

Naturalizing SUSY with the relaxion and the inflaton

Tony Gherghetta



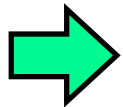
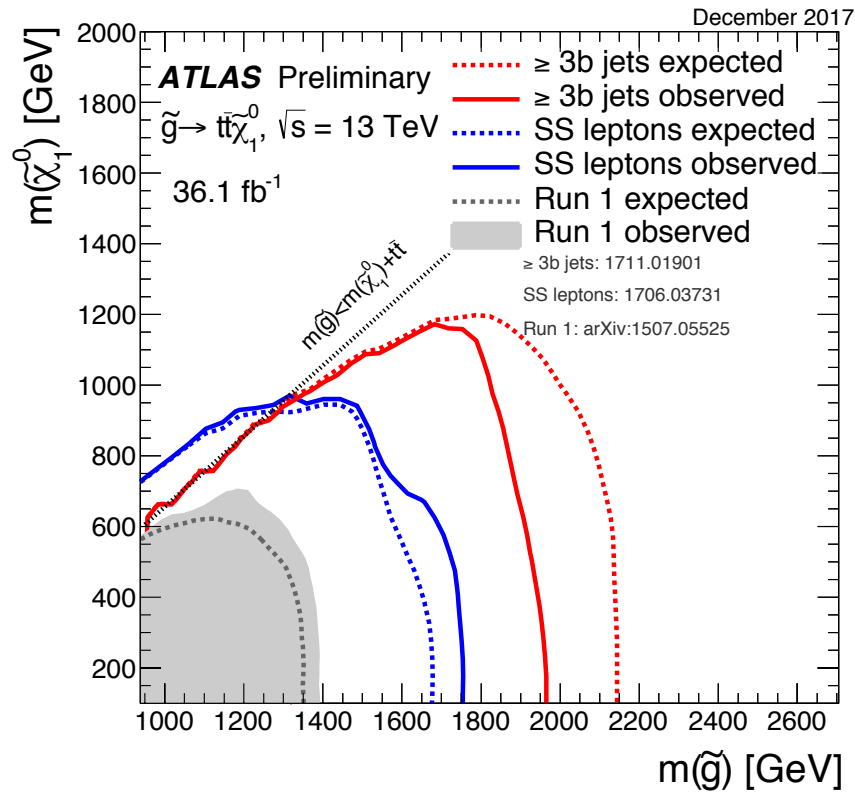
UNIVERSITY OF MINNESOTA

***7th Workshop on Flavour Physics and the Future of BSM,
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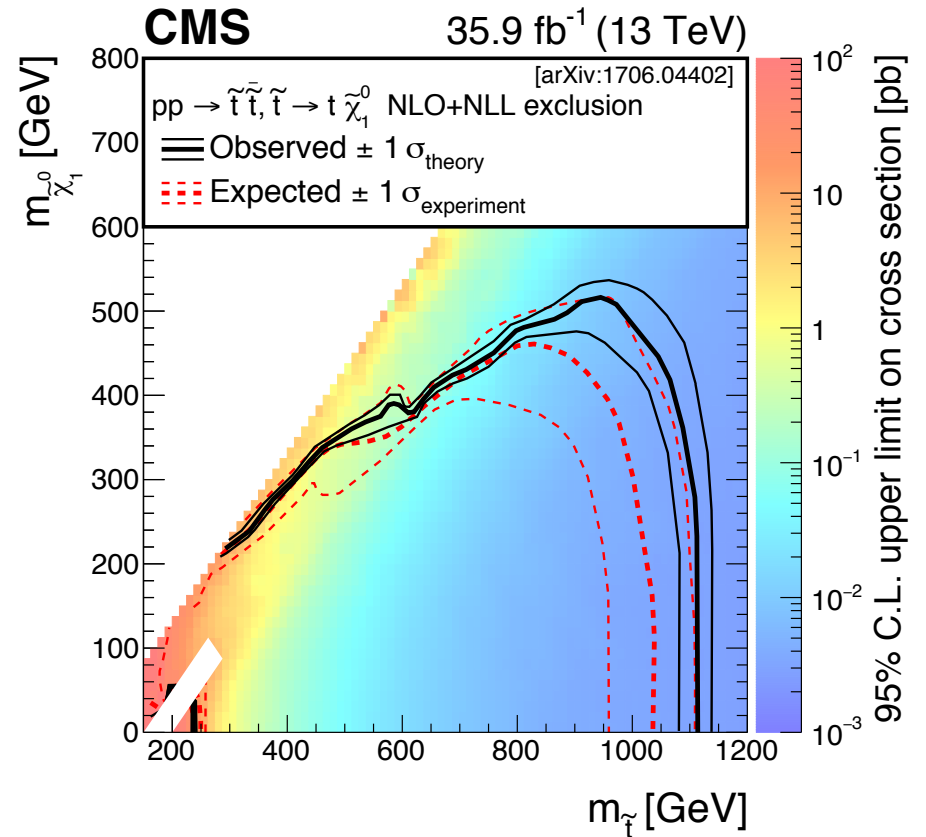
[Jason Evans, TG, Natsumi Nagata, Zach Thomas, arXiv:1602.04812]

[Jason Evans, TG, Natsumi Nagata, Marco Peloso, arXiv:1704.03695]

LHC Run 2 Limits:



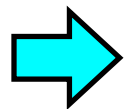
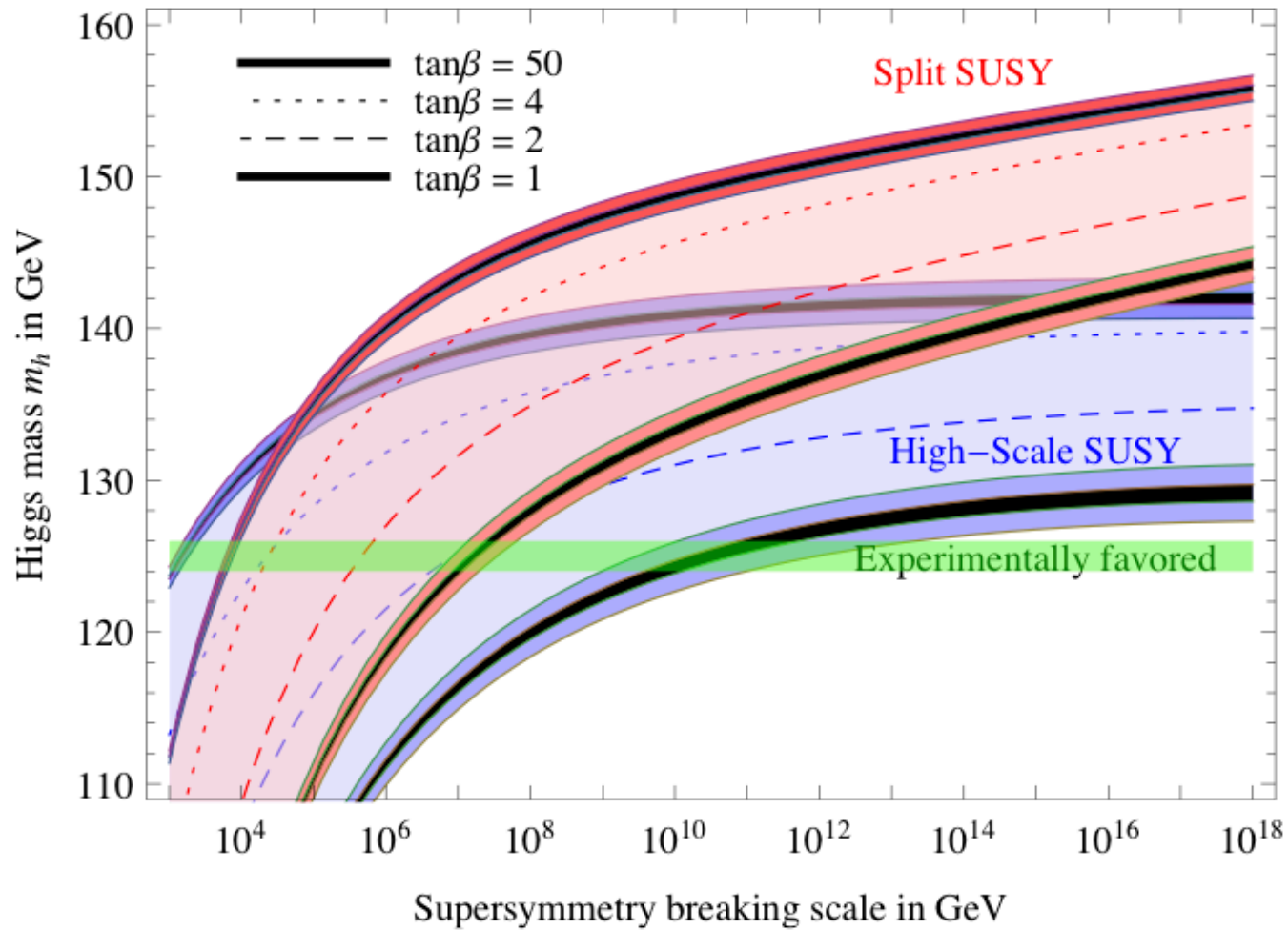
$$m_{\tilde{g}} \gtrsim 1900 \text{ GeV}$$



$$m_{\tilde{t}} \gtrsim 1120 \text{ GeV}$$

Predicted range for the Higgs mass

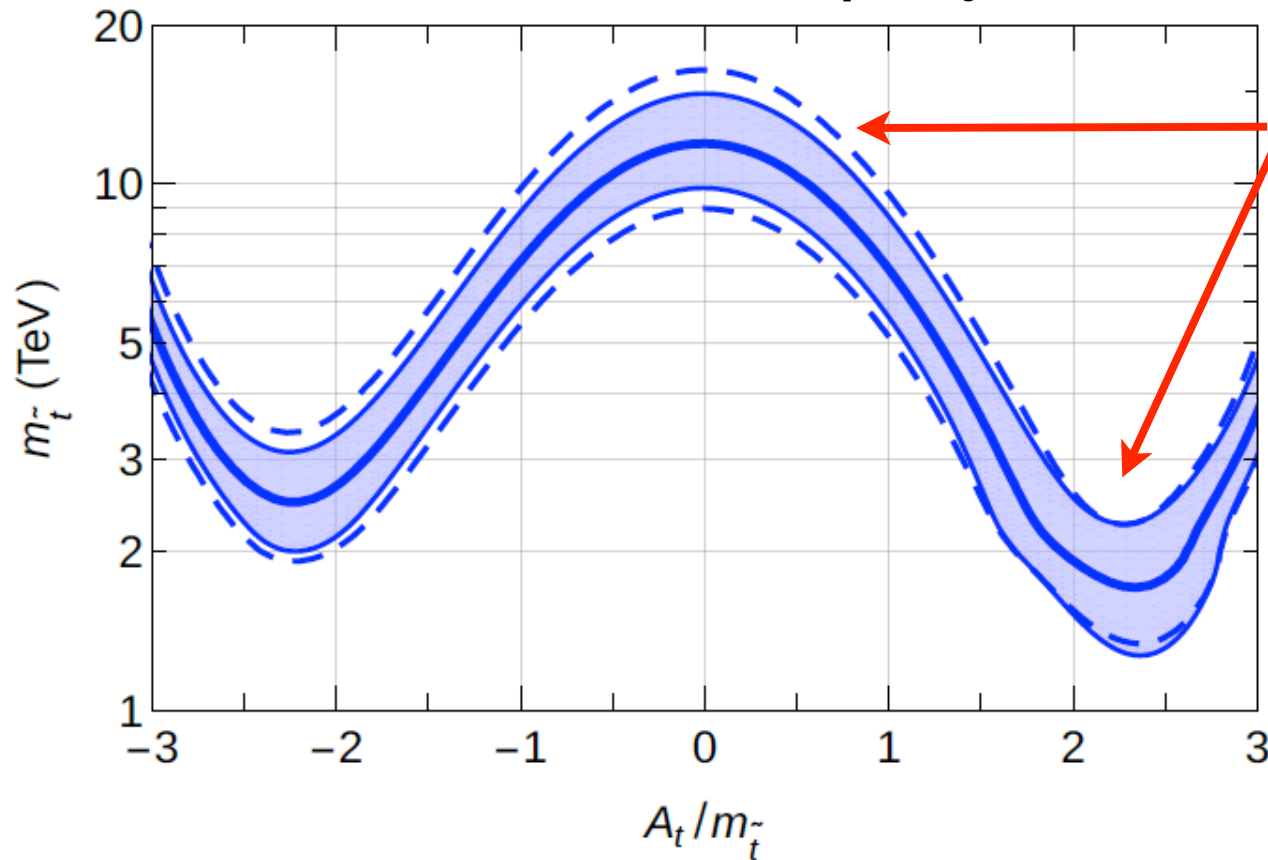
[Giudice, Strumia 1108.6077]



SUSY breaking scale $\lesssim 10^7$ GeV

Higgs mass in minimal SUSY

[Pardo Vega, Villadoro 1504.05200]



➡ Increases tuning in supersymmetric models



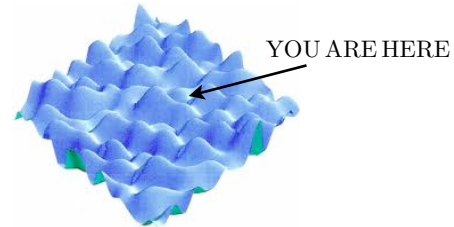
Why is $m_{\tilde{t}} \gtrsim \text{TeV}$ and not near electroweak scale?

- There is no *low-energy* supersymmetry



- SUSY top partners are uncolored e.g. Folded SUSY “Neutral Naturalness”

- Anthropic - we live in a multiverse



Is there an alternative possibility? Yes!

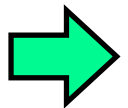
Special point in parameter space:

$m_H^2 = 0$ **not** related to a symmetry

e.g. supersymmetry $m_H^2 \simeq \Lambda^2 - \Lambda^2 + \dots$

Instead, $m_H^2 \simeq 0$ related to
early-universe dynamics!

e.g. self-organized criticality



Dynamical evolution sets the SUSY scale!

→ explains why $m_{\tilde{t}} \gg v$!

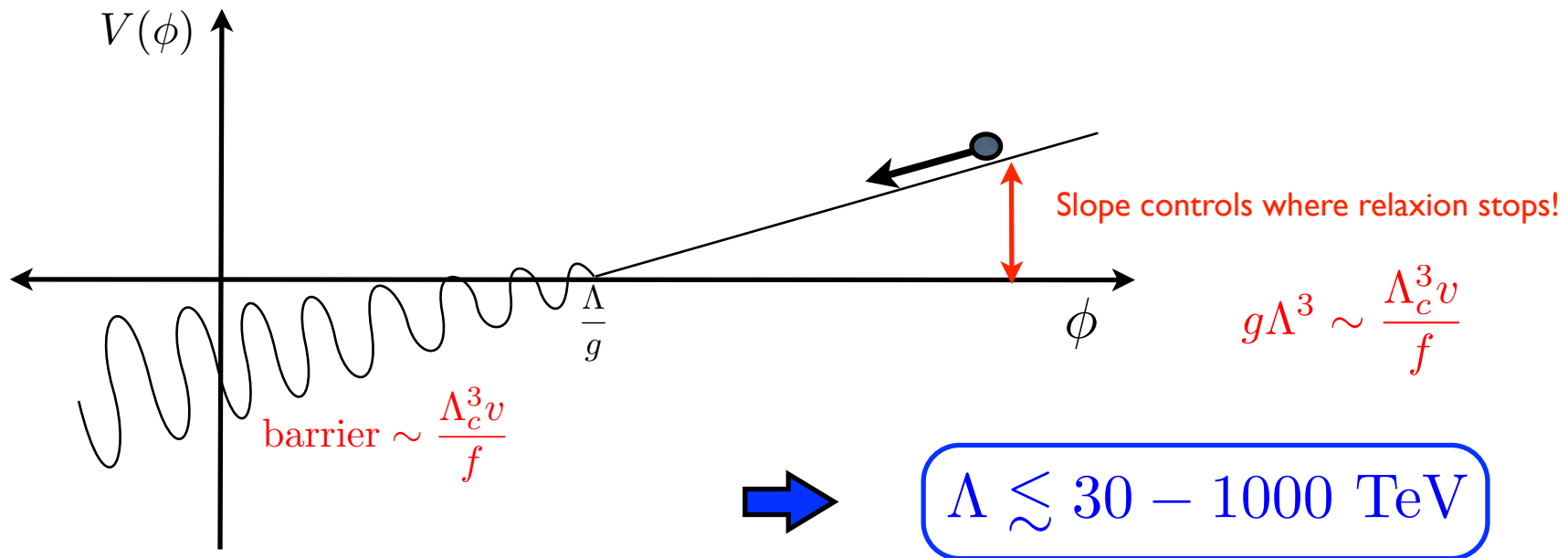


← This talk
“Hidden” Naturalness

Relaxion mechanism [Graham, Kaplan, Rajendran 1504.07551]

Introduce scalar field (relaxion): ϕ

$$V(\phi, h) = \underbrace{g\Lambda^3\phi}_{\text{breaks shift symmetry: } \phi \rightarrow \phi + c} - \Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)|H|^2 + \lambda_h|H|^4 + \underbrace{\Lambda_c^3 v \cos \frac{\phi}{f}}_{\text{back reaction from strong dynamics}}$$



However:

- Relaxion = QCD axion $\longrightarrow$ large θ_{QCD} !
- Alternatively, non-QCD dynamics requires new fermions near electroweak scale $\longrightarrow$ coincidence?

In general:

$$V(\phi, h) = g\Lambda^3\phi + \Lambda^2\left(1 - \frac{g\phi}{\Lambda}\right)|H|^2 + \lambda_h|H|^4 + \Lambda_c^{4-n}v^n \cos \frac{\phi}{f}$$

$n = 1$ Requires new source of EWSB e.g. QCD

$n = 2$ $\Lambda_c^2|H|^2 \cos \frac{\phi}{f}$ Gauge invariant - new source not required!

→ However, quantum corrections generate: $\Lambda_c^4 \cos \frac{\phi}{f}$, $\Lambda_c^3 g\phi \cos \frac{\phi}{f}$ Large potential barriers!

Introduce second field, σ [Espinosa et al, 1506.09217]

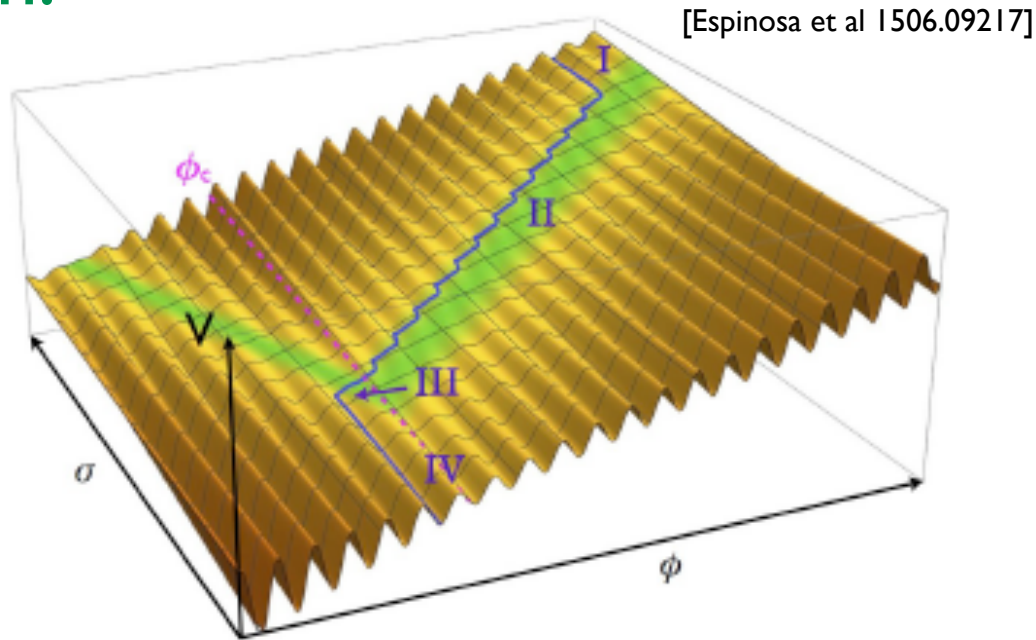
$$V(\phi, \sigma, h) = g\Lambda^3\phi + \underbrace{g_\sigma\Lambda^3\sigma}_{\text{new term}} + \Lambda^2\left(\alpha - \frac{g\phi}{\Lambda}\right)|H|^2 + \lambda_h|H|^4 + \mathcal{A}(\phi, \sigma, H) \cos \frac{\phi}{f}$$

where $\mathcal{A}(\phi, \sigma, H) = \epsilon \left(\beta\Lambda^4 + c_\phi g\Lambda^3\phi - \underbrace{c_\sigma g_\sigma\Lambda^3\sigma}_{\text{new term -- cancels large potential barrier!}} + \Lambda^2|H|^2 \right)$

new term -- cancels large potential barrier!

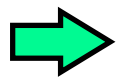
[Assuming no $\sigma|H|^2$ coupling and $\epsilon^2\Lambda^4 \cos^2 \frac{\phi}{f}$ terms]

Obtain:



Cosmological Evolution Stages:

- I. ϕ trapped, σ rolls
- II. $\mathcal{A} = 0$; both ϕ and σ roll
- III. EWSB barrier appears
- IV. ϕ stops, σ continues to roll



$$\Lambda \lesssim 2 \times 10^9 \text{ GeV}$$

$$\text{for } g_\sigma = 0.1g \simeq 10^{-27}$$

But $\Lambda \ll M_P$, so still require a UV completion....

Instead, apply to SUSY “little” hierarchy! 😊

Supersymmetric *two-field* relaxion mechanism

[Evans, TG, Nagata, Thomas | 602.04812]

Embed ϕ, σ into chiral superfields S, T

$$S = \frac{s + \overset{\text{relaxion}}{i\phi}}{\sqrt{2}} + \sqrt{2}\tilde{\phi}\theta + F_S\theta\theta$$

$$T = \frac{\tau + \overset{\text{"amplitudon"}}{i\sigma}}{\sqrt{2}} + \sqrt{2}\tilde{\sigma}\theta + F_T\theta\theta$$

Shift symmetries:

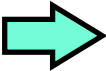
$$\begin{aligned} \mathcal{S}_S : \quad S &\rightarrow S + \overset{\phi = \text{NG boson}}{i\alpha f_\phi}, \\ T &\rightarrow T, \\ Q_i &\rightarrow e^{iq_i\alpha} Q_i, \\ H_u H_d &\rightarrow e^{iq_H\alpha} H_u H_d, \end{aligned}$$

$$\begin{aligned} \mathcal{S}_T : \quad S &\rightarrow S, \\ T &\rightarrow T + \overset{\sigma = \text{NG boson}}{i\beta f_\sigma}, \\ Q_i &\rightarrow Q_i, \\ H_u H_d &\rightarrow H_u H_d, \end{aligned}$$

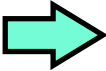
where Q_i = MSSM matter superfields, f_ϕ, f_σ = decay constants
 H_u, H_d = MSSM Higgs superfields

Break shift symmetry to generate potential for ϕ, σ

Superpotential: $W_{S,T} = \frac{1}{2}m_S S^2 + \frac{1}{2}m_T T^2$ $m_S, m_T = \text{mass parameters}$

 $V(\phi, \sigma) = \frac{1}{2}|m_S|^2 \phi^2 + \frac{1}{2}|m_T|^2 \sigma^2$

Kahler potential: $K = K(S + S^*, T + T^*)$ shift invariant

 no renormalisable coupling of σ to H_u, H_d !

But ϕ can couple to MSSM Higgs fields via $U(S + S^*, T + T^*) e^{-\frac{q_H S}{f_\phi}} H_u H_d$

mu-term: $W_\mu = \mu_0 e^{-\frac{q_H S}{f_\phi}} H_u H_d$

Scanning of soft mass parameters

[Batell, Giudice, McCullough
1509.00834]

Assume large initial ϕ, σ field value and $\sigma \sim \phi, f_\sigma \sim f_\phi, m_T \sim m_S$

$$\Rightarrow m_\phi \sim m_\sigma \sim m_S \quad \boxed{F_S \sim F_T \sim m_S \phi}$$

SUSY is broken by relaxion!

Soft terms:

$$\int d^4\theta \frac{1}{M_*^2} [(S + S^*)^2 + (T + T^*)^2] \Phi^\dagger \Phi \quad \Rightarrow \quad \tilde{m} \sim B \sim A_{ijk} \sim \frac{m_S \phi}{M_*}$$

$$\int d^2\theta \frac{c_a S}{16\pi^2 f_\phi} \text{Tr} W_a W_a \quad \xrightarrow{f_\phi \sim M_*} \quad M_a \sim \frac{\alpha_a}{4\pi} \frac{m_S \phi}{M_*}$$

only S shift symmetry induces chiral anomaly

varies as relaxion evolves!

Electroweak symmetry breaking

Obtain: $m_{H_u}^2 = c_u |m_S|^2 \phi^2$, $m_{H_d}^2 = c_d |m_S|^2 \phi^2$, [assuming $m_T \ll m_S (F_T \ll F_S)$]

$$\mu = c_{\mu 0} \mu_0 + c_{\mu} m_S^* \phi, \quad B\mu = c_{B0} \mu_0 m_S \phi + c_B |m_S|^2 \phi^2 + \cancel{\frac{\lambda \Lambda^3}{M_L} \cos \frac{\phi}{f}}$$
assume subdominant $\lesssim v^2$

Order parameter: $\mathcal{D}(\phi) \equiv (m_{H_u}^2 + |\mu|^2) (m_{H_d}^2 + |\mu|^2) - |B\mu|^2$

decreases until $\mathcal{D}(\phi) < 0 \longrightarrow$ EWSB

Critical value: $\mathcal{D}(\phi_*) = 0$ occurs when $\mu_0 \sim \frac{m_S \phi_*}{f_\phi} \sim m_{SUSY}$

For: $m_S \sim 10^{-7} \text{ GeV} \times \left(\frac{m_{SUSY}}{10^5 \text{ GeV}} \right) \left(\frac{f_\phi}{10^5 \text{ GeV}} \right) \left(\frac{10^{17} \text{ GeV}}{\phi_*} \right)$

$\Rightarrow \mu \sim m_{SUSY}, \quad m_{H_u}^2 \sim m_{H_d}^2 \sim B\mu \sim m_{SUSY}^2$

Solves little hierarchy problem!

The Inflaton-Relaxion model [Evans, TG, Nagata, Peloso 1704.03695]

Identify “amplitudon” σ as the inflaton!

D-term inflation

$$D = g \underbrace{(|\phi_+|^2 - |\phi_-|^2)}_{\text{U(1) gauge coupling}} - \underbrace{\xi}_{\text{F-I term}}$$

where $\Phi_{\pm}$ charged under U(1)

$$W = \kappa T \Phi_+ \Phi_- + \frac{1}{2} m_T T^2 + \frac{1}{2} m_S S^2 + \left(m_N + i g_S S + i g_T T + \frac{\lambda}{M_L} H_u H_d \right) \underbrace{N \bar{N}}_{\text{generates periodic potential}}$$

Local minimum: $\phi_{\pm} = 0 \quad \left(\sigma \gg \sigma_c = \frac{g}{\kappa} \sqrt{2\xi} \right)$



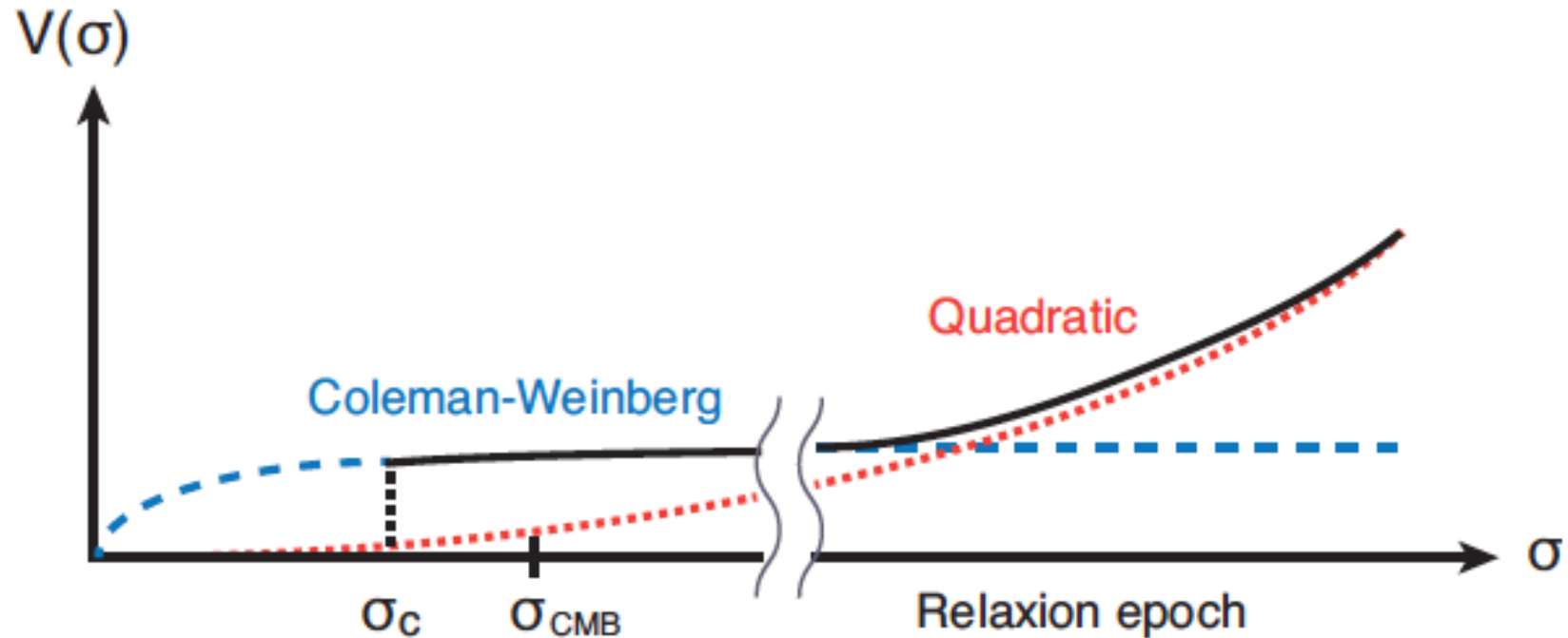
$$V \simeq \underbrace{\frac{g^2 \xi^2}{2}}_{\text{vacuum energy}} \left[1 + \frac{g^2}{8\pi^2} \ln \left(\frac{\kappa^2 \sigma^2}{2Q^2} \right) \right] + \frac{1}{2} |m_T|^2 \sigma^2 + \frac{1}{2} |m_S|^2 \phi^2$$

vacuum energy

CMB epoch

relaxion epoch

Scalar potential



Slow-roll parameters: $\epsilon \equiv \frac{M_P^2}{2V^2} \left(\frac{\partial V}{\partial \sigma} \right)^2 \simeq \frac{g^4}{32\pi^4} \left(\frac{M_P}{\sigma} \right)^2$, $\eta \equiv \frac{M_P^2}{V} \frac{\partial^2 V}{\partial \sigma^2} \simeq -\frac{g^2}{4\pi^2} \left(\frac{M_P}{\sigma} \right)^2$

➡ $\epsilon_{\text{CMB}} = \frac{g^2}{16\pi^2} \frac{1}{N_{\text{CMB}}}$, $\eta_{\text{CMB}} = -\frac{1}{2N_{\text{CMB}}}$ i.e. $\epsilon \ll |\eta|$

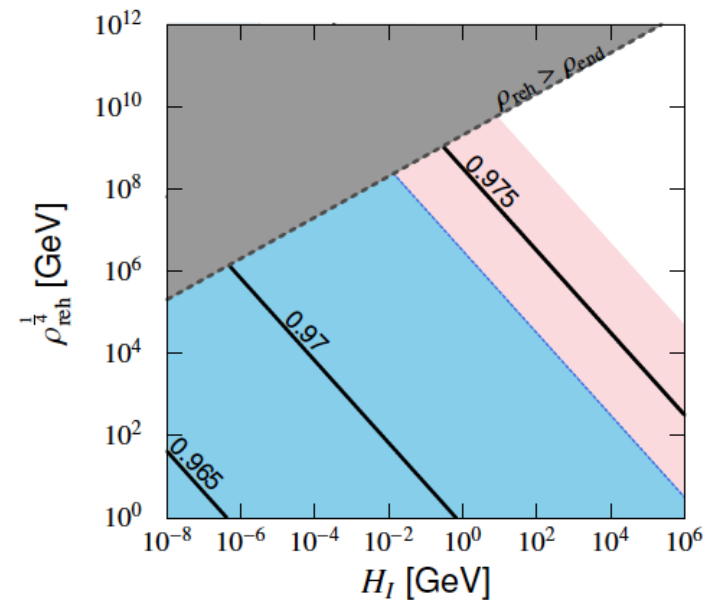
Inflation check list

✓ Spectral tilt

$$n_s - 1 \simeq 2\eta = -0.026 \left(\frac{39}{N_{\text{CMB}}} \right)$$

$$N_{\text{CMB}} \simeq 38.9 + \frac{1}{3} \ln \left(\frac{H_I}{10^5 \text{ GeV}} \right) + \frac{1}{3} \ln \left(\frac{\rho_{\text{reh}}^{1/4}}{100 \text{ GeV}} \right)$$

$$\Rightarrow H_I \lesssim 10^5 \text{ GeV} \quad \boxed{\text{Low-scale inflation!}}$$



✓ Density perturbations

$$\sqrt{\xi} \simeq 9.0 \times 10^{15} \text{ GeV} \times \left(\frac{1 - n_s}{0.03} \right)^{\frac{1}{4}} \left(\frac{A_s}{2.1 \times 10^{-9}} \right)^{\frac{1}{4}} \Rightarrow g \simeq 7.4 \times 10^{-9} \times \left(\frac{H_I}{10^5 \text{ GeV}} \right) \left(\frac{1 - n_s}{0.03} \right)^{-\frac{1}{2}} \left(\frac{A_s}{2.1 \times 10^{-9}} \right)^{-\frac{1}{2}}$$

✓ Cosmic strings

U(1) broken during inflation via dynamical D-terms

✓ Reheating

Energy stored in ϕ_+ requires cancellation of large $\langle \phi_+ \rangle$: $-\mathcal{L} \supset \frac{1}{2} \kappa_1 \kappa_2 \sin 2\beta \langle M_- \rangle \phi_+ h^2$

$$\Rightarrow T_R = 485 \text{ GeV} \times \left(\frac{106.75}{g_\rho} \right)^{1/4} \left(\frac{\langle M_- \rangle}{10^{16} \text{ GeV}} \right) \left(\frac{10^8 \text{ GeV}}{m_{\phi_+}} \right)^{1/2} \left(\frac{\kappa_1}{10^{-9}} \right) \left(\frac{\kappa_2}{10^{-8}} \right)$$

Constraints

- Inflaton, relaxion slow roll

$$|m_S| \ll H_I \quad \frac{d\sigma}{dt} = -\frac{1}{3H_I} \frac{\partial V}{\partial \sigma} = -\frac{1}{3H_I} \left[m_T^2 \sigma - \frac{g_T}{\sqrt{2}} \Lambda_N^3 \cos\left(\frac{\phi}{f_\phi}\right) \right] \quad \Rightarrow \quad m_T^2 \sigma \gg \frac{g_T}{\sqrt{2}} \Lambda_N^3$$

- Stability of relaxion minimum $|m_S| \lesssim \frac{v^4}{m_{\text{SUSY}} f_\phi^2}$

- Classical rolling $\frac{d\phi}{dt} H_I^{-1} > H_I \quad \Rightarrow \quad H_I^3 \ll \frac{g_S}{|g_T|} |m_T|^2 \phi_*$

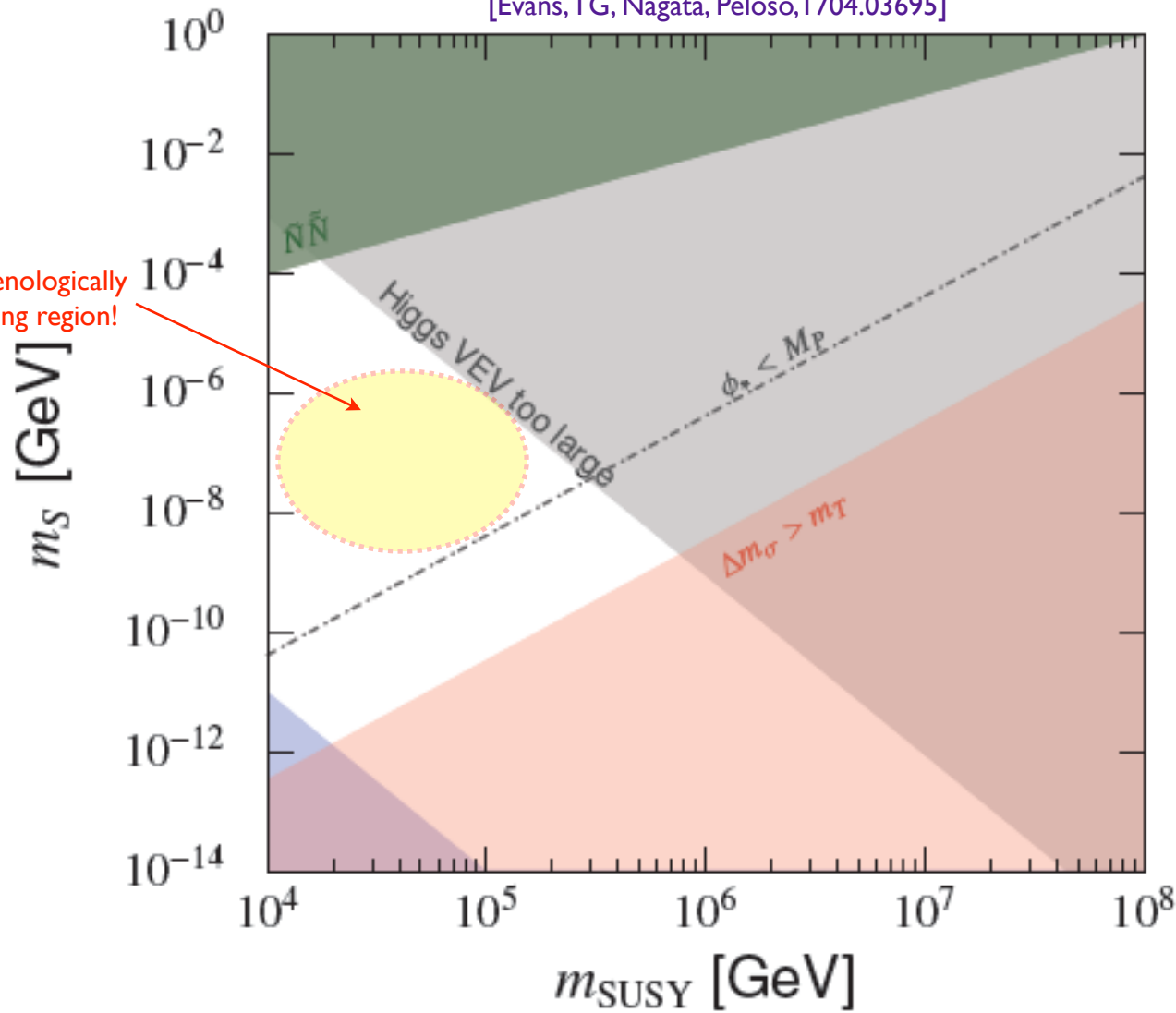
- Sufficient number of e-folds $N_e \simeq \frac{H_I \Delta\phi}{\left| \frac{d\phi}{dt} \right|} \gtrsim \frac{H_I^2}{|m_S|^2} = 10^{14} \times \left(\frac{H_I}{1 \text{ GeV}} \right)^2 \left(\frac{10^{-7} \text{ GeV}}{|m_S|} \right)^2$

- Loop corrections to inflaton mass

$$\Delta K \simeq \frac{\kappa^2}{16\pi^2} |T|^2 \quad \Rightarrow \quad \Delta m_\sigma \simeq 3.3 \times 10^{-12} \text{ GeV} \times \left(\frac{\kappa}{10^{-2}} \right) \left(\frac{m_{\text{SUSY}}}{10^5 \text{ GeV}} \right) \left(\frac{f_\phi}{10^5 \text{ GeV}} \right) < m_T$$

[Evans, TG, Nagata, Peloso, I 704.03695]

Phenomenologically
interesting region!



$$g_S = \zeta \frac{m_S}{f_\phi}, \quad g_T = \zeta \frac{m_T}{f_\sigma}, \quad f \equiv f_\phi = f_\sigma,$$

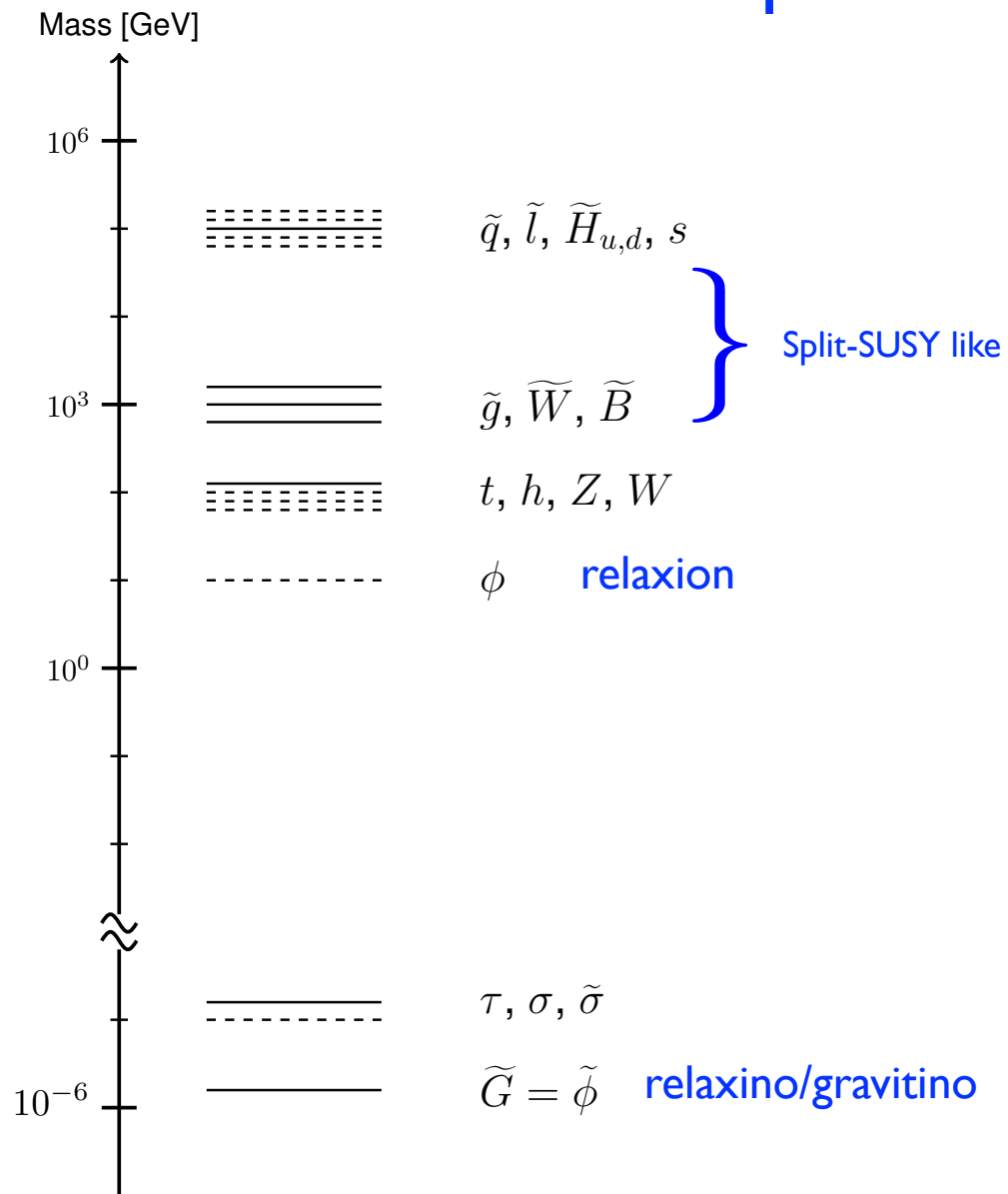
$$r_{TS} \equiv \frac{m_T}{m_S}, \quad r_\Lambda \equiv \frac{\Lambda_N}{f}, \quad r_{\text{SUSY}} \equiv \frac{m_{\text{SUSY}}}{f},$$

$$\zeta = 10^{-8}, r_{TS} = 0.1$$

$$r_\Lambda = 1, r_{\text{SUSY}} = 1, \kappa = 10^{-2}$$

$$\max \left\{ |m_S|, 4 \times 10^{-9} \text{ GeV} \times \left(\frac{m_{\text{SUSY}}}{10^5 \text{ GeV}} \right)^2 \left(\frac{1}{r_{\text{SUSY}}} \right) \right\} < H_I < 4.6 \text{ GeV} \times \left(\frac{r_{TS}}{0.1} \right)^{\frac{1}{3}} \left(\frac{1}{r_{\text{SUSY}}} \right)^{\frac{1}{3}} \left(\frac{|m_S|}{10^{-7} \text{ GeV}} \right)^{\frac{1}{3}} \left(\frac{m_{\text{SUSY}}}{10^5 \text{ GeV}} \right)^{\frac{2}{3}}$$

Generic particle spectrum:



Features:

- i) No SUSY flavor problem
- ii) Preserves gauge coupling unification
- iii) Relaxino/Gravitino dark matter
- iv) Collider signal long-lived NLSP decay
- v) Higgs-relaxion mixing

Long-lived Particles [in preparation]

- Higgs can mix with the relaxion-inflation sector

Higgs-scalar: $\mathcal{L} \supset 2\lambda' \sin\left(\frac{\phi_0}{f} + \delta\right) \frac{v^2}{f} h^2 \phi$ relaxion $-\mathcal{L} \supset \kappa'_2 m_{\phi_+} \phi_+ h^2 + h.c.$ “waterfall” field

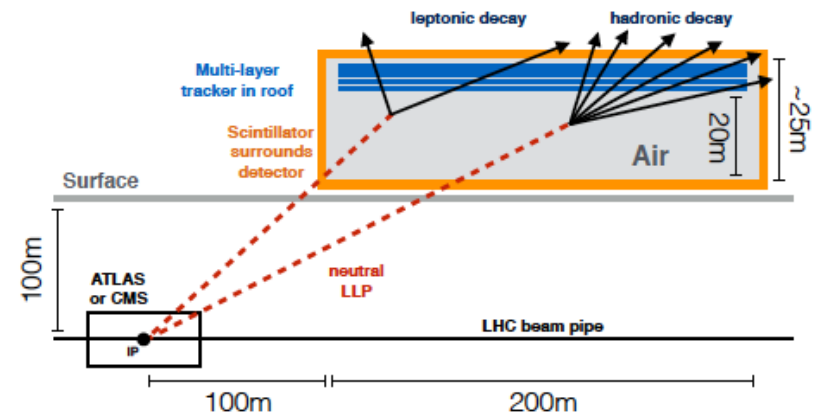
Higgsino-relaxino: $\mathcal{L} \supset \kappa_2 \tilde{R} (v_u \tilde{H}_d + v_d \tilde{H}_u) + h.c.,$

⇒ $\Gamma_{\lambda_3 \rightarrow h \tilde{R}} \simeq (150 \text{ m})^{-1} \left(\frac{|\kappa_2|}{10^{-7}}\right)^2 \left(\frac{10 \text{ TeV}}{\mu}\right)^2 \left(\frac{c_{2\beta}}{0.5}\right)^2 \left(\frac{M_2}{1 \text{ TeV}}\right)$

- Direct axion-like coupling: $\mathcal{L} \supset \int d^2\theta \frac{C}{32\pi^2} \frac{S}{f} W^{a\alpha} W_\alpha^a + h.c.$

⇒ $\Gamma_{\lambda^a \rightarrow \tilde{S} + \gamma/Z} = (171 \text{ m})^{-1} \left(\frac{|C|}{1}\right)^2 \left(\frac{M_{\lambda^a}}{1 \text{ TeV}}\right)^3 \left(\frac{3 \times 10^8 \text{ GeV}}{f}\right)^2$

Possible signal at
MATHUSLA experiment!



Summary

- Inflaton-relaxion dynamics can explain SUSY scale up to 10^6 GeV
 - preserves QCD axion solution to strong CP problem
 - “naturalizes” supersymmetry
- Predicts split-SUSY-like + “invisible” spectrum
 - relaxino/gravitino = dark matter
 - “invisible” spectrum = sign of dynamical relaxation!
- UV completion possible with multi-axion/clockwork fields

Questions/Future Work

- What fixes the scale of the explicit breaking?
 - PeV scale: $10^{-10} \text{ GeV} \lesssim m_S \lesssim 10^{-4} \text{ GeV}$
- Alternate ways to generate periodic potential?
- Other ways to incorporate inflation? e.g. F-term inflation
- Cosmological constant
 - how to reconcile large number of vacua?
 - requires non-anthropropic solution?
- Ways to search for “invisible” sector
 - beam dump experiments, superradiance, pulsar timing array, MATHUSLA....

Extra slides

Generation of periodic potential

Assume SU(N) gauge theory with singlet superfields $N, \bar{N}$

$$W_N = m_N N \bar{N} + i g_S S N \bar{N} + i g_T T N \bar{N} + \frac{\lambda}{M_L} H_u H_d N \bar{N}$$

➡ $\mathcal{L}_N = -m_N \bar{\psi}_N \psi_N - \frac{i}{\sqrt{2}} g_S (s + i\phi) \bar{\psi}_N \psi_N - \frac{i}{\sqrt{2}} g_T (\tau + i\sigma) \bar{\psi}_N \psi_N - \frac{\lambda}{M_L} H_u H_d \bar{\psi}_N \psi_N + \text{h.c.}$

Fermion condensate: $\langle \bar{\psi}_N \psi_N \rangle \simeq \Lambda_N^3$ $\Lambda_N = \text{confinement scale}$

$$\bar{\psi}_N \psi_N \rightarrow e^{i \frac{\phi}{f_\phi}} \bar{\psi}_N \psi_N \quad (\text{eliminates } \frac{\phi}{f_\phi} G'_{\mu\nu} \tilde{G}'^{\mu\nu})$$

➡ $V_{\text{period}} = \mathcal{A}(\phi, \sigma, H_u H_d) \Lambda_N^3 \cos\left(\frac{\phi}{f_\phi}\right)$

where $\mathcal{A}(\phi, \sigma, H_u H_d) = \bar{m}_N - \frac{g_S}{\sqrt{2}} \phi - \frac{g_T}{\sqrt{2}} \sigma + \frac{\lambda}{M_L} H_u H_d$ g_S, g_T real
 $\bar{m}_N = \text{effective mass}$

Supergravity effects

For super-Planckian field excursions

$$V = e^{K/M_P^2} \left(\underbrace{D^i W^* D_i W}_{\sim m_T^2 \sigma^2} - \underbrace{\frac{3|W|^2}{M_P^2}}_{\sim \frac{m_T^2 \sigma^4}{M_P^2}} \right)$$

Requires no-scale SUSY breaking with field X

$$V = e^{K/M_P^2} \left(W^{*i} W_i + \frac{1}{M_P^2} \underbrace{(W^{*i} K_i W + \text{h.c.})}_{W_X \simeq 0} + \underbrace{(K^i K_i - 3M_P^2)}_{\simeq 0} \frac{|W|^2}{M_P^4} \right)$$

Gravitino

$$m_{3/2} = \frac{F}{\sqrt{3} M_P} \simeq 2 \times \left(\frac{F}{F_S} \right) \left(\frac{m_{\text{SUSY}}}{10^5 \text{ GeV}} \right) \left(\frac{f_\phi}{10^5 \text{ GeV}} \right) \text{ eV}$$

sub-Planckian $F = F_S$

relaxino eaten by gravitino

super-Planckian $F > F_S$

relaxino, no longer Goldstino, remains light

}

Can be dark matter!

UV completion

[Based on Kaplan, Rattazzi:1511.01827]

Consider set of chiral superfields $\phi_i, \bar{\phi}_i, S_i (i = 0, \dots, N)$

$$W_{UV} = \sum_{i=0}^N \lambda_i S_i (\underbrace{\phi_i \bar{\phi}_i - f_i^2}_{\text{spontaneous breaking}}) + \epsilon \sum_{i=0}^{N-1} (\underbrace{\bar{\phi}_i \phi_{i+1}^2 + \phi_i \bar{\phi}_{i+1}^2}_{\text{explicitly breaks } U(1)^{N+1} \text{ to } U(1)})$$

$$\phi_i = f_i e^{\frac{\Pi_i}{f_i}}, \quad \bar{\phi}_i = f_i e^{-\frac{\Pi_i}{f_i}}$$

Massless mode:

relaxion $\phi \supset S = c_N \sum_{i=0}^N \frac{f_i}{2^i f_0} \Pi_i$

Identify remnant $U(1)$ as shift symmetry $\mathcal{S}_S$

$y \phi_0 \bar{\psi}_0 \psi_0$ coupling $\Rightarrow f_\phi \sim f_0 \quad V_\phi \sim V_0 \propto \cos \frac{\phi}{f_\phi}$

$y' \phi_N \bar{\psi}_N \psi_N$ coupling $\Rightarrow V_N \propto \tilde{\Lambda}_N^4 \cos \left(\frac{\phi}{2^N f_\phi} \right) \simeq \tilde{\Lambda}_N^4 - \frac{1}{2} \frac{\tilde{\Lambda}_N^4}{4^N f_\phi^2} \phi^2 + \dots$
 $= |m_S|^2 !$

Similarly:

$i \frac{\kappa}{\tilde{M}_N^2} \int d^4 \theta N \bar{N} \Xi^* \bar{\Xi}^* + \text{h.c.} \Rightarrow i \frac{\kappa}{\tilde{M}_N^2} \int d^2 \theta \tilde{\Lambda}_N^3 e^{\frac{\Pi_N}{f_N}} N \bar{N} + \text{h.c.} \simeq \int d^2 \theta \frac{i \kappa \tilde{\Lambda}_N^3}{f_\phi 2^N \tilde{M}_N^2} S N \bar{N} + \text{h.c.}$
 $= g_S !$