TEORIE DI CAMPO QUANTISTICHE

un approccio teorico alla descrizione dei fenomeni fondamentali della natura

dalle interazioni fondamentali alla fisica dello stato condensato e i fenomeni critici

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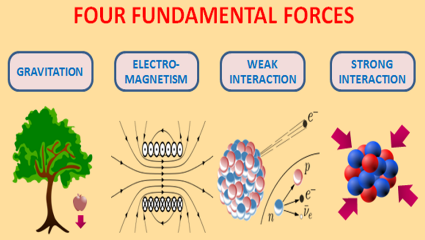
They are general frameworks to describe physical laws and fundamental paradigma in theoretical physics.

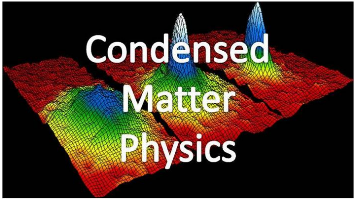
They are discussed, emphasizing their general principles, and their striking successes to describe fundamental interactions, condensed-matter physics, quantum many-body systems, statistical mechanics, critical phenomena, etc...

What is a Quantum Field Theory (QFT)?



QFT is the framework* in which the theories of fundamental interactions and particles are formulated

QFT provides powerful tools for condensed matter physics, quantum many-body problems in condensation phenomena, theory of metals, superconductivity, quantum liquids, quantum Hall effects, ...

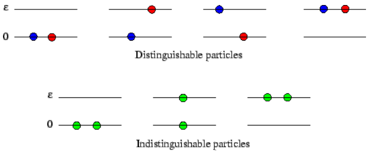


$\Longrightarrow$ Theoretical framework for quantum mechanical models of subatomic particles and quasiparticles or collective modes.

* Framework: class of theories sharing some characterizing features, which have important implications, leaving however the freedom of many different realizations/theories

How does QFT describe particles?

QFTs treat particles as indistinguishable excitations of an underlying physical field (like ocean waves)



For example:

- quantum electrodynamics has an electron field and a photon field
- in metals there is an atomic displacement field that gives rise to phonon particles

Quantization of Fields (functions of space-time)

- **Classical Particles** $\longrightarrow$ $m\vec{a} = \vec{F}$, $H = \frac{1}{2m}p^2 + V(x)$

- **Classical Electromagnetic Field Theory** $\longrightarrow$ Electromagnetic forces among charges, introducing electric and magnetic fields, $\vec{E}$ and $\vec{B}$,

$$\vec{\nabla} \cdot \vec{E} = \rho, \quad \vec{\nabla} \times \vec{B} - \partial_t \vec{E} = \vec{J}, \qquad \vec{E} = -\vec{\nabla} A_0 - \partial_t \vec{A}, \quad \vec{B} = \vec{\nabla} \times \vec{A},$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + J_\mu A^\mu \qquad \longrightarrow \qquad \partial_\mu F^{\mu\nu} = J^\nu$$

- **Quantum Mechanics of a Particle**, promoting x, p to operators:

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(\hat{x}), \qquad [\hat{x}, \hat{p}] = i\hbar$$

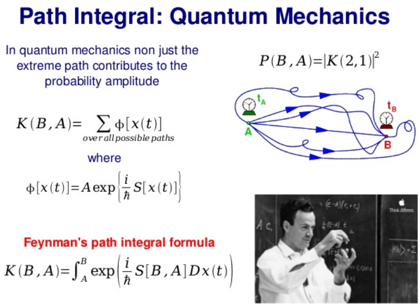
- **Quantization of a Field** $\phi(x)$ promoting it and $\pi = \partial\mathcal{L}/\partial_t\phi$ to operators, by imposing the canonical commutation relations

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi(x,t)\partial^\mu \phi(x,t) - \frac{1}{2}m^2 \phi(x,t)^2, \qquad [\phi(x,t), \pi(y,t)] = i\hbar\delta(x-y)$$

- **Quantization of the Electromagnetic Field**: from electromagnetic waves to photons (particles with zero mass, $E = h\nu = c|\vec{k}|$)

QFT and Statistical Field Theories are generally related by an analytic continuation of the time, from Minkowski to Euclidean space

- in **quantum mechanics** the amplitude $\langle \Psi' | e^{-itH} | \Psi \rangle$ can be obtained by a sum over paths $\longrightarrow$ PATH INTEGRAL



- in **statistical physics** the matrix density $\rho = \langle \Psi' | e^{-\beta H} | \Psi \rangle$ and the partition function $Z = \mathrm{Tr}\rho$ can be written as a PATH INTEGRAL with $t \to -i\tau, \quad \tau = \beta = 1/T$

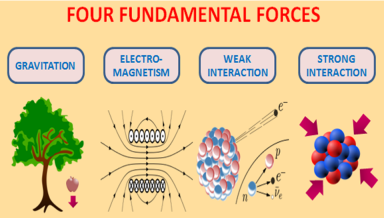
$\Longrightarrow$ Quantum Field Theories and Statistical Field Theories provide a unified framework which allows us to investigate issues related to quantum and thermal fluctuations

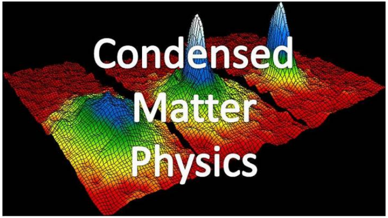
QFT and SFT provide powerful frameworks for high-energy physics and condensed-matter physics

Fundamental interaction theories: To describe microscopic phenomena at the most fundamental level

← **QFT** →

Condensed matter theories: To describe emergent macroscopic phenomena arising from microscopic laws of physics





We look for more and more fundamental theories able to explain phenomena at higher and higher energy, corresponding to smaller and smaller distance, from which all physical phenomena arise, driven by REDUCTIONISM

Big question: Does the ULTIMATE THEORY exist?

PRAGMATISM suggests: Theories should be considered as effective theories valid up to the actual physical scale of observation.

New discoveries and theories move such scale further and further

Molecules → Atoms → Electrons and Protons → quarks → ??

like moving further a wall, without being able to tear it down to look beyond

We know the interactions among the electrons and ions of a generic metal or insulator

$$H = H_{\rm e} + H_{\rm i} + H_{\rm ei}, \qquad H_{\rm e} = \sum_i \frac{p_i^2}{2m_e} + \sum_{i<j} V_{\rm ee}(x_i - x_j),$$

$$H_{\rm i} = \sum_I \frac{P_I^2}{2m_I} + \sum_{I<J} V_{\rm ii}(X_I - X_J), \qquad H_{\rm ei} = \sum_{iI} V_{\rm ei}(x_i - X_J).$$

thus the microscopic *Theory of Everything* is assumed known.

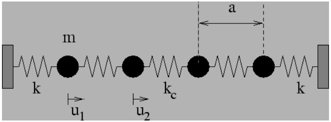
But there is a long way to describe collective behaviors of many $O(10^{23})$ interacting particles, characterized by universal macroscopic behaviors like liquid, solid, superfluids, superconductivity, ... $\longrightarrow$ EMERGENCE

EMERGENT PHENOMENA: new qualitative features appear when we go from microscopic to macroscopic systems.

the sound: from particles to field and then back to particle-like excitations

Dynamics of the ions at the sites of a crystal at macroscopic length scales $\ell \gg a$:

$$H = \sum_{i=1} \left[\frac{p_i^2}{2m} + \frac{\kappa}{2}(x_{i+1} - x_i - a)^2 \right]$$



$\phi_i = x_i - x_{\mathrm{eq},i}$, then continuum limit $a \to 0$, $\phi_i \to \phi(x) \implies$ classical field theory describing sound excitations with sound speed $v_s \sim \sqrt{\kappa/m}$

$$\mathcal{L} = \pi_i \dot{\phi}_i - H = \frac{1}{2}[\partial_t \phi(x,t)]^2 - \frac{v_s^2}{2}[\partial_x \phi(x,t)]^2 \qquad \implies \qquad \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi$$

After quantization, analogous to quantization of coupled oscillators, excitations above the ground state appear as massless scalar particles

$$p_\mu p^\mu = 0, \qquad c \to v_s$$

EMERGENCE of a Lorentz invariant QFT: we start from coupled oscillators and we end up with massless particles PHONONS

$\implies$ The low-energy physics of many-body systems may qualitatively differ from the microscopic one

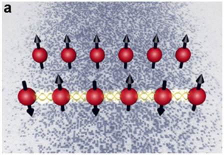
EMERGENCE: Simple interactions may give rise to complex phenomena

CRITICAL PHENOMENA $\longrightarrow$ physical systems develop long-distance correlations by an appropriate tuning some of the parameters, although the microscopic interactions are short ranged and very simple,

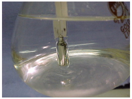
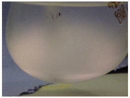
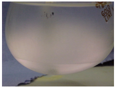
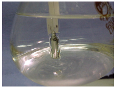
such as *agree/disagree* between nearest-neighbors at the sites of a lattice, but also ferromagnetism, liquid-vapor transition, neuron connections, ...

$$H_{\rm spin} = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j , \quad \sigma_i = \pm 1$$

$$H_{\rm gas} = -J \sum_{\langle ij \rangle} \rho_i \rho_j - \mu \sum_i \rho_i , \quad \rho_i = 0, 1$$



Critical opalescence in liquid systems, at their liquid-gas continuous transition, light diffusion when the correlation length increases from 10^{-9} m to 10^{-6} m

Phenomena described by a **QFT**: $\mathcal{L} = |\partial_\mu \varphi|^2 + r\,|\varphi|^2 + u\,|\varphi|^4$

How can QFT work in both high-energy and condensed matter physics?

The QFT approach to Condensed Matter Physics does not generally need the knowledge of the microscopic interactions,

but only some global properties, like the nature of the long-distance modes, symmetries, dimension, etc...

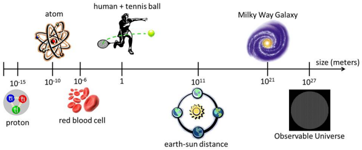
The resulting QFT effectively describes the behavior of the low-energy degrees of freedom.

Leaving aside the IDEA OF AN ULTIMATE THEORY, even the applications to fundamental interactions may be considered as emergent phenomena from unknown physics laws at smaller distance scales, once assuming some general symmetry properties

WHY do QFT frameworks work?

Hints from effective theories:

In the presence of very different scales of distance and energy,

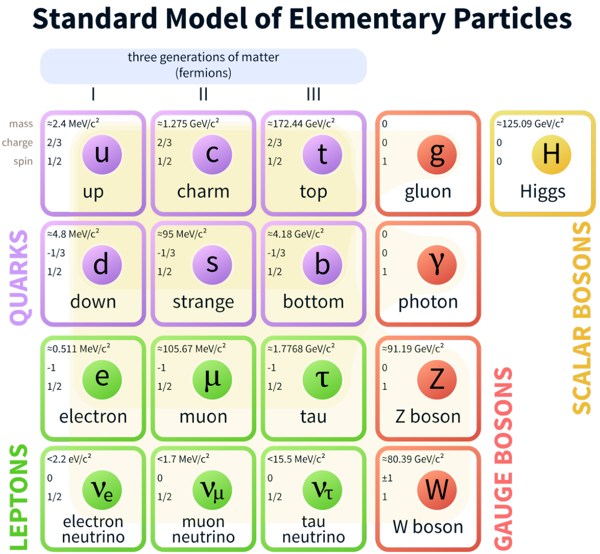


we focus on a limited number of degrees of freedom relevant at the scale of the phenomena considered, constraining their dynamics by appropriate symmetry principles.

QFT is a formidable framework to disentangle *relevant low-energy features* from the *irrelevant high-energy* degrees of freedom.

QFT works when different scales exist, providing effective descriptions at low energy, even though the microscopic mechanism remains unknown. It generally assumes that the scale of the unknown physics is much larger (close to the Planck scale?)

FUNDAMENTAL PARTICLES



Elementary particles associated with quantum fields

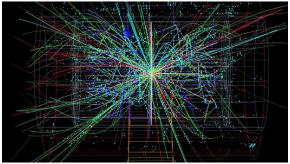
- STRONG INTERACTIONS, **QCD** (quarks and gluons)
$\mathcal{L}_{\rm QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{q}_f(i\gamma^\mu\partial_\mu - g\gamma^\mu A_\mu + m_f)q_f$

- ELECTRODYNAMICS, **QED** (+ photons, electrons, μ,τ)
$\mathcal{L}_{\rm QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}_f(i\gamma^\mu\partial_\mu - e\gamma^\mu A_\mu + m_f)\psi_f$

- **ELECTROWEAK** INTERACTIONS (+ neutrinos, W, Z, Higgs)
$\mathcal{L}_{\rm sm} = -\frac{1}{4}\vec{A}_{\mu\nu}\vec{A}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\psi}_f[(i\gamma^\mu\partial_\mu + m_f)\psi_f + eJ^\mu_\gamma A_\mu + g_W J^\mu_W W_\mu + g_Z J^\mu_Z Z_\mu + ... + (\partial_\mu\partial^\mu + M^2_H)H^2 + ...$

- **GRAVITY** (+ gravitons) $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu} + \rho_\Lambda g_{\mu\nu})$

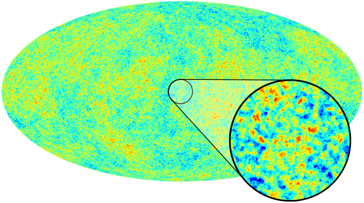
High-energy QFTs perform extremely well

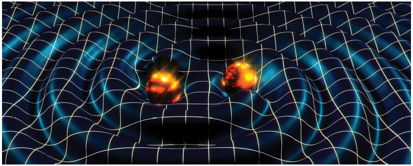


checked down to $\ell \approx 10^{-20}$ m

in high-energy experiments at colliders

and to large scales by observations of the universe





Essential features of high-energy QFTs

- **infinitely many degrees of freedom**, many-body systems

- the dynamical degrees of freedom are **field operators** $\Phi(\vec{x}, t)$, giving rise to **indistinguishable particles** (excitations over the ground state)

- **forces associated with particle exchanges**, e.g., photons in QED

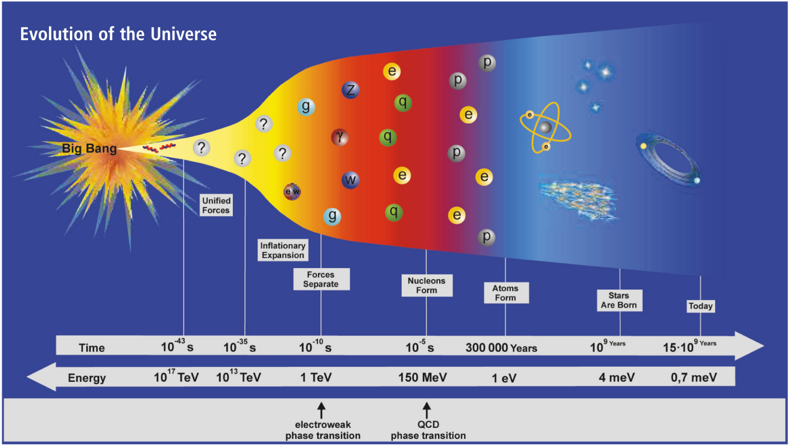
- **interactions are local**, they describe many-body processes with creation and destruction of elementary excitations (particles), **causality**

- **Lorentz invariance**: $\longrightarrow$ spin and statistics theorem: **particles of spin-1/2 are fermions, those with integer spin are bosons**
$\longrightarrow$ CPT theorem: **existence of antiparticles**

- **based on abelian and nonabelian gauge principles**

- **renormalizability** of the lowest-order polynomial interactions ensures predictivity of the perturbative (loop) expansion (for $E \ll \Lambda_{\text{new physics}}$)

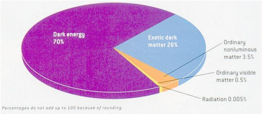
The theories of fundamental interactions allow us to sketch the story of the universe from **BIG BANG** to today (after 13.7 billion years)



However, only 4% is made of known particles such as barions, leptons, lights $\longrightarrow$

26% is DARK MATTER

70% is DARK ENERGY



the muon magnetic moment $\vec{M}_\mu = g_\mu \frac{e}{2m_\mu}\vec{S}$

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 0.001\,165\,918\,0(7) \qquad \text{theory}$$

$$a_\mu \equiv \frac{g_\mu - 2}{2} = 0.001\,165\,920\,9(8) \qquad \text{experiments}$$

the available experimental results are so precise to probe the quantum nature of the theory, requiring high-loop calculations for the comparison with theory (up to five-loop calculations within QED, supplemented with two-loop *weak* contributions, and hadrons contributions to next-to-leading order)

small (significant?) difference $\Delta a_\mu = 2.9(1.1) \times 10^{-9}$

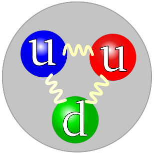
hint of new physics? adding a further 5-dim Pauli interaction

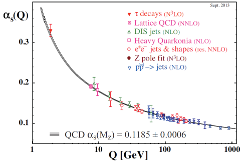
$$\frac{e}{2\Lambda}\bar{\psi}[\gamma_\mu, \gamma_\nu]\psi F^{\mu\nu} \quad \longrightarrow \quad \Lambda \gtrsim 10^8\text{Gev}$$

Quantum Chromodynamics:

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu,a} + \bar{q}_f (i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu + m_f) q_f$$

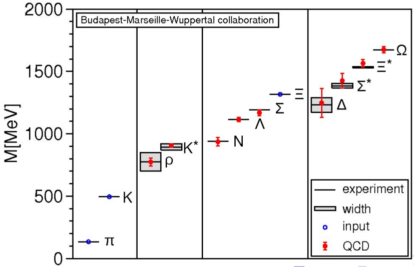
from quarks, gluons to hadrons, nuclei





Perturbative running coupling constant compared with experiments

Hadron spectrum from numerical simulations of lattice QCD compared with experiments



Are the fundamental interactions emergent phenomena?

Are fundamental interactions emerging phenomena arising from unknown mechanisms at a much larger scale Λ?

Why not?

Such mechanisms may appear drastically different from QFT

An analogous idea from string theory $\longrightarrow$ the *ultimate theory* is a string theory, while the standard QFT model of fundamental interactions is a low-energy emergent phenomenon.

Condensed matter example: superconductivity can be described by a QFT featuring the Higgs mechanism; however we know that its microscopic models are based on interacting Cooper pairs of electrons.

QFT and condensed-matter physics

Effective QFTs for the low-energy properties allow us to describe many features of the phenomena arising in condensed-matter physics:

theory of metals, superconductivity, ^{3}He and ^{4}He quantum liquids, quantum Hall effects, optical lattices, etc...

QFTs are powerful tools for quantum many-body problems:

• universality $\longrightarrow$ different systems may show common collective behaviors although they differ microscopically

• large number of degrees of freedom, $N = O(10^{23})$, so that problems can be addressed using statistical approaches

• space-time and internal symmetries which entail conservation laws

• Wilson renormalization-group theory (RG flow in the Hamiltonian space) further supports the use of QFT

• the degrees of freedom describing the low-energy excitations are often very different from the microscopic fundamental ones $\rightarrow$ emergence

Liquid-gas and magnetic transitions, superfluid transitions, Bose-Einstein condensation, deconfinement transition in QCD, ...

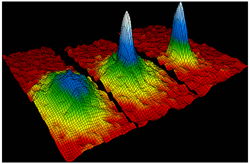
Bose-Einstein condensation in atomic gases and liquid ^{4}He

BEC in bosonic gases, when

$$\lambda_{\text{de Broglie}} = \left(\frac{2\pi\hbar^2}{mT}\right)^{1/2} \approx d_{\text{atoms}} = (N/V)^{-1/3} \quad ,$$

recently observed in weakly interacting gases, made of alkali atoms, rubidium, sodium, lithium

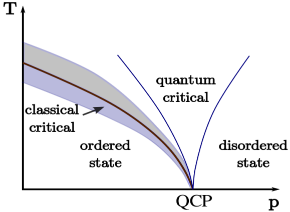
velocity distribution of rubidium atoms $\longrightarrow$



A quantum $T = 0$ critical point separates different quantum phases.

Scaling laws describe the interplay between quantum and thermal fluctuations at the QCP

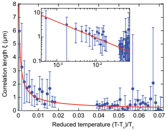
$\longrightarrow$ new interesting phenomena in quantum magnetism, high-T superconductors, quantum particle systems, new matter states, such as spin liquids



Bose-Einstein condensation in atomic gases and liquid ^{4}He

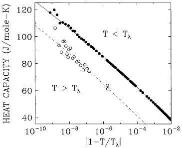
Characterized by a diverging correlation length ξ or the critical modes related to the coherence properties: $\xi \sim (T - T_c)^{-\nu}$,

and also $C_V \sim |T - T_c|^{-\alpha}$, $G(x) \sim |x|^{-1-\eta}$, etc...



$\longleftarrow$ the correlation length ξ of critical modes at BEC of a trapped atomic gas

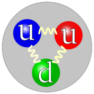
C_V at the normal-superfluid transition in ^{4}He (microgravity conditions on the Shuttle), up to a few nK from T_c $\longrightarrow$



Quantum amplitude of atoms as order parameter $\longrightarrow$ complex field $\varphi(x)$ with U(1) symmetry $\longrightarrow$ QFT: $\mathcal{L} = |\partial_\mu \varphi|^2 + r\,|\varphi|^2 + u\,|\varphi|^4$

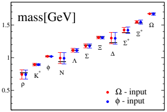
		α	ν	η
Experiment	^{4}He	$-0.0127(3)$	**0.6709(1)**	
QFT	6,7-loop MZM exp	$-0.011(4)$	**0.6703(15)**	0.035(3)
	6-loop ϵ exp	$-0.007(3)$	**0.6690(10)**	0.0380(6)
	conformal bootstrap	$-0.0157(33)$	**0.6719(11)**	0.0385(6)
Stat mech	MC+HT	$-0.0151(3)$	**0.6717(1)**	0.0381(2)

- RG theory and nonperturbative formulations of QFTs



Strong-interaction QCD defined from the critical regime of 4D statistical systems

⟸ from quarks to baryons ⟹



- Symmetry breaking and Goldstone bosons

• Goldstone particles: massless excitations in broken phases (pions in hadronic physics)

• analogous excitations in magnets, superfluids, etc...

- Higgs mechanism

• model of superconductivity in the case of U(1) gauge theories

• mechanism to provide mass to elementary particles in standard model

- RG theory of critical phenomena and effective field theories
physical interpretation of renormalizability, suppressed *new-physics* in terms of a large scale Λ, analogously to irrelevant perturbations at continuous transitions

- topological structures such as solitons, instantons, vortices, etc

- conformal field theory developed within QED and string theory, and used to described 2D critical phenomena, more recently holography approach

- anomalies $\pi_0 \to \gamma + \gamma$, topological insulators

- Dirac fermions describing fundamental particles and the spectrum of graphene

Summarizing .....

Quantum and Statistical Field theories play a central role in our description of natural phenomena.

Their applications range from fundamental interactions to many-body physics in condensed matter, and critical phenomena

Is QFT completely understood? several issues within QFTs are not understood yet! research projects to develop their methods are still important: wide potentiality and applications

Nonperturbative methods require further developments, summation of perturbative expansions, bound states, extended topological objects, off-equilibrium quantum real-time evolutions, ...

- **dark energy** why is it so small? is it constant?
- **dark matter** are they light axions or heavy WIMPs?
- **inflation after Big Bang** what drove it?
- **quantum gravity** to go beyond the actual nonrenormalizable theory?
- **matter-antimatter asymmetry** how did it arise?
- **new physics** what is the large scale Λ parametrizing the high-order polynomial interactions in the standard model?
- **neutrino mass** is it a Majorana or a Dirac particle?
- **phase diagram of quark matter** quark-gluon plasma
- θ **dependence of QCD** does the topological parameter θ vanish?
- **new quantum state of matters** such as spin liquid, deconfined transitions, topological phases, etc...
- **high-T superconductors** related to quantum phase transitions?
- **off-equilibrium QFT** to describe off-equlibrium phenomena
- ...

Let us go back to the initial question: WHY do QFTs work so well?

Fundamental interaction theories: To describe microscopic phenomena at the most fundamental level

$\leftarrow$ **QFT** $\rightarrow$

Condensed matter theories: To describe emergent macroscopic phenomena arising from microscopic laws of physics

QFT turns out to be a formidable framework to interpret the physical phenomena, and provide predictions for new observations,

disentangling relevant low-energy features from high-energy degrees of freedom, using symmetry as a guide

QFT works when different scales exist, providing effective descriptions at low energy, even though the microscopic mechanism remains unknown.