_R\bar{q}_R G}(w,\mu) + \bar{\boldsymbol{C}}^{\dagger}_{Q_L\bar{Q}_L G}(w,\mu)\, \boldsymbol{Y}_q(\mu) \right]$$

$$\mu \frac{d}{d\mu} \boldsymbol{C}_{q\bar{q}G}(u, M_S, M, \mu) = \int_0^1 dw\, \Gamma_{q\bar{q}G}(u, w, M_S, \mu)\, \boldsymbol{C}_{q\bar{q}G}(w, M_S, M, \mu) \qquad \text{with } q = Q_L \text{ or } q_R$$

Example: QCD evolution at O(λ³)

- Relevant diagrams:



mixing of $O(\lambda^2)$ operators into $O(\lambda^3)$ operators via subleading interactions in the SCET Lagrangian

Example: QCD evolution at O(λ³)

- with:

$$\Gamma_{Q_L\bar{q}_R} = C_F\,\gamma_{\rm cusp}^{(3)}\left(\ln\frac{M_S^2}{\mu^2} - i\pi\right) + 2\gamma^q$$

$$\Gamma_{\rm mix}(u,w,\mu) = \frac{C_F\alpha_s(\mu)}{\pi}\,\frac{\theta(1-u-w)}{1-u} + \mathcal{O}(\alpha_s^2)$$

$$\Gamma_{q\bar{q}G}(u,w,M_S,\mu) = \left[C_F\left(\ln\frac{\bar{u}M_S^2}{\mu^2} - i\pi - \frac{3}{2}\right) + \frac{C_A}{2}\left(\ln\frac{u}{\bar{u}} + 1\right)\right]\gamma_{\rm cusp}^{(3)}\,\delta(u-w) - \bar{w}\left[V_1(\bar{u},\bar{w}) + V_2(\bar{u},\bar{w})\right] + \mathcal{O}(\alpha_s^2)$$

and:

$$V_1(\bar{u},\bar{w}) + V_2(\bar{u},\bar{w}) = -\frac{C_A}{2}\,\frac{\alpha_s}{\pi}\left\{\frac{1}{\bar{u}\bar{w}}\left[\bar{u}\,\frac{\theta(u-w)}{u-w} + \bar{w}\,\frac{\theta(w-u)}{w-u}\right]_+ + \left(\frac{w}{\bar{w}} - \frac{1}{u}\right)\theta(u-w) + \left(\frac{u}{\bar{u}} - \frac{1}{w}\right)\theta(w-u)\right\} + \left(C_F - \frac{C_A}{2}\right)\frac{\alpha_s}{\pi}\left[\left(2 - \frac{\bar{u}\bar{w}}{uw}\right)\theta(u+w-1) + \frac{uw}{\bar{u}\bar{w}}\,\theta(1-u-w)\right]$$

see also: Hill, Becher, Lee, MN 2004

Conclusions

- If a new heavy particle — the first of a new sector — is discovered at the LHC, its interactions with SM particles can be described using an effective field theory
- This should be done consistently!
- SCET_{BSM} offers the systematic framework for separating the scales M_S and v and resumming large Sudakov logs of this ratio, while correctly treating the dependence on the mass ratio M/M_S, where M refers to the masses of yet undiscovered particles


The end!