

Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ?

Let me list the contents of the title:

- **Dark Matter** We have our own model for dark matter being cm-size pearls, that are droplets of new speculated phase of vacuum, with ordinary matter under white dwarf like pressure inside.
- **Double Supernova Neutrino Explosion** One of our speculated effects of our model is that a supernova will send the largest neutrino bursts out in **two** bunches with some hours between the bursts.

Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ? (Title explanation continued)

- **Degenerate Vacua** This alludes to our proposed new law of nature saying that there are several vacua with very small energy density(=cosmological constant). We call it Multiple Point Principle (MPP) and announce it as connected with the concept “slush”. We used this MPP to PREDict the mass of the Higgs prior to its discovery with only 10 GeV deviation!
- **Predetermination** To obtain the “degenerate vacua” it is called for to have a theory in which there is some “organization of the future” or “influence from the future”.

Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ? (Title explanation yet continued)

- **Why the Standard Model Group?** I want to tell about a feature that is characterizing the gauge group of the Standard Model: especially **small** representation. (This subject is a separate one, almost no connection to the other ones)

Plan for “Dark Matter, Double Supernova ..., Why the Standard Model Group?”

- **Main** Main picture: Only deviations from Standard Model in practice: nonperturbative effects and our MMP.
- **Vacua** 3 vacua, we suggest in our picture.
- **Dark** Application of one of the vacuum-phases to form dark matter bubbles.
- **Observational** Observational effects: Extra neutrino burst in SN 1987A, kimberlite pipes, Tunguska event in 1908, keV-X-ray from dark matter **unifications**.
- **Conclusion** for Pearls and MPP,...
- **Why SMG** A separate work: The reason Nature choose just $S(U(2) \times U(3))$ as the gauge **group** is a “smaller” representation relative to its “Adjoint” than for any other Lie group.

Collaboration with: **Colin D. Froggatt, ...**

Colin Froggatt is the main collaborator on most of the work of the present talk especially on the Dark matter pearls and their phenomenological effects.

On the “solving” the scale (and in a way hierarchy problem too) problem also **Larisa Laperashvili** participated. And in the crude calculational estimate that the Multiple Point Principle Principle in pure Standard Model with three vacuum-phases she and **Chitta Das** were also with.

But the origin of the Multiple Point Principle was first of all with **Don Bennett** and again with **Colin Froggatt** based on developments starting with Niels Brene and a bit with Ivica Picek. Some works on Multiple Point Principle which will only be shortly mentioned are with **Roman Nevzorov** and one with **Yasutaka Takanishi**.

Multiple Point Principle, Three Mass-estimates of Bound State of $6t + 6\bar{t}$ Agree Dark Matter Pearls

“Multiple Point Principle” is the name given to our proposal for a **new law of nature**, saying that the coupling constants are fine tuned to be at a **critical point** rather analogously to say a mixture of ice and water (slush) necessarily being at temperature 0^0 Celsius.

Collaborators: **D. Bennett, C.D. Froggatt, L.V. Laperashvili, C. Das**

The ideas go also back to work involving also Niels Brene, I. Picek, ...and at later stage Yasutaka Takanishi

Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ?

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Havana , April, 2017

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Main ideas

- **Strongly Bound State:** We speculate that mainly due to exchange of Higgs bosons a system of 6 top plus 6 anti top quarks bind so strongly as to make a **bound state** with appreciably lower mass than that of 12 separate quarks and anti quarks.
- **Multiple Point Principle:** We propose a **new law of nature(MPP)** saying, that - somewhat mysteriously may be - the **coupling constants** and other parameters, such as the Higgs mass square, **get adjusted** so as to guarantee that there are *several vacua all with very small energy density(=cosmological constant).*

Main Ideas (continued):

- **Dark Matter** One of the postulated - in Standard Model 3 - vacua, the “Condensate vacuum”, is supposed to exist in small pearls of cm-size, bubbles of the “condensate vacuum”, filled on top of this vacuum with normal matter under very high pressure. This pressure is due to the surface tension of the wall/skin between the “condensate vacuum” and the “present vacuum”. Due to this high pressure the specific density of the inside ordinary matter is about 10^{14} kg/m^3 and the pearls have masses of the order of 100000 t to a million t.

Yet an Important Point: Scale Problem

The “multiple point principle” would if true give an explanation to the otherwise rather mysterious fact (the scale problem):

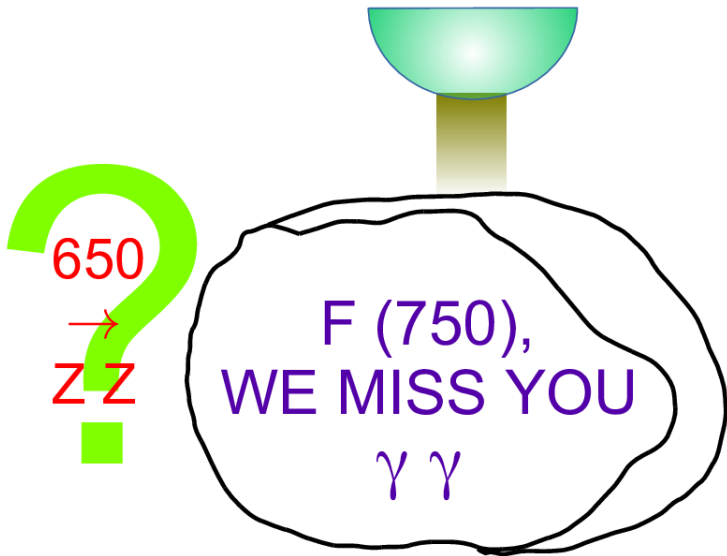
Why is the Higgs mass and thus the weak scale so very small compared to the Planck energy scale?

We obtain in our Multiple Point Principle model with pure Standard Model even **a good order of magnitude for the ratio of the weak to the Planck scale**, on the basis of known coupling constants entering the renorm group running for the top-Yukawa coupling.

Plan of talk:

- **Heading** F(750), We miss you!
- **Introduction** “New” Law of Nature, and a Bound State
- **MPP** The New law, “Multiple Point (Criticality) Principle”.
- **Bound** Bound state of 6 top and 6 anti top.
- **Reasons** Attempts to explain why MPP.
- **High** The mass of the bound state that could arrange the stability of our vacuum to be just borderline stable w.r.t. the Higgs field.
- **Condensate** Mass of bound state making the “condensate vacuum” degenerate with “present vacuum”
- **Bag** Bag model estimation of the mass of the bound state.
- **Dark** Dark Matter as Pearls of the “condesate vacuum”.
- **Scale problem** Almost “Hierarchy problem”
- **Conclusion** Telling that you should now believe our MPP law!





Multipel Point Principle(=MPP):

Our proposal for a “new” law of nature - Multiple Point Principle(=MPP) - (first by Don Bennett and myself) means that **there shall exist several vacua with very small energy density.**

Three Vacua in Standard Model:

For simplicity and **trustability** we in this talk restrict ourselves to **pure Standard Model** and only the following **three** vacua:

- **Present** The vacuum, in which we live, in the sense, that, if we in practice find a place with zero density of material, then that region is in the state of the “present vacuum”.
- **High (Higgs) field vacuum** This vacuum is a state, in which the Higgs field is at a minimum in the Higgs-effective potential $V_{\text{eff}}(\phi_H)$ having a value of the Higgs field near $\phi_H \sim 10^{18}$ GeV. It is known, that with pure Standard Model it seems, that the energy density of this vacuum is slightly negative (with 3 standard deviations from being just zero).
- **Condensate vacuum** This third vacuum is a very speculative possible state inside the pure Standard Model, which contains a lot of strongly bound states, each bound from 6 top + 6 anti top quarks.

Can use Multiple Point Principle together with Any Model (side-remark)

In the present talk I shall concentrate on the version “several vacua, that all have very **small energy densities**” = MPP.

But older version had it: “Several vacua all have the **same energy density** (with some accuracy, that can be discussed)” = MPP.

Also one does not need to assume just Standard Model as I shall do in the present talk. For instance with Roman Nevzorov some of us (Froggatt and me) assumed a supersymmetry only broken tinily in one vacuum, but much stronger broken in e.g. the present vacuum.

The with Roman Nevzorov works extend Standard Model with Susy etc.

(Otherwise in the present talk I only keep Standard Model.)

Assuming some of the vacua to have only tinily broken susy, a tiny **cosmological constant** in the almost susy unbroken vacuum could be **transferred - by means of MPP** - to the present vacuum, and a rather successfull fitting/derivation of the astronomically determined cosmological constant could be achieved!

Also the original idea that we - Don Bennett and I - should invent the “multiple point principle” was based on a model called AntiGUT, which extends the Standard Model, although first having new physics only rather close to the Planck scale, actually by letting each family of fermions have its own system of gauge groups. The gauge bosons also in families!

Also Application in Extensions of Standard Model by Kawana et al.

Multiple Point Principle of the Standard Model with Scalar Singlet Dark Matter and Right Handed Neutrinos Kiyoharu Kawana
Prog. Theor. Exp. Phys. (2015) 023B04

Finetuning of Parameters, Couplings

Our “multiple point principle” is really just an assumption about the coupling constants - in the Standard Model, if we, as in this talk, take the model to be pure Standard Model - being **finetuned** so as to make the three vacua proposed have just zero energy density $V_{present}, V_{condensate}, V_{high\ field}=0$ (with say the accuracy of the order of the astronomically found energy density in the “present vacuum” $\sim 75\%$ of the total energy density in the present universe.)

I.e. MPP provides 3 restrictions between the parameters of the model in question, here the Standard Model, from which the \sim zero energy densities in all three vacua follows.

Multiple Point Principle means Relations between the couplings and other parameters:

$$V_{\text{present}}(\Lambda_{CC}, g_t, m_H^2, \Lambda_{QCD}, \dots) = 0 \quad (1)$$

$$V_{\text{condensate}}(\Lambda_{CC}, g_t, m_H^2, \Lambda_{QCD}, \dots) = 0 \quad (2)$$

$$V_{\text{high field}}(\Lambda_{CC}, g_t, m_H^2, \Lambda_{QCD}, \dots) = 0 \quad (3)$$

Here we wrote explicitly the following parameters of the Standard Model:

$$\Lambda_{CC} : \text{The cosmological constant} \quad (4)$$

$$g_t : \text{The top Yukawa coupling} \quad (5)$$

$$m_H^2 : \text{Higgs mass squared} \quad (6)$$

$$\Lambda_{QCD} : \text{The scale parameter of QCD} \quad (7)$$

Whether these parameters are renormalized or bare does not matter so much here.

$V_{present}$, $V_{condensate}$, $V_{high\ field}$ are the vacuum energy densities for the three speculated vacua.

Use of Multiple Point Principle:

Taking the experimental values for all the Standard Model parameters except for say Λ_{CC} , m_H^2 , and g_t we could look at it, that e.g. $V_{present} = 0$ fixes the cosmological constant Λ_{CC} to essentially zero. (It is very small indeed). Then $V_{high\ field} = 0$ (meaning the energy density of the vacuum having the very high Higgs field $\phi_H \approx 10^{18} GeV$) could be taken to predict the Higgs mass, and the $V_{condensate} = 0$ to predict, say, the g_t Yukawa coupling. In fact Colin Froggatt and I (H.B.N.) PREDICTED the Higgs mass many years ago to $135 GeV \pm 10 GeV$ from such an MPP-assumption.



Other PREDictions of Higgs Mass: Kane et al., bound...

Higgs mass prediction for realistic string/M theory vacua Gordon Kane, Piyush Kumar, Ran Lu, and Bob Zheng Phys. Rev. D 85, 075026 Published 30 April 2012

SUSY theories put an upper bound on the Higgs mass:

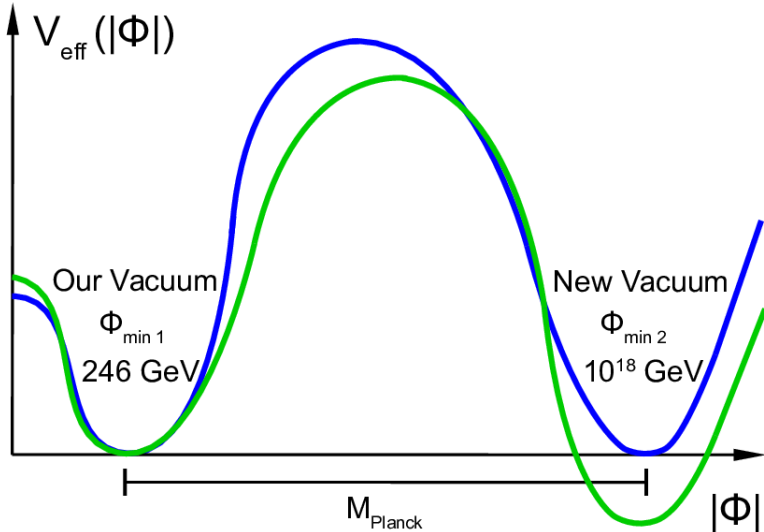
Very close to the found mass actually:

D. Abbaneo et al. [ALEPH Collaboration], arXiv:hep-ex/0112021.

Y. Okada, M. Yamaguchi and T. Yanagida, Prog. Theor. Phys.

85, 1 (1991); M. Carena, M. Quiros, and C. E. M. Wagner, Nucl.

Phys. B 461, 407 (1996) [arXiv:hep-ph/9508343];



A Remarkable Side-Point:

The second minimum in the Higgs effective potential corresponding to what, we call the “High field” vacuum, has an expectation value for the Higgs field ϕ_H , which is **remarkably close** (order of magnitudewise) **to the Planck energy scale!** This should not be an accident, but rather explained: Planck energy scale is the “fundamental physics scale” for both energy and Higgs fields; they have the same dimension.

Two of the vacua, which we discuss to day, have for some reason exceptionally small, say, Higgs field, while the “high field” vacuum has the “normal” order of unity in Plack units value for *its* Higgs expectation value. So rather ask the question: **Why do the two vacua, “present vacuum” and “condensate vacuum”, not have Planck scale, say, Higgs fields?** (Let me for the moment postpone, that indeed we have an explanation from “multiple point principle”, that these two vacua have exceptionally small Higgs expectation values scale.)



But Why should we believe in the postulate of “Multiple Point Principle” ?

- **Need Coupling Explanations** Even with great effort e.g. Graham Ross could not get the factor Δ with which the Higgs is too light further down than about $1/20$. Cosmological constant ?...
- **“Derivations”** In models which allow somehow influence from the future to adjust coupling constants one may make some “derivations” of MPP.
- **Empirical:** But really it is the main point of today's talk to deliver some empirical support (our PRediction of Higgs mass, and two derivations of same bound state mass).

Universe does Finetune; Multilocal Action

Natural solution to the naturalness problem Universe does fine-tuning Yuta Hamada , Hikaru Kawai, and Kiyoharu Kawana

1. Reason: “ Plural of Cosmological Constant”

- 1 We have to assume that the energy density in the “present vacuum” is very small compared say to Planck energy density, because it has been known small long before the measurements with supernovae A1 settled non-zero.
- 2 This assumption remains beautiful or not so complicated by “putting it in plural”: **Several vacua have very small energy density/cosmological constant** compared to say the Planck energy density or the Higgs energy density or most high energy physics contributions.


(Private conversation with L. Susskind.)

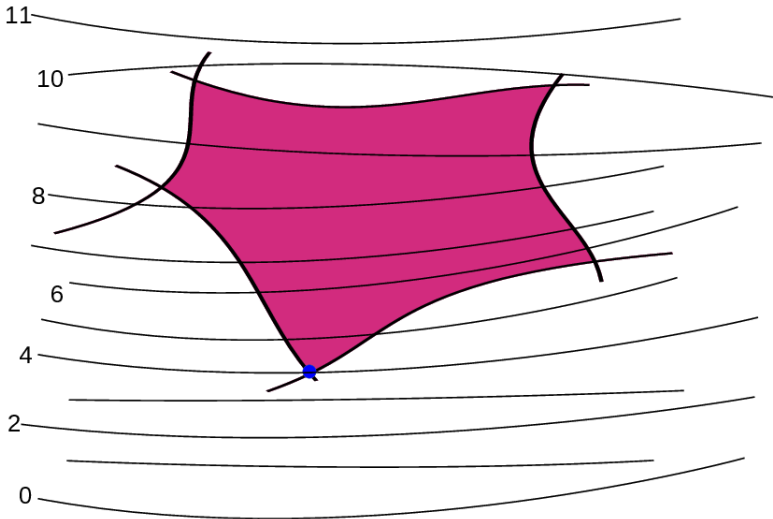
But each time you fix the energy density of one more vacuum you get one more relation between the parameters/couplings of the theory.

Even more beautiful to talk about several/all vacua than just a special one.

2. Reason: Extremizing Something, Positive Energy

- A. Assume that energy-density should be positive or zero. i.e. bottom in hamiltonian density is at least zero. **This will restrict the coupling constants and other parameters - e.g. Higgs mass - to some polyhedron-like figure with curved sides, where the sides correspond to one possible vacuum or another one having just zero energy density.**
- B. Assume that the couplings and parameters inside the positivity restriction are selected by **minimizing something / some function of these couplings and parameters**(a generic or “random” function).
Using antropic principle the function could suggestively be the number of human beings in the universe resulting with the couplings etc. in the point in parameter-space considered.

On the figure I have drawn the cote curves -really cote surfaces-. 



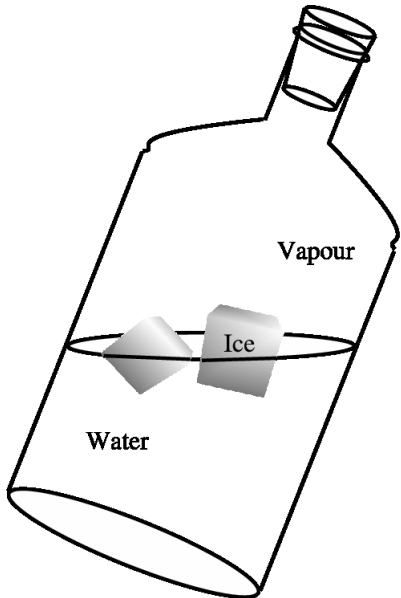
Very often Minimum of Function occurs in Corners

The crucial point is, that - especially in a high dimensional coupling-constant and parameter space - will very often the minimum fall in a **corner** (where several sides cross) of the polyhedron-like region with curved sides (violet).

But the “sides” correspond to different vacua having zero energy density. So a corner corresponds to several vacua all having zero energy density → “Multiple Point Principle”!

3. Reason; Our, Bennett's and Mine, Original Explanation

One assume that some **extensive quantities / commodities** i.e. some integrals over space time of say fields raised to some powers etc. - say Higgs field squared - are **fixed** by "God" / some law, rather than as I think we would usually think, it is the couplings themselves, that are selected by "God".







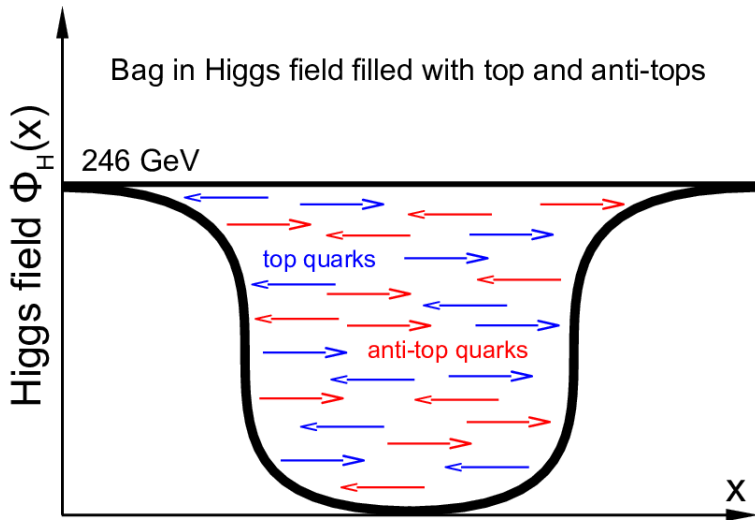
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Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ?

The Difficulty of the Bound State

When we - as we now want - want to check if the “multiple point principle” is true/valid law of nature, we have the difficulty, that an important role is played by a bound state, with which the vacuum, which we call “condensate vacuum”, is filled.

Fundamentally one cannot calculate completely perturbatively, when one calculates on a bound state!



Lucky Overdetermined Situation

Luckily we are with “multiple point principle” (to be tested) in the very good situation computationally, that in addition to know already from experiment all the parameters of the Standard Model to day, we have the ca. three extra equations, if MPP assumed, and so we can use this information to help us through the calculational problems with the bound state.

Checking MPP by Calculating Mass of the Bound State in Several Ways

Since it is non-perturbative and thus either difficult or very crude only, that we can compute say the mass of the bound state, it is suggestive to take it as a parameter. Then we may formulate testing the “multiple point principle” as evaluating by *different* assumptions inside MPP the mass of the bound state.

Really Check Bound State Mass Obtained from Degeneracy of Vacuum Pairs

Our technique - to day - is to estimate/calculate the value of the mass of the bound state of 6 top and 6 anti top quarks speculated to exist in our picture/model. (Since we have two relative energy density predictions - ignoring the absolute smallness of cosmological constant - we get two such bound state mass fits.) In addition we can seek to obtain the bound state mass by building a bag-model-like ansatz for the bound state and estimates its mass. Thus we get using our mutiple point principle two a priori different mass predictions for the $6\text{top} + 6\text{ anti top}$ bound state.

Our Three Bound State Mass Fits:

- **High field fit:** We fit to get a tiny correction to the Higgs mass relative to the running selfcoupling so as to ensure the MPP-requirement, that the “present vacuum” be degenerate with the “high field vacuum”: Fitting mass $m_{from\ high\ field\ fit} \approx 700\text{GeV}$ to 800GeV .
- **condensate vacuum fit:** We fit the mass to the binding between the bound states in a region filled with such particles to lowest energy density just gets zero/same energy density as the present vacuum. With a simple but accidentally almost true assumption we fit the mass to $m_{from\ condensate\ fit} \approx 4m_t = 692\text{GeV} \pm 100\text{GeV}$, say.
- **Ansatz calculation:** We make a bag-model-like crude ansatz for the bound state of the 6 top + 6 anti top and seek the minimum energy/mass by varying bag radius R . With very crude inclusion of various corrections we reach the mass estimate $m_{bag-model} \approx 5m_t = 865\text{GeV} \pm 200\text{GeV}$, say.

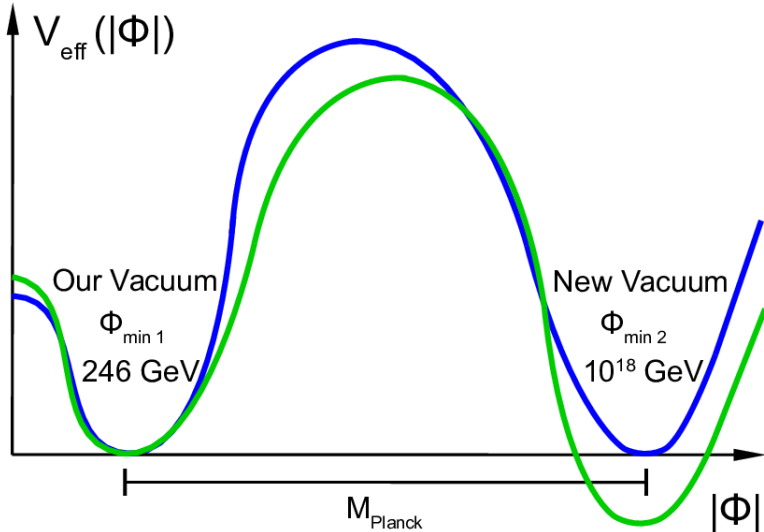
Small Correction by Laperashvili, Das, and me to “High field Vacuum” Energy Density

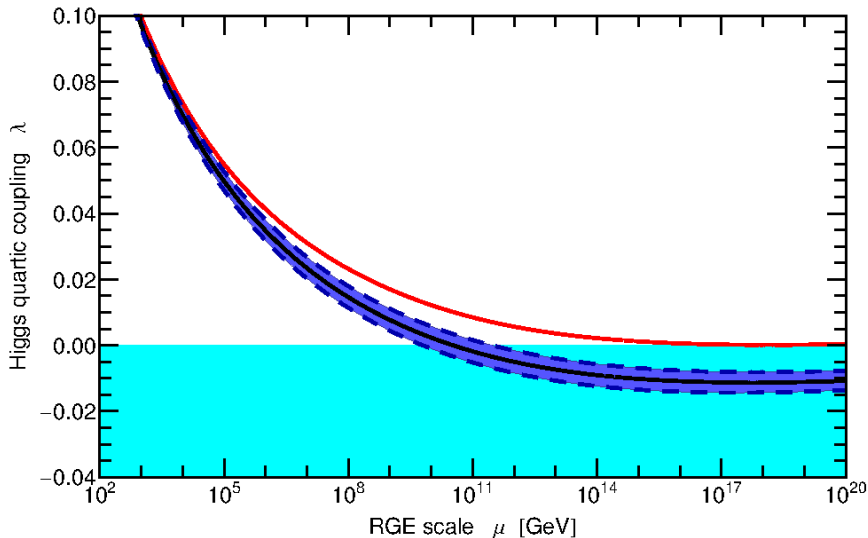
From De Grassi et al.'s calculation of the effective Higgs field potential $V_{eff}(\phi_H)$ there is a minimum in this potential, but it goes **slightly** under 0 so that the present vacuum is **unstable** for the experimental Higgs mass 125.09 ± 0.24 , while the value, that would have made the second minimum just degenerate with the present vacuum energy density would be rather

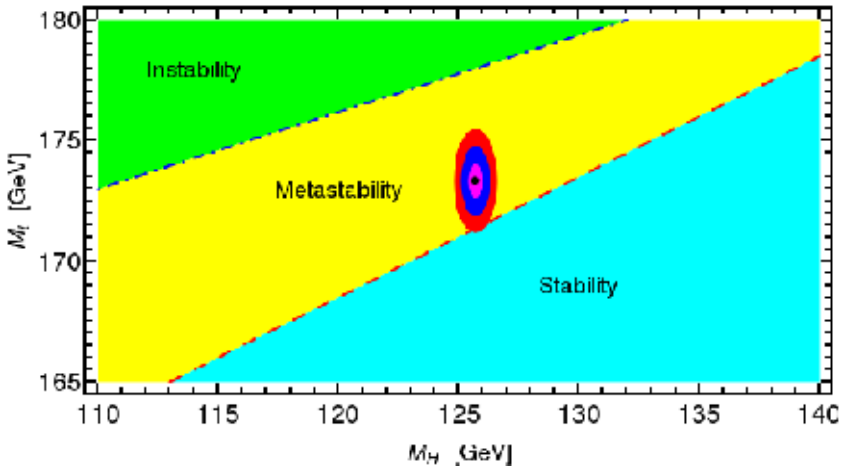
$$m_H|_{\text{from MPP De Grassi...}} = 129.4 \text{ GeV.}$$

We claimed that with a bound state e.g. with the mass 750 GeV we would get corrected the De Grassi et al. calculation so as to be consistent with exact MPP.

Basically we claim: The leading diagrams treating the bound state as an “elementary particle” (i.e. with formal Feynmann rules) though modified by including estimated form-factors, we can fit the mass of the bound state, so that the diagrams just cancel the instability and make the energy density of the high Higgs field minimum exactly zero, $V_{eff}(\phi_H \sim 10^{18} \text{ GeV}) = 0$. That comes for a mass ~ 700 to 800 GeV.







$$\lambda_S = \text{[Square Diagram]} + \text{[Triangle Diagram]} + \dots$$

The diagram shows the expansion of the scalar self-energy λ_S . It consists of two terms:

- First term:** A square loop diagram. The four vertices of the square are connected by thick black lines, representing fermion (F) loops. From each vertex, a thin black line extends outwards, representing a Higgs (H) line. The labels 'H' are placed above the top-left and top-right vertices, and below the bottom-left and bottom-right vertices. The label 'F' is placed in the center of each of the four thick edges.
- Second term:** A triangle loop diagram. The three vertices of the triangle are connected by thick black lines, representing fermion (F) loops. From each vertex, a thin black line extends outwards, representing a Higgs (H) line. The labels 'H' are placed above the top-left vertex, below the bottom-left vertex, and to the right of the rightmost vertex. The label 'F' is placed in the center of each of the three thick edges.

The Small Instability, Negative Self-coupling at the High field Minimum:

Extrapolated using DeGrassi et al. without our correction one gets the following value of the running self coupling $\lambda_{run}(10^{18} GeV)$:

$$\lambda(\phi \text{ "high field" }) = -0.01 \pm 0.002. \quad (8)$$

at the high field scale $\phi \text{ "high field"}$. However, a value very accurately zero is required by Multiple Point Principle(=MPP). Since bound state F is an extended object we must include a formfactor, when using it in Feynman diagrams.

Defining a quantity b denoting the radius of the bound state measured with top quark Compton wave length $1/m_t$ as unit by:

$$\langle \bar{r}^2 \rangle = 3r_0^2, \quad (9)$$

$$r_0 = \frac{b}{m_t}, \quad (10)$$

Approximating the Bound State as were it an Elementary Particle, since so Strongly Bound

The dominant diagram/correction - the first and quadratic of the diagrams on the figure just above - is

$$\lambda_S \approx \frac{1}{\pi^2} \left(\frac{6g_t}{b} \frac{m_t}{m_S} \right)^4$$

where we have the estimated or measured values

$$g_t = 0.935; \quad m_t = 173 \text{ GeV}; \quad b \approx 2.34 \text{ or } 2.43$$

Using the after all rather small deviation from perfect MPP

$$\lambda_{\text{high field}} = -0.01 \pm 0.002$$

and requiring it to be cancelled by the correction from the bound state we get the requirement

$$\lambda_S = \frac{1}{\pi^2} \left(\frac{6g_t}{b} * \frac{m_t}{m_F} \right)^4 * (\sim 2) \approx 0.01 \pm 0.002, \quad (12)$$

where $g_t = .935$, $m_t = 173\text{GeV}$, $b \approx 2.43$ and the factor “(~ 2)” were taken in to approximate some neglected diagrams.

If a nearer study should show that the next diagrams add up to roughly as much as the first one should include the factor ~ 2 to take into account the neglected Feynman diagrams correcting the Higgs self coupling.

The solution w.r.t. the mass of the bound state m_F gives

$$\begin{aligned}
 m_F &\approx \frac{6g_t m_t}{b} \left(\frac{\sim 2}{\pi^2 * 0.01 \pm 0.002} \right)^{1/4} \\
 &\approx 2.31 * 173\text{GeV} * 2.1 = 4.9 * 173\text{GeV} = 850\text{GeV} \pm 20\% \\
 &\quad \text{or without the } \sim 2: \\
 m_F &= 2.31 * 173\text{GeV} * 1.8 = 4.1 * 173\text{GeV} = 710\text{GeV} \pm 20\%
 \end{aligned} \tag{13}$$

Three Agreeing Fits of the Bound State Mass:

In this way we got even two calculations for the bound state mass - using in addition crude estimation -

$$m_F(\text{from "high field vacuum"}) \approx 850 \text{ GeV} \pm 30\% \text{ with } \sim 2 \quad (14)$$

$$m_F(\text{from "high field vacuum"}) \approx 710 \text{ GeV} \pm 30\% \text{ without } \sim 2 \quad (15)$$

$$m_F(\text{"condensate vac."}) \approx 692 \text{ GeV} \pm 40\% \quad (16)$$

$$m_F(\text{"bag estimate"}) \approx 5m_t = 865 \text{ GeV} (\text{very uncertain}) \quad (17)$$

The agreement of the value “692 GeV” with the estimate(s) from the completely different vacuum with the high Higgs field “850GeV” or “710 GeV” and with the mass by estimating how strong the top and anti tops can bind m_F (“*bagestimate*”) $\approx 865\text{GeV}$ is encouraging and a support of our “Multiple Point Principle”!

Fitting Bound State Mass to the “Condensate Vacuum” having Same Energy Density as the “Present” one.

For calculational purpose we approximate the “condensate vacuum” with a crystal (but it should at least be a fluid, but that may not matter much for our crude energy density estimate) made from the bound states sitting each with 4 neighbors, the top and anti tops of which are in approximate main quantum numbers $n=2$ seen from the bound state considered.

The MPP-requirement may be written

$$0 = m_F - \text{"binding per } F'' \quad (18)$$

$$= m_F - \frac{\# \text{neighbors}}{2} * \text{"binding to neighbor } F'' \quad (19)$$

$$\approx m_S - \frac{4}{2} * \text{"binding of } F \text{ in } n=2 \text{ around another } F'' \quad (20)$$

$$\approx m_F - \frac{4}{2} * \text{"binding of } F'' * \frac{1/2^2}{1/1^2} \quad (21)$$

$$= m_F - \frac{1}{2} * \text{"binding of } F'' \quad (22)$$

$$= m_F - \frac{1}{2} * (12m_t - m_F) \quad (23)$$

$$= \frac{3}{2}m_F - 6m_t \quad (24)$$

We shall indeed follow an appendix of our earlier work[?] and assume, that the structure of the condensate can be approximated

We made then the approximation, that we can effectively consider it, that the neighboring top quarks and anti topquarks contained in an F neighboring to another one are *in effect in the $n=2$ orbit* of the latter. Thus we can take the binding energy of a neighboring F to a given one “binding to neighbor F(750)” to be as, if the top and anti tops were in an $n=2$ orbit or some superposition thereof. Thus the binding of the neighbors occur with binding energy “binding of F in $n=2$ around another F”.

As long as we can take the effective Higgs mass for the two lowest orbits $n = 1$ and 2 to be zero, we can count, that the binding energy, for top say, in the orbit $n=2$ is just one quarter of that in the $n=1$ orbit, provided we can use the same potential of the form $\propto 1/r$. But now that were, what our above discussion “accidental cancellation” in section ?? should ensure, and so even for an F-particle, which consists of tops and anti tops the ratio of the binding energies should be $1/2^2 = 1/4$.

From the last step in (24) we easily derive of course

$$m_F = \frac{2}{3} * 6m_t = 4m_t = 173\text{GeV} * 4 = 692\text{GeV} \text{ agreeing well with } 710\text{GeV} \quad (25)$$

Excusing the Simplifying Accident

Basically the accidental cancellation, that simplifies the calculation so much that we get the mass of bound state F needed to make the “condensate vacuum” degenerate with the “present one” to be $m_F = \frac{12m_t}{3} = 4m_t = 692\text{GeV}$ is the following: Realistically the top and antitop do not bind to an ideal point Higgs coupling object, but rather to all the other 11 top or anti tops, and these form a smeared out group of particles, not surrounded by an ideal Yukawa potential but a field emitted from a smeared out source! Very crudely this means that a accidental top or anti feels the field from only about half the other 11 or 12 constituents. **But the mistake by ignoring this reduction by a factor 2 due to the smear out is rather well compensated for by the following two then also to be ignored corrections:**

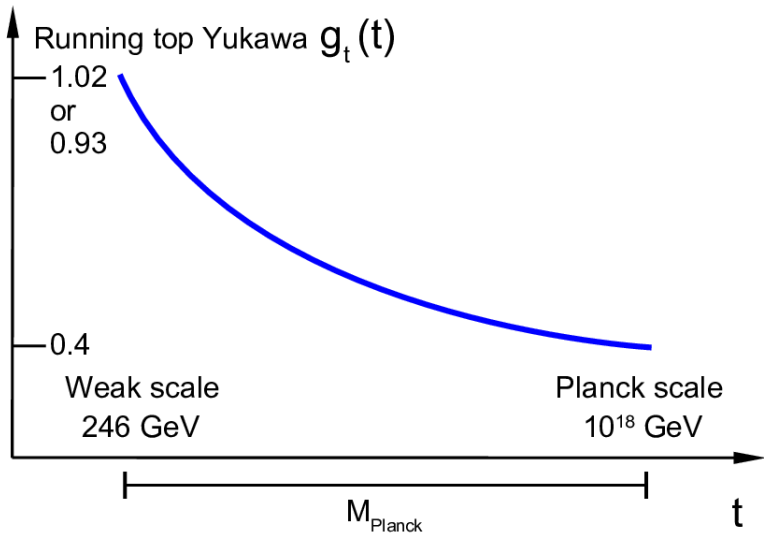
- The exchange of W and Z.
- The gluon exchange only active for quarks inside the F's

Estimating How 6 Top + 6 Antitop Bind

Imagining that the top-Yukawa-coupling g_t were gradually screwed up, the Higgs field inside an ansatz bound state of 6 top + 6 anti tops at say the typical distance of the quarks themselves from the center, would gradually be lowered compared to the usual vacuum expectation value.

“Solve” Hierarchy problem

Having made a fine-tuning theory/model/rule we have at least the chance to have our fine-tuning theory MPP give the experimentally observed order of magnitude for the Higgs mass say. And indeed we predict the right order for the logarithm of the scale range over which μ has to run to get the running top-yukawa-coupling $g_{t\text{ run}}(\mu)$ go from 0.4 at the 10^{18} GeV to the 0.935 needed at the weak scale from the requirement of the “condensate vacuum ” being degenerate with the ‘present one’.



Dark Matter Pearls

Work mainly by

Colin D. Froggatt and Holger Bech Nielsen

but also other as Bennett, Laperashvili,...Takanishi have been very important mainly in the ideas behind the dark matter itself.

This Dark Matter contribution is a continuation of my talk about “Multiple Point Principle” recently presented in Toyama.

Short review of “Multiple Point Principle”

- Postulate “A new law of Nature”, [Multiple Point Principle \(MPP\)](#) saying: There are several different vacua, all having very small energy densities (= cosmological constants), i.e. coupling constants, parameters, adjust themselves so as to achieve this degeneracy.
- If one just uses pure Standard Model, there are just [three](#) such small energy density [vacua](#):
 - Present vacuum,
 - High field vacuum,
 - F(750)-condensate vacuum.
- Arranging especially the “F(750)-condensate vacuum” requires that 6 top quark plus 6 anti-top bind themselves together under exchange of (Standard Model) Higgs and helped by other exchanges, gluons, weak gauge bosons,...to form [a very strongly bound bound state, F\(750\) say.](#)

Different Versions of Multiple Point Principle Formulations:

- **Meta-formulation** Instead of requiring just the energy density small or degenerate, you require that realistically a series of successive vacua could be realized in nature. With Yasutaka Takanishi and Froggatt we considered such meta-stability $\rightarrow m_{Higgs} = 121 GeV$.
- **Inclusion or not of Present Cosmological constant** one can formulate MPP in two slightly different ways even after having chosen meta- or just energy density requirements:
 - There are several vacua and they have the **same energy density**, or
 - There are several vacua and they all have **very small energy density**, of the order say of the 3/4 of the present total energy density (including the normal matter etc.).

Three Agreeing Fits of the Bound State Mass:

In this way we got even two calculations for the bound state mass -
using in addition crude estimation -

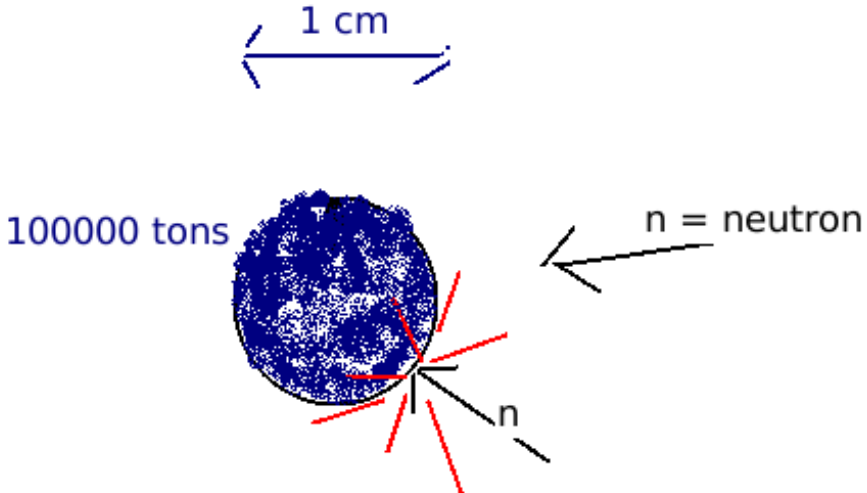
$$m_F(\text{from "high field vacuum"}) \approx 780 \text{ GeV} \pm 40\% \quad (26)$$

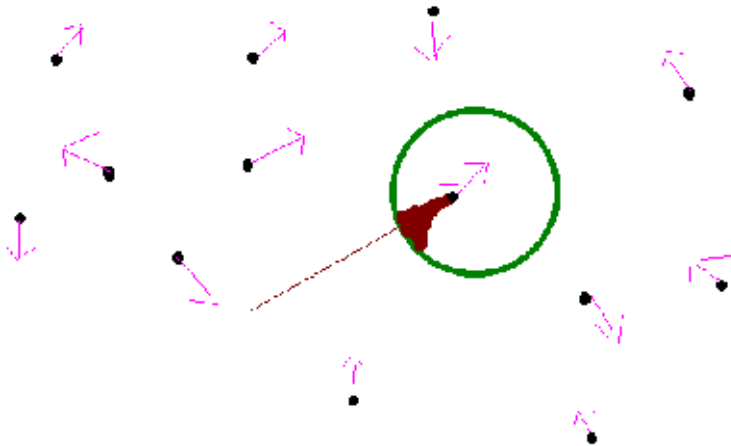
$$m_F(\text{"condensate vac."}) \approx 692 \text{ GeV} \pm 40\% \quad (27)$$

$$m_F(\text{"bag estimate"}) \approx 5m_t = 865 \text{ GeV} (\text{veryuncertain}) \quad (28)$$

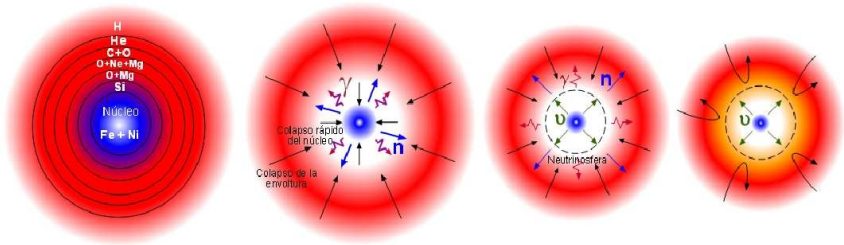
Dark Matter in pure Standard Model with our MPP and bound state:

- The “condensate vacuum” can be used for a model for dark matter as pearl size balls of the “condensate vacuum” surrounded necessarily by a skin - the transition surface - that is then pumped up by ordinary matter, carbon say, to a pressure of the order of that in a white dwarf star. Such pearls may be useful for
 - Dark matter
 - making supernovae explode so as to throw sufficient material out so that we can observe it.
 - Explaing the two bursts of neutrinoes observed with ~ 5 hours time difference in SN1989A in the Big Maggelanic Cloud.
 - helps r-process fit ?
 - explain ratio of dark to normal matter being of order 6.
 - the 3.5 keV X-ray line (from dark matter?)





Supernova development

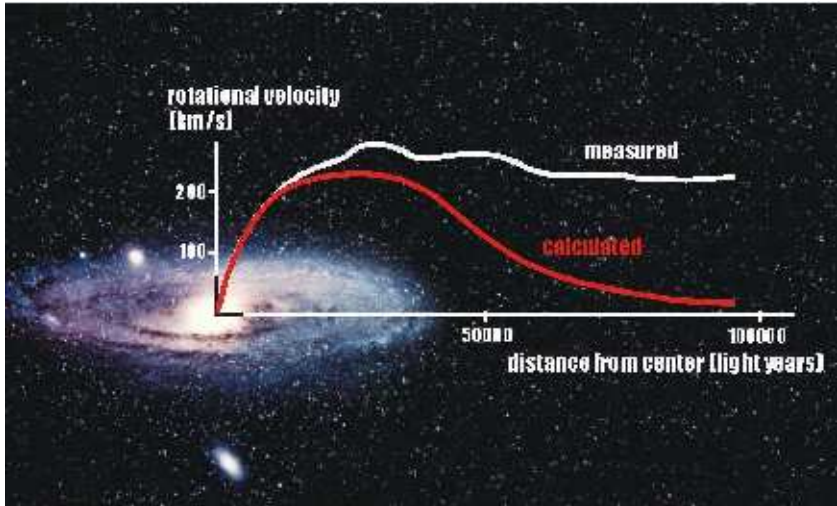


Achievements yet via dark matter as bubbles of “condensate vacuum”:

Dark matter pearls in the mass range best fitting our picture of them as bubbles of alternative vacuum will hit with time intervals of 100 years the earth and cause significant volcanoes, and they will reach so deep that such volcanoes will bring up the only in the deep stable diamonds. It will be Kimberlite pipes. Could also be the Tunguska event.

The Cold Dark Matter out in Space

Most of you think - I guess - that the dark matter means that the dark matter needed to allow the stars to run fast enough around the galaxy requires modification of the Standard Model. Only the black hole type theory and that of Colin Froggatt and me can explain dark matter without changing Standard model. Even **we** though need the finetuning from say our MPP. The top Yukawa g_t has to be sufficiently strong from a bound state F and successively a new vacuum “condensate vacuum”.



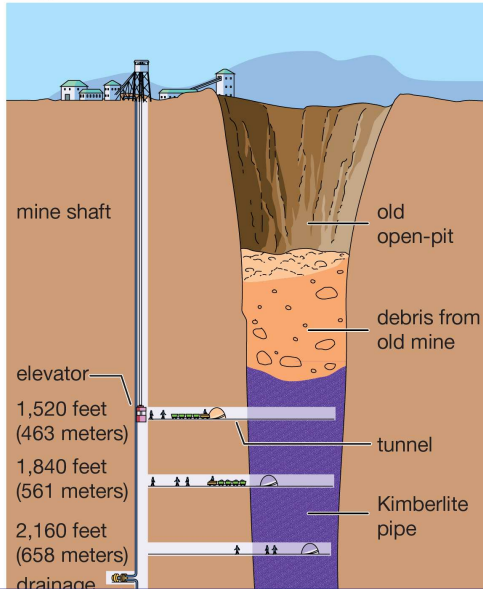
L. Laperashvili(ITEP), H.B. Nielsen(Copenhagen), and C. D. Froggatt, C. Das, Takanishi

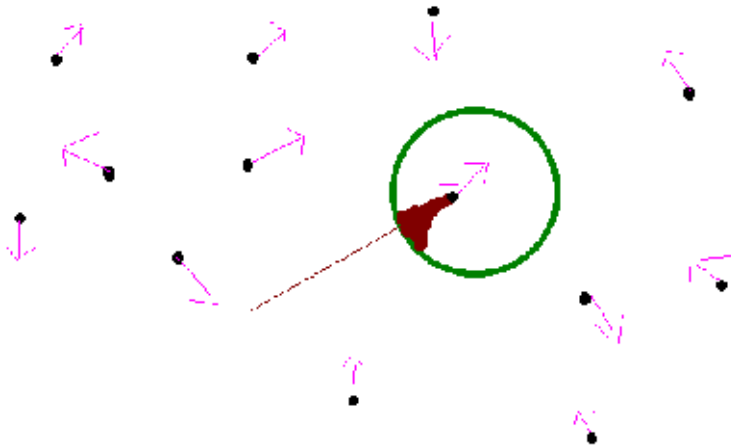
Dark Matter, Double Supernova Neutrino Explosion, Degenerate Vacua, Pre-determination ? Why the Standard Model Group ?

Fritz Zwicky, Opdager af Mørkt Stof



A typical diamond mine

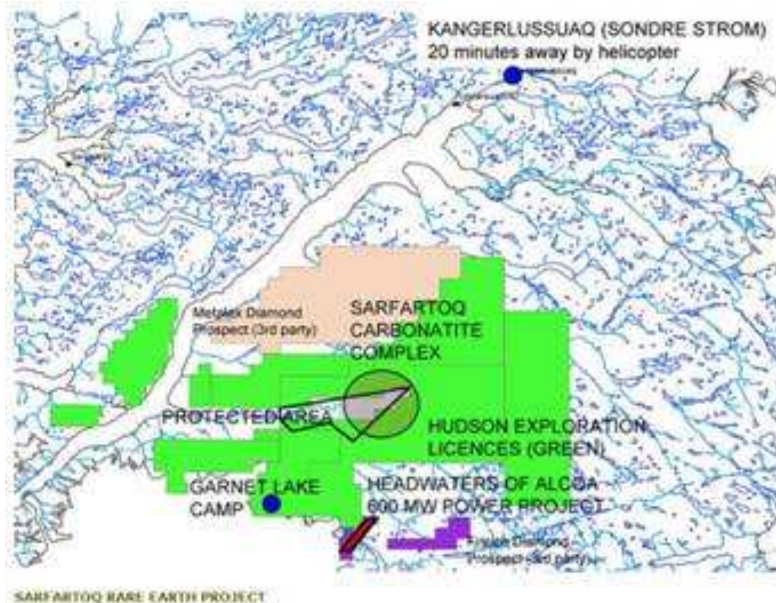


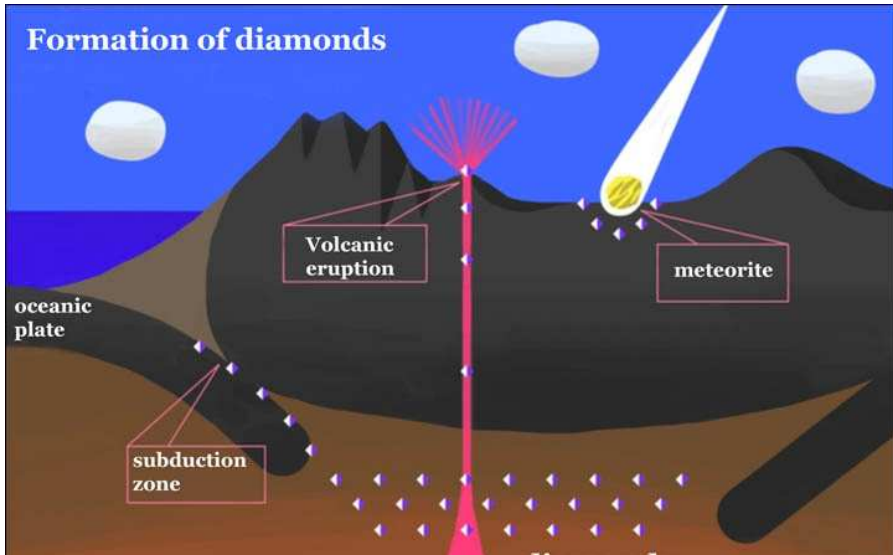


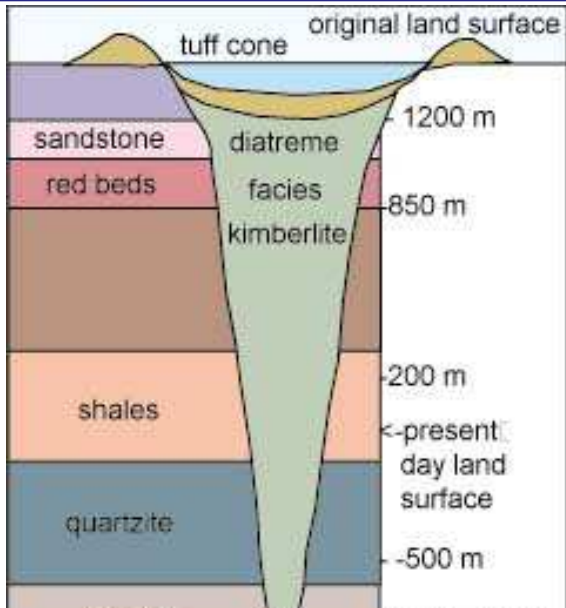


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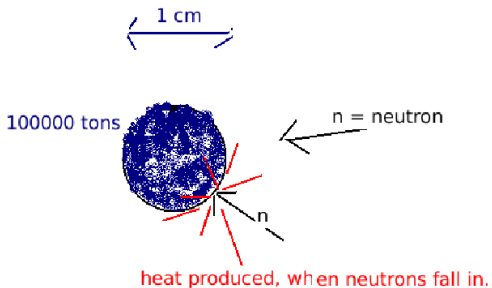
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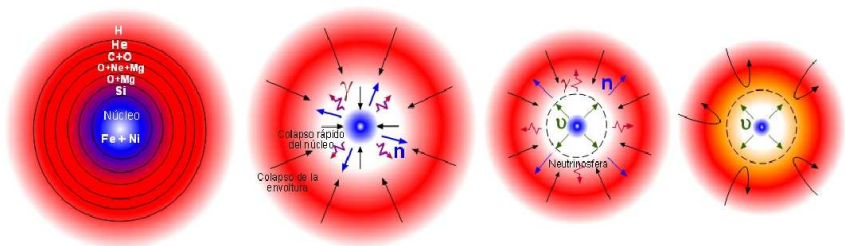




Pearl produce about 10 MeV energy per neutron caught



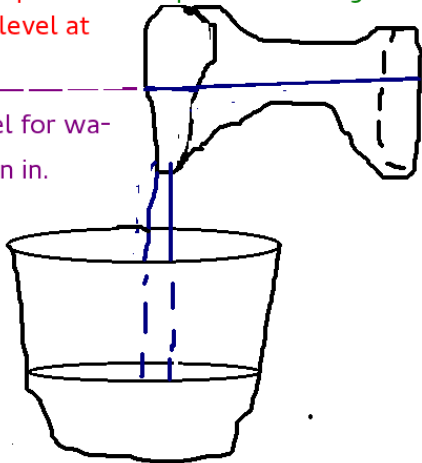
Supernova development



Will any droplet be above top-level at start ? ○

Supernova analogue:

Upper level for water thrown in.



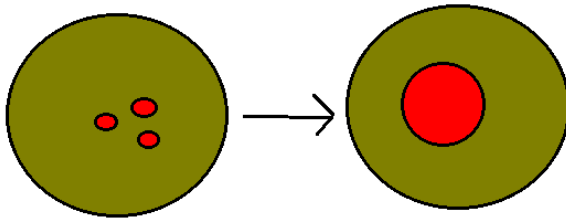
Yes, droplets can come above the start-level !

The Problem for Supernova Explosion Visibly:

- If you calculate “one dimensionally” (i.e. keeping the rotational symmetry) you easily see that the shock wave **cannot** come out so as to produce a **outblow** into space outside, much in analogy to the argument that water keeping its energy cannot go higher than at the start.
- But **turbulence** expected from computer studies **can** help the situation and bring about a sufficiently strong shock wave ? Really often calculate too little, also!
- Only about 1 % ($\approx 1 \text{ Behte} = 1 \text{ foe} = 10^{51} \text{ erg}$) of the total gravitational energy is needed to go out in space.
- But an **explosion** would help - that is **our model**.

Our Dark Matter Pearl(s) Explode and Heat up a lot a big part of the Star.

Then when the shock-wave comes from the collapsing neutron star - in the second step - a **region in the star material was already heated** up a few hours ago (**by our dark pearl explosion**) but is still sufficiently hot that it can help the **shock wave** go all through **to the outside** of the star.



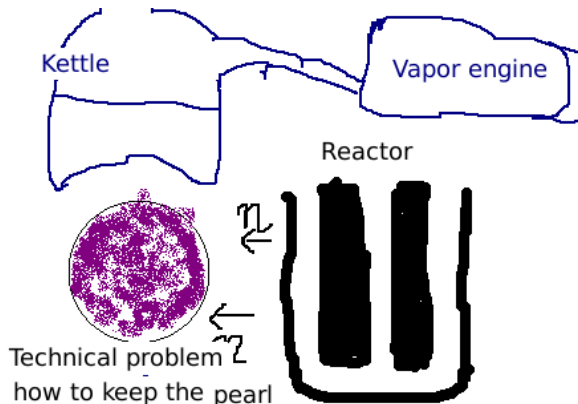
When enough neutrinos present, our **dark matter pearls grow very large**, and may fill a big region of the interior of the star. This growing of our pearl provides a heat of the order of 10 MeV per nucleon absorbed. **Explosion!**

Our Picture of the Late Stages of Supernova Development:

- When interior became iron and further contracts because cooling, but no more fusion energy produced to damp the contraction, the interior begins to fall towards being a neutron star,...
- but now neutrons in big amounts appear and get absorbed by **our pearls under huge heat production.**
- That **stops** the further **contraction** (for a time)!
- But after some hours (we got 14 hours with our fitted numbers, while the observed interval between the neutrino bursts in SN 1987A were ~ 5 hoers.) especially the most central part gets cooled down by neutrino emission,...
- and then the contraction can **start again!** (Second neutrino burst comes then.)

Our picture of Supernova Development (contnued)

- Now an intermediate layer (still very deep, probably iron) is still **hot**, so that is material may (almost? or fully) have energy enough to escape from the star with high temperature still.
- When now the - in usual theory - shock-wave comes, it can easily work its way out to the surface, because the material is so **hot**! (So the problem of the difficulty in getting the visible explosion is solved!)



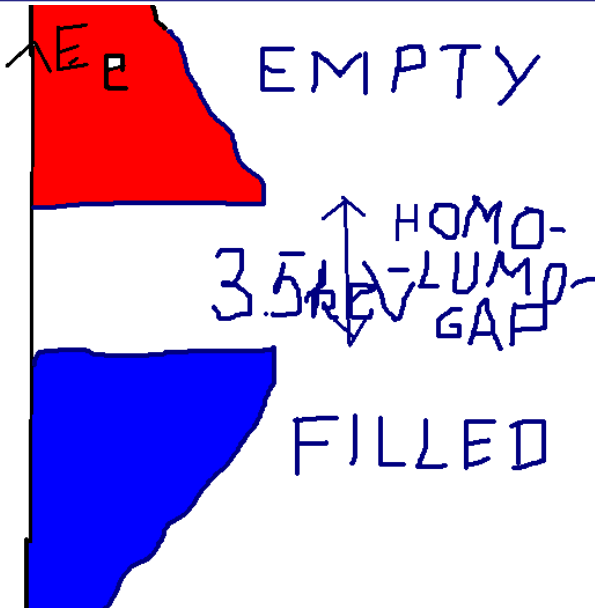
Our pearl may be used to get more energy out of a reactor by their property of heating up by 10 MeV per neutron, when hit by neutrons.

The 3.5 keV X-ray line

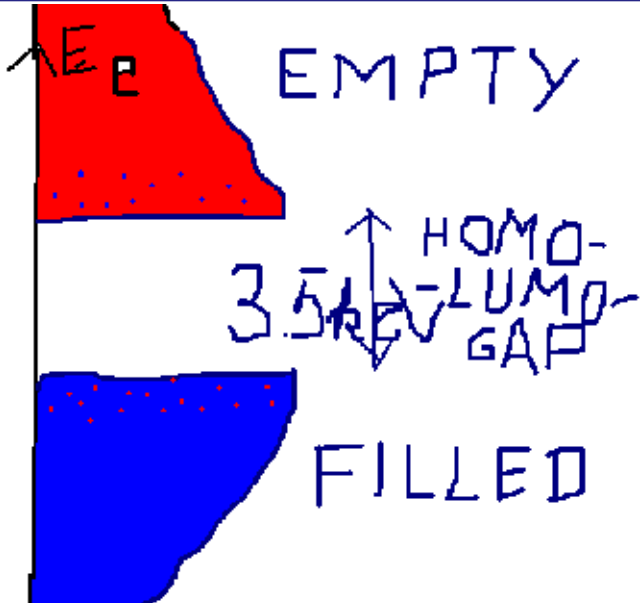
From the Perseus Cluster and from the Center of Our Milky Way and collecting statistics from galaxy clusters... the astronomers have found a hard to identify X-ray line, which corrected for the redshift from the supposed cluster of galaxies had the photon energy 3.5 keV.

This 3.5 keV-line is suspected to come from dark matter.

3.5-line



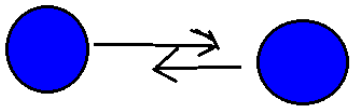
3.5-line



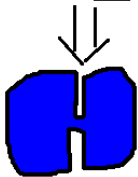
Collisions of Our Pearls as Source of Energy for e.g. the 3.5 keV line.

Somewhat analogous to annihilation there can in our pearl model be released a lot of energy when the dark matter particles meet/collide:

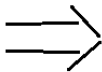
We expect them to unite and then the surface / the **skin** (can) **contract** and thereby **release energy**, in fact a lot, about the Einstein energy of a tenth of the mass of the pearl.



↓ NEXT



WARM



Self Interaction of Our Pearls

We fitted our model parameters to match that the rate of the earth being hit by one of our dark-matter pearls was about once every 100 years or 200 years.

Now the ratio of the radius of our pearl 0.6cm to the radius of the earth is

$$\frac{r_{\text{pearl}}}{r_{\text{earth}}} = \frac{0.6\text{cm}}{6000\text{km}} \quad (29)$$

$$= 10^{-9} \quad (30)$$

giving (31)

$$\frac{\text{area}_{\text{pearl}}}{\text{area}_{\text{earth}}} = (10^{-9})^2 = 10^{-18} \quad (32)$$

Estimating Rate of 3.5-line from Pearl Collissions

Because the area for hitting another pearl is 10^{-18} times that for the earth, a pearl - a selected one, we think of - will hit another pearl every:

$$\text{"hitting interval"} = 100 \text{ years} * 10^{18} = 10^{20} \text{ years} = 3 * 10^{27} \text{ s.} \quad (33)$$

When a collision between two of our pearls occurs an energy of the order of the energy in **bubble surface** is released, and likely a large fraction of that becomes **excitons** and thereby gives the **3.5 keV radiation**.

Ratio of Surface to Bulk Energy

The surface tension of our pearls was supposed to be of the order of magnitude as given by the weak interaction physics, say given in terms of W -masses by dimensional arguments. By the unification after collision of two pearls the total surface area for the uniting bubbles get reduced by of order unity. Taking crudely the weak interaction length scale to be $10^{-18} \text{ m} = 10^{-16} \text{ cm}$ and the energy to be $100 \text{ GeV} = 100 * 10^9 * 1.6 * 10^{-19} \text{ J} \sim 10^{-8} \text{ J}$ the energy in the tension of the pearl surface becomes

$$\text{“surface energy”} = (10^{16})^2 * 10^{-8} \text{ J} \quad (34)$$

$$= 10^{24} \text{ J} \quad (35)$$

$$\sim 10^{24} / 10^{17} \text{ kg} = 10^7 \text{ kg} \quad (36)$$

This is about $\frac{1}{10}$ of the mass of the whole pearl 10^8 kg .

Simulated Life- time $10^{28} s$.

Since about one tenth of the Einstein energy of the pearl sits in the surface tension and gets released by collision of two balls, the life time of a simulating sterile neutrino model particle would be 10 times the “hitting time” $=10^{27} s$, i.e $10^{28} s$ simulated life time. agreeing fine with the earlier fits!

The Energy Comes in Bunches of $\sim 10^7 \text{ kg} * c^2$

With our story that the dark matter consists of pearls with a surface energy getting released when two of them collide and unite to one we have got the dark matter pearls function as **bombs** releasing 10^5 **times more energy than the one from just colliding.**

By the pure collision one only gets the energy of the event in Tunguska, which lead down the trees in a region of order of 70 km, but with the unification of droplets we get with the enormous surface tension about 100000 times as much energy release ! The temperature may raise to $\sim 50 \text{ MeV}$ and corresponding γ -rays would be emitted, a candidate for a gamma-ray burst? That the radiation from the dark matter in this way comes in **pulses**, may be **experimentally accessible.**

Excitons in our Dark Matter Pearls Correpond to Band Gap agreeing order of magnitudewise with 3.5 keV

We have estimated our dark matter pearls to have a specific density of 10^{14} kg/m^3 corresponding to pearl with diameter 1 cm and a mass of 10^8 kg . So they are compressed reltive to ordinary matter having specific densities rather 10^3 to 10^4 kg/m^3 , by a factor 10^{11} to 10^{10} in volume or 5000 to 2000 times in length scale.

We shall argue shortly that such a compression dimishing the lattice constant -if it were a lattice - by a factor 2000 to 5000 would lead to an increase in the band gap giving an exciton decay energy by such a factor. Thus e.g. a band gap in say diamond of 5.5 eV or in Germanium 0.67 eV would be scaled up to respectively 10 keV to 28 keV for diamond and 1.3 keV to 3 keV for Germanium.

How does the Band Gap Scale Under Compression?

Let us obtain an idea about how the band gap in an insulator or semiconductor changes with pressure for extremely high pressure (for ordinary “low” pressures one finds somewhat different results in literature) by the following assumptions/derivations:

- At very high pressure the dispersion relation for the electrons is dominated by the kinetic energy of the electron.
- If only there were the kinetic energy the band gaps would be zero.
- The smallest thus likely to be where we have in first approximation the zero gap.
- The potential energy providing a level splitting - by level repulsion - is given by the Coulomb potential behaving like $1/r$ and thus for dimensional goes as the inverse of the lattice constant a i.e. as $1/a$.

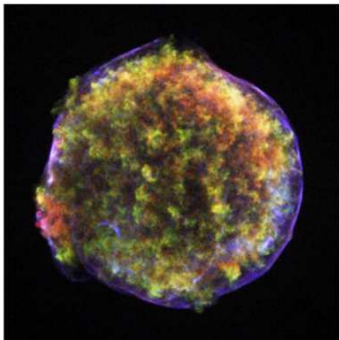
Derivation of Scaling Behavior for Band gap under compression (continued):

- Thus if the lattice constant a is compressed by a factor being the cubic root of the volume compression factor 10^{11} say, the band gap goes up under the compression by the cubic root of 10^{11} , which is 5000.
- So say 1 eV band gap would scale 5 keV. (Within our crude estimate that could well be the 3.5 keV!)



Systematics?

Tycho Supernova Remnant



Credit: NASA/CXC/Rutgers/Warren, Hughes et al.

175 ksec XMM observations

Line at 3.55 keV detected:

- potassium with high abundance?
- systematics in line flux?
- *NOT dark matter*

Fusion Explosion in Pearls to Estimate Isotope Composition of the Ordinary Matter Inside Our Pearls

We suppose that at first the pearl had become filled with Helium, but that then **explosively** a further fusion reaction took place converting this helium into heavier elements. For our hope to understand the 3.5-line hopefully also some uranium and thorium. An understanding from such nuclear explosion may also be helpful for estimating type of ordinary matter under high compression we have got inside the pearl and thereby a bit more accurate estimate of the band gap leading to the 3.5-line radiation.

Importance of the Fusion Explosion

The fusion explosion is important for:

- What is now being the ordinary - from which we are made - was emitted by the explosion due to the heat development, when helium fused into carbon and heavier elements.
- The energy of the fusion to higher elements of helium liberates for heat about 1.5 MeV per nucleon in as far as the binding in Helium is 7 MeV while in the heavier elements it is rather 8 to 9 MeV. Thus there is excess energy to emit totally out of the part one nucleon in 5 when the helium fusions.
- We hope some uranium or thorium is formed (by some neutron capture process, essentially r-process)

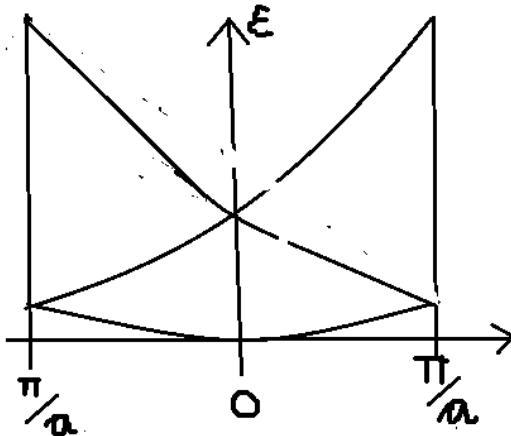
The 3.5keV line is energized by Radioactive thorium or uranium inside the pearls

The pearls must be around for the order of the age of the Universe 13.6 milliard years, and thus only the isotopes with life times of this order or longer will still be present in our times. Uranium and can cope with such long times and still be active to excite electrons inside our pearls. These (quasi)electrons and the holes then form excitons which may go to have just the band gap energy and finally decay under emission of photons (in the case of the huge gap X-rays). Some line will appear with exciton energy equal to the band gap energy. **3.5 keV ?**

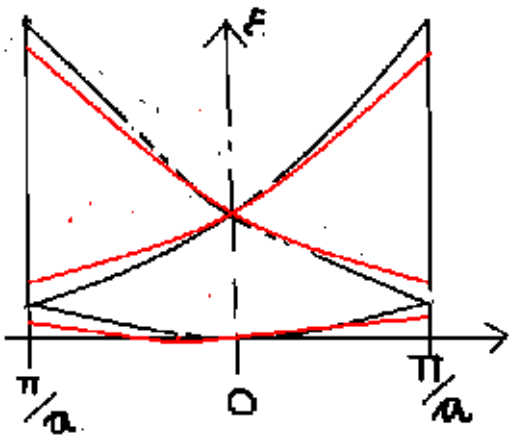
Degenerate Electron states get split by the Potential term treated as a perturbation to the purely kinetic energy

When the first order approximation has two electron states - namely with momenta $p = \pm \frac{\pi}{a}$, where a is the lattice constant - with the same energy $E = \frac{\pi^2}{a^2 2m}$ and the same quasi momentum $p_Q = \frac{\pi}{a} (\text{mod } \frac{2\pi}{a})$, the perturbation term, namely the potential term is to be diagonalized in the two-dimensional subspace of the state space for the electron corresponding to these two states. The energy of the states resulting from these two states after the perturbation will be split just as the splitting of the eigenvalues for the perturbation/the potential term.

Dispersionrelation Only with Kinetic Energy in One Dimension



Dispersionrelation Perturbed by Potential Term



Conclusion on Dark Matter Pearls

- Have a Dark Matter Model with **Only Standard Model !**
- Our Dark Matter consists of cm-size pearls of mass 500000 ton, inside which there is some ordinary matter carbon ... uranium sitting on a background of a new vacuum called “condensate vacuum” .
- The pearls should have formed after the time when the temperature passed the weak interaction temperature scale 100 GeV. First they should have formed in random shapes and first after the antibaryons were annihilated away they would have had time to contract. Now they would then stopped from totally contracting - if sufficiently large - by including nucleons carried by a 10 MeV barriere from going out of the pearls.
- First there would form helium inside the pearls,

- but at some moment an **explosion** of helium being fused to heavier elements would take place. A so high temperature would arise that by essentially a rapid neutron capture process, r-process, would take place, so that even uranium and thorium would be formed.
- After full contraction till the ordinary chemicals inside could stop further contraction of the star separating the condensate and the present vacuum the pressure inside is so high that the density of the ordinary matter there becomes 10^{14} kg/m^3 .
- The atoms inside are then compressed so that the lattice constant or the distance between the atoms is of the order of 3000 times smaller than in ordinary matter under normal conditions.

- This compression raise the band gap between the filled and empty electronic bands by a factor being the cubic root of the volume compression factor 10^{11} . So an ordinary band gab in the eV range goes up by a factor of the order of 5000.
- An elctron in the empty bound to a hole in the uppermost filled band makes up an exciton, which after relaxation has the energy of the band width meaning 5000 times an “ordinary” band gap of eV order of magnitude.
- We want to identify radiation from the decay of such excitons with the by the astronomers observed 3.5 keV X-ray radiation, suspected to come from dark matter.
- We estimate that cosmic rays as a source of energy to produce the observed 3.5 keV radiation falls short of the observed intensity by a factor 10^5 but that radioactivity from uranium and or thorium inside the pearls themselves might provide sufficient energy for the radiation.

- The line has been seen in the supernova remnant from Keplers supernova. This could be explained in our model as being due to the very high 3.5 keV emission due to the energy provided by the cosmic radiation, which is of course much stronger in the supernova remnant than at a random place in the dark matter. Very few dark matter models could match that a dark matter line should be seen especially from a supernova remnant.

Conclusion

The remarkable **coincidence**, that our three mass estimations coincide is an evidence in favour of the **truth of our model, with the Multiple Point Principle, and the bound state!**

Some Achievements of our Model MPP and Strongly Bound State of 6 Top + 6 Anti top, (in pure SM)

I must mention the following achievements most of which I did not have time for:

- **Hierarchy problem** The fine-tuning caused by our MPP requirements combined with the assumption, that the Higgs field in the “high field vacuum” is of the order of the Planck scale (or only a bit under) leads to the scale problem being solved in the sense, that the Higgs mass and weak scale get fixed to be *exponentially* much lower than the Planck scale, and that in fact very closely by the right size for the logarithm.

Achievements of Multiple Point Principle(=MPP)(in pure SM) Continued:

- g_t Froggatt and I estimated the value of the top-Yukawa-coupling g_t needed for MPP in the sense, that it represents a phase transition value between the “condensate vacuum” and the “present vacuum”. We found the phase transition $g_t \text{ phase transition} = 1.02 \pm 14\%$, agreeing with experiment $g_t \text{ exp} = 0.935$.
- **Stability** Explaining the that the Higgs mass just puts our vacuum on the borderline of being meta-stabel.
- **Correction to Stability** ...even very accurately, if we take seriously the very small correction due to the bound state by Laperashvili, Das, and myself.

Acievements Still in pure Standard Model of our MPP and bound sate:

- The “condensate vacuum” can be used for a model for dark matter as pearl size balls of the “condensate vacuum” surrounded necessarily by a skin - the transition surface - that is then pumped up by ordinary matter, carbon say, to a pressure of the order of that in a white dwarf star. Such pearls may be useful for
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Propose to solve some Astronomical Mysteries

We propose to solve some astronomical mysteries in the sense that you may explain the phenomena by some model, but you do NOT expect things to go like they do in the first crude thinking. Such “mysteries”, “we solve”:

- The supernovas show up as external visible explosions in spite of that the energy comes by material falling deeper in, and computers often does not give enough energy thrown out.
- **two** strong neutrino bursts in supernova. (It were seriously proposed that first were formed a “planet” around the neutron star, which then hours later fell in and united with the neutron star. Alternatively the star might **first become neutron star** and **then a black hole**, a few hours later to make two neutrino bursts.)

Achievements of MPP in Extended Models (i.e. not only SM):

- **Value of Cosmological Constant** With Roman Nevzorov we got values for the CC using “same version” of MPP and an almost supersymmetric vacuum state.
- **Number of families** Prior to having formulated MPP we fitted finestructure constants in an extension of the Standard Model “AntiGUT” in which each family of fermions has its own set of gauge bosons. We - including Brene and Don Bennett and me - **predicted** the number of families, which was not known yet at that time.

Encouragement for Theoreticians to Calculate More Accurately This Bound State

At the end I would stress: **Since Our picture is PURE STANDARD MODEL, everything can in principle be CALCULATED!** So it is only a question of better techniques - Bethe Salpeter Equation ? - or better computers and use of them - lattice theory with Higgsfield on the lattice ? - to obtain more solid and accurate checks of MPP and calculation of the bound state mass than my crude estimates.

And this is just a work for the theoreticians (among the students say).

Then there should pop up some peaks - like the joke of Pich's - by themselves, when the experimentalists make the plots.

If not it would mean that Standard Model were not right also nonperturbatively.

Many in the audience should be able to do a better calculation involving our phantasy-bound state:

If you can make lattice calculations on a theory with many scalars in addition to gauge fields, it should be easy to do it for the Standard Model with the Higgs field as the important exchange between top quarks. Or if you have a frame wherein the bound state can be studied by a method ending up mathematically similar to the Schrödinger-equation - just for mass square - it should not be so difficult to add the Higgs-exchange and investigate if our 6 top + 6 anti top will bind as a superexcotic resonance.

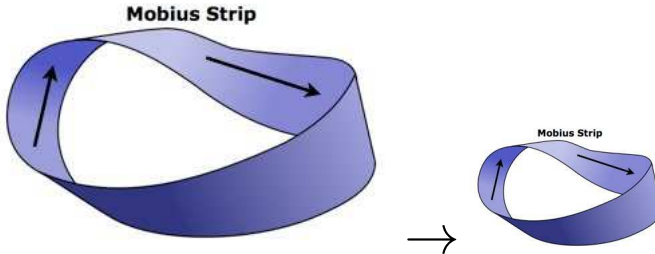
Why did Nature Choose Just the Standard Model Group $S(U(2) \times U(3))$?

The **group** (rather than only the Lie **algebra**) has a phenomenological meaning according to a way of making sense of it by O'Raifeartaigh:

The point is that various **groups** with the same Lie **algebra** have different systems of **representations**.

The covering group has all the representations possible for the Lie algebra, but the other Lie groups with a given algebra have fewer representations. The suggestion would be to call that **group** which has all the representations found experimentally, but among the groups with the right representations is the one with fewest possible representations, for the phenomenologically found **group**.

**Two Different Representations of the Same Group
Look Like Almost the One being just an Enlarged
Image of the Other One, at most a bit different
scale in different directions.**



Main Point of the Characterizing Feature of the Standard Model Group:

When we ignore a very important trouble with the **Abelian** components of the groups, we explain our suggested property, meant to specify the Standard Model Group $S(U(2) \times U(3))$, as:

- Between (unitary) matrices one can define a very simple distance concept

$$\text{dist}(\mathbf{U}_1, \mathbf{U}_2) = \sqrt{\frac{\text{Tr}(|\mathbf{U}_1 - \mathbf{U}_2|^2)}{\text{Tr}(\mathbf{1})}}, \quad (37)$$

or use this expression for infinitesimally close matrices on say a unitary representation of a group, and then extend that to a distance concept over the whole representation manifold, by intergating up the infinitesimal one.

Explaining the Standard-Model-Group Characteristic Property (Continued)

- Our by trace of unitary representation matrices defined distance concept obviously induce a **distance** concept on the Lie group **for each representation** considered.
- Such representation induced distance concepts are both left and right invariant; i.e. for three elements in the group g_1, g_2, g_3 we have

$$dist(g_1, g_2) = dist(g_3 g_1, g_3 g_2), \quad (38)$$

whatever the representation applied to define $dist$. This is called left invariance, while analogously, if $dist(g_1 g_3, g_2 g_3) = dist(g_1, g_2)$, it is called right invariant.

- This left and right invariance specify a Riemann metric up to an over all scaling **along any simple subgroup**.

Explaining our Standard Model Group Characteristic Feature (Continued)

- For a simple Lie group the invariant Riemann-space distance is unique **up to an overall scale**.
- For an only semisimple Lie group the scale factor can be different for the different simple factors.

Adjoint / "Simplest" Quadratic Casimirs C_A/C_F

$$\frac{C_A}{C_F} \Big|_{A_n} = \frac{2(n+1)^2}{n(n+2)} = \frac{2(n+1)^2}{(n+1)^2 - 1} = \frac{2}{1 - \frac{1}{(n+1)^2}} \quad (39)$$

$$\frac{C_A}{C_{F \text{ vector}}} \Big|_{B_n} = \frac{2n-1}{n} = 2 - \frac{1}{n} \quad (40)$$

$$\frac{C_A}{C_{F \text{ spinor}}} \Big|_{B_n} = \frac{2n-1}{\frac{2n^2+n}{8}} = \frac{16n-8}{n(2n+1)} \quad (41)$$

$$\frac{C_A}{C_F} \Big|_{C_n} = \frac{n+1}{n/2 + 1/4} = \frac{4(n+1)}{2n+1} \quad (42)$$

$$\frac{C_A}{C_{F \text{ vector}}} \Big|_{D_n} = \frac{2(n-1)}{n-1/2} = \frac{4(n-1)}{2n-1} \quad (43)$$

$$\frac{C_A}{C_{F \text{ spinor}}} \Big|_{D_n} = \frac{2(n-1)}{\frac{2n^2-n}{8}} = \frac{16(n-1)}{n(2n-1)} \quad (44)$$

The Adjoint/“smallest” Squared Distance Ratios for Special Lie Groups:


$$\frac{C_A}{C_F}|_{G_2} = \frac{4}{2} = 2 \quad (45)$$

$$\frac{C_A}{C_F}|_{F_4} = \frac{9}{6} = \frac{3}{2} \quad (46)$$

$$\frac{C_A}{C_F}|_{E_6} = \frac{12}{\frac{26}{3}} = \frac{18}{13} \quad (47)$$

$$\frac{C_A}{C_F}|_{E_7} = \frac{18}{\frac{57}{4}} = \frac{72}{57} = \frac{24}{19} \quad (48)$$

$$\frac{C_A}{C_F}|_{E_8} = \frac{30}{30} = 1 \quad (49)$$

For calculation of this table seek help in Rittenberg, MacFarlane. 



Adjoint / "Simplest" Quadratic Casimirs C_A/C_F

$$SU(n+1) : \frac{C_A}{C_F} |_{A_n} = \frac{2}{1 - \frac{1}{(n+1)^2}} \quad (50)$$

$$[SO(2n+1) : \frac{C_A}{C_F \text{ vector}} |_{B_n} = 2 - \frac{1}{n}] \quad (51)$$

$$SO(2n+1) : \frac{C_A}{C_F \text{ spinor}} |_{B_n} = \frac{16n-8}{n(2n+1)} \quad (52)$$

$$Sp(2n) : \frac{C_A}{C_F} |_{C_n} = \frac{4(n+1)}{2n+1} \quad (53)$$

$$[SO(2n) : \frac{C_A}{C_F \text{ vector}} |_{D_n} = \frac{4(n-1)}{2n-1}] \quad (54)$$

$$SO(2n) : \frac{C_A}{C_F \text{ spinor}} |_{D_n} = \frac{16(n-1)}{n(2n-1)} \quad (55)$$

The Case $SU(2) = A_1$ is easily Checked

The smallest representation F of $SU(2)$ is one with $j_F = \frac{1}{2}$, while the adjoint has $j_A = 1$.

The quadratic Casimir is (normalization is not welldefined) $C = j(j + 1)$ since it is the eigenvalue of the sum \vec{J}^2 of one operator for each Lie algebra generator.

Indeed

$$C_A/C_F = \frac{j_A(j_A + 1)}{j_F(j_F + 1)} = \frac{1 * 2}{1/2 * (1 + 1/2)} = \frac{8}{3}. \quad (56)$$

For Semi-simple group Use the with the dimensions of the simple factors of the logarithm of our C_A/C_F .

Generally for a semisimple group we take the effective/replacement ratio

$$C_A/C_F = \exp\left(\frac{d_1 \ln((C_A/C_F)_1) + \dots + d_k \ln((C_A/C_F)_k)}{d_1 + \dots + d_k}\right) \quad (57)$$

$$= \left(\frac{C_A}{C_F}\right)_1^{\frac{d_1}{d_1+\dots+d_k}} \dots \left(\frac{C_A}{C_F}\right)_k^{\frac{d_k}{d_1+\dots+d_k}} \quad (58)$$

Example of Calculating a C_A/C_F -ratio.

For example for the group $SU(2) \times SU(3) = A_1 \times A_2$ we take the effective or replacement for C_A/C_F to be

$$(C_A/C_F)_{SU(2) \times SU(3)} = \exp\left(\frac{3 * \ln(8/3) + 8 * \ln(9/4)}{11}\right) \quad (59)$$

$$= \exp\left(\frac{3 * 0.98082925301 + 8 * 0.81093021621}{11}\right) \quad (60)$$

$$= \exp\left(\frac{9.42992948877}{11}\right) \quad (61)$$

$$= \exp(0.85726631716) = 2.3567093838 \quad (62)$$

provided we can use the spinor representations.

We Need Covering group... to get Allowed to use Spinor Representations.

The quadratic Casimir for an Abelian group must be the **square of the charge**.

For specific representations there is welldefined (square of) charge, but there is no welldefined “Adjoint” representation for an Abelian group.

But we can e.g. in Standard Model define two charge-quanta:

- The smallest weak hyper-charge $y/2$ allowed with the ORaifaitaigh Standard Model Group (= the smallest $y/2$ found on any quark or lepton etc.)
- The smallest allowed in a representation with a “trivial”, “vector” or composed of vector representations representation of the nonabelian groups. (here “vector” really should be Adjoint)

To Argue for What to use for C_A/C_f in Case of an Abelian Group is Slightly a Definitional Choice, Weak Point.

Define for a certain $U(1)$ in the group:

Q_{min} = Smallest $U(1)$ -charge (in model)

Q_{alone} = smallest $U(1)$ -charge allowed, when the other group factors out

Then define for the $U(1)$:

$$(C_A/C_F)_{U(1)} = \frac{Q_{alone}^2}{Q_{min}^2}.$$

E.g. in Standard Model the weak hypercharge $U(1)$ has

$$(C_A/C_F)_{U(1)} = 36.$$

C_A/C_F for Standard Model Group.

The dimensionality weighted logarithms of the quadratic Casimir ratios C_A/C_F for the respective Lie algebras in the Standard Model group SMG becomes

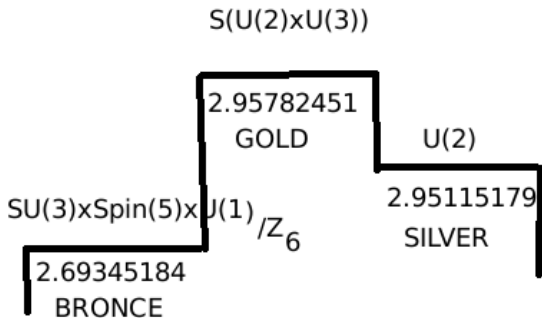
$$(C_A/C_F)_{SMG} = \exp\left(\frac{1 * \ln(36) + 3 * \ln(8/3) + 8 * \ln(9/4)}{12}\right) \quad (67)$$

$$= \exp\left(\frac{1 * 3.58351893846 + 11 * 0.85726631716}{12}\right) \quad (68)$$

$$= \exp(1.0844540356) \quad (69)$$

$$= 2.95782451094 \quad (70)$$

This 2.95782451094 is the score of the SMG and according to our calculations no Lie **group** can get more.



This figure illustrates the three Lie groups getting in our game the highest scores for our “goal quantity” as were the sportsmen winning gold silver and bronze medals