Quantifying theoretical uncertainties

the case of the inclusive Higgs cross section

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LHC: a new precision machine?

LHC: built to be discovery machine

- Higgs boson: discovered
- BSM particles: nothing yet

Direct discovery:
requires a decent signal to be identified over a huge background

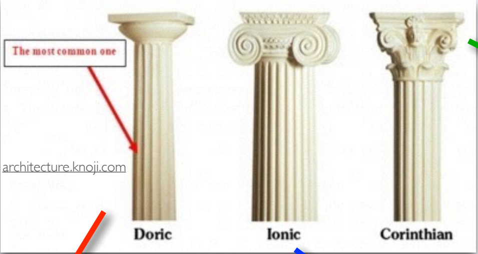
Indirect discovery:
identify small deviations from the Standard Model expectations

In both cases, the keyword is: **precision**
Need very precise measurements *and* theoretical predictions!
Partly, the first depend on the second

LHC $\rightarrow$ protons $\rightarrow$ QCD $\rightarrow$ precision is a challenge!

Simone Marzani

The three pillars of LHC pheno



Resummation

- well defined and improvable accuracy (examples up to N^3LL)
- provide insights and understanding
- feasible for a limited number of observables (low automation)

Parton showers

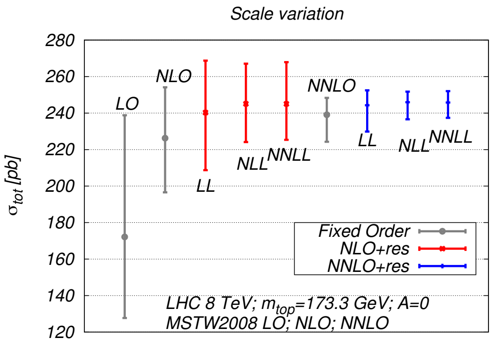
- powerful general-purpose tools
- provide fully differential events
- interfaced with non-perturbative models to give a realistic description
- all-orders, but theoretical accuracy beyond leading-log difficult to assess

Fixed-order

- exploit QCD pert. expansion
- NLO highly automated
- can provide fully differential events
- NNLO revolution on its way
- N^3LO examples

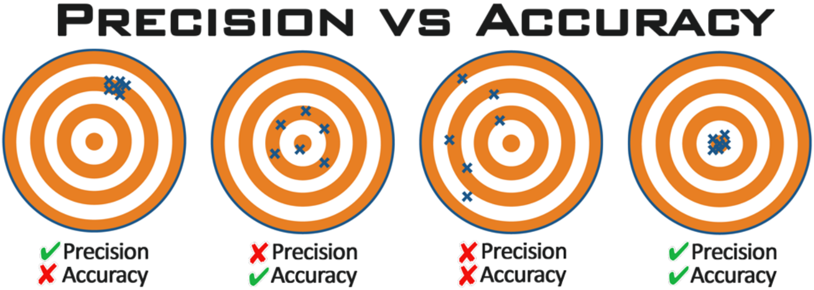
A successful example: $t\bar{t}$ total cross section

[Czakon,Fiedler,Mitov,Rojo 1305.3892]



Perturbative expansion converges nicely
Theoretical uncertainty shrinks increasing the order

Precision is nothing without accuracy



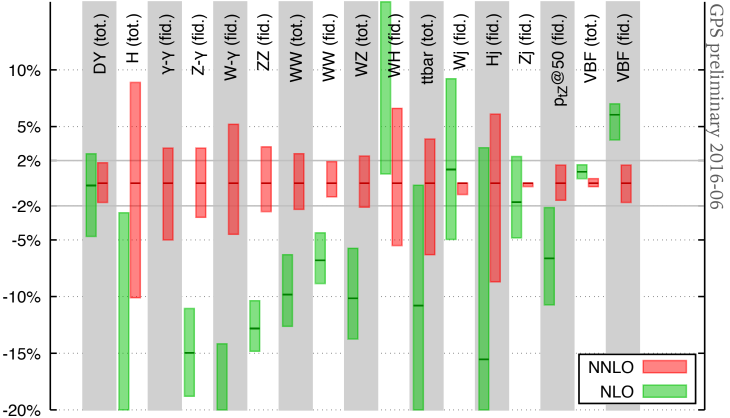
A very precise (small uncertainty) determination of a cross section which is far from the "true" value is not good for anyone...

A realistic determination of the theory uncertainties is preferred/mandatory!

How accurate are current theoretical predictions?

WHAT PRECISION AT NNLO?

Slide from Gavin Salam, PSR 2016



For many processes NNLO scale band is ~±2%

Though only in 3/17 cases is NNLO (central) within NLO scale band…

How do we determine theory uncertainties?

Fact: theory computations depend on unphysical scales (factorization $\mu_{\rm F}$, renormalization $\mu_{\rm R}$, ...)

$$\sigma(m_H) = \sum_{i,j} C_{ij}\left(\alpha_s(\mu_{\rm R}), \frac{\mu_{\rm R}}{m_H}, \frac{\mu_{\rm F}}{m_H}\right) \otimes f_i(\mu_{\rm F}) \otimes f_j(\mu_{\rm F})$$

These scales are fictitious, and the scale dependence is formally higher order (i.e., it vanishes to all orders)

Idea: vary the scale(s) about a given arbitrary central value to probe higher orders

Canonical method: **Scale variation**

Scale variation

Pros:

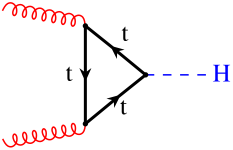
- simple
- applicable to all processes

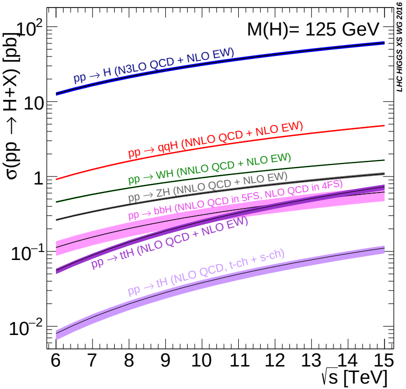
Caveats:

- which central scale? → no large logs in C
- how much should I vary the scale? → canonical factor 2
- the variation only probes a limited class of higher order terms: is it significant? → it depends (e.g., other channels)
- how do I construct an uncertainty from the variations?
  → envelope, or symmetrized max deviation
- how do I interpret the uncertainty? → good question...
- what if I'm close to a stationary point? → good luck!

Higgs production in gluon fusion at LHC

Dominant production mechanism



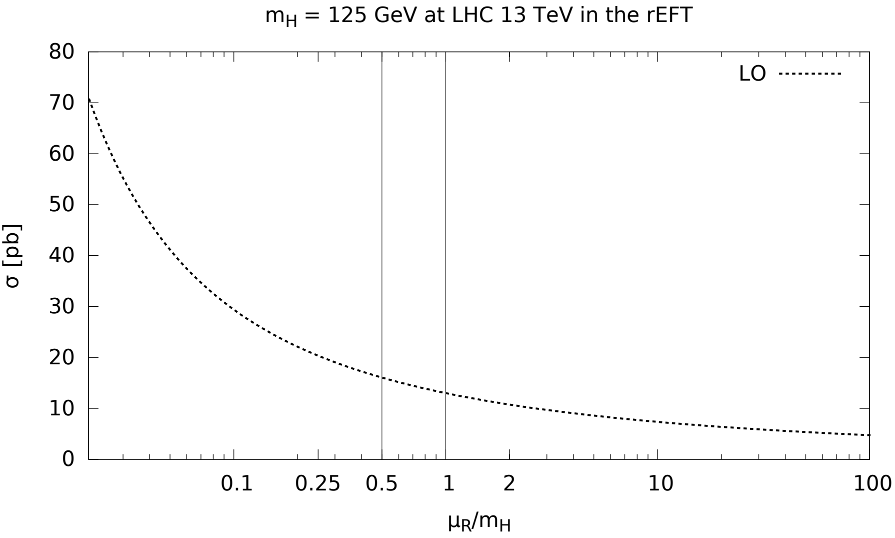


(one order of magnitude larger than vector-boson fusion)

Recently computed to N^{3}LO

[Anastasiou,Duhr,Dulat,Furlan,Gehrmann,Herzog,Lazopoulos,Mistlberger 1602.00695]

Higgs in gluon fusion at LHC: perturbative (in)stability



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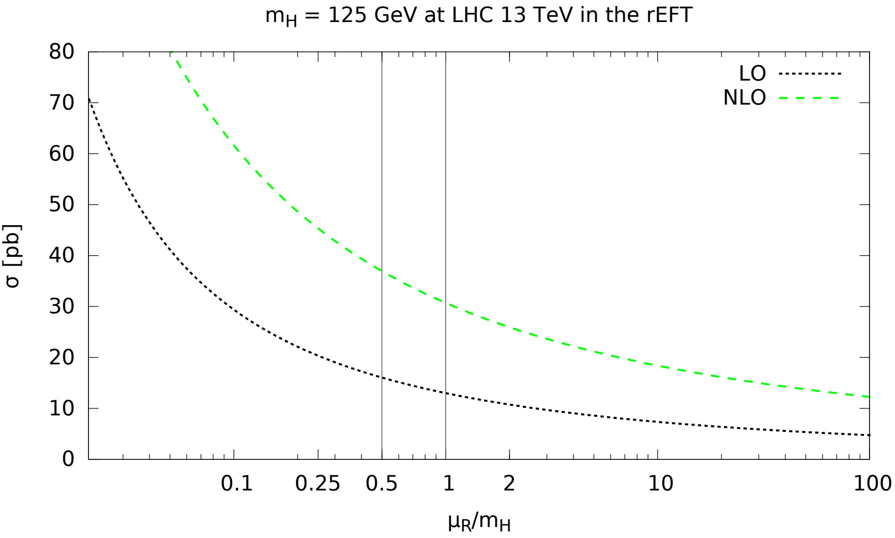

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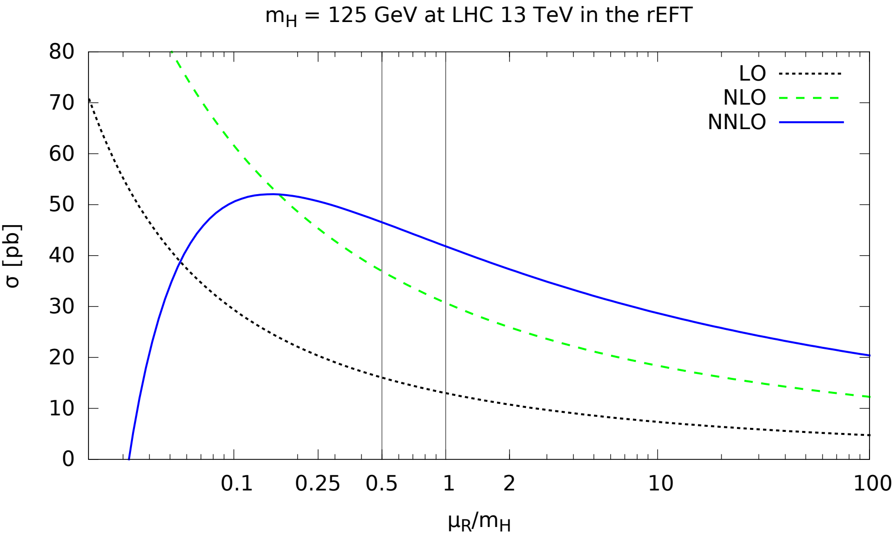

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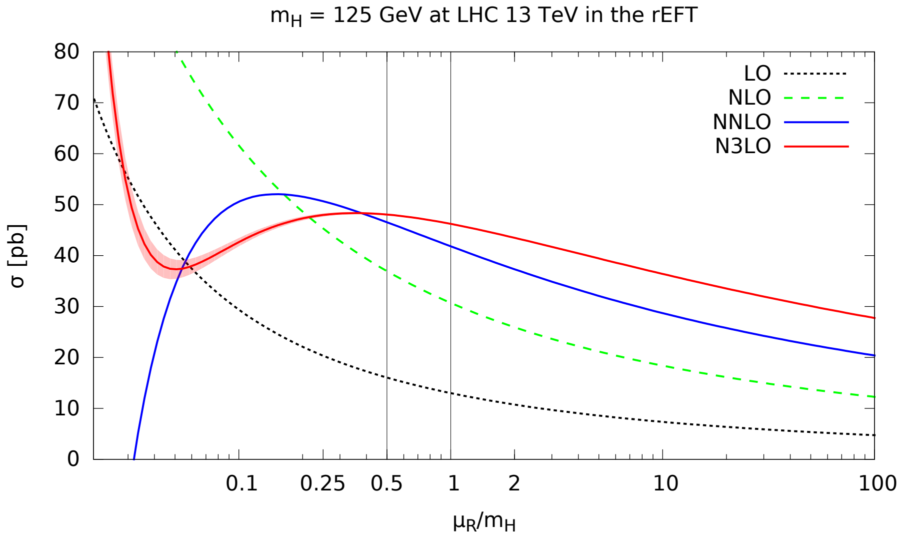
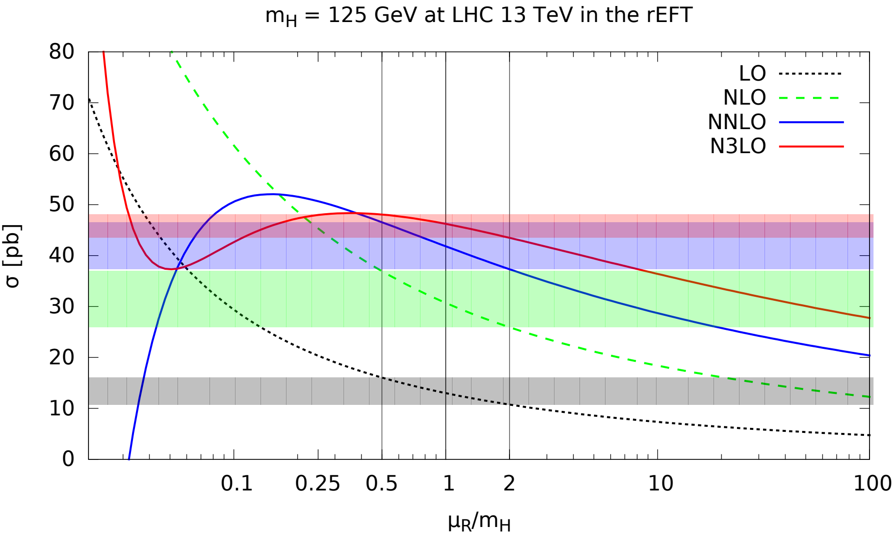
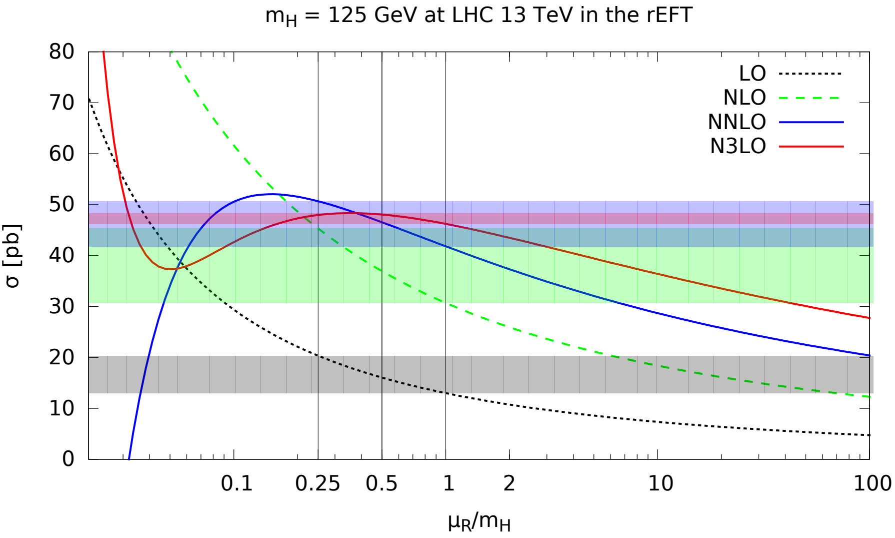


Higgs in gluon fusion at LHC: perturbative (in)stability



$$1/2 < \mu_{\mathrm{R}}/m_H < 2$$

Higgs in gluon fusion at LHC: perturbative (in)stability



$$1/4 < \mu_{\mathrm{R}}/m_H < 1$$

Scale variation not promising...

Canonical scale variation by a factor of 2 fails for ggH and many others

The problem seems to lie on the large perturbative corrections compared with the size of scale dependent terms.

Possible solutions:

- symmetrising is always better (in vicinity of a stationary point)
- increase the scale variation range (factor 4? 10?)
- revert to a different perturbative expansion $\leftarrow$
- consider other sources of uncertainty from missin higher orders $\leftarrow$
- attack the problem in a completely different way $\leftarrow$

Comment: having a statistical interpretation is useful for including theory uncertainties in PDF fits.

Reverting to a different perturbative expansion

The fixed order expansion is just a way to organize the perturbative series

$$C(\alpha_s) = \sum_k \alpha_s^k \, C^{(k)}$$

Nothing prevents us to reshuffle the terms in the series

I'll consider two options:

- resummation: it's an expansion in α_s at fixed $\alpha_s \log(\text{something})$
- non-linear transformations for convergence acceleration

Why resummation?

In general, perturbative coefficients contain logarithms of scaleless ratios. Sometimes, these are logarithmically enhanced:

$$\begin{aligned} \alpha_s &: \quad \log X & X = E_1/E_2 \\ \alpha_s^2 &: \quad \log^2 X,\ \log X \\ \alpha_s^3 &: \quad \log^3 X,\ \log^2 X,\ \log X \\ \alpha_s^n &: \quad \log^n X, \quad \ldots, \quad \log X \end{aligned}$$

If/when $\alpha_s \log X \sim 1 \rightarrow$ fixed-order expansion no longer predictive!

Resum the logs, and convert to a "logarithmic-order" expansion:

$$g_{\rm LL}(\alpha_s \log X) + \alpha_s\, g_{\rm NLL}(\alpha_s \log X) + \alpha_s^2\, g_{\rm NNLL}(\alpha_s \log X) + \ldots$$

Leading Log (LL), Next-to-Leading Log (NLL), Next-to-Next-to-Leading Log (NNLL)...

Threshold resummation

Inclusive cross section: the scales are

- m_H: Higgs mass
- $\sqrt{\hat{s}}$: partonic center of mass energy

Scaleless ratio:

$$z = \frac{m_H^2}{\hat{s}} \qquad \begin{cases} z\hat{s} \to \text{energy that flows into the Higgs} \\ (1-z)\hat{s} \to \text{energy for extra radiation} \end{cases}$$

Multiple gluon emissions induce terms

$$\alpha_s^n \left(\frac{\ln^k(1-z)}{1-z}\right)_+, \qquad 0 \le k \le 2n-1$$

large in the *partonic threshold (soft) limit* $\hat{s} \sim m_H^2$, when the remaining available energy for gluon radiation $(1-z)\hat{s}$ is low (*soft* gluons).

In Mellin N space (soft region: $N \to \infty$)

$$\alpha_s^n \ln^k N, \qquad 0 \le k \le 2n$$

Improved threshold resummation

Standard dQCD resummation gives

$$C_{gg}(N,\alpha_s) \overset{N\to\infty}{=} g_0\Big(\alpha_s, \tfrac{m_H}{m_t}\Big) \times \exp \mathcal{S}(\alpha_s, \ln N) + \mathcal{O}(1/N)$$

$$\alpha_s \mathcal{S}(\alpha_s, \ln N) = g_1(\alpha_s \ln N) + \alpha_s g_2(\alpha_s \ln N) + \alpha_s^2 g_3(\alpha_s \ln N) + \alpha_s^3 g_4(\alpha_s \ln N) + \ldots$$

[Sterman 1987] [Catani, Trentadue 1989] [Forte, Ridolfi 2003]

Resums $\ln^j N$, contains constants (corresponding to $\delta(1-z)$), and nothing else.

Resummation doesn't fix subleading contributions suppressed by $1/N$!

Can be improved taking into account [MB,Marzani 2014]

- exact single gluon emission kinematics

$$\ln N \to \psi_0(N)$$

- collinear contributions from the full splitting function P_{gg}

$$N \to N+1 \qquad \text{(in its simplest form)}$$

- exponentiation of (some) constants

Benefits of using a resummed expansion

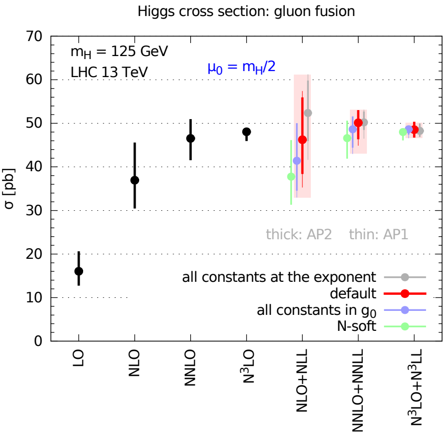
Resummed expansion (valid in the limit of large log) is then matched to the fixed-order expansion (valid, hopefully, elsewhere).

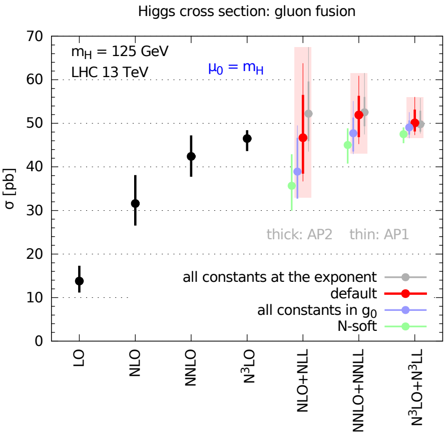
The resummed and matched series allows to:

- speed up perturbative convergence
- estimate theory uncertainty from
  - scale variation
  - subleading-log (all-order!) contributions (moving constants in the exponent)
  - subleading-power (all-order!) contributions (changing $1/N$ contributions)

Our proposal: [MB,Marzani,Muselli,Rottoli 2016]
vary all these things (42 variations) and take an envelope

Higgs resummed expansion





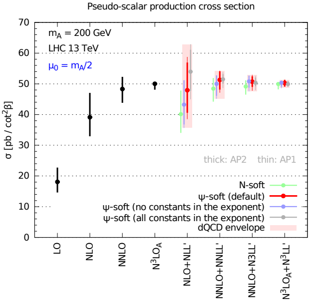
Perturbative convergence sped up!
Reduction of theory uncertainty increasing the order
Less sensitivity to central scale
More reliable uncertainty estimate (statistical interpretation still missing...)

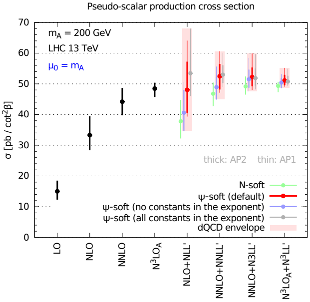
Results

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
LO	$18.6^{+5.8}_{-3.9}$	$16.0^{+4.3}_{-3.1}$	$13.8^{+3.2}_{-2.4}$	$11.9^{+2.5}_{-1.9}$
NLO	$44.2^{+12.0}_{-8.5}$	$36.9^{+8.4}_{-6.2}$	$31.6^{+6.3}_{-4.8}$	$27.5^{+4.9}_{-3.9}$
NNLO	$50.7^{+3.4}_{-4.6}$	$46.5^{+4.2}_{-4.7}$	$42.4^{+4.6}_{-4.4}$	$38.6^{+4.4}_{-4.0}$
N^3LO	$48.1^{+0.0}_{-7.5}$	$48.1^{+0.1}_{-1.8}$	$46.5^{+1.6}_{-2.6}$	$44.3^{+2.5}_{-2.9}$
LO+LL	$24.0^{+8.9}_{-6.8}$	$20.1^{+6.2}_{-5.0}$	$16.9^{+4.5}_{-3.7}$	$14.3^{+3.3}_{-2.8}$
NLO+NLL	$46.9^{+15.1}_{-12.6}$	$46.2^{+15.0}_{-13.2}$	$46.7^{+20.8}_{-13.8}$	$47.3^{+26.1}_{-15.8}$
NNLO+NNLL	$50.2^{+5.5}_{-5.3}$	$50.1^{+3.0}_{-7.1}$	$51.9^{+9.6}_{-8.9}$	$54.9^{+17.6}_{-11.5}$
N^3LO+N^3LL	$47.7^{+1.0}_{-6.8}$	$48.5^{+1.5}_{-1.9}$	$50.1^{+5.9}_{-3.5}$	$52.9^{+13.1}_{-5.3}$

Same method applied to pseudo-scalar Higgs production

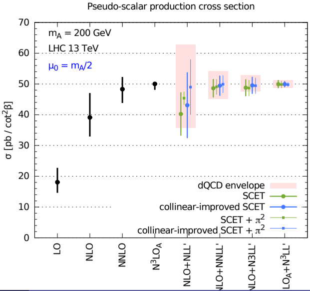
dQCD→

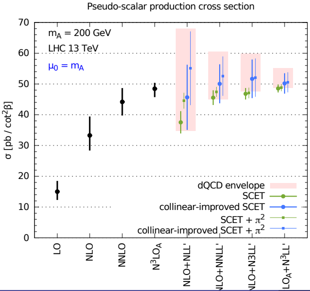




[Ahmed,MB, Kumar,Mathews, Rana,Ravindran, Rottoli 2016]

SCET→





Non-linear sequence transformations

NONLINEAR SEQUENCE TRANSFORMATIONS FOR THE ACCELERATION OF CONVERGENCE AND THE SUMMATION OF DIVERGENT SERIES

Ernst Joachim WENIGER

Given a sequence $s_1, s_2, ..., s_n, ...$ (partial sums of a perturbative expansion) with

$$\lim_{n \to \infty} s_n = s$$

(all-order cross section) one can speed up the convergence by applying a non-linear transformation to the sequence. Used e.g. for determination of special functions from series representations.

Caveat: if you know all the terms in the expansion, you can select the best algorithm, and check the convergence; but we only know the first few terms of the perturbative series...

One formula to describe them all

$$\mathcal{G}_k^{(n)}(q_m, s_n, \omega_n) = \frac{\displaystyle\sum_{j=0}^{k} (-1)^j \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_m}{n+k+q_m} \frac{s_{n+j}}{\omega_{n+j}}}{\displaystyle\sum_{j=0}^{k} (-1)^j \binom{k}{j} \prod_{m=1}^{k-1} \frac{n+j+q_m}{n+k+q_m} \frac{1}{\omega_{n+j}}}$$

From different forms of q_m and ω_n many algorithms are generated/recovered.

$$s \stackrel{?}{=} \lim_{k+n\to\infty} \mathcal{G}_k^{(n)}$$

Application to the Higgs cross section

Idea 1: Choose some "good" algorithms and compute a guess for the all-order cross section [David,Passarino 2013]

Idea 2: Choose many algorithms and compute many guesses (~ 100) for the all-order cross section. Do it for several choices of the central scale (the sum must be the same). Observe. Decide. [MB,Marzani,Muselli,Rottoli 2016]

Note: statistical interpretation still missing, but not impossible...

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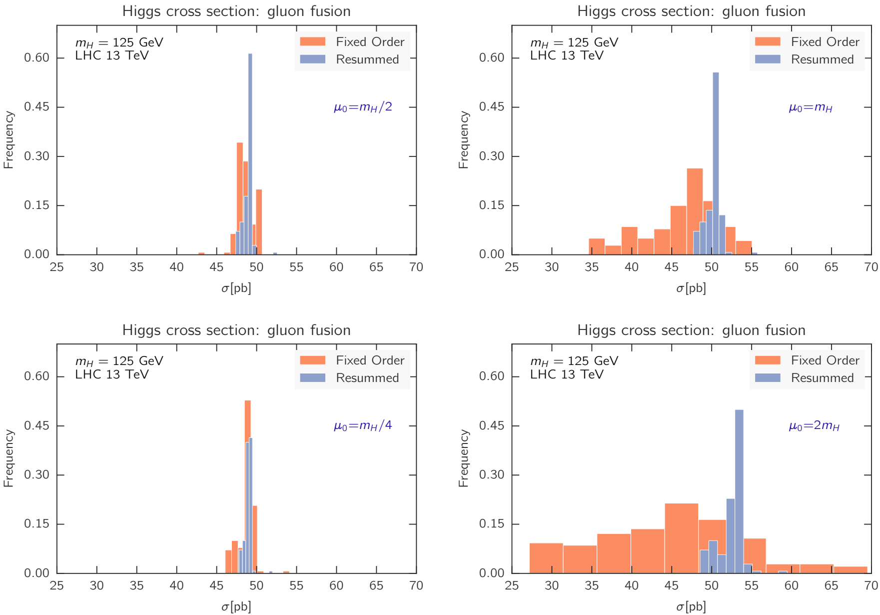
	$\mu_0 = m_{\rm H}/4$	$\mu_0 = m_{\rm H}/2$	$\mu_0 = m_{\rm H}$	$\mu_0 = 2m_{\rm H}$
Fixed-order expansion	48.7 ± 1.0	48.7 ± 1.2	46.3 ± 4.6	44.6 ± 9.3
Resummed expansion	48.9 ± 0.5	48.9 ± 0.6	50.2 ± 1.0	52.6 ± 1.6

Accelerated Higgs cross section



A different approach based on Bayesian statistics

Cacciari and Houdeau (2011) proposed a statistical model for the interpretation of theory uncertainties, from which one can compute the uncertainty on the truncated perturbative series for a given degree of belief (DoB) given the first terms in the expansion.

$$\sigma = \sum_n c_n \alpha_s^n$$

Quoting from the original paper:
We make the assumption that all the coefficients c_n in a perturbative series share some sort of upper bound $\bar{c} > 0$ to their absolute values, specific to the physical process studied. The calculated coefficients will give an estimate of this $\bar{c}$, restricting the possible values for the unknown c_n.

The model assumes a prior for $f(\bar{c})$ and a prior for $f(c_n|\bar{c})$. Since a small number of c_n are known, a good prior is quite useful

The method outputs a probability density for σ: statistical interpretation!

Improvements of the CH method

To account for a possible power growth (CH)

$$\sigma = \sigma_{\rm LO} \sum_{k=0}^{\infty} c_k(\lambda) \left(\frac{\alpha_s}{\lambda}\right)^k$$

Considering also a factorial (renormalon) growth ($\overline{\rm CH}$)

[Bagnaschi,Cacciari,Guffanti,Jenniches 2014]

$$\sigma = \sigma_{\rm LO} \sum_{k=0}^{\infty} b_k(\lambda, k_0)\, (k+k_0)! \left(\frac{\alpha_s}{\lambda}\right)^k$$

Problem: determining λ

Solution 1: [Bagnaschi,Cacciari,Guffanti,Jenniches 2014]
survey over several observables (assumes λ is process-independent)

Solution 2: $\leftarrow$ [Forte,Isrgò,Vita 2013]
fit λ requiring the first known coefficients $c_k(\lambda)$ are of the same size

Solution 3: [MB,Borroni *(work in progress)*]
making λ a parameter of the model, with its own prior

CH: results

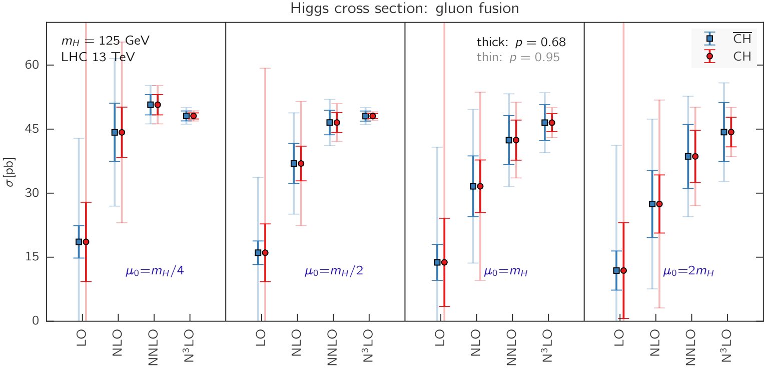


Figure 4. The CH (red) and $\overline{\text{CH}}$ (blue) errors on the LO, NLO, NNLO and N^{3}LO cross sections for the four scales $\mu_F = \mu_R = m_H/4, m_H/2, m_H, 2m_H$ (from left to right). For the four values of the scales, the fitted values of λ are respectively 0.44, 0.46, 0.24, 0.17 for CH and 1.08, 1.14, 0.58, 0.41 for $\overline{\text{CH}}$. Thicker bands correspond to 68% DoB, while thinner bands correspond to 95% DoB.

	$\mu_0 = m_H/4$	$\mu_0 = m_H/2$	$\mu_0 = m_H$	$\mu_0 = 2m_H$
CH	$48.1 \pm 0.7(1.2)$	$48.1 \pm 0.6(1.0)$	$46.5 \pm 2.1(3.5)$	$44.3 \pm 3.5(5.8)$
$\overline{\text{CH}}$	$48.1 \pm 1.2(1.9)$	$48.1 \pm 1.2(2.0)$	$46.5 \pm 4.2(7.0)$	$44.3 \pm 6.9(11.5)$

Something else?

Summary

- A careful treatment of theory uncertainties is mandatory for precision physics at LHC.
- A statistical definition of theory uncertainties will be also useful for PDF fits and experimental analyses
- Options:
  - Canonical scale variation often fails, and has no statistical interpretation
  - Non-canonical scale variation might not be the best way
  - Reverting to a different expansion (resummed, accelerated) is promising
  - Estimate other sources of unknown higher orders other than scale dependent terms
  - CH Bayesian approach gives a statistical interpretation of theory uncertainties, but room for improvements
- Future: construct a new statistical model, along the lines of CH, which uses more information to provide a better estimate