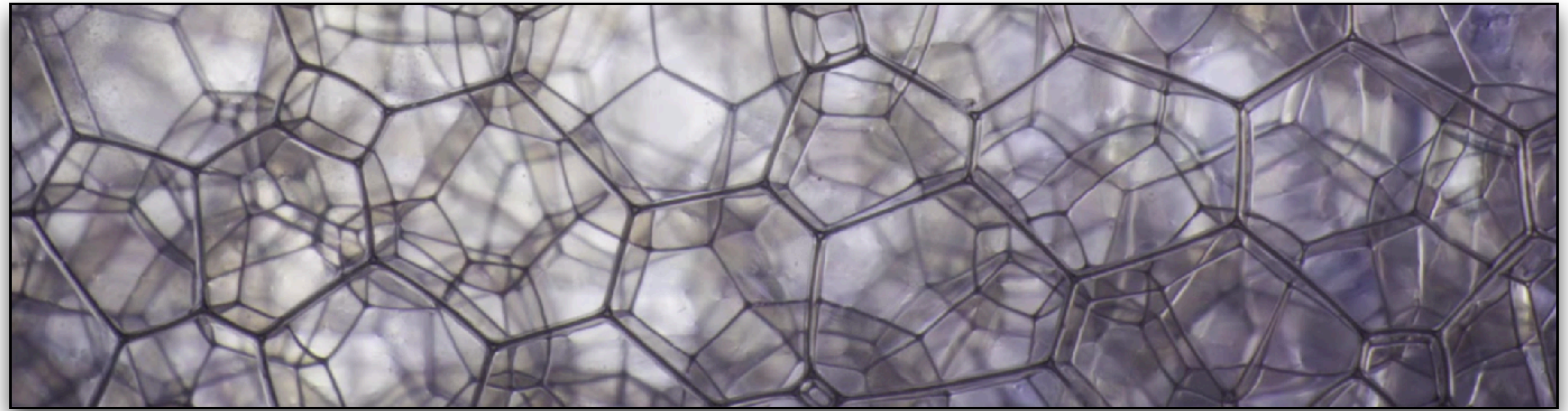
Entanglement in loop quantum gravity

Eugenio Bianchi

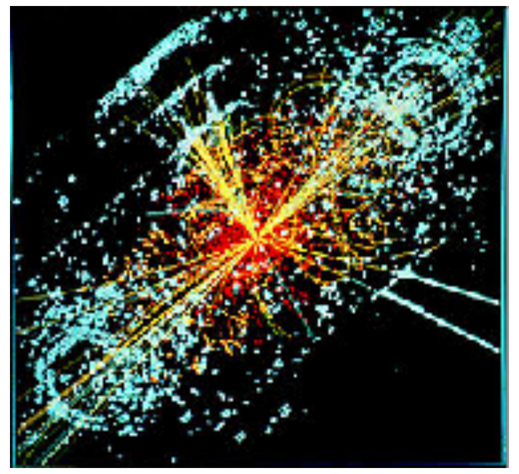
Institute for Gravitation and the Cosmos
& Physics Department, Penn State



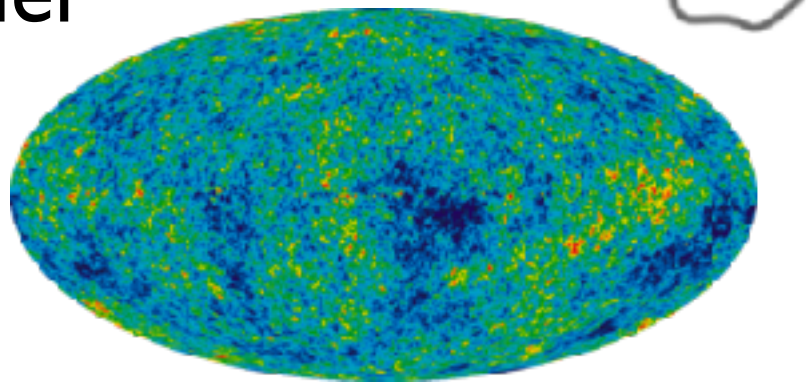
[soap foam - microphotography by Pyanek]

Laboratori Nazionali di Frascati
20 Dicembre 2017

High Energy
Frontier



Large Scale
Frontier



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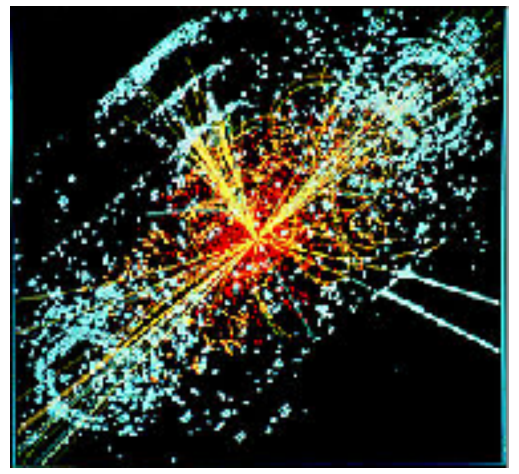
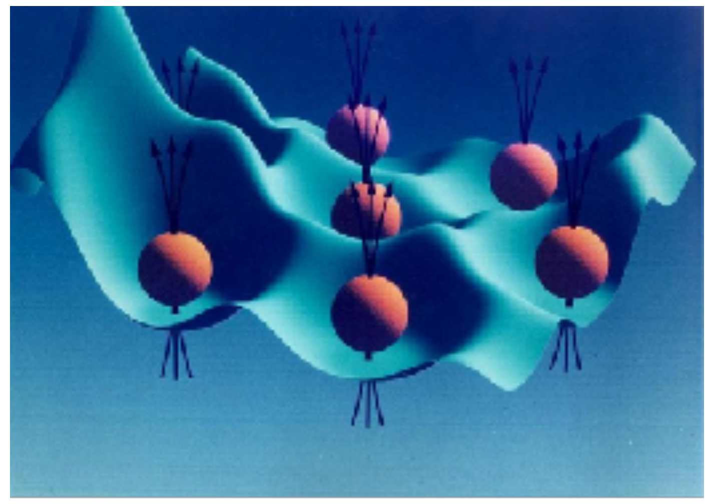
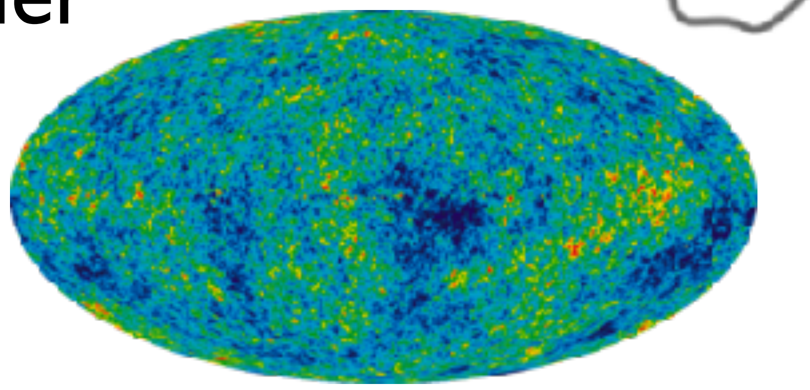


The Entanglement Frontier



- complex quantum systems

Large Scale
Frontier



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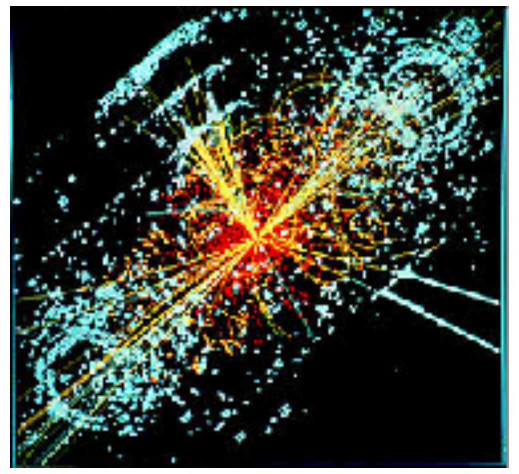

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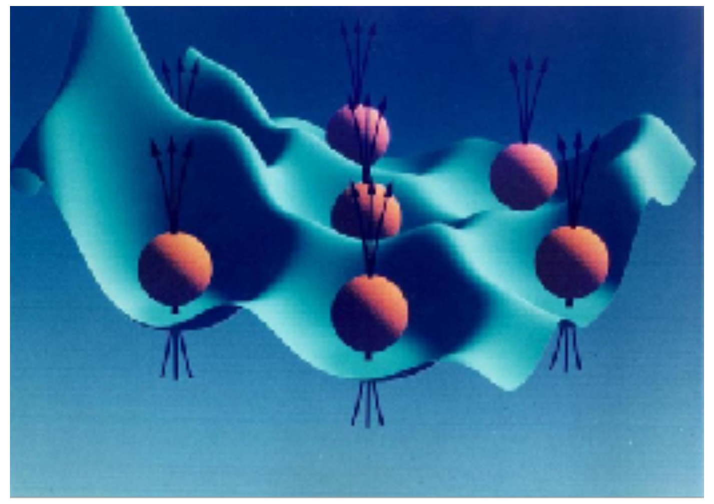

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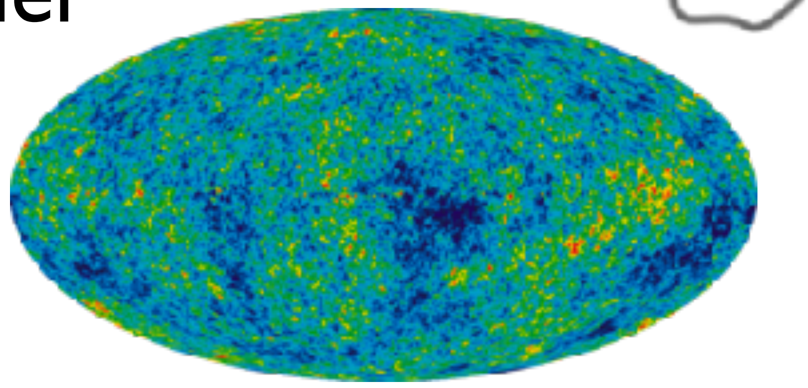

- complex quantum systems
- many-body entanglement
- phases of quantum matter
- quantum computing

...

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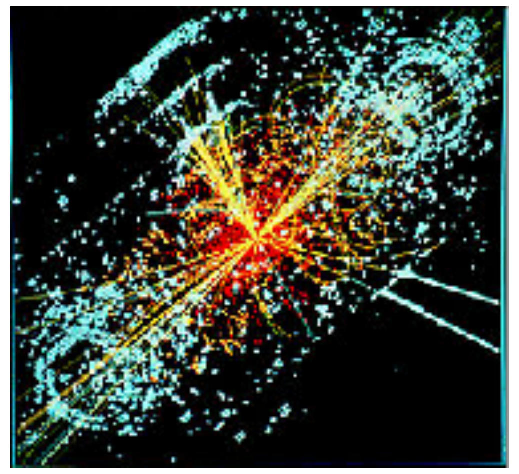

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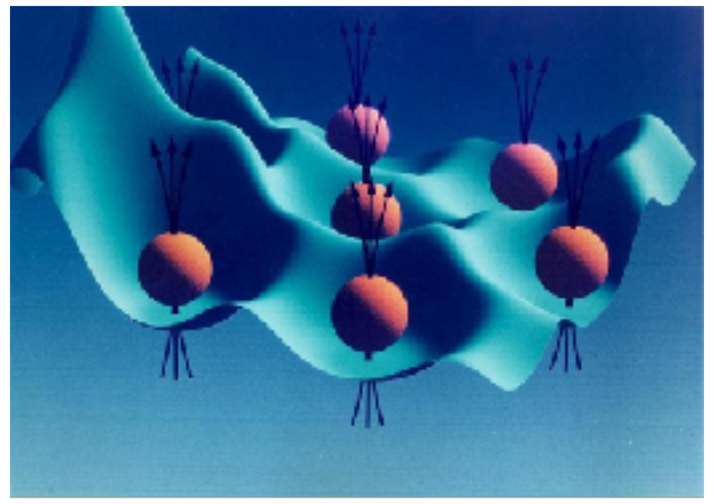

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- thermalization in isolated quantum systems

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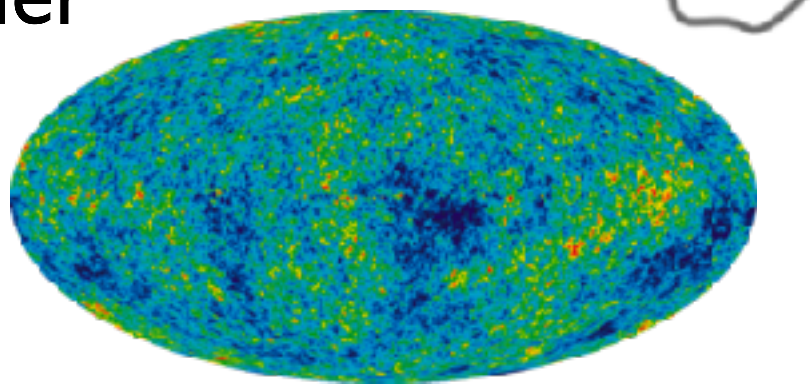

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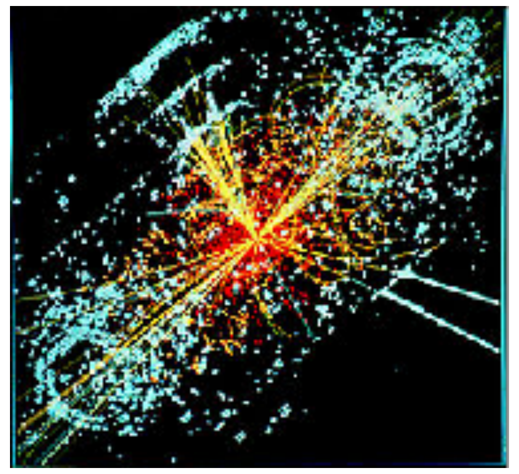

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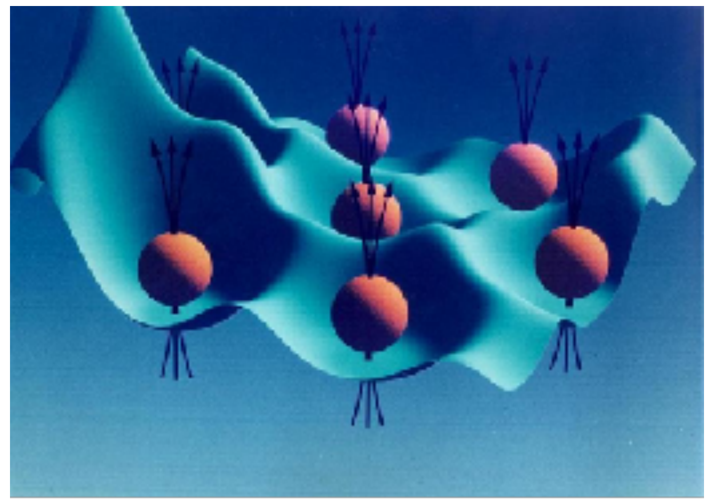

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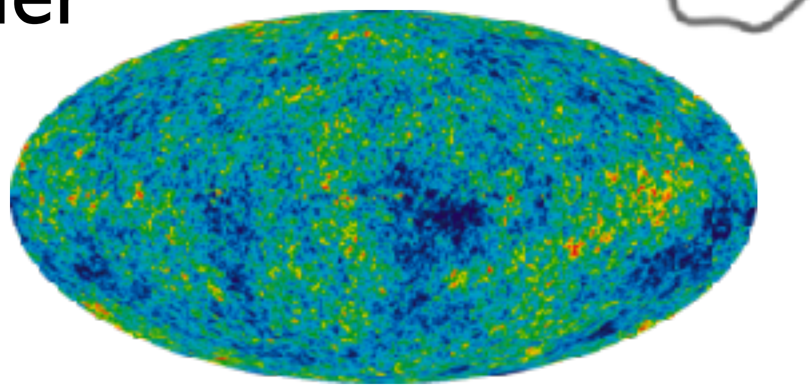

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- black hole evaporation

High Energy
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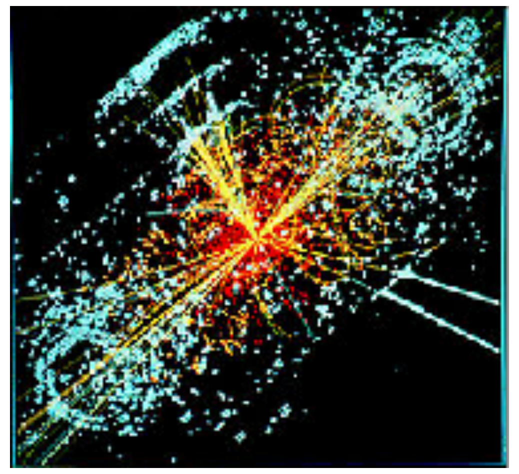

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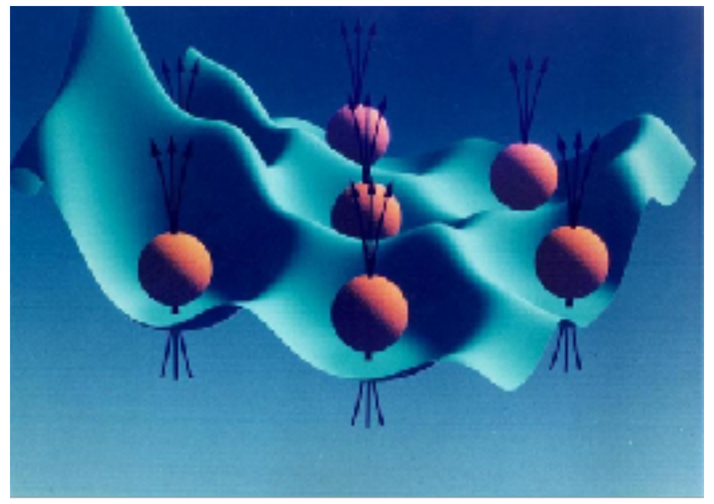

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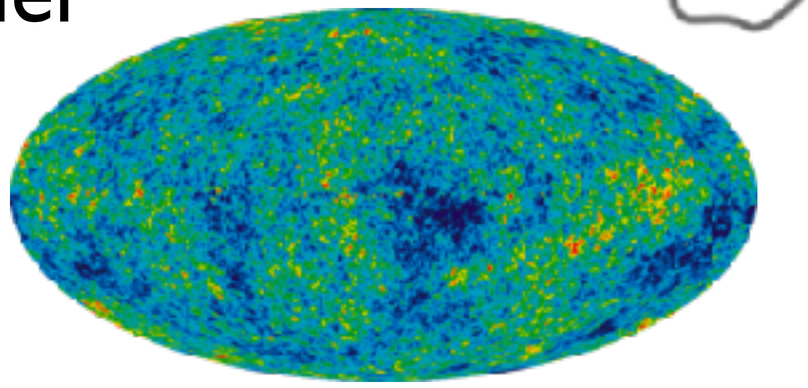

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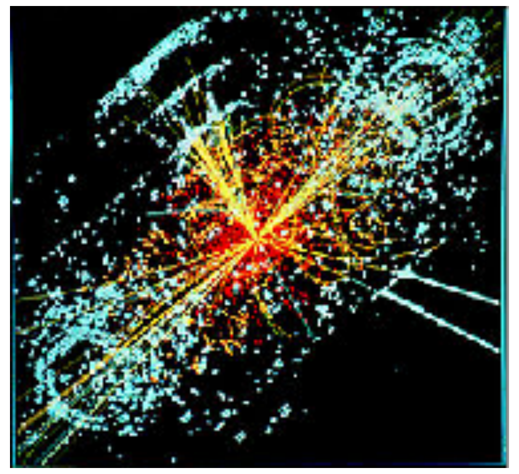
- black hole evaporation
- entanglement and the architecture of spacetime

High Energy Frontier



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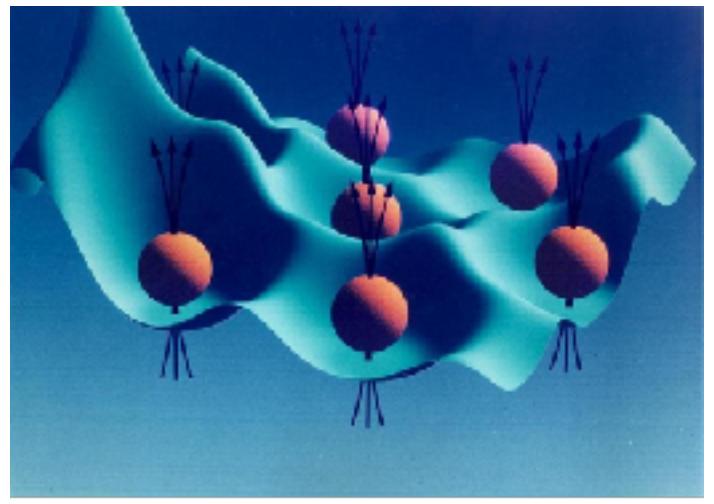

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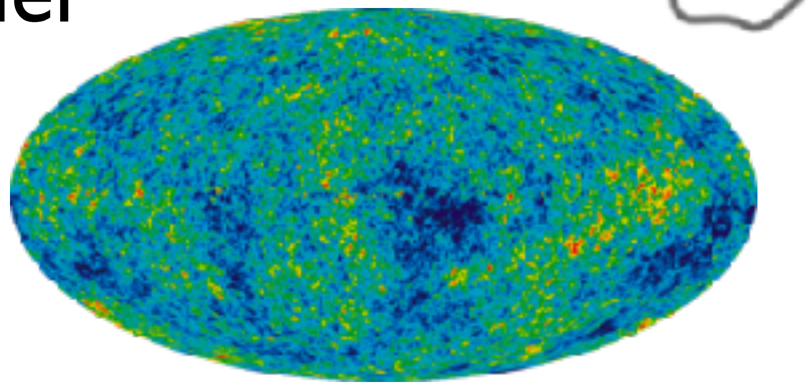

- complex quantum systems
- many-body entanglement
- phases of quantum matter
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- black hole evaporation
- entanglement and the architecture of spacetime
- quantum correlations in the very early universe

Plan:

I) Entanglement in simple systems

II) Building space from entanglement

III) Entanglement in the sky

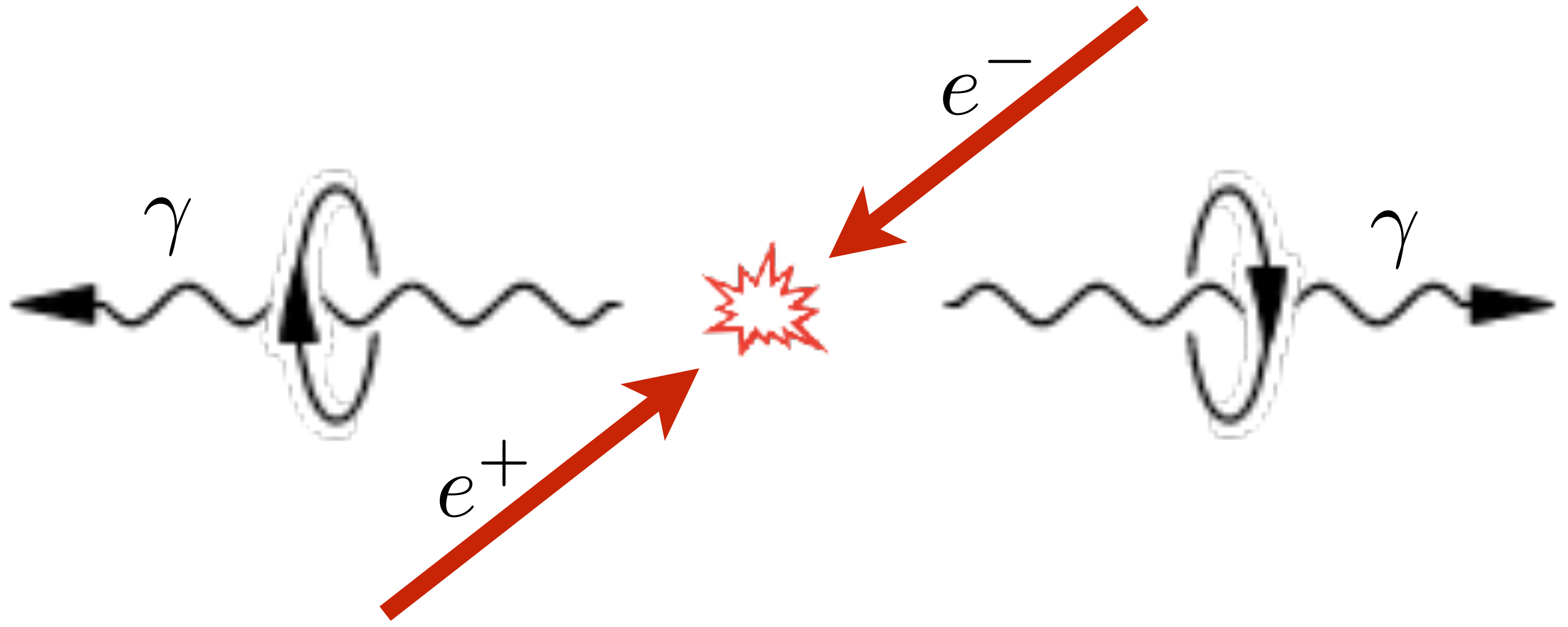
Entangled state

Einstein-Podolsky-Rosen, 1935
Schrödinger, 1935

Singlet state of two spins: $$|s\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle\Big)$$

How entanglement is produced:
E.g., electron-positron annihilation into two gamma rays

$$e^- + e^+ \to \gamma + \gamma$$



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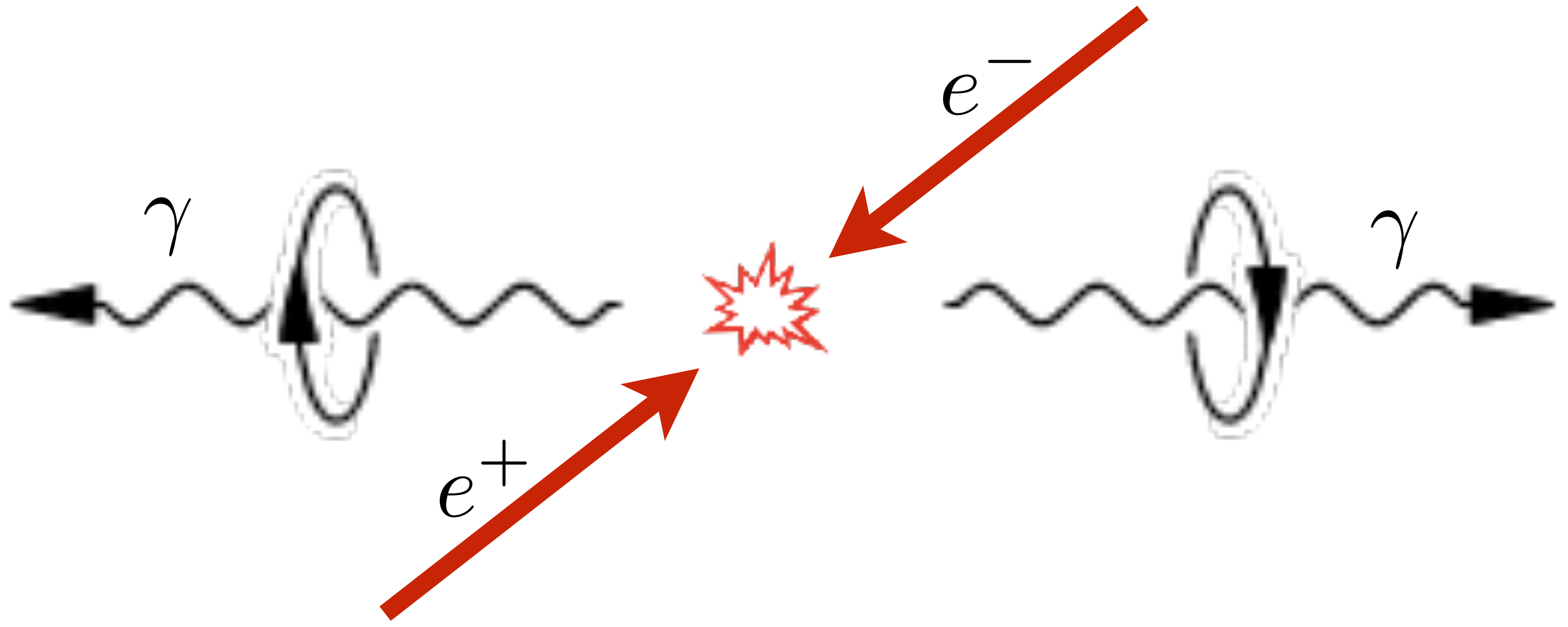
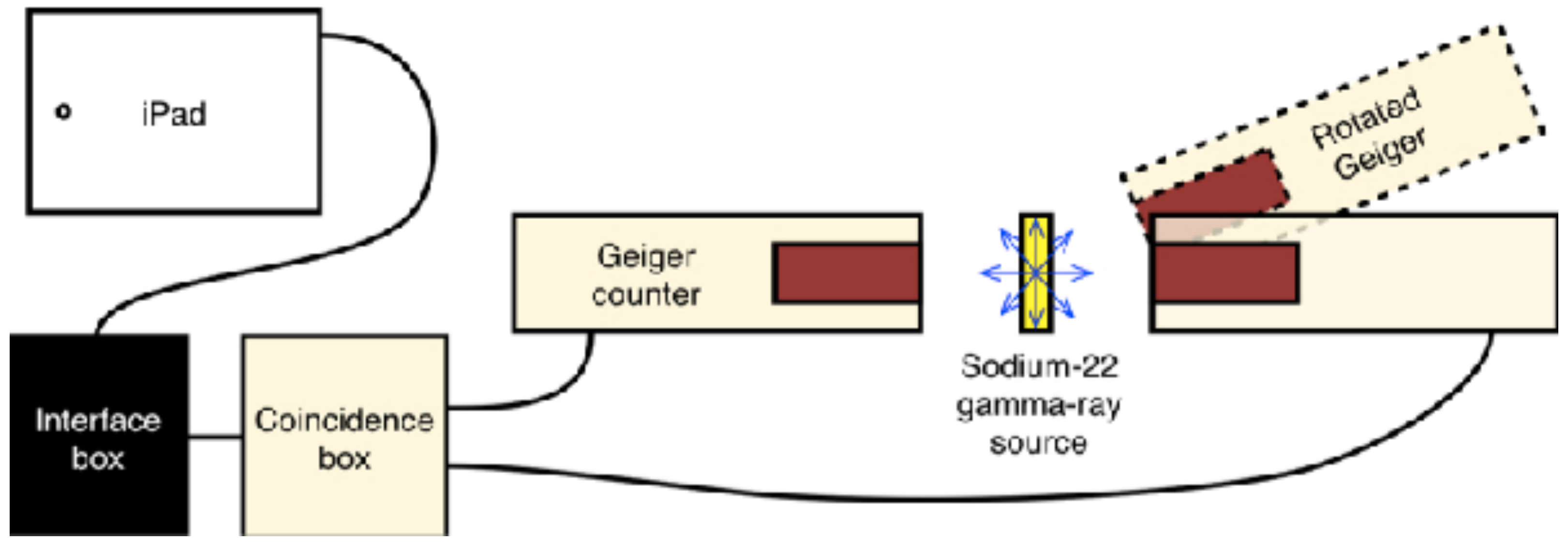
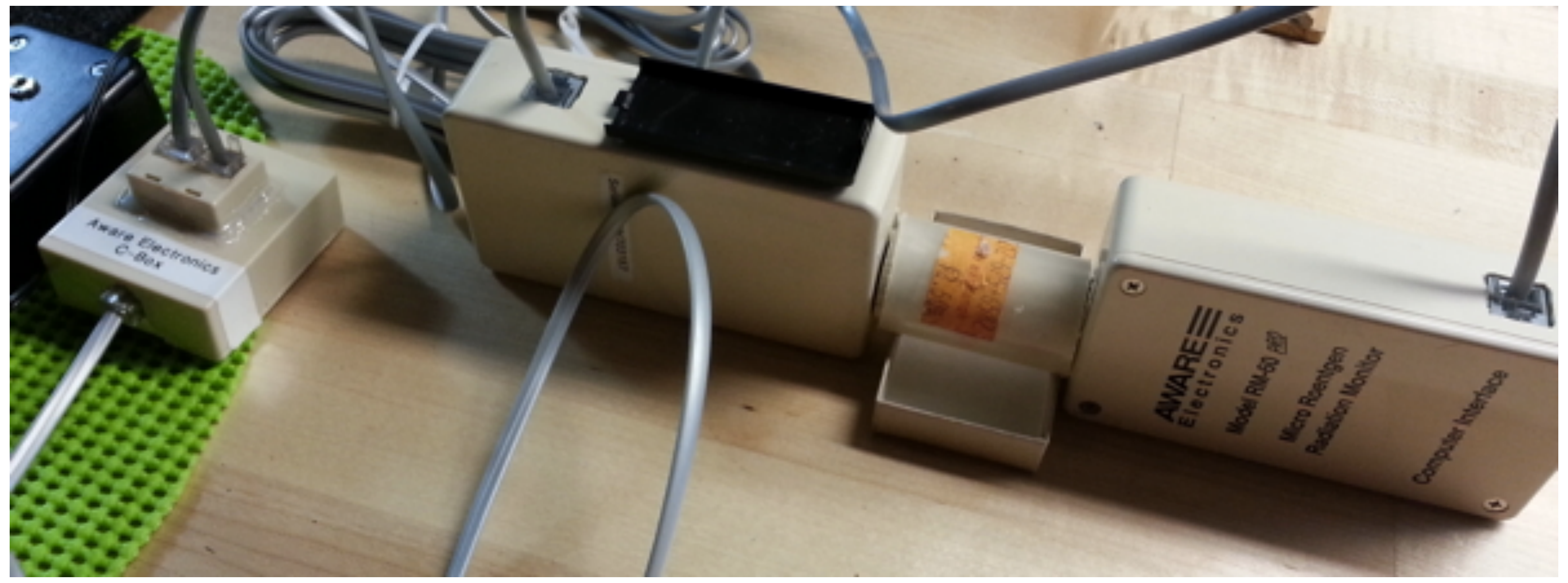


How to build your cheap entanglement experiment at home:

you need
- two Geiger counters
- a disk of radioactive Sodium 22
- a tablet running a GeigerBot app





See G. Musser (2013)
http://blogs.scientificamerican.com/critical-opalescence/how-to-build-your-own-quantum-entanglement-experiment-part-1-of-2/

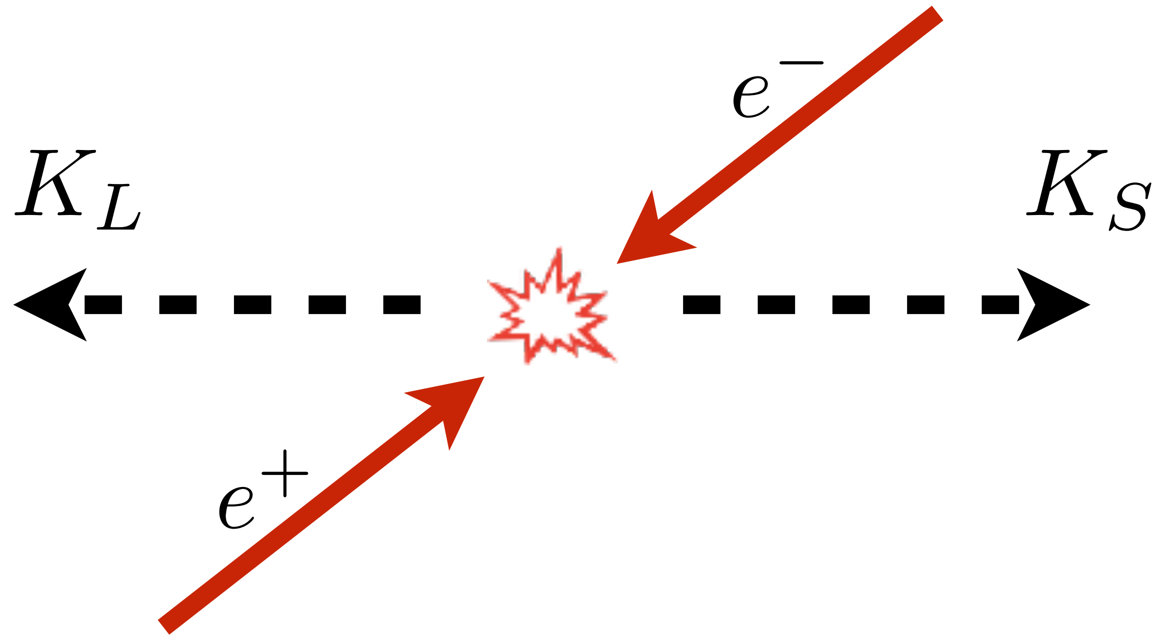
Entangled state

Singlet state of strangeness: $$|f\rangle = \frac{1}{\sqrt{2}}\Big(|K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle\Big)$$

How entanglement is produced:

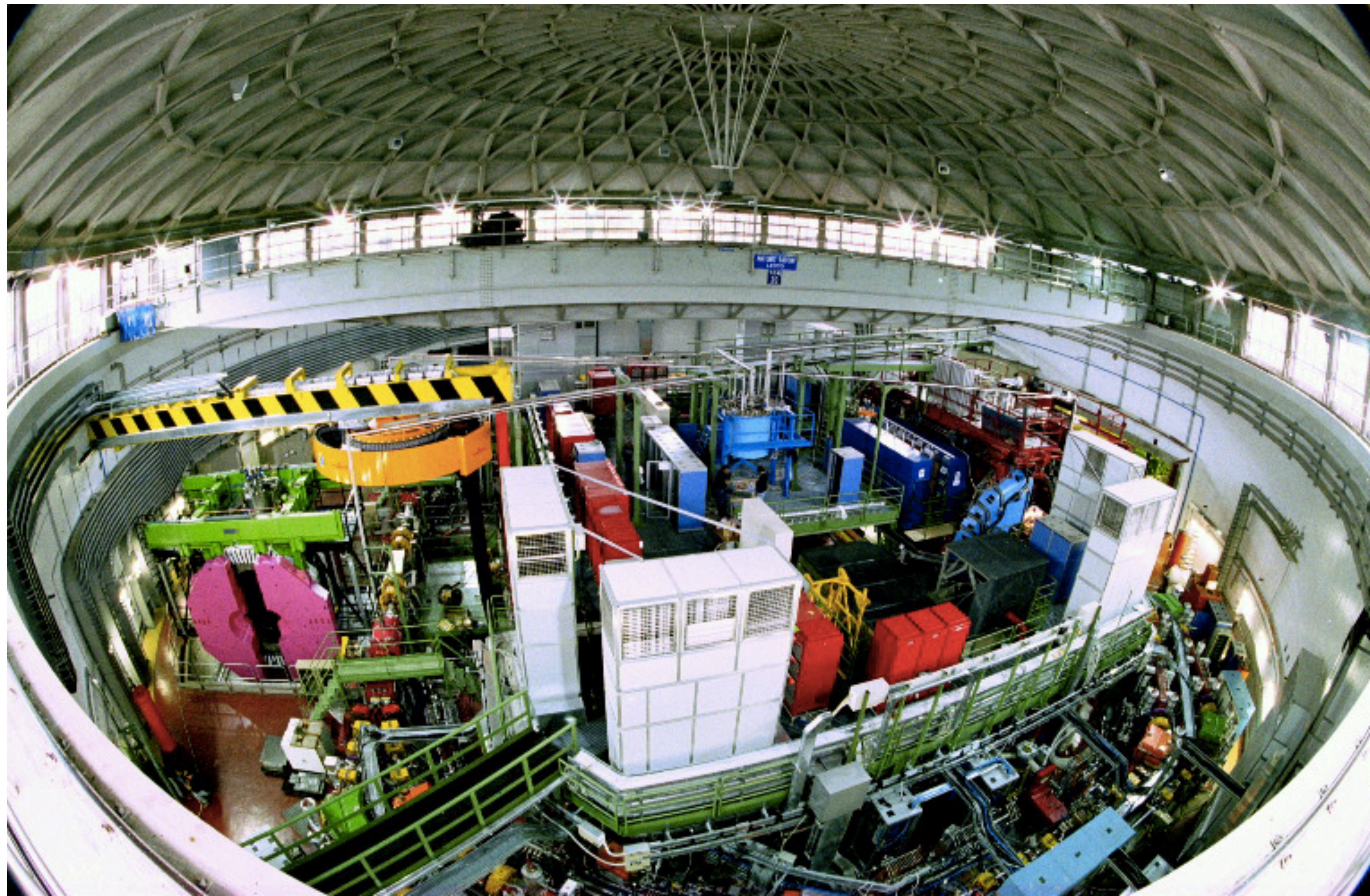
E.g., resonant production of neutral Kaons

$$e^- + e^+ \rightarrow \phi(1024) \rightarrow K^0 + \bar{K}^0$$

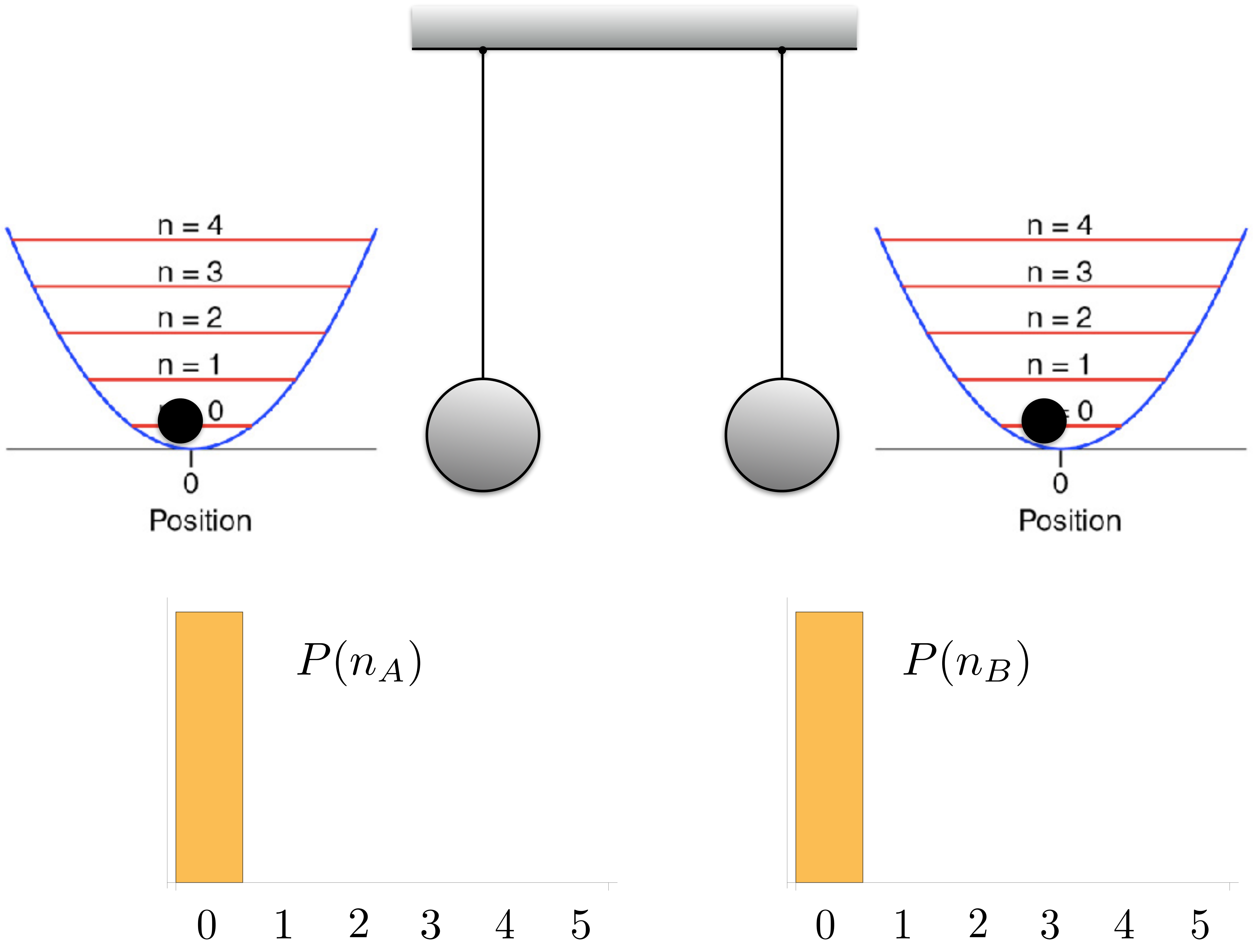


DAFNE @ LNF

a Double Annular Φ Factory for Nice Experiments



Entangled oscillators

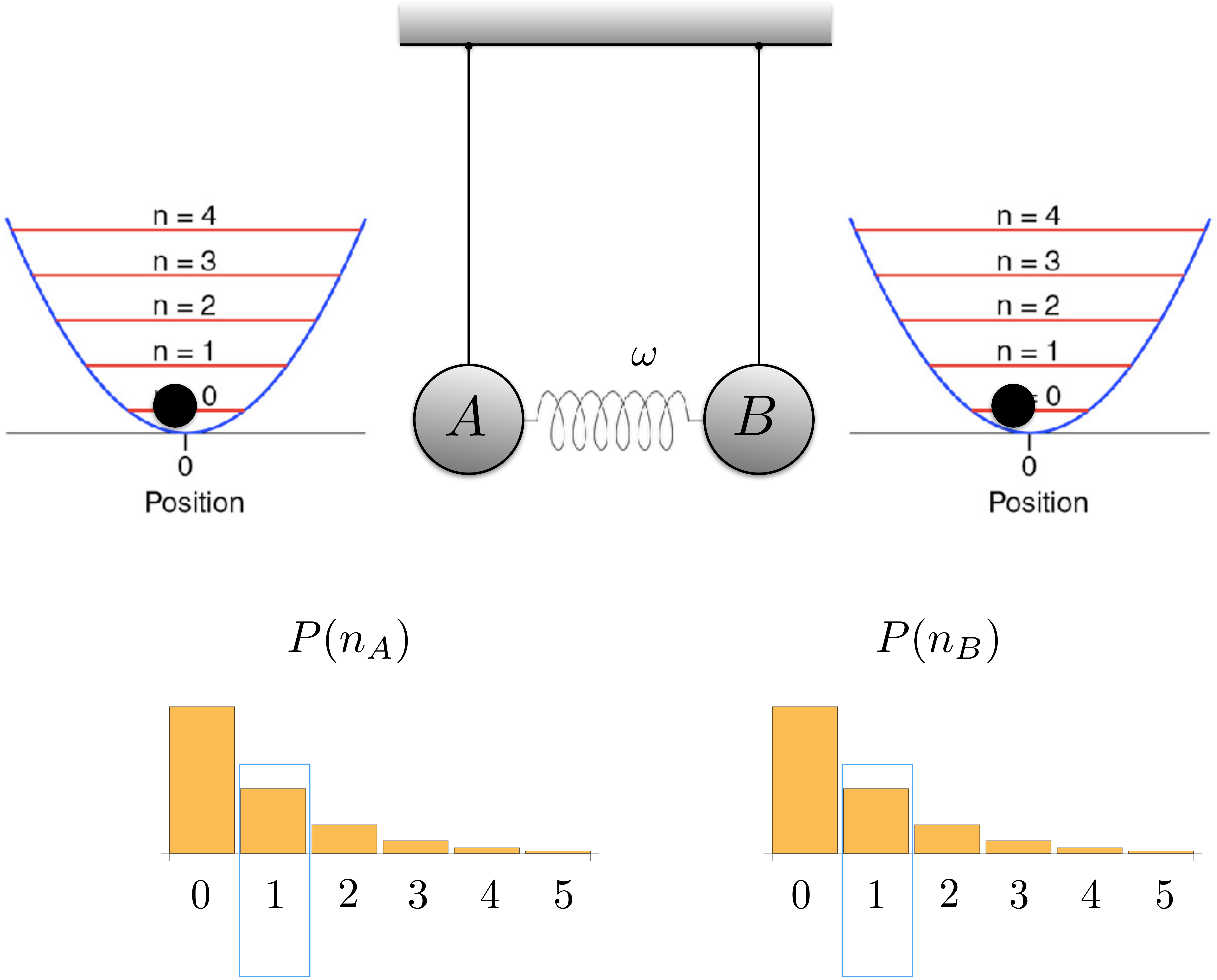


Ground state

without the spring

$$|0\rangle_A \, |0\rangle_B$$

Entangled oscillators



Ground state

without the spring

$$|0\rangle_A \, |0\rangle_B$$

with the spring

$$|\psi_0\rangle = \sum_{n=0}^{\infty} \sqrt{p_n} \;\; |n\rangle_A \, |n\rangle_B$$

If we make measurements on *A* only,

Mixed state from entanglement

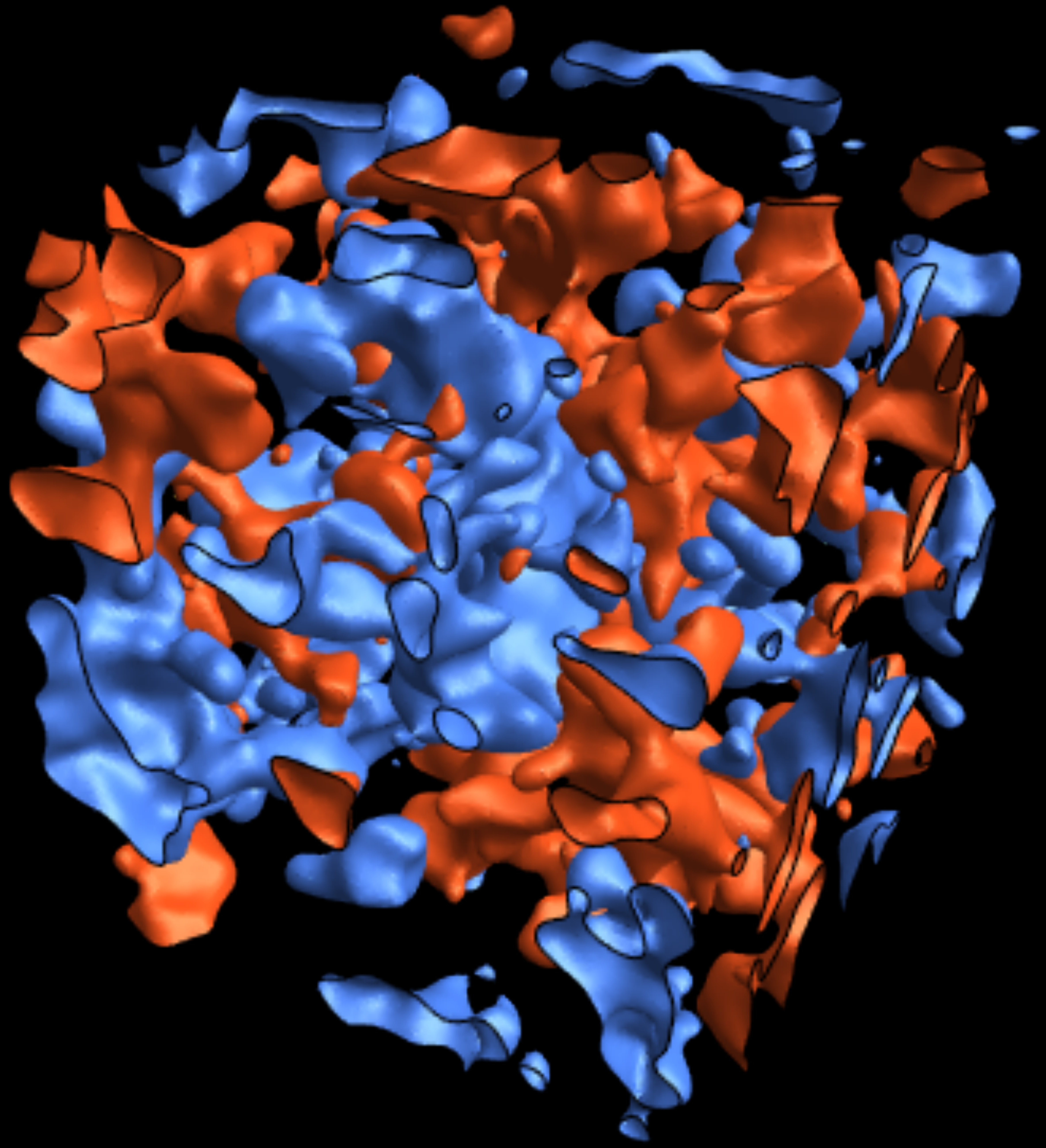
$$\rho_A = \mathrm{Tr}_B(|\psi_0\rangle\langle\psi_0|) = \sum_{n=0}^{\infty} p_n \;\; |n\rangle_A \, \langle n|_A$$

Entanglement entropy

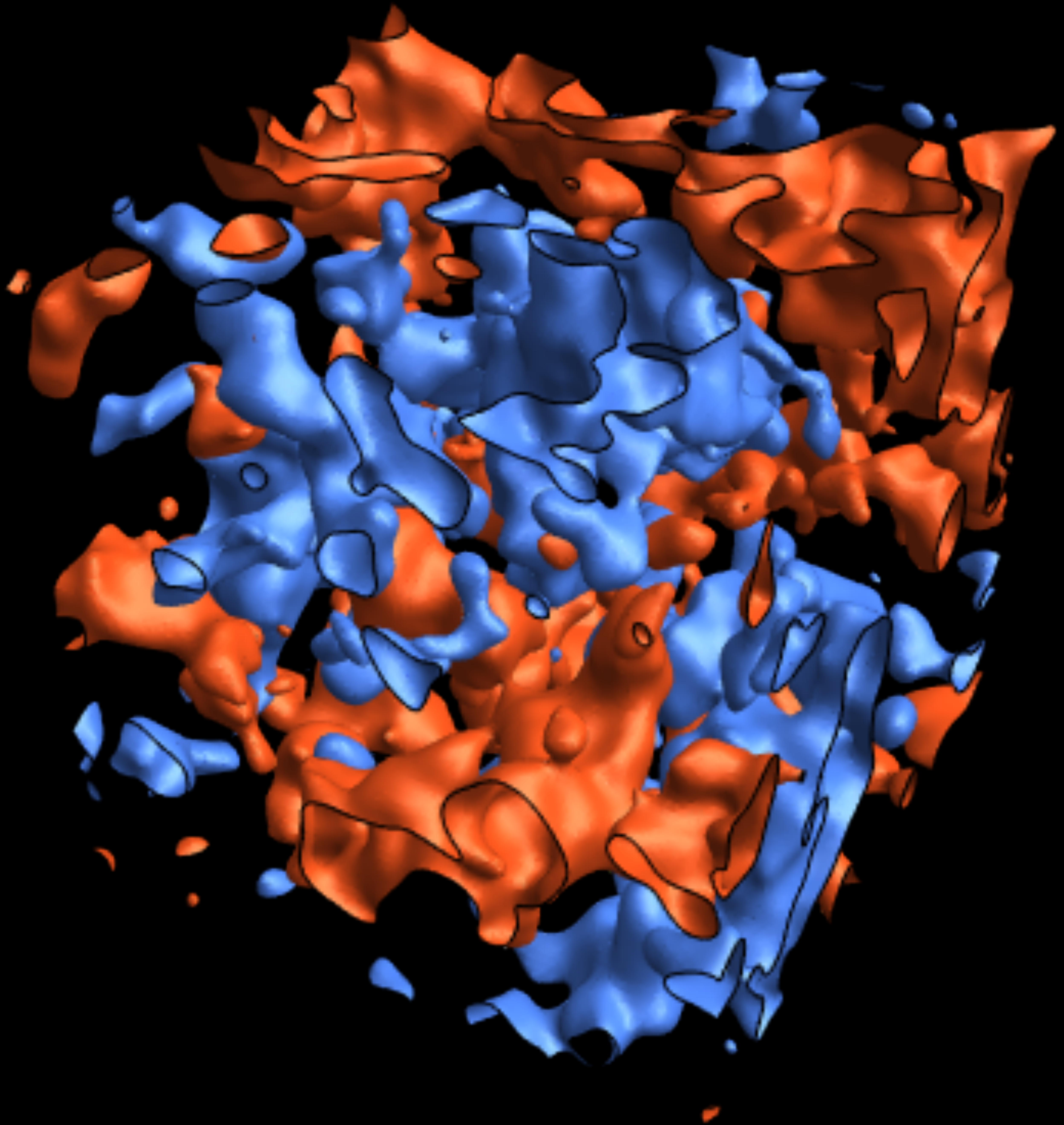
$$S_A = -\mathrm{Tr}_A(\rho_A \log \rho_A) \; = -\sum_n p_n \log p_n$$

The Vacuum

The Vacuum State of a Quantum Field



The vacuum state of a quantum field is highly entangled



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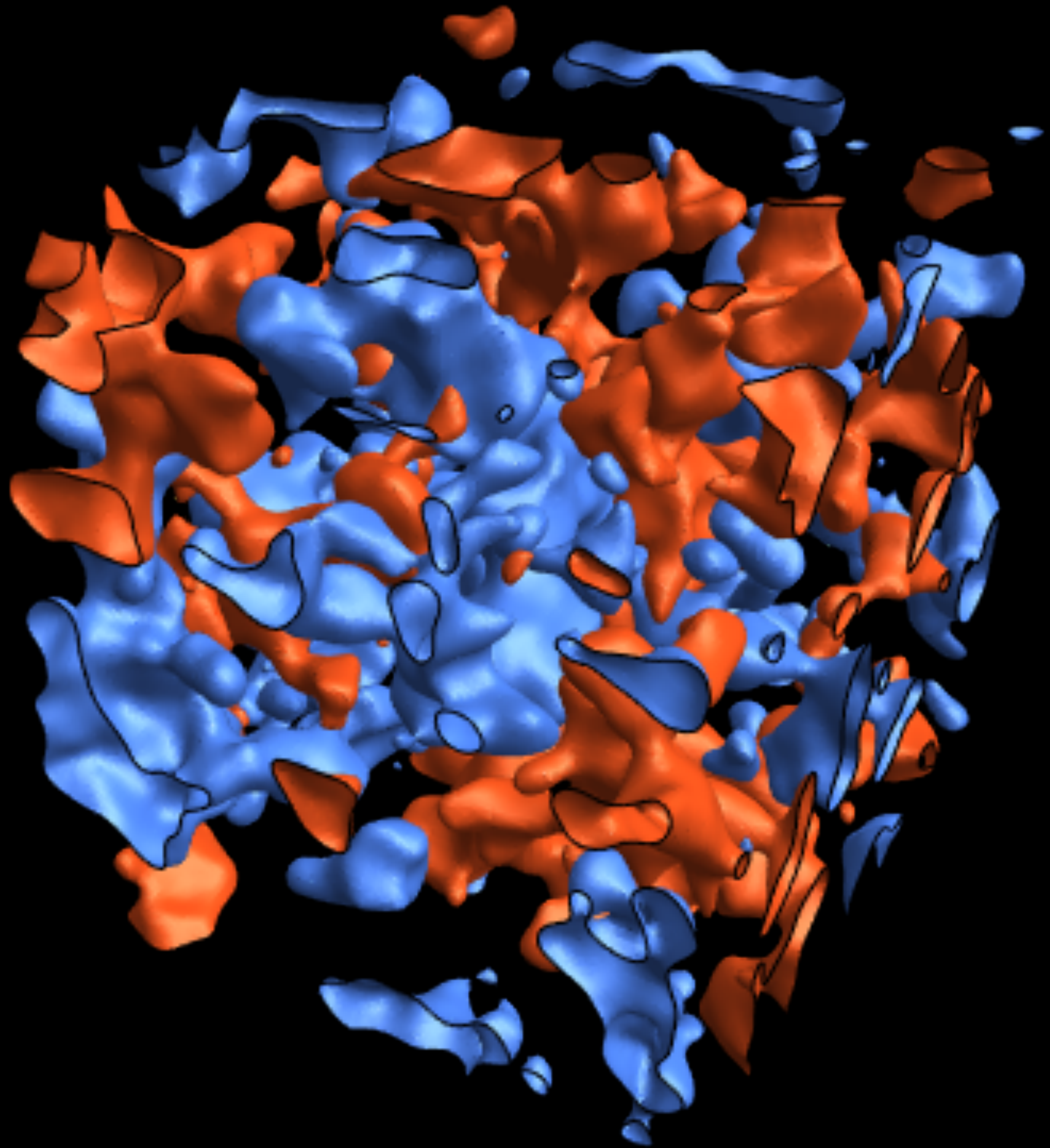

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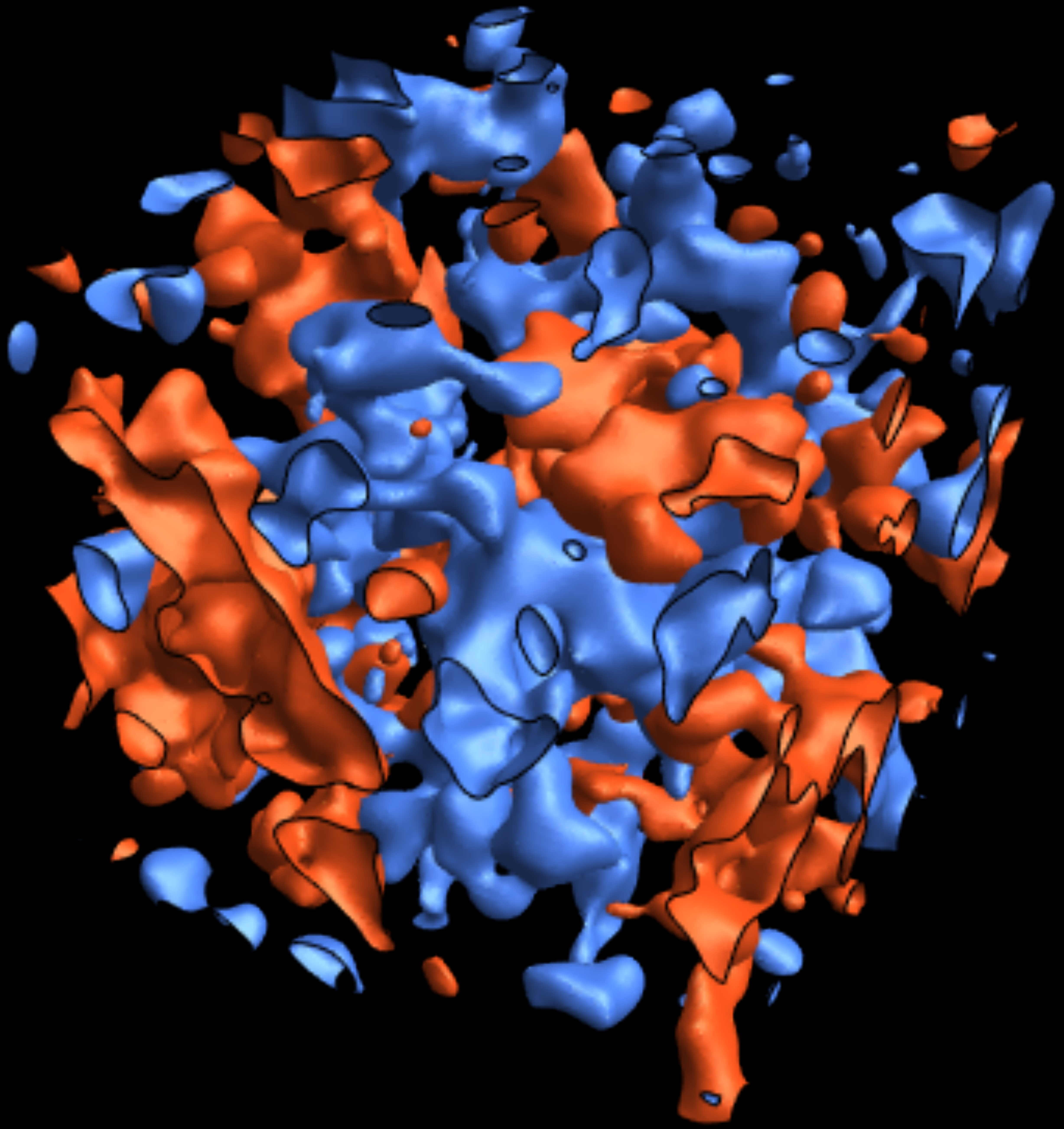

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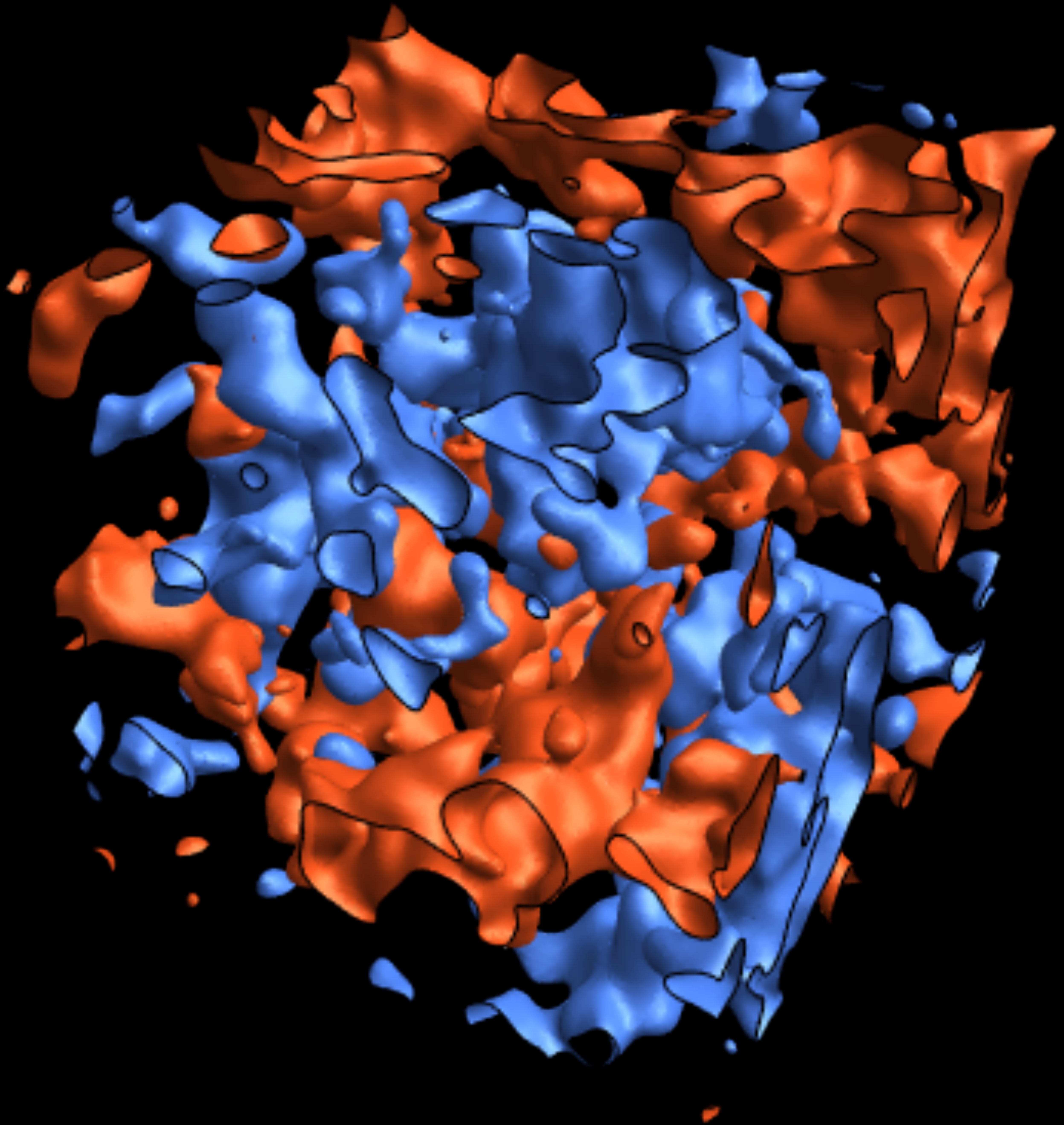
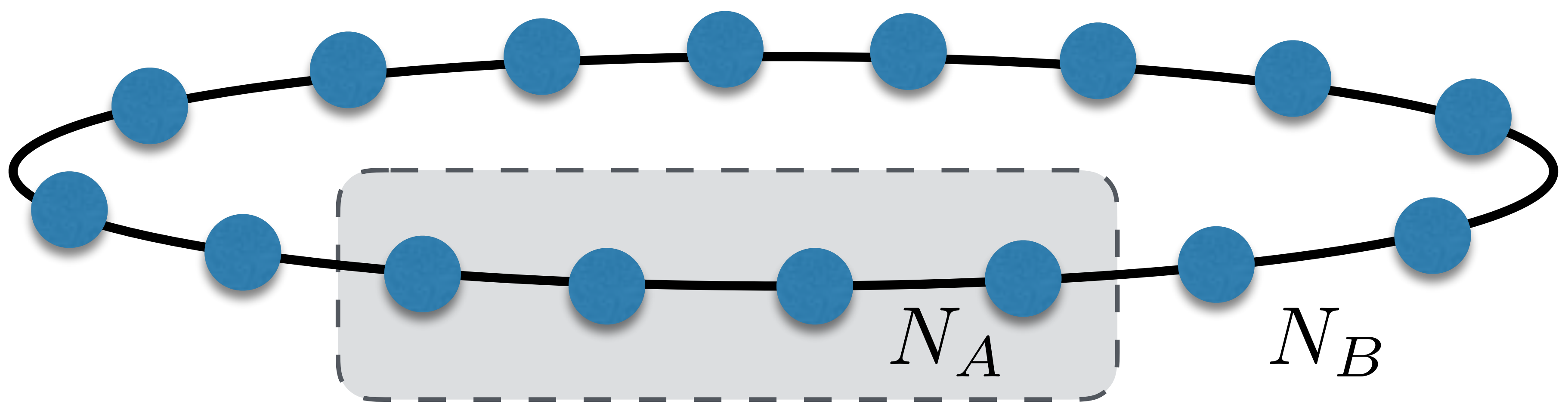


Entanglement as a probe of locality - e.g. $1\mathrm{d}$ fermionic chain



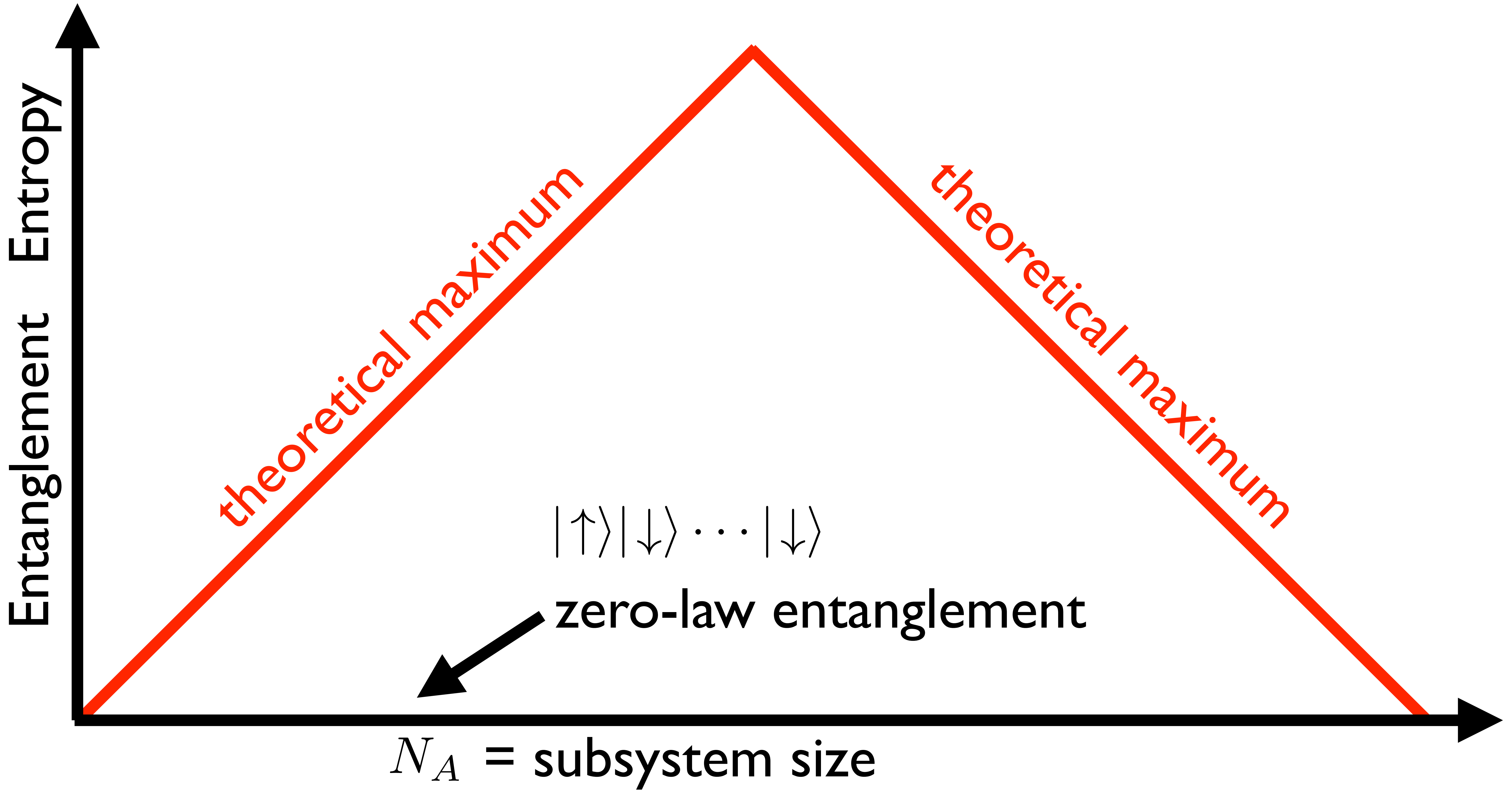
Hilbert space: 2^N dimensional $\mathcal{H} = \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2$

Geometric subsystem $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$

Entanglement entropy $S_A(|\psi\rangle) = -\mathrm{Tr}_A(\rho_A \log \rho_A)$

1) Factorized basis states $|\uparrow\rangle|\downarrow\rangle\cdots|\downarrow\rangle$

zero law



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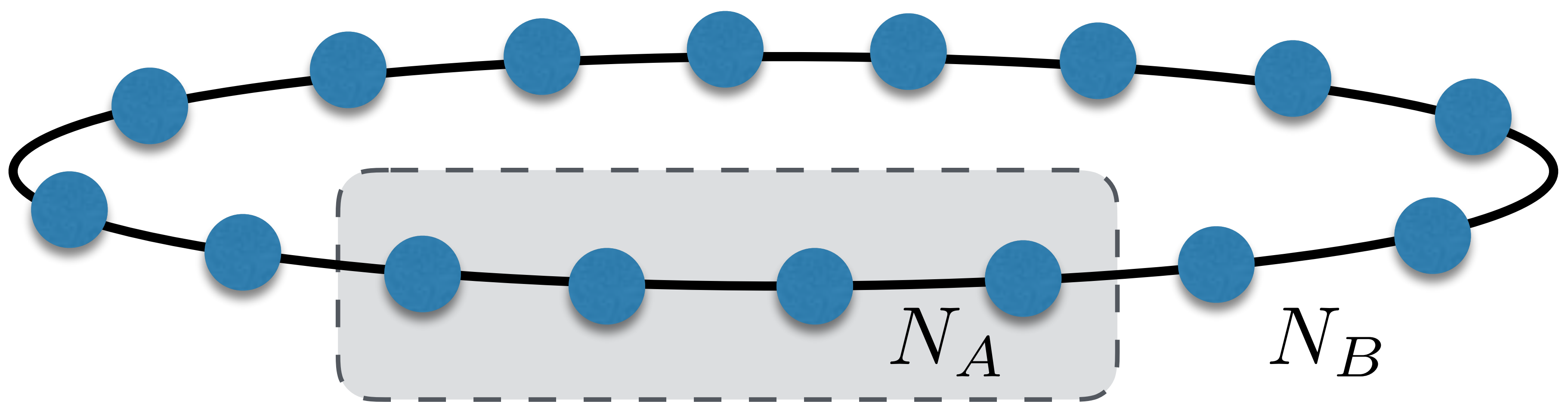

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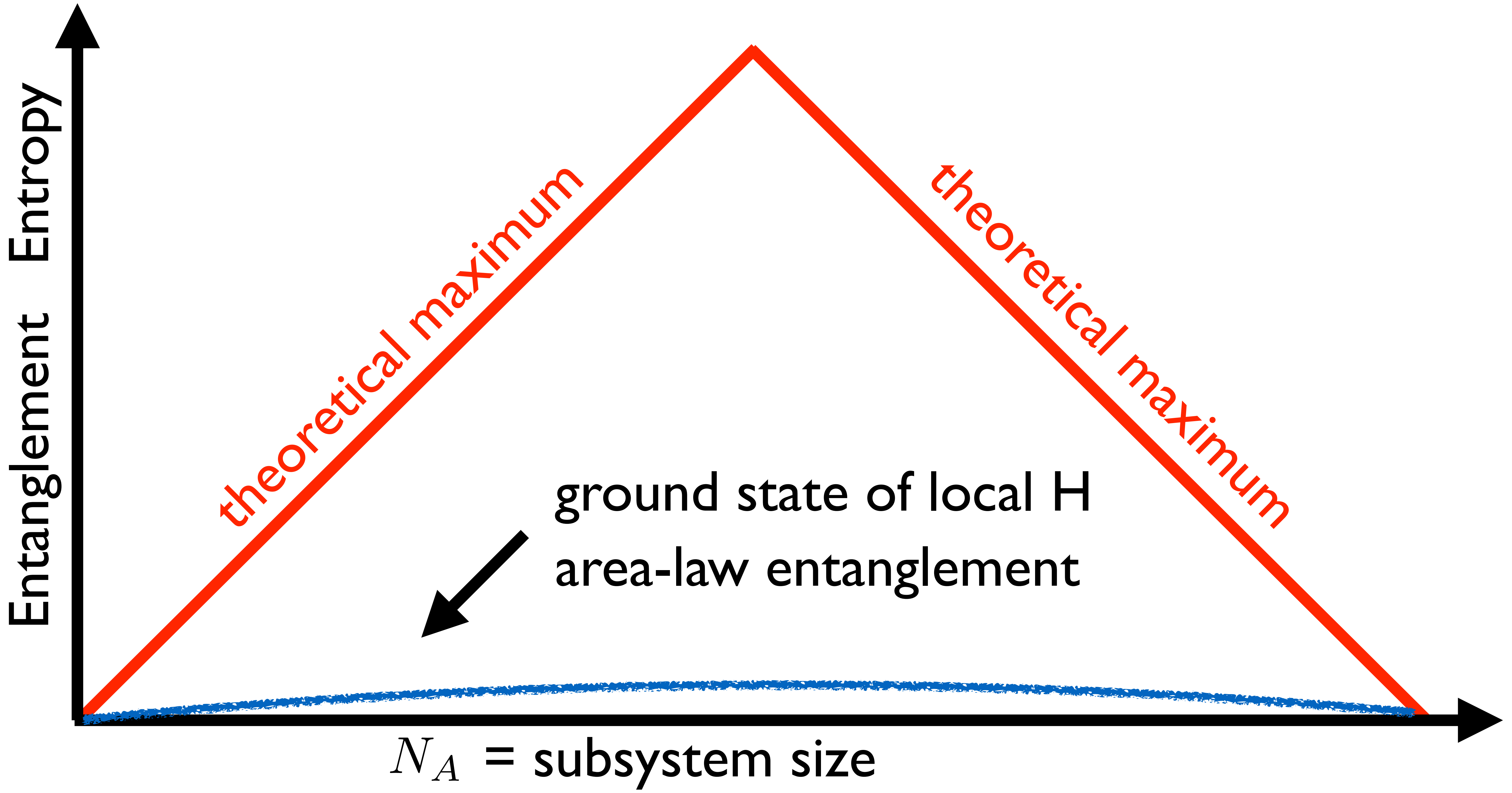
2) Ground state of a local Hamiltonian
area law



[Sorkin (10th GRG) 1985]
[Srednicki, PRL 1993]

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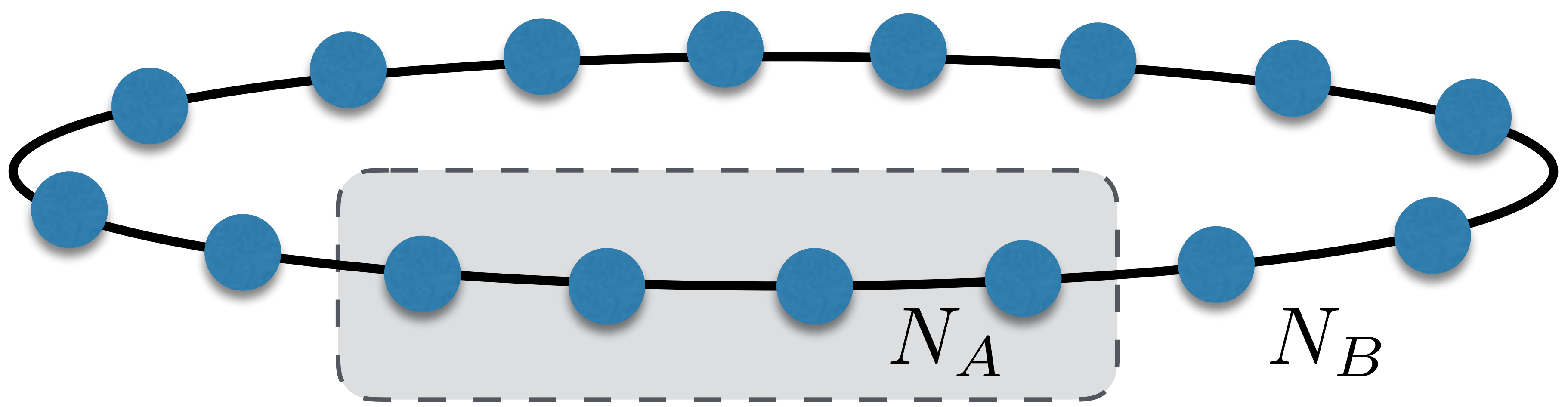

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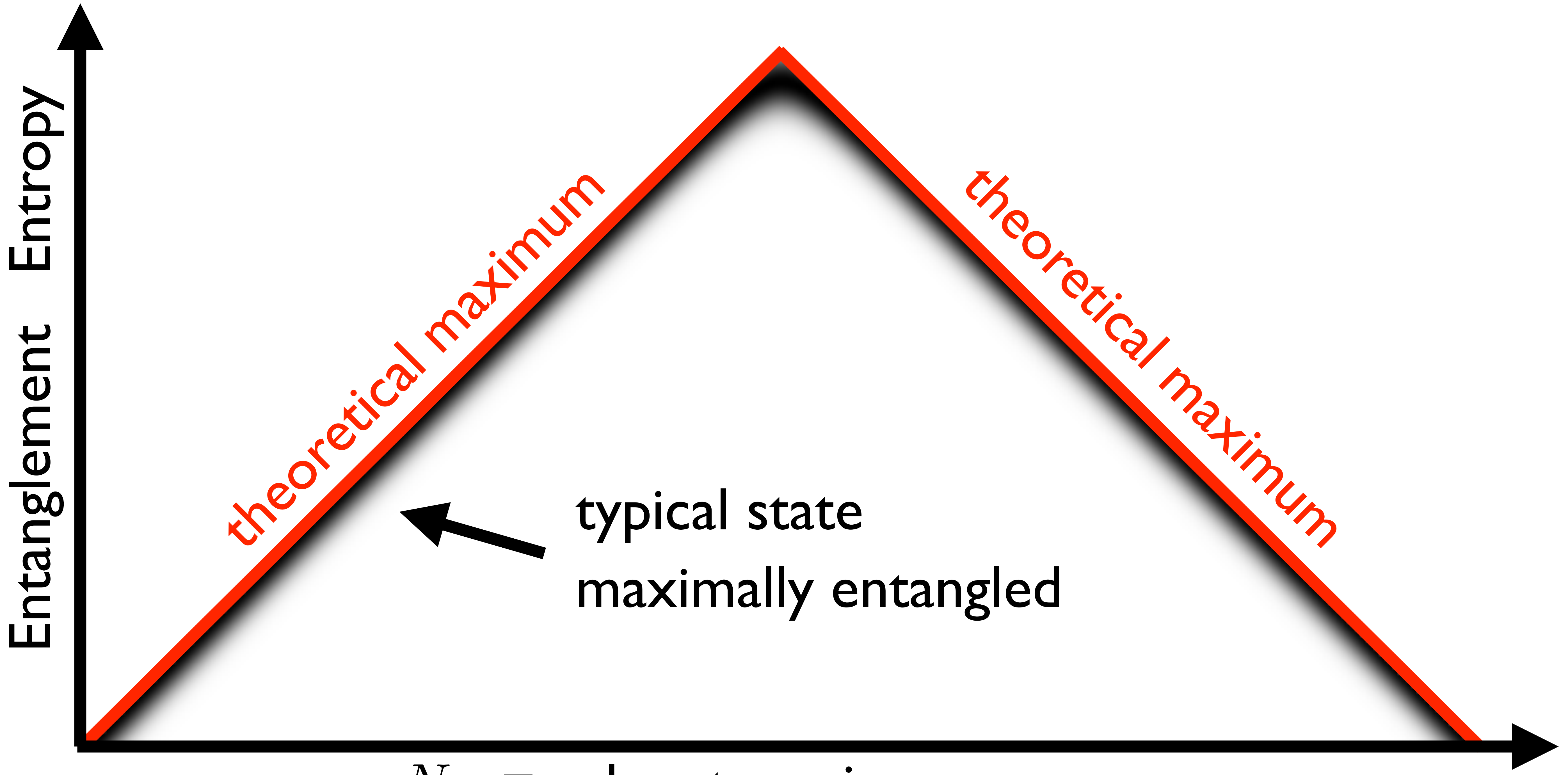

[Page, *PRL* 1993]

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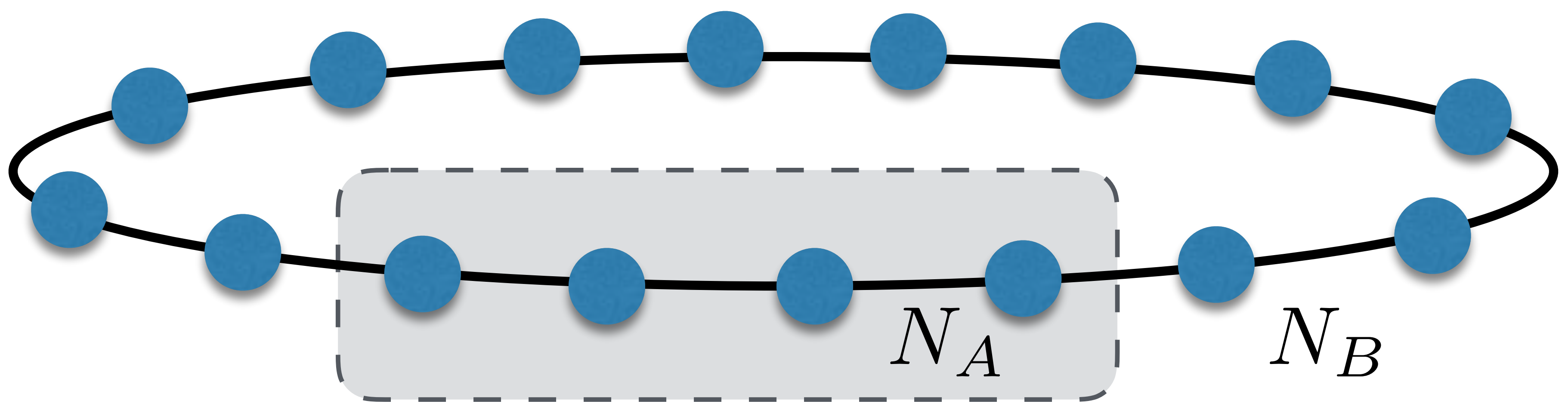

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4) Typical excited state of a local Hamiltonian
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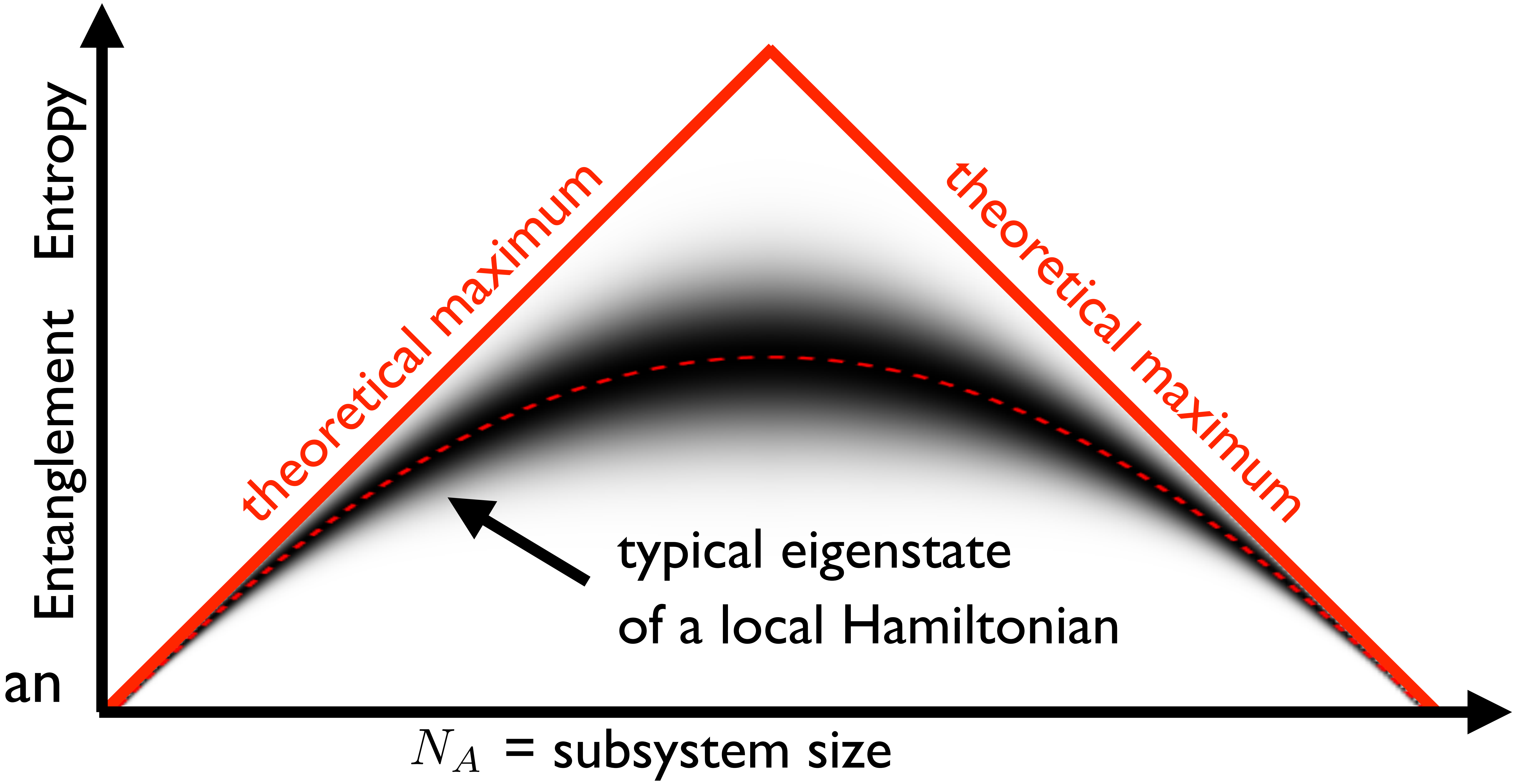

[E.B.-Hackl-Rigol-Vidmar, *PRL* 2017]

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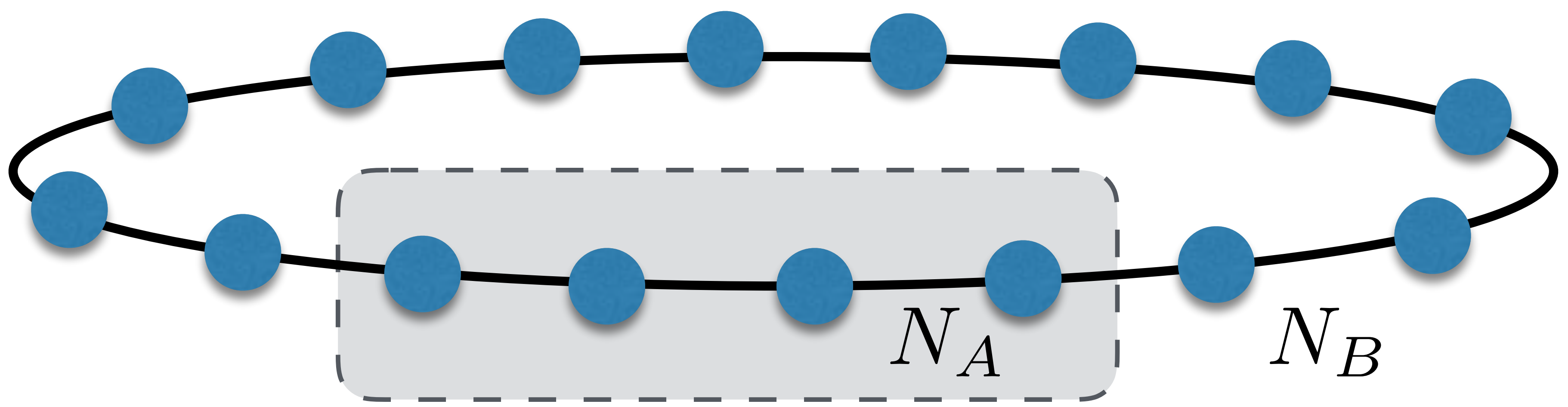

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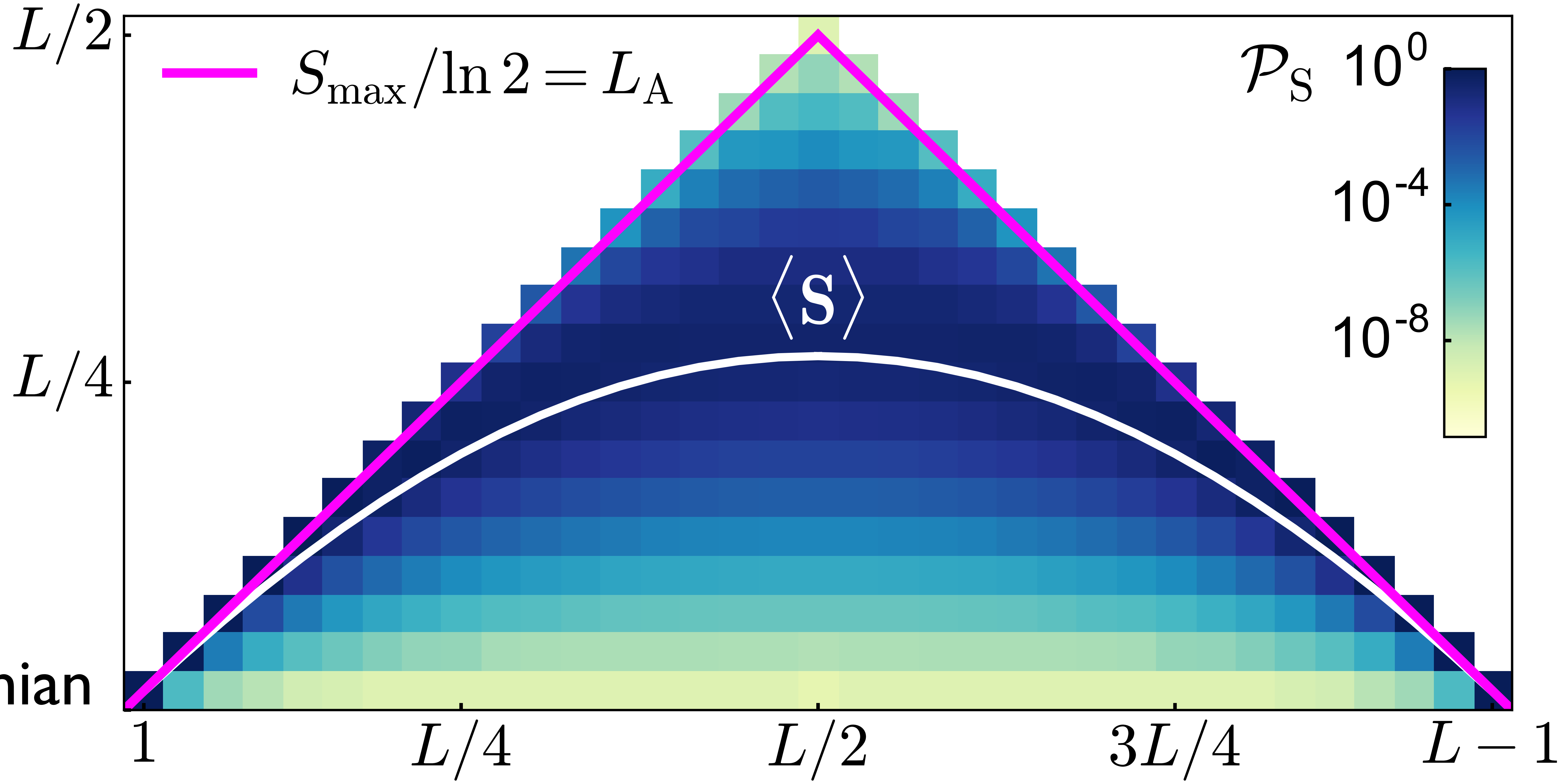

[E.B.-Hackl-Rigol-Vidmar, *PRL* 2017]

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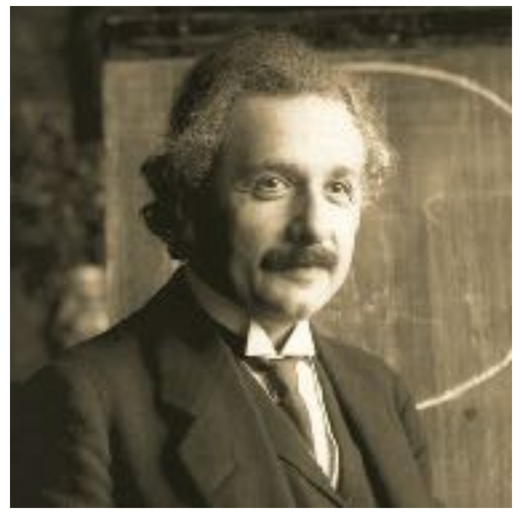
General Relativity 1915



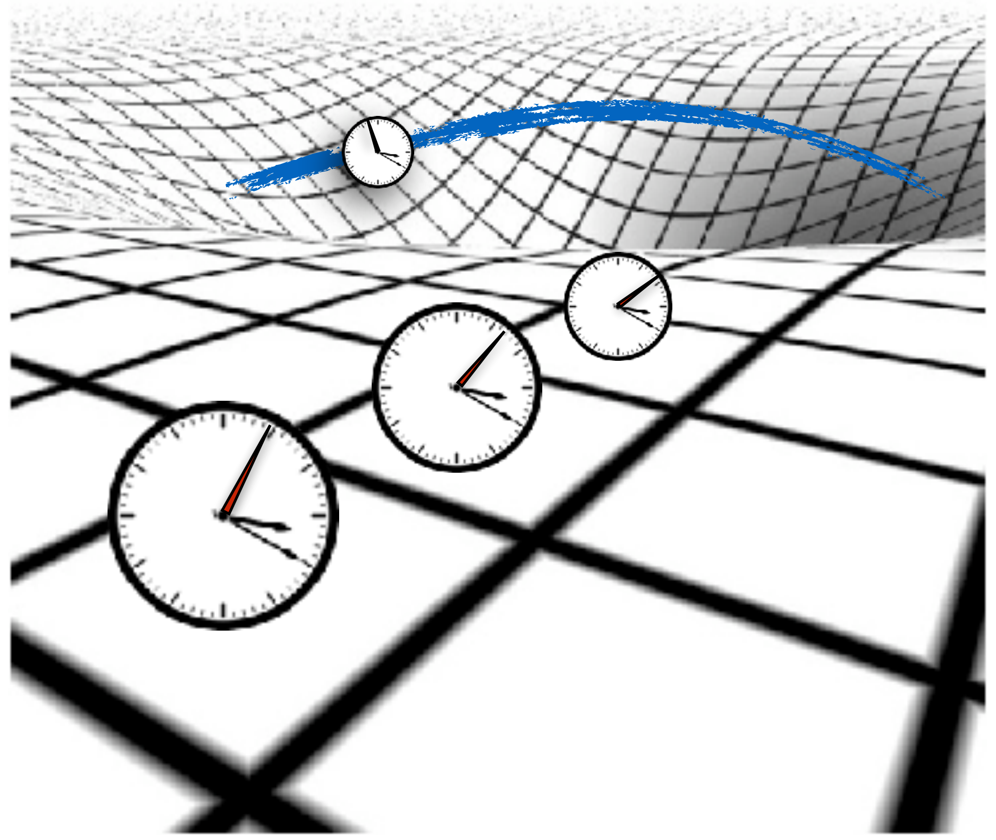
Quantum Mechanics ~1925

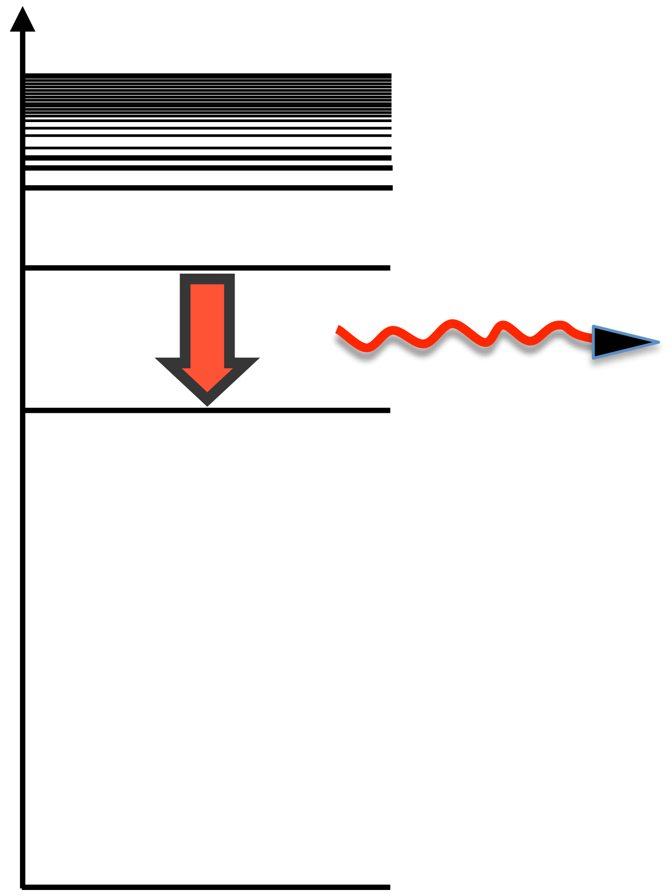
Degrees of freedom of gravity:

- Geometry of spacetime

Degrees of freedom:

- Discrete spectra
- Entangled





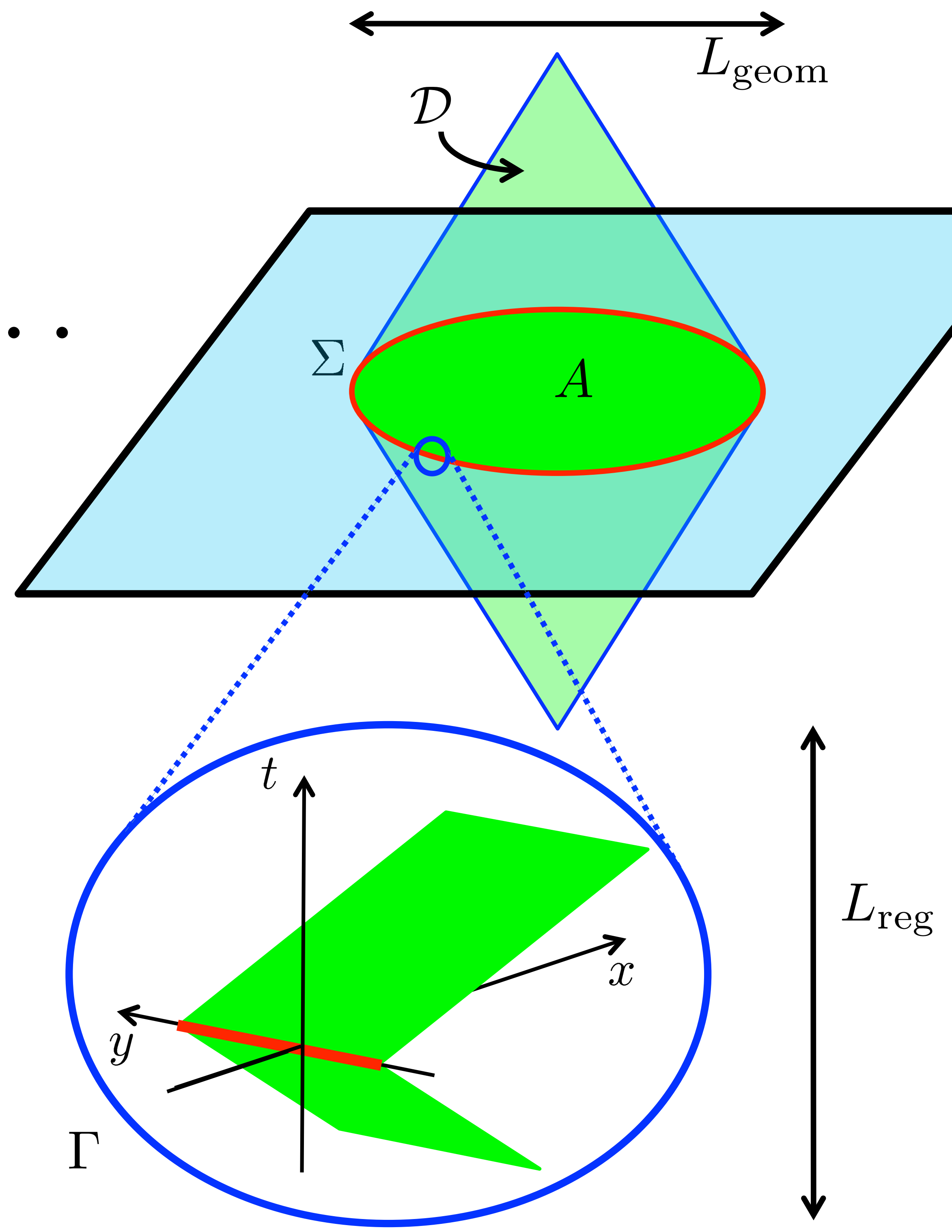
Two fundamental descriptions of the world: an unfinished revolution

Entanglement and the architecture of a spacetime geometry

- Entanglement entropy as a probe of the architecture of spacetime
  Area-law not generic, property of semiclassical states

EB and R.Myers, CQG (2012)
"On the Architecture of Spacetime Geometry"

$$S_A(|0\rangle) = 2\pi \frac{\text{Area}(\partial A)}{L^2_{Planck}} + \ldots$$



Arguments from: Black hole thermodynamics (Bekenstein, Hawking, Sorkin,…)
Holography and AdS/CFT (Maldacena,.. Van Raamsdonk,.. Ryu, Takayanagi,…)
Entanglement equilibrium (Jacobson)
Loop quantum gravity (EB)

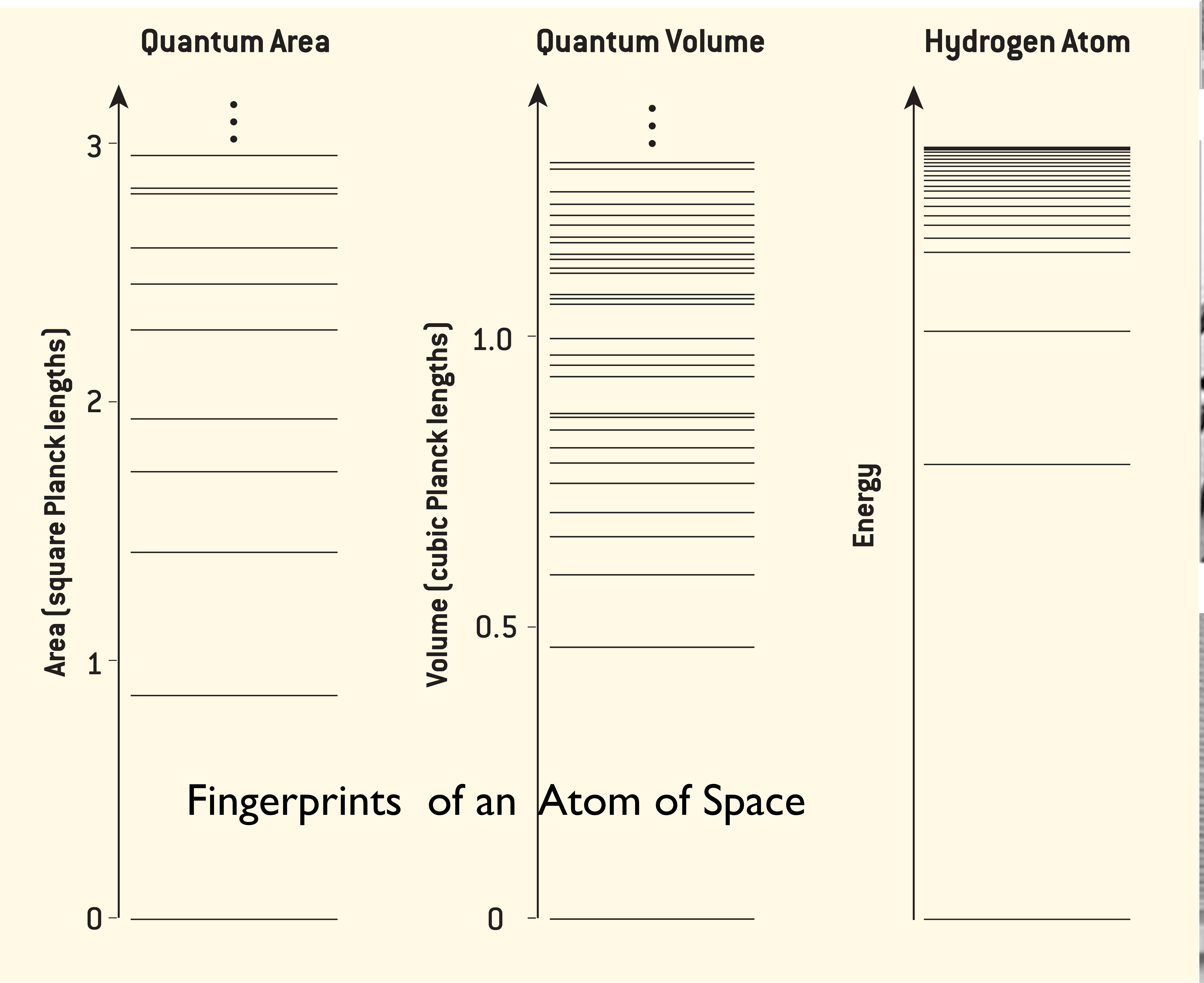
Loop Quantum Gravity

1986 - New Variables for General Relativity - *Abhay Ashtekar*

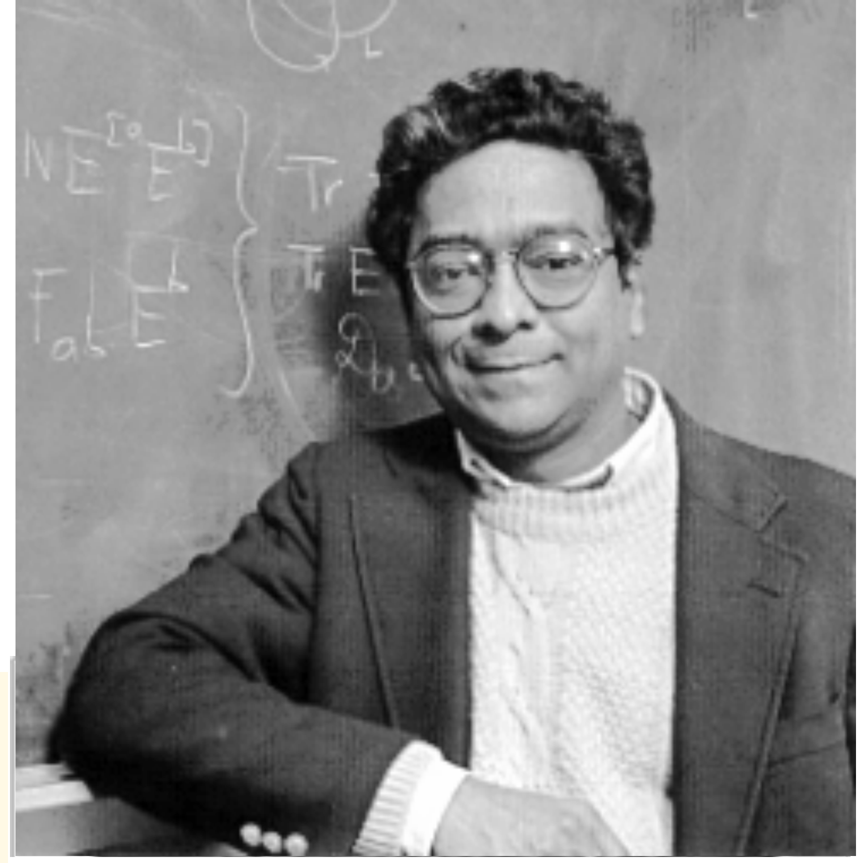
1987 - The Loop Representation - *Carlo Rovelli and Lee Smolin*

1992 - Discrete Quanta of Space - *Ashtekar-Rovelli-Smolin*

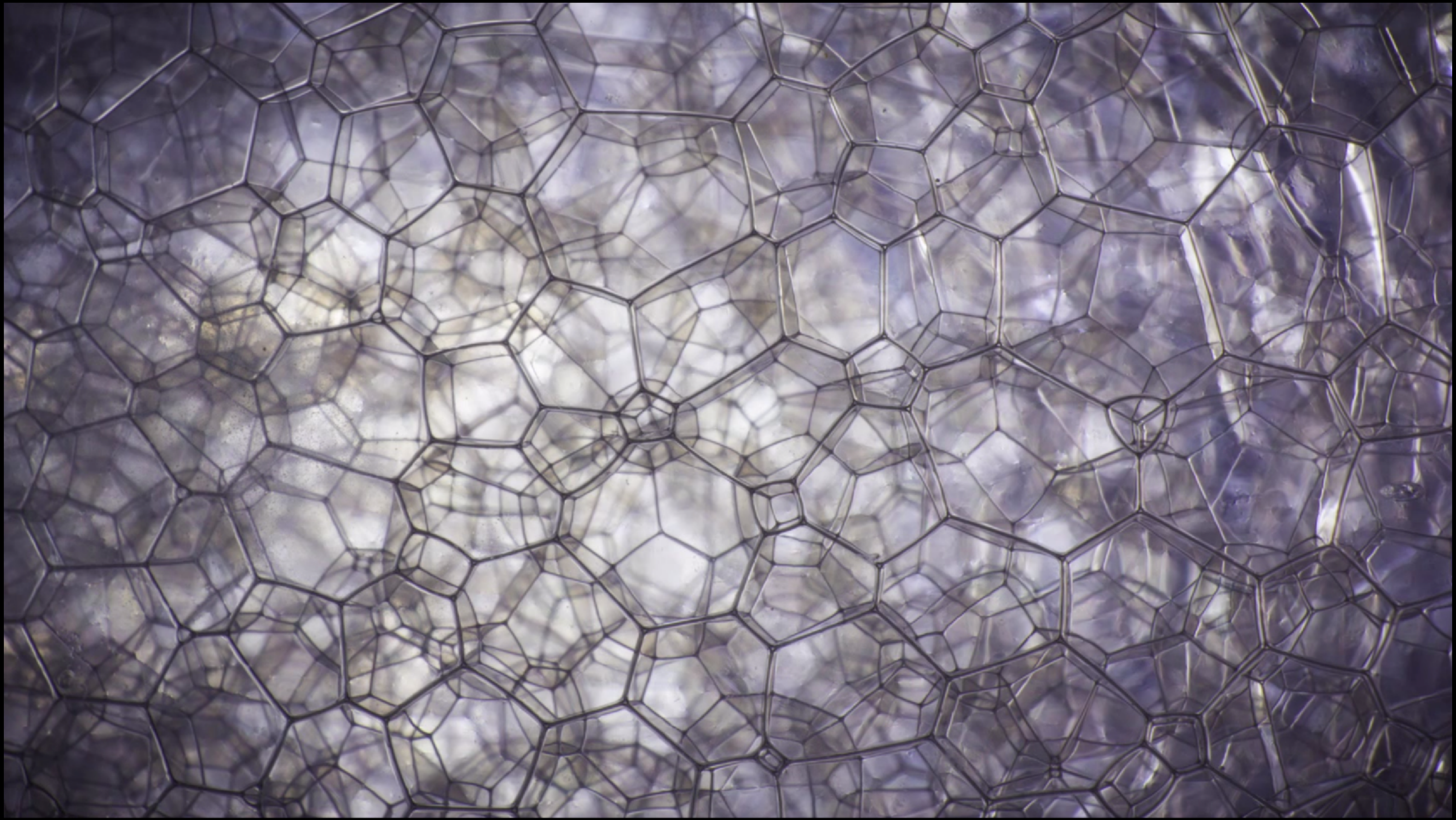
…



Fingerprints of an Atom of Space



Quantum geometry of spacetime: discrete, non-commutative, entangled.



[soap foam - microphotography by Pyanek]

Degrees of freedom of covariant loop quantum gravity (aka *spin-foams*)

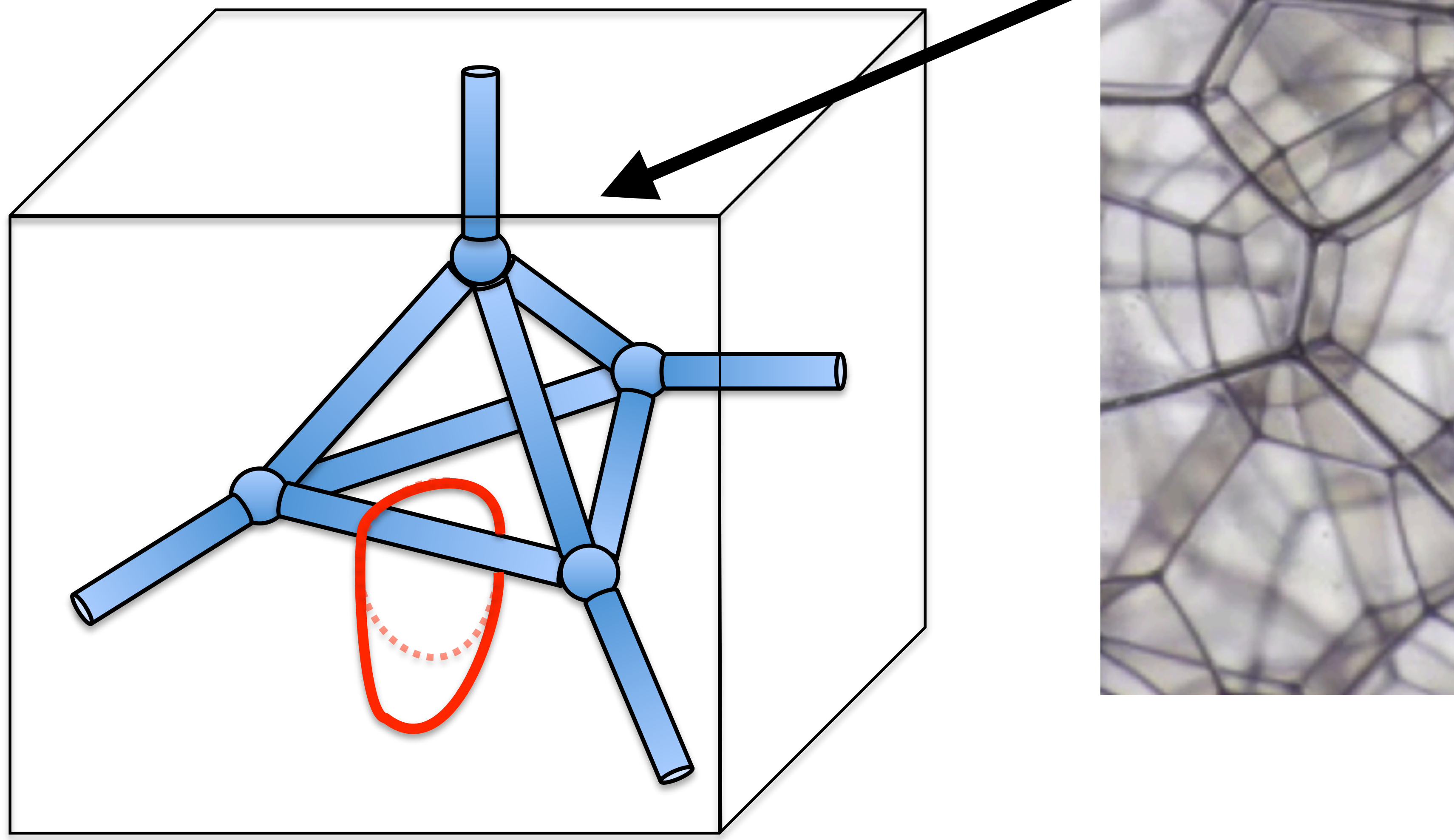
Spacetime manifold and the notion of 2d-foam

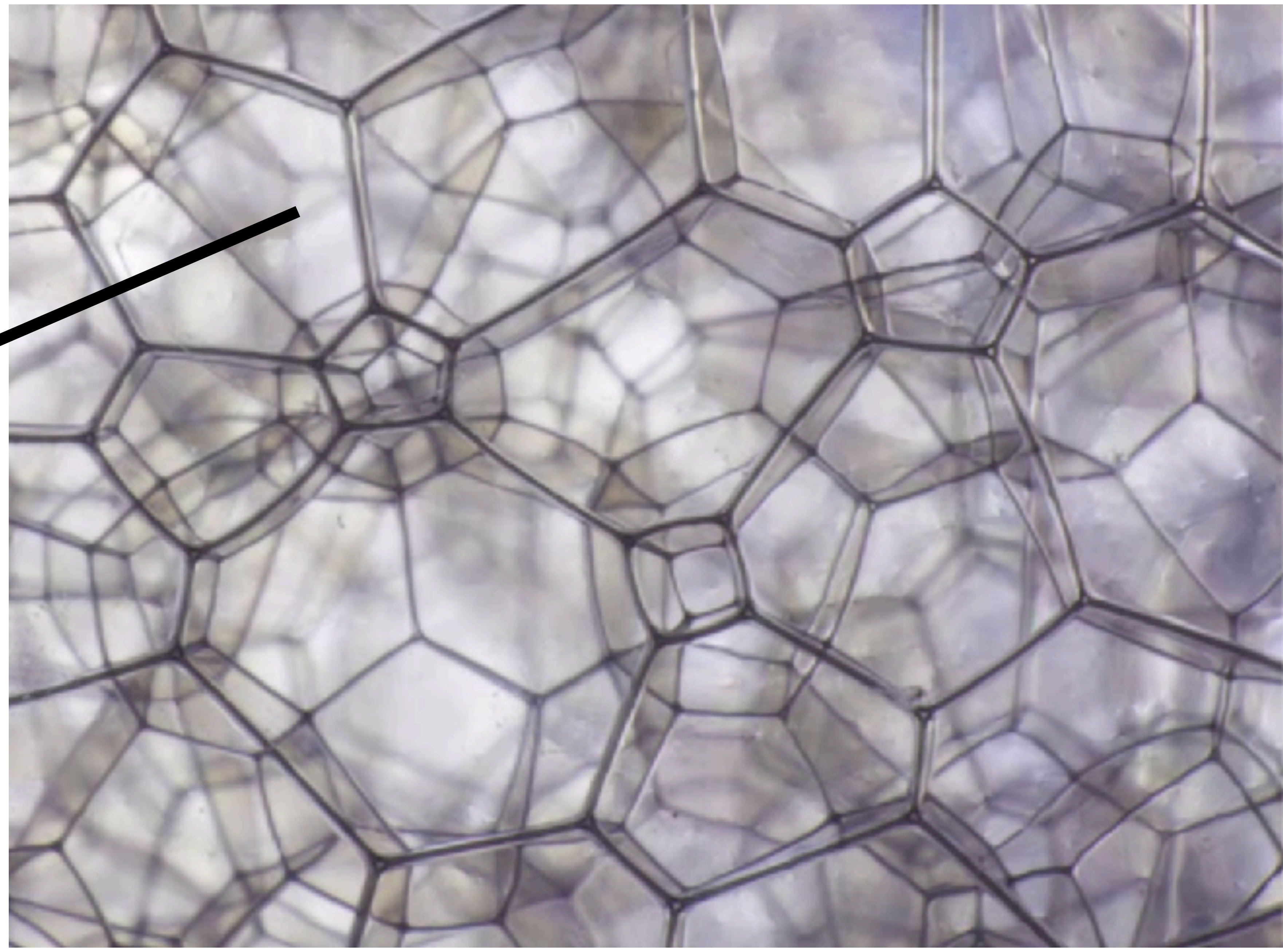
- M = 4d manifold of trivial topology
- Δ = Topological decomposition of M

4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2





- Set $\{\Delta_2\}$ = 2-skeleton of (M, Δ) = *2d-foam*

* The manifold $M' = M - \{\Delta_2\}$ is non simply-connected, non-trivial π_1

non-contractible loops around Δ_2

Rovelli-Reisenberger '96

Barrett-Crane '98

Engle-Pereira-Rovelli-Livine '08

EB '09

…

Dynamics of covariant loop quantum gravity (aka *spin-foams*)

- Gravity: Einstein-Cartan action + Holst term $\gamma \in \mathbb{R}$ = Barbero-Immirzi parameter

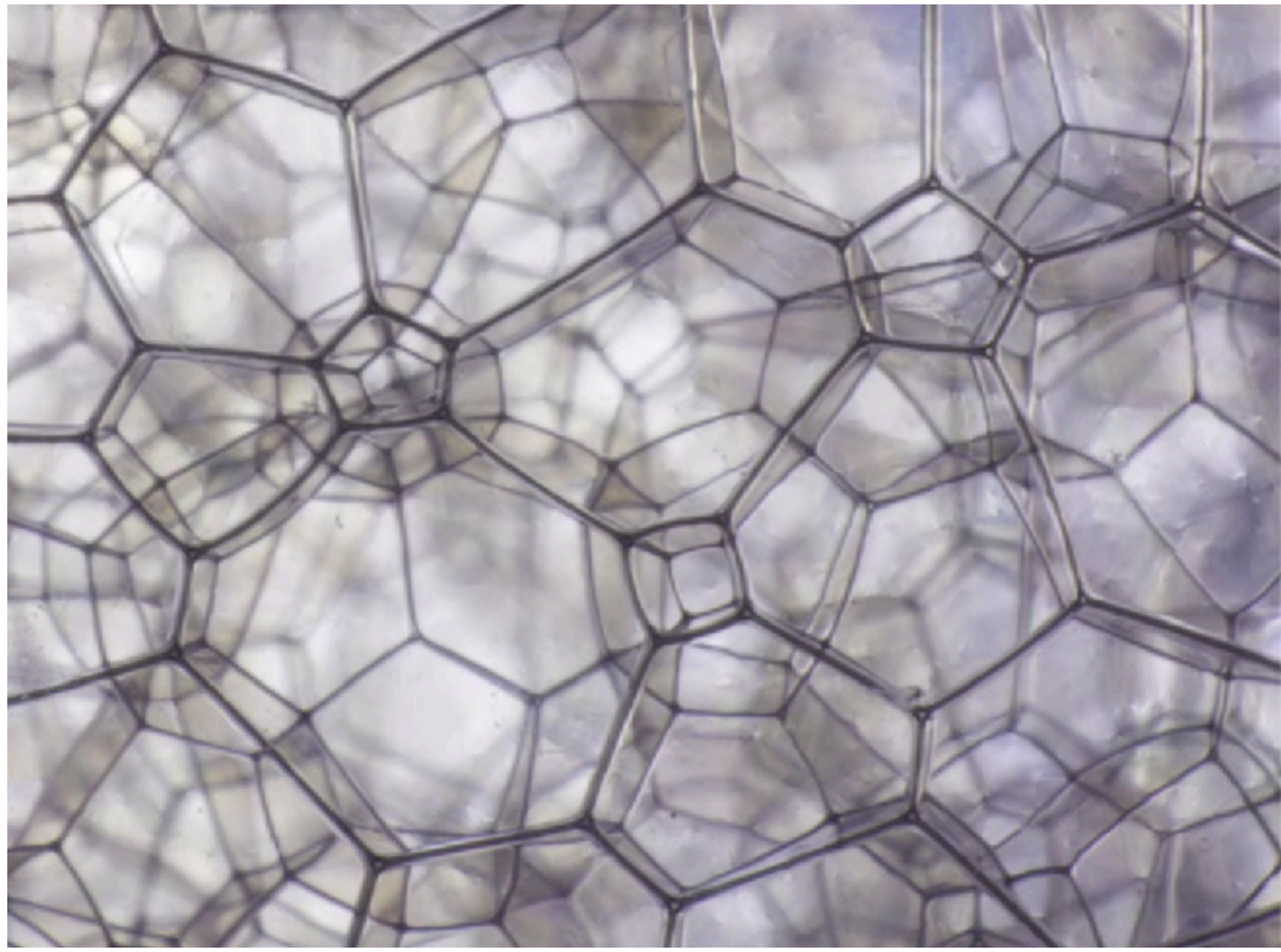
$$S[e,\omega] = \frac{1}{16\pi G}\int \frac{1}{2}\epsilon_{IJKL}e^I \wedge e^J \wedge F^{KL}(\omega) \;+\; \frac{1}{\gamma} e_I \wedge e_J \wedge F^{IJ}(\omega)$$

- Topological Field Theory: BF action B^{IJ} = two-form field

$$S[B,\omega] = \frac{1}{2}\int_{M_4} (\frac{1}{2}\epsilon_{IJKL}B^{KL} + \frac{1}{\gamma}B_{IJ}) \wedge F^{IJ}(\omega) \quad \Rightarrow \quad F^{IJ}(\omega) = 0$$

* Gravity as a Topological Theory with constrained B-field:

Constraint $B^{IJ} = \frac{1}{8\pi G}\, e^I \wedge e^J$ unfreezes $F^{IJ}(\omega)$



Constraint imposed on
- everywhere on *M* ➡ General Relativity
- on a 2d-foam in *M* ➡ Spin Foam action

2d-foam allows to unfreeze a finite number of gravitational degrees of freedom:
- quantization straightforward
- perspective: General Relativity as Effective field theory description

Rovelli-Reisenberger '96
Barrett-Crane '98
Engle-Pereira-Rovelli-Livine '08
EB '09
…

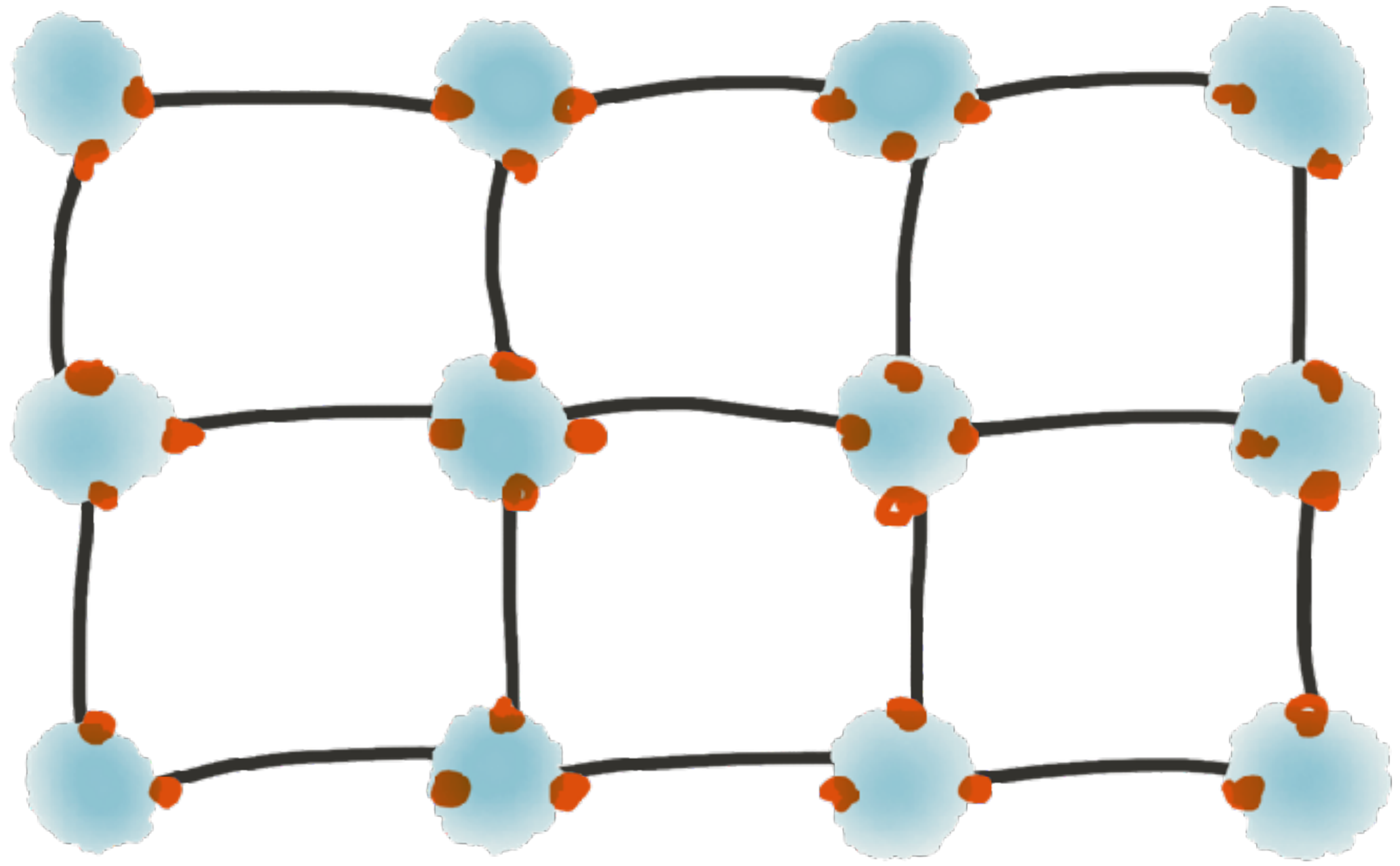
Bosonic formulation of LQG on a graph

[also known as the *twistorial* formulation]

- Two oscillators per end-point of a link

spin from oscillators $$|j,m\rangle = \frac{(a^{0\dagger})^{j+m}}{\sqrt{(j+m)!}}\frac{(a^{1\dagger})^{j-m}}{\sqrt{(j-m)!}}|0\rangle$$

[Schwinger 1952]



- Hilbert space of LQG and the bosonic Hilbert space

$$L^2(SU(2)^L/SU(2)^N) \;\subset\; \mathcal{H}_{\text{bosonic}}$$

$$|\psi\rangle = \sum_{n_i=1}^{\infty} c_{n_1\cdots n_{4L}}\; |n_1,\ldots,n_{4L}\rangle$$

[Girelli-Livine 2005] [Freidel-Speziale 2010]
[Livine-Tambornino 2011] [Wieland 2011]

[EB-Guglielmon-Hackl-Yokomizo 2016]

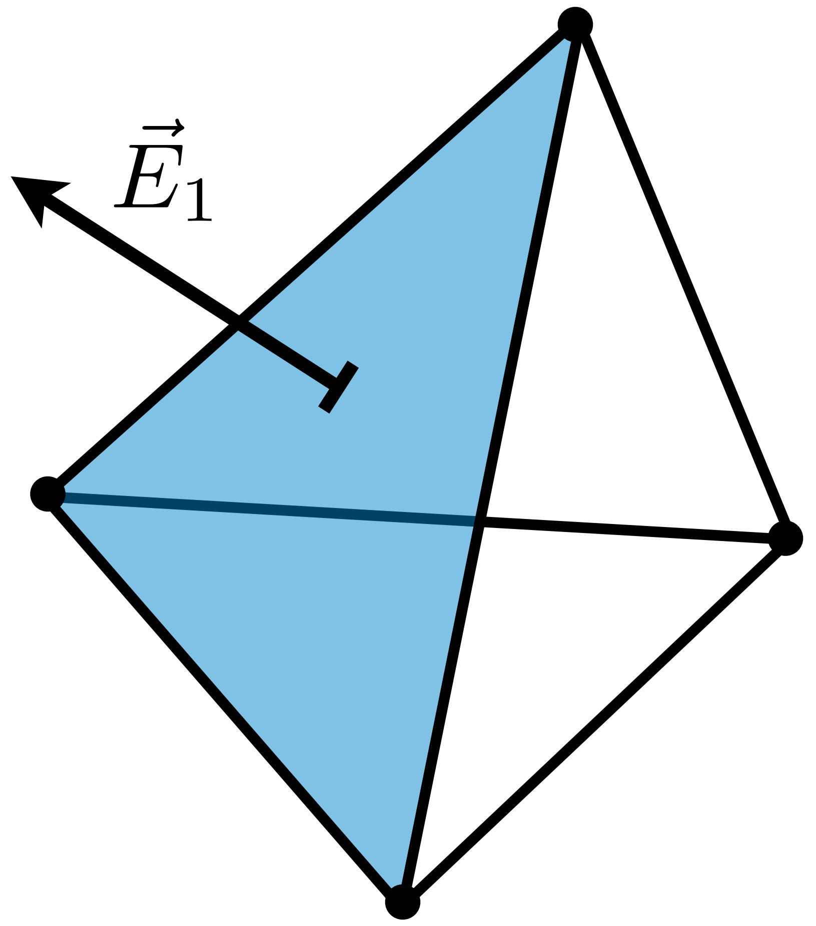
- The bosonic Hilbert space factorizes over nodes: easy to define and compute the entanglement entropy

- Geometric operators in a region *R* of the graph generate a subalgebra $\mathcal{A}_R^{\text{LQG}} \subset \mathcal{A}_R^{\text{bosonic}}$

Classical geometry of a tetrahedron in $\mathbb{R}^3$ - area vectors

Area vectors $\vec{E}_a \qquad a = 1, 2, 3, 4$

Closure $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = 0$

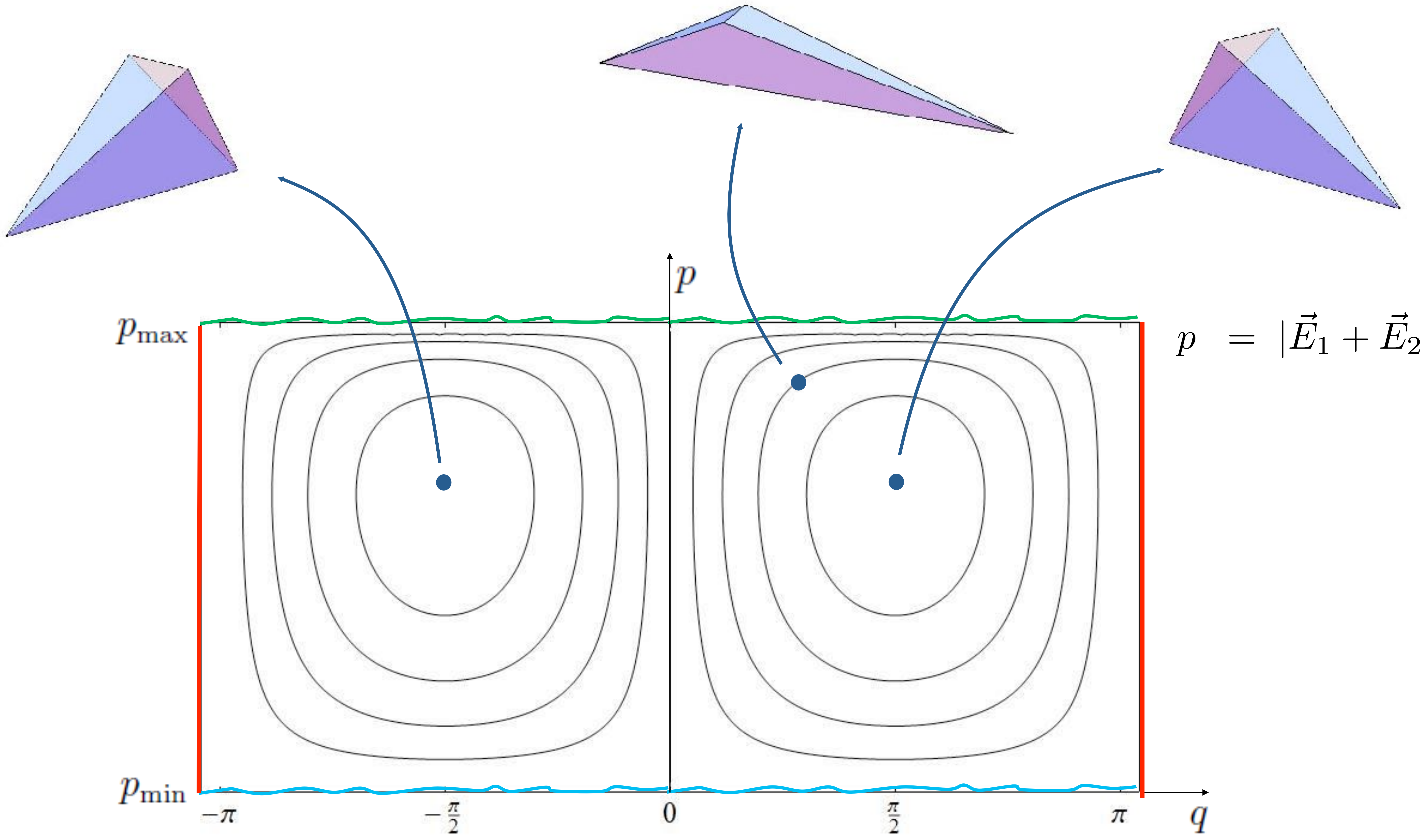


- area of a face $A_a = |\vec{E}_a|$

- angle between two faces $\vec{E}_a \cdot \vec{E}_b = A_a A_b \cos\theta_{ab}$

- volume of the tetrahedron $V = \frac{\sqrt{2}}{3}\sqrt{|\vec{E}_1 \cdot (\vec{E}_2 \times \vec{E}_3)|}$

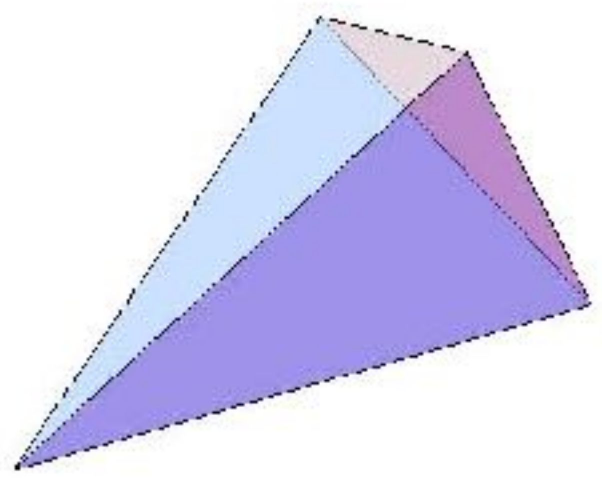
The phase space of a tetrahedron (face-areas A_a fixed)



$$p = |\vec{E}_1 + \vec{E}_2|$$

Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

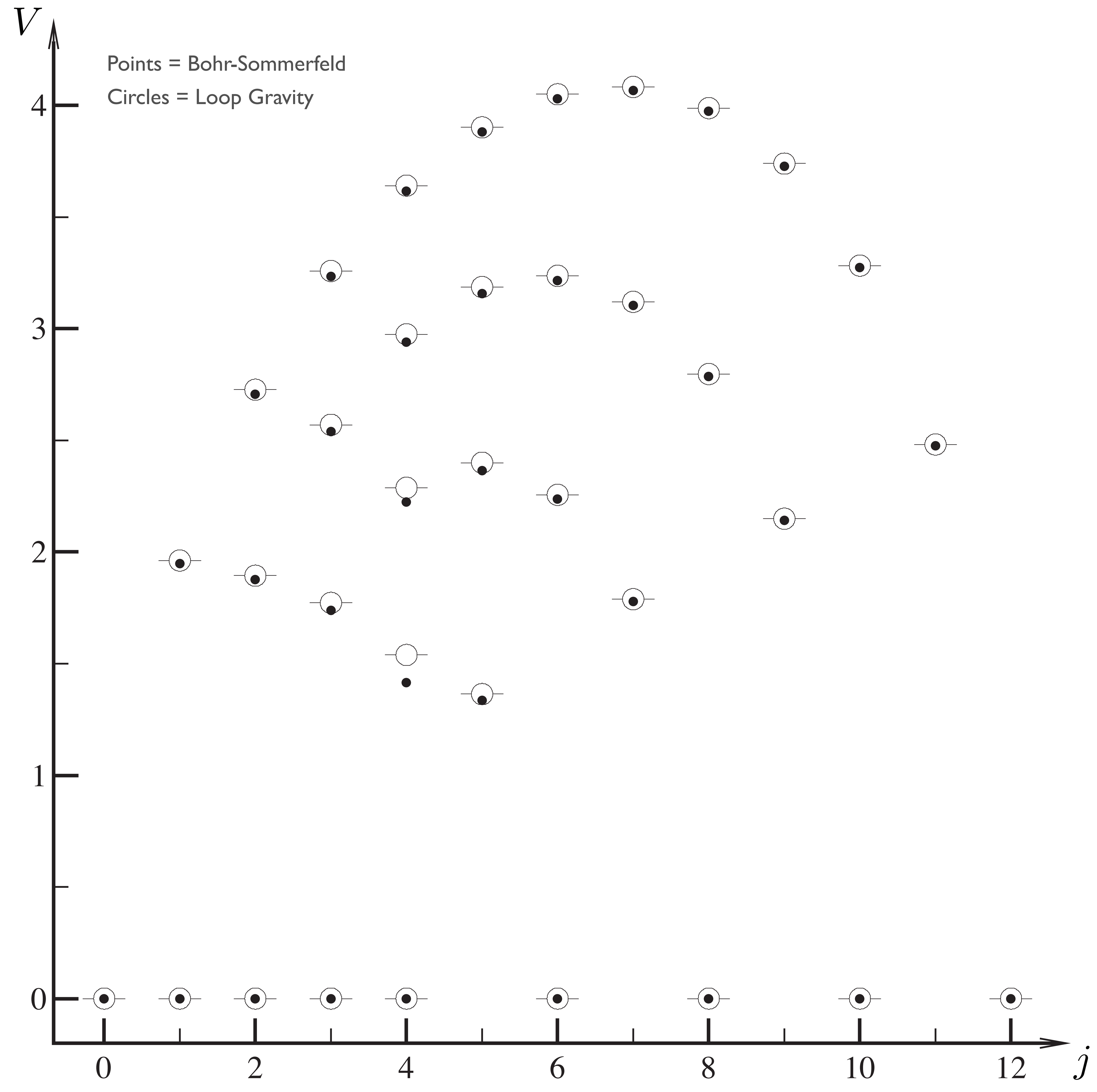


$$A_1 = 9/2$$

$$A_2 = 9/2$$

$$A_3 = 9/2$$

$$A_4 = j + 1/2$$



Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

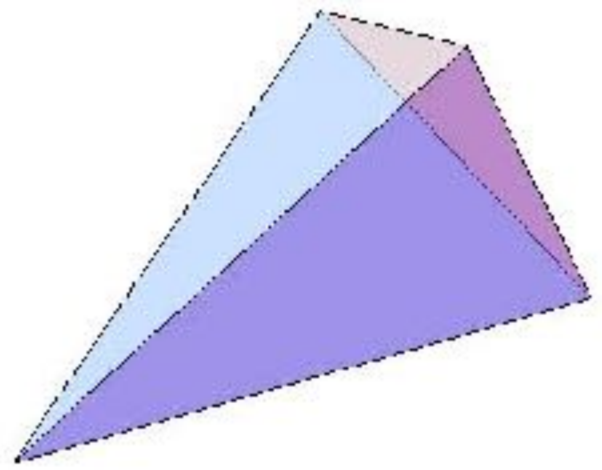
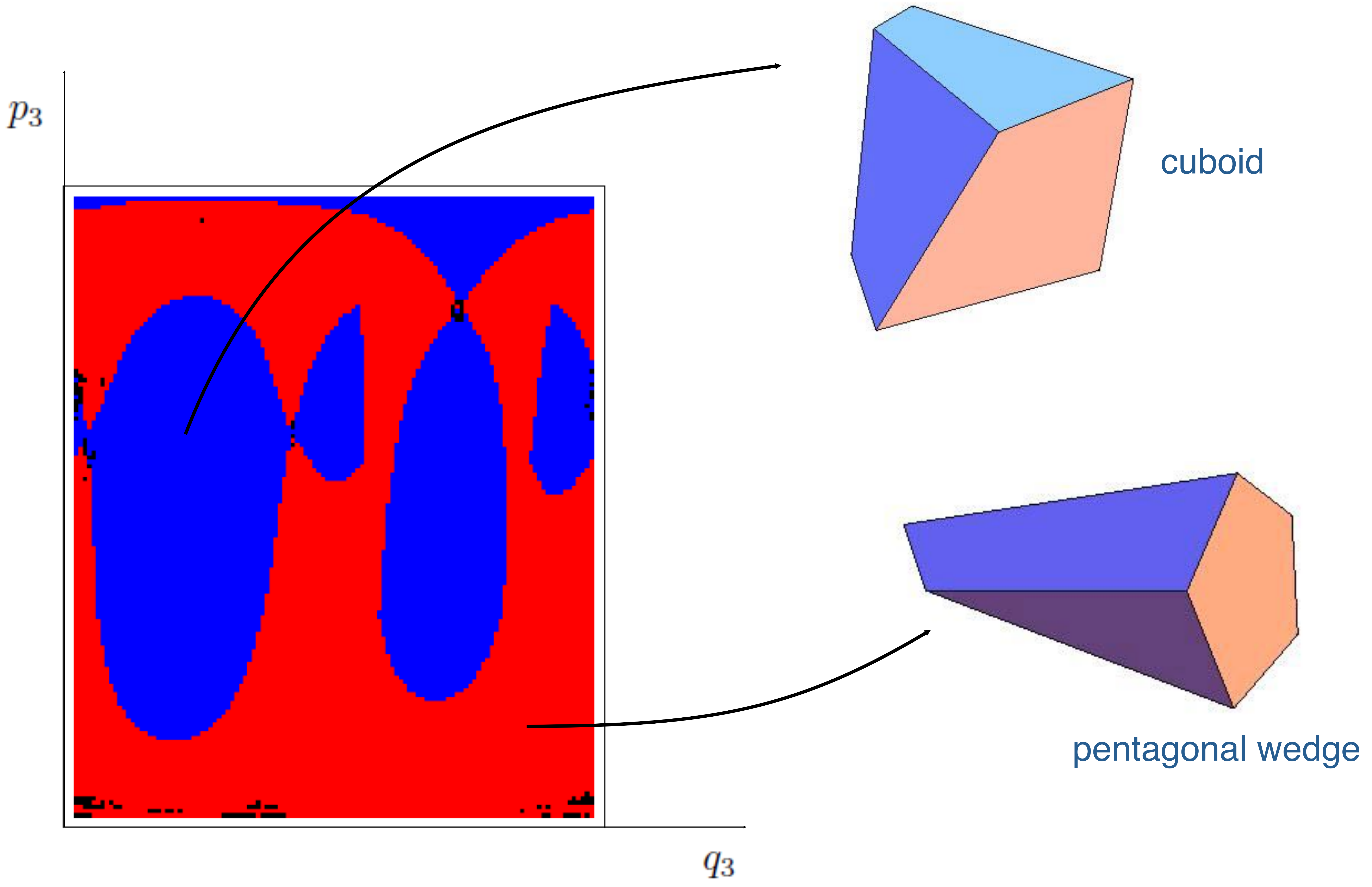


Table: Volume spectrum

j_1 j_2 j_3 j_4	Loop gravity	Bohr-Sommerfeld	Accuracy
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	0.310	0.252	19%
$\frac{1}{2}$ $\frac{1}{2}$ 1 1	0.396	0.344	13%
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$	0.464	0.406	12%
$\frac{1}{2}$ 1 1 $\frac{3}{2}$	0.498	0.458	8%
1 1 1 1	0	0	exact
	0.620	0.566	9%
$\frac{1}{2}$ $\frac{1}{2}$ 2 2	0.522	0.458	12%
$\frac{1}{2}$ 1 $\frac{3}{2}$ 2	0.577	0.535	7%
1 1 1 2	0.620	0.598	4%
$\frac{1}{2}$ $\frac{3}{2}$ $\frac{3}{2}$ $\frac{3}{2}$	0.620	0.598	4%
1 1 $\frac{3}{2}$ $\frac{3}{2}$	0	0	exact
	0.753	0.707	6%
...			
6 6 6 7	1.828	1.795	1.8%
	3.204	3.162	1.3%
	4.225	4.190	0.8%
	5.133	5.105	0.5%
	5.989	5.967	0.4%
	6.817	6.799	0.3%

Beyond tetrahedra: $F = 6$, the space of shapes of ...

Bianchi-Dona-Speziale *PRD*'10



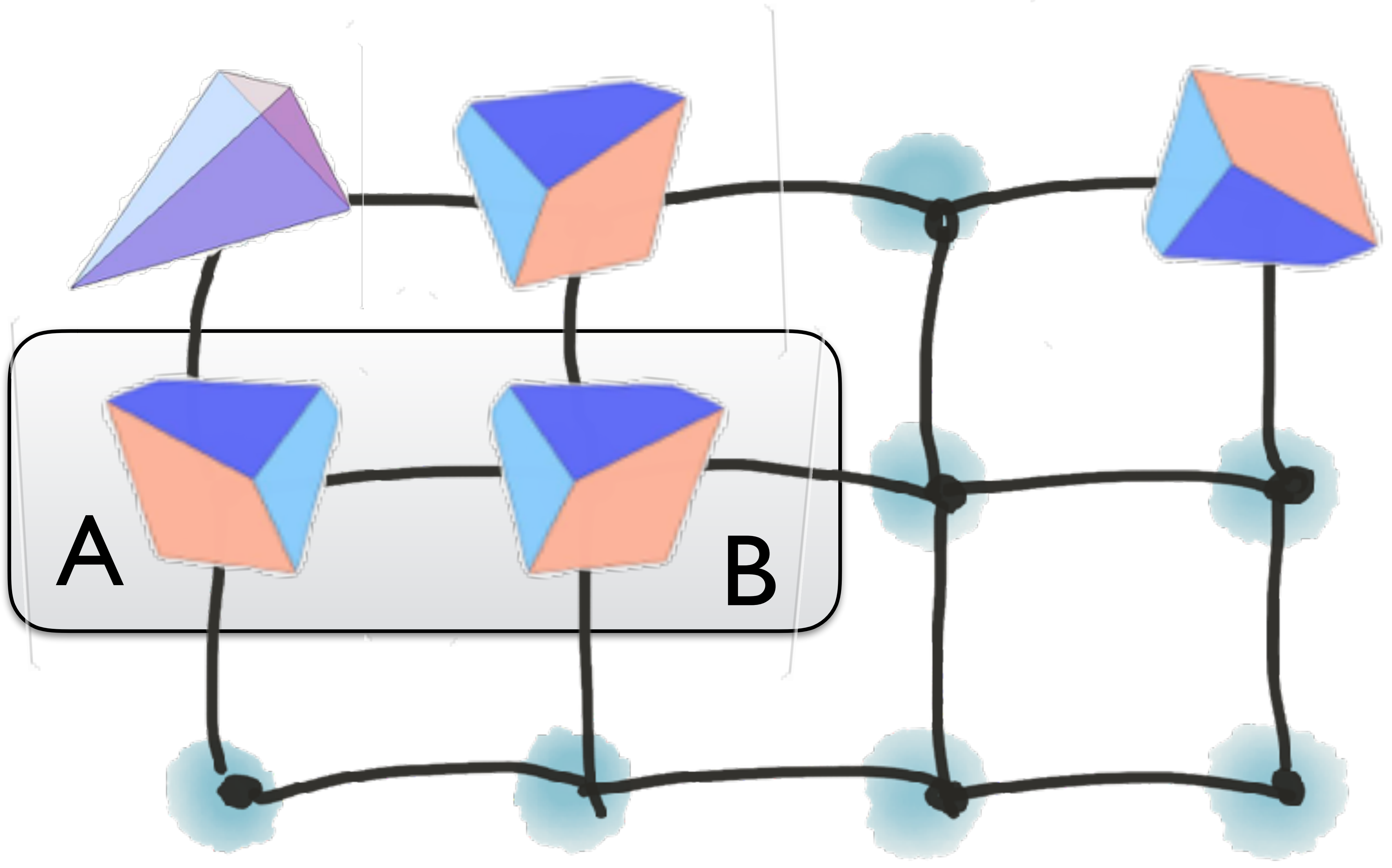
Volume spectrum with Quantum Chaos behavior

Haggard *PRD*'13
ColemanSmith-Muller *PRD*'13

Gluing quantum polyhedra with entanglement

- Fluctuations of nearby quantum shapes are in general uncorrelated: twisted geometry

[Dittrich-Speziale 2008] [EB 2008]
[Freidel-Speziale 2010]
[EB-Dona-Speziale 2010]
[Dona-Fanizza-Sarno-Speziale 2017]



- Saturating uniformly the short-ranged relative entropy

$$\frac{\Big(\langle \mathcal{O}_A\, \mathcal{O}_B\rangle - \langle \mathcal{O}_A\rangle\langle \mathcal{O}_B\rangle\Big)^2}{2\,\|\mathcal{O}_A\|^2\,\|\mathcal{O}_B\|^2} \leq I(A,B)$$

where

$$I(A,B) \equiv S(\rho_{AB}|\rho_A \otimes \rho_B) \;= S_A + S_B - S_{AB}$$

correlates fluctuations of the quantum geometry

State with $\max \sum_{\langle A,B\rangle} I(A,B)$ Glued geometry from entanglement [EB-Baytas-Yokomizo, to appear]

Plan:

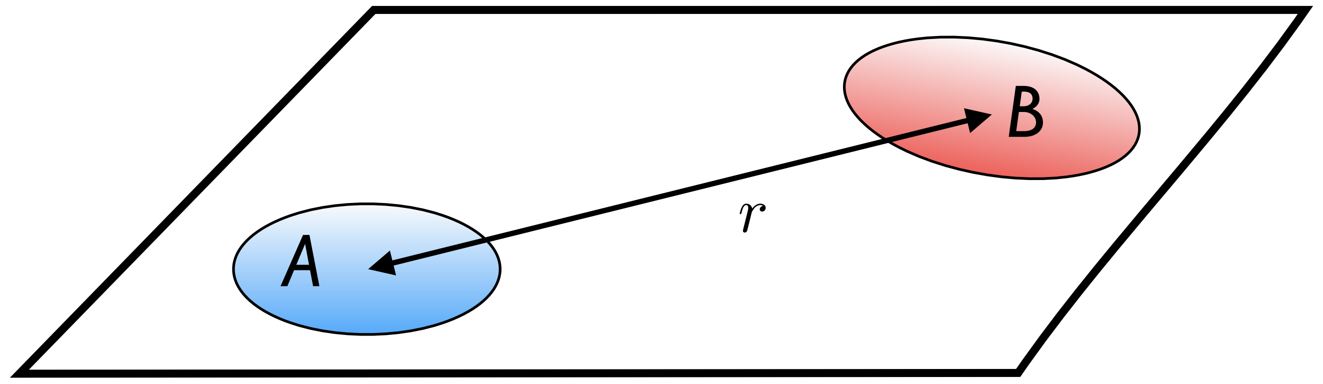
I) Entanglement in simple systems

II) Building space from entanglement

- a) Entanglement, mutual information and bosonic correlators
- b) Gluing quantum polyhedra with entanglement
- c) Entanglement and Lorentz invariance

III) Entanglement in the sky

Correlations at space-like separation



- *In quantum field theory*

Fock space $\mathcal{F} = \mathbb{C} \oplus \mathcal{H} \oplus\ S(\mathcal{H} \otimes \mathcal{H}) \oplus \cdots$

contains

~~(i) states with *no* space-like correlations~~

(ii) states *with* specific short-ranged correlations (e.g. Minkowski vacuum)

crucial ingredient for quantum origin of cosmological perturbations

- *In loop quantum gravity*

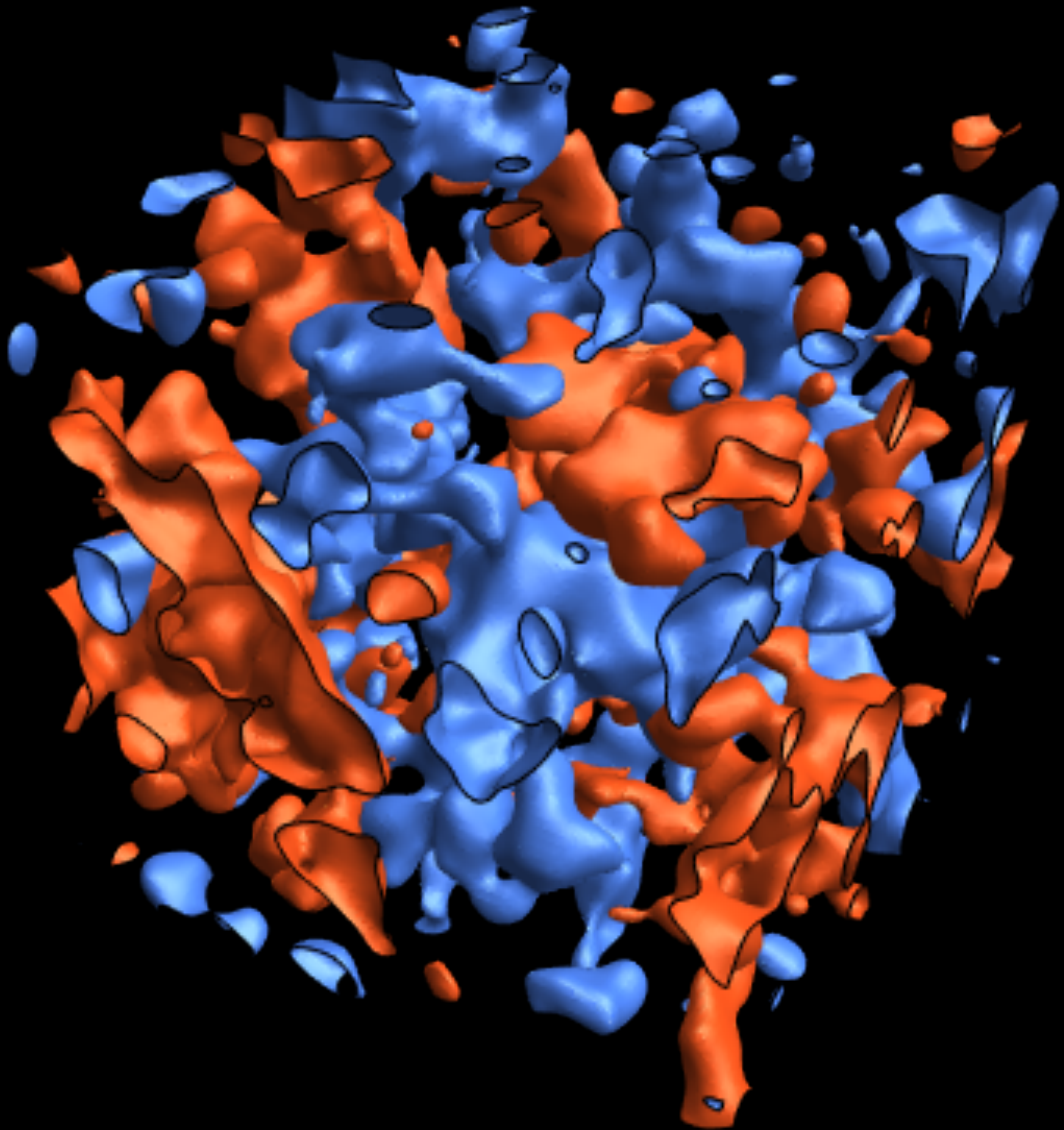
Hilbert space $\mathcal{H}_\Gamma = L^2(SU(2)^L / SU(2)^N)$

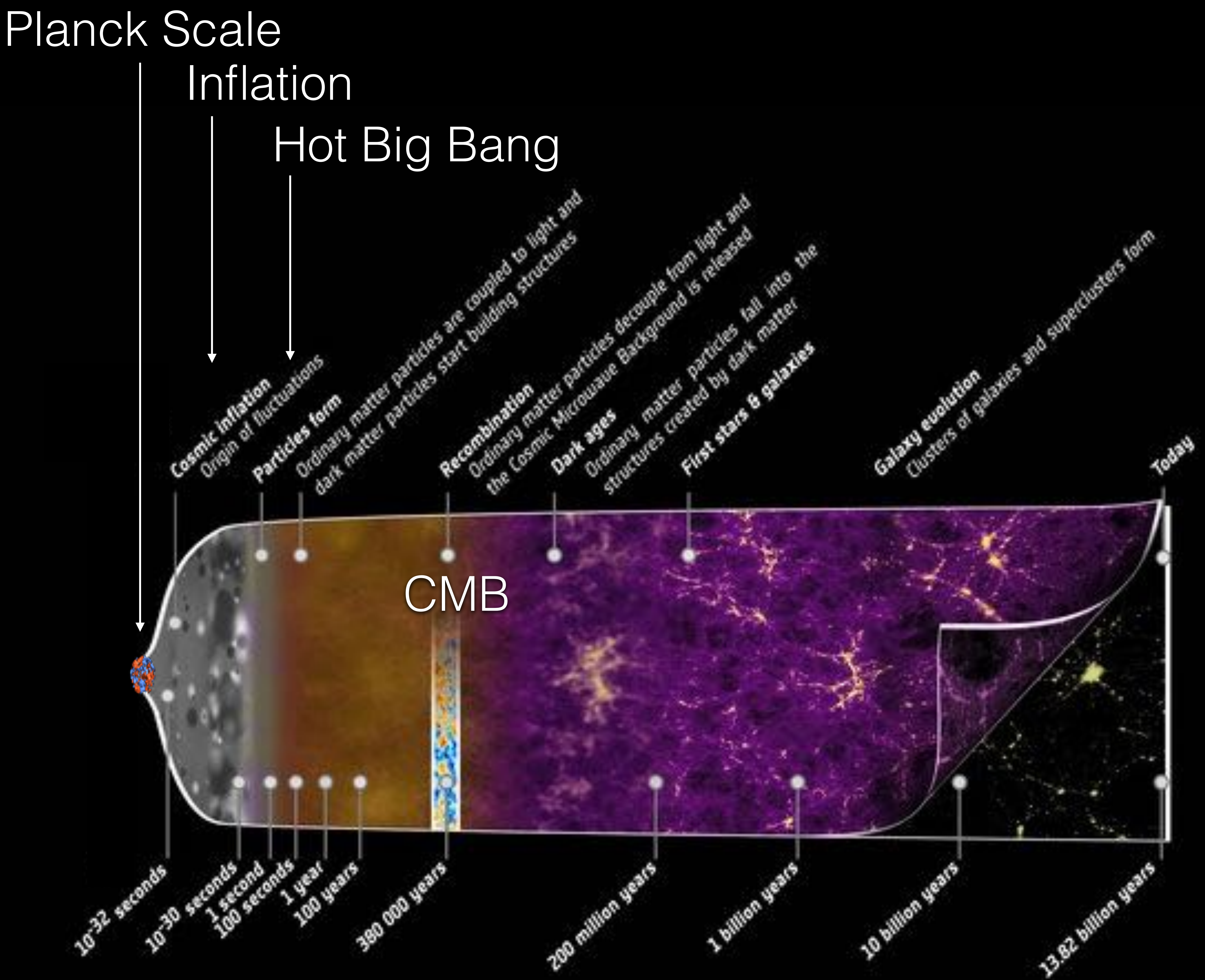
contains

(i) states with *no* space-like correlations (e.g. spin-networks)

(ii) states *with* long-range space-like correlations (e.g. squeezed vacua)

The vacuum state of a quantum field is highly entangled



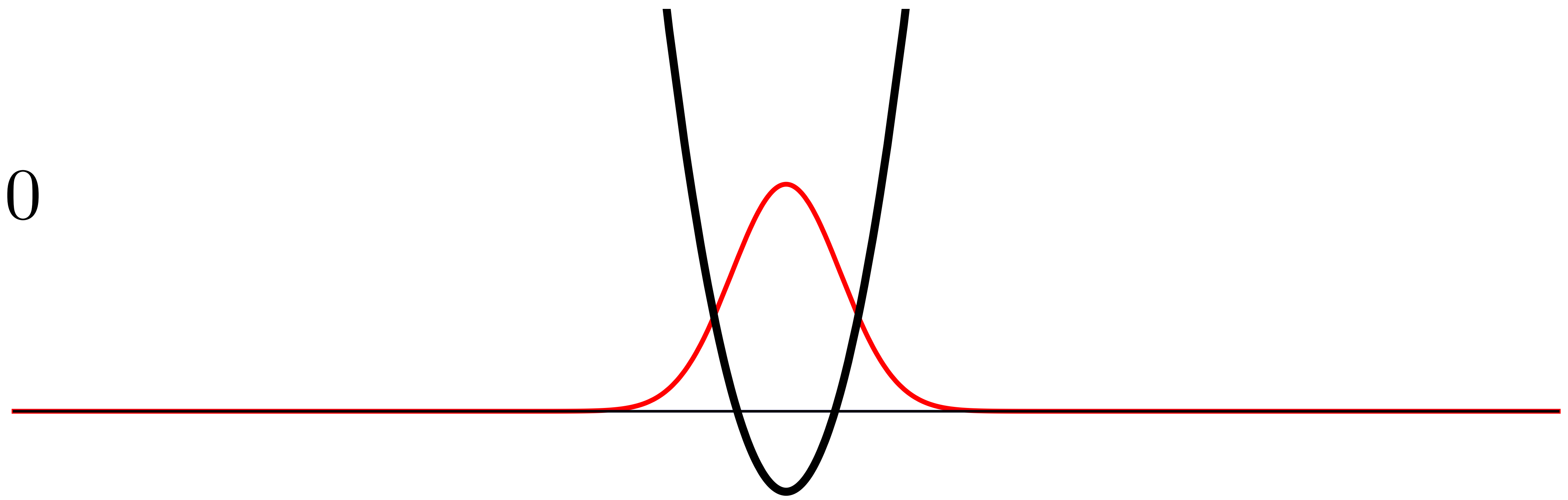

Planck Scale
Inflation
Hot Big Bang
CMB
Cosmic inflation
Origin of fluctuations
Particles form
Ordinary matter particles are coupled to light and dark matter particles start building structures
Recombination
Ordinary matter particles decouple from light and the Cosmic Microwave Background is released
Dark ages
Ordinary matter particles fall into the structures created by dark matter
First stars & galaxies
Galaxy evolution
Clusters of galaxies and superclusters form
Today
10^{-32} seconds
10^{-30} seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years

The Vacuum State of a Quantum Field

No particles $$a(\vec{k})\,|0\rangle \;=\; 0$$

Vanishing expectation value $$\langle 0|\,\varphi(\vec{x})\,|0\rangle \;=\; 0$$

but non-vanishing fluctuations



Uncorrelated momenta

$$\langle 0|\,\varphi(\vec{k})\,\varphi(\vec{k}')\,|0\rangle \;=\; P(|\vec{k}|)\,(2\pi)^3\delta(\vec{k}+\vec{k}')$$

with power spectrum $$P(k) = \frac{1}{2k}$$

Non-vanishing correlations at space-like separation

$$\langle 0|\,\varphi(\vec{x})\,\varphi(\vec{y})\,|0\rangle \;=\; \int_0^\infty \frac{k^3\,P(k)}{2\pi^2}\,\frac{\sin(k\,|\vec{x}-\vec{y}|)}{k\,|\vec{x}-\vec{y}|}\,\frac{dk}{k} \;=\; \frac{1}{(2\pi)^2}\,\frac{1}{|\vec{x}-\vec{y}|^2}$$

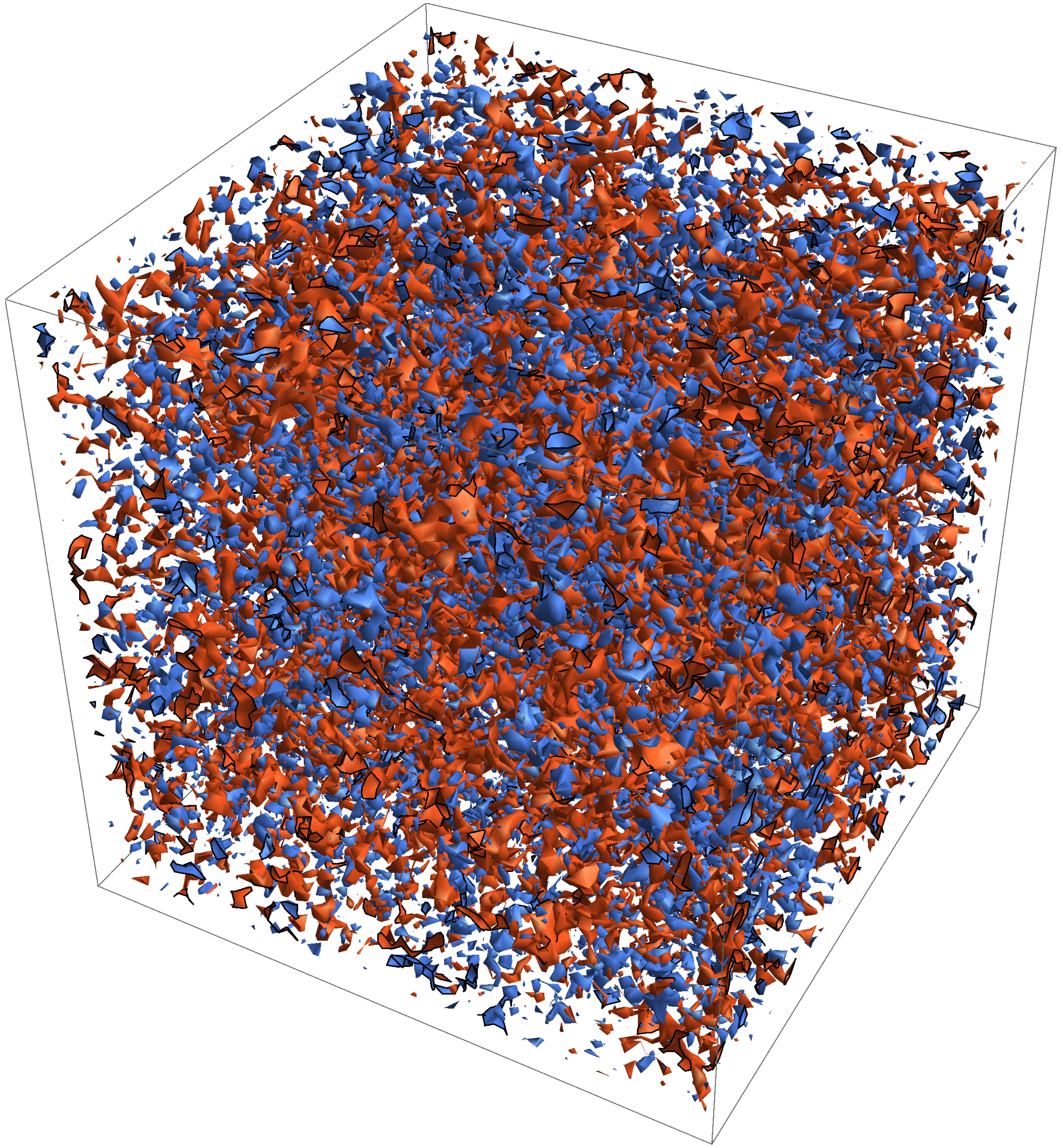
Fluctuations of the field averaged over a region of size R

$$(\Delta\varphi_R)^2 \equiv \langle 0|\varphi_R\,\varphi_R|0\rangle \;-\; (\langle 0|\varphi_R|0\rangle)^2 \;\sim\; \frac{1}{R^2}$$

The vacuum of a quantum field after inflation

Minkowski

$$P(k) = \frac{1}{2k}$$



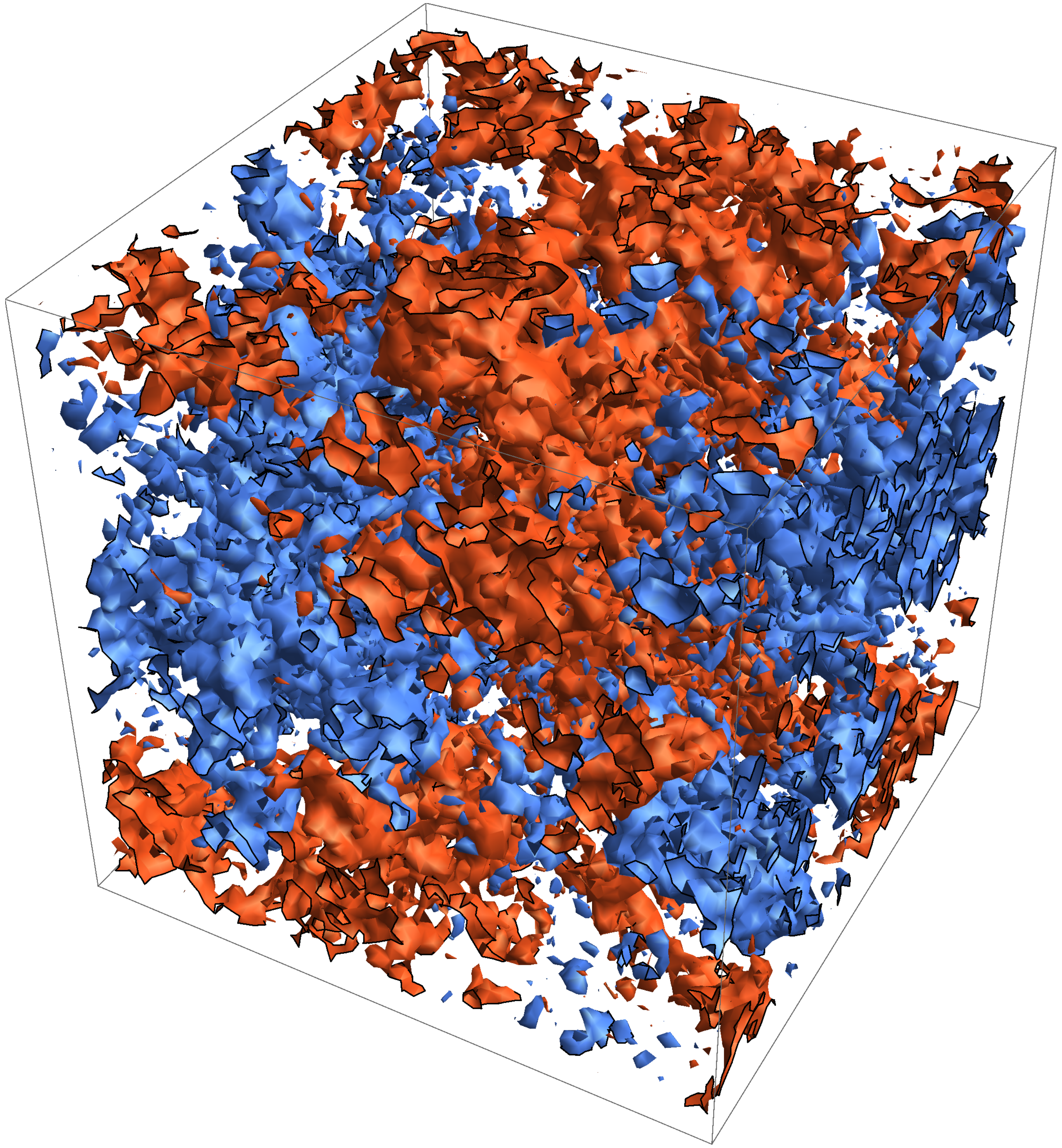
The vacuum of a quantum field after inflation

Minkowski

$$P(k) = \frac{1}{2k}$$

de Sitter

$$P(k) = \frac{1}{2k} e^{-2H_0 t} + \frac{H_0^2}{2\,k^3}$$



The vacuum of a quantum field after inflation

Mukhanov-Chibisov (1981)

Minkowski

$$P(k) = \frac{1}{2k}$$

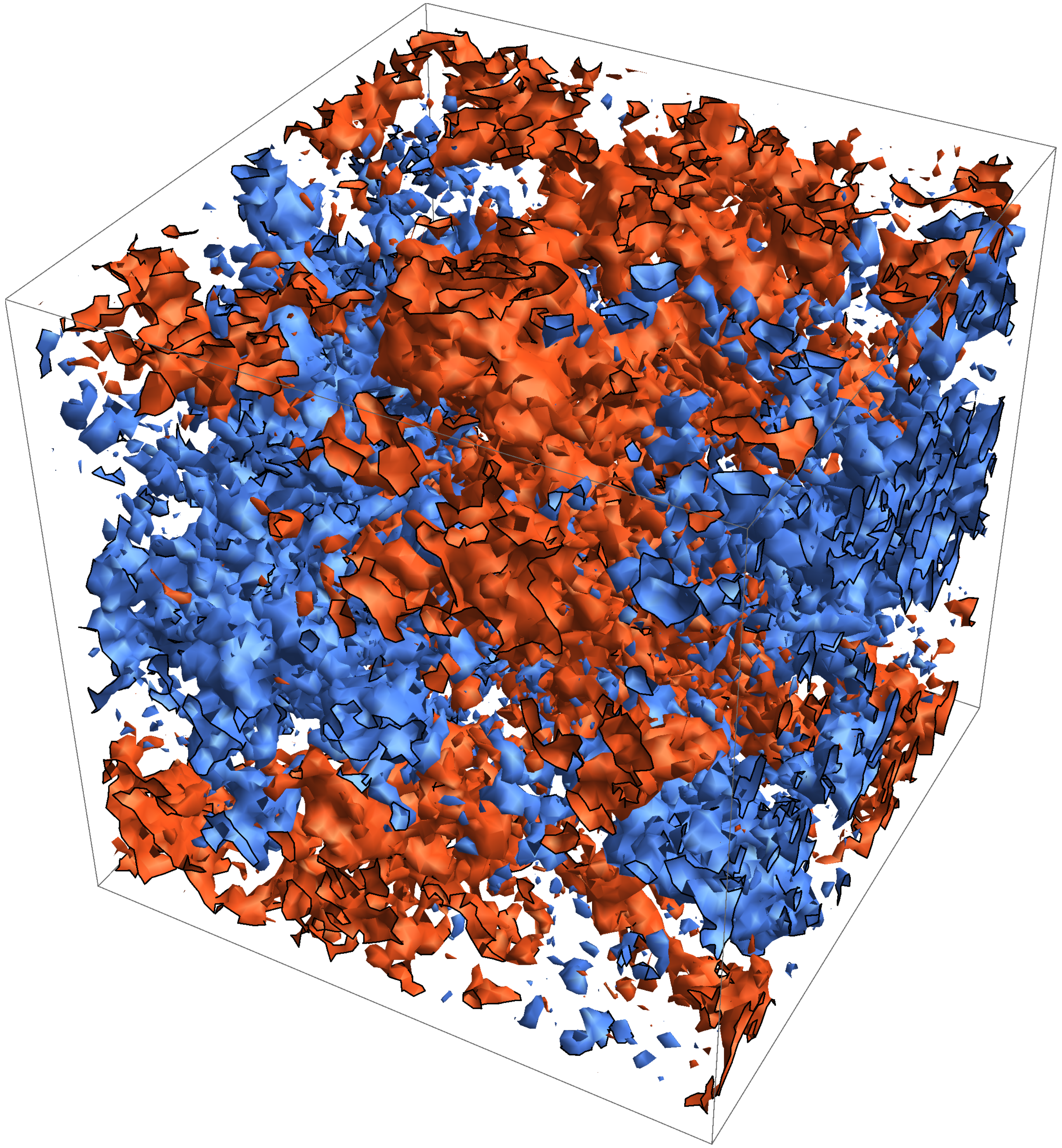
de Sitter

$$P(k) = \frac{1}{2k} e^{-2H_0 t} + \frac{H_0^2}{2\,k^3}$$

Inflation (quasi-de Sitter)

$$P_s(k) \approx \frac{2\pi^2 A_s}{k^3} \left(\frac{k}{k_*}\right)^{n_s - 1}$$

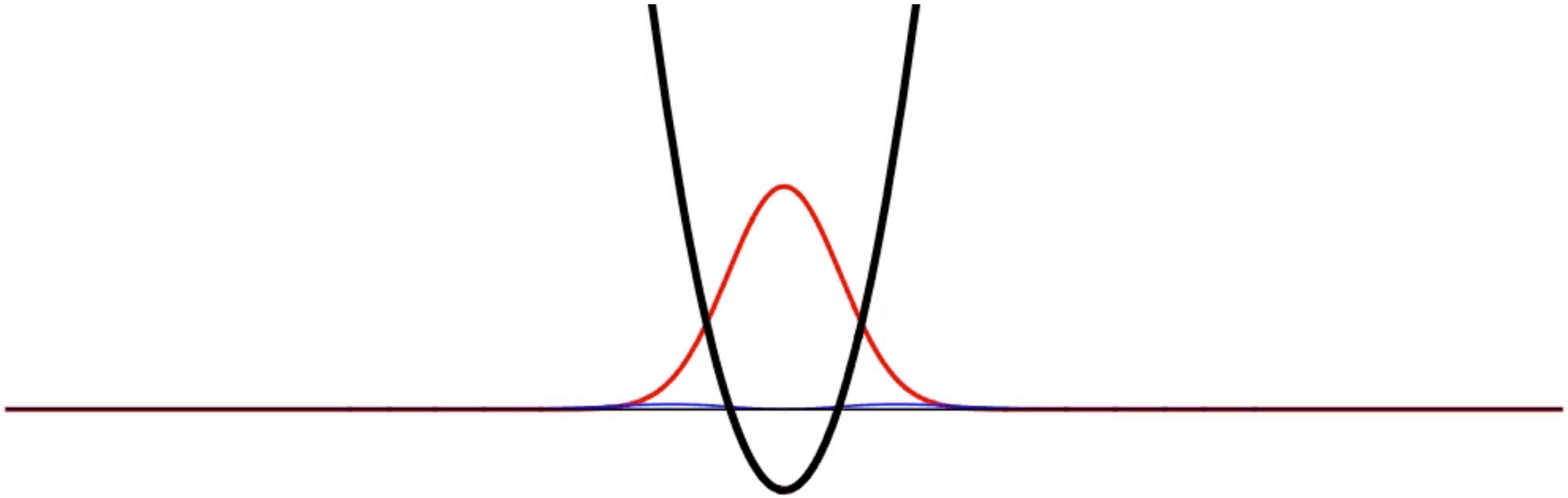
with $A_s \sim \frac{G\hbar\, H_*^2}{\varepsilon_*}$



Planck 2015

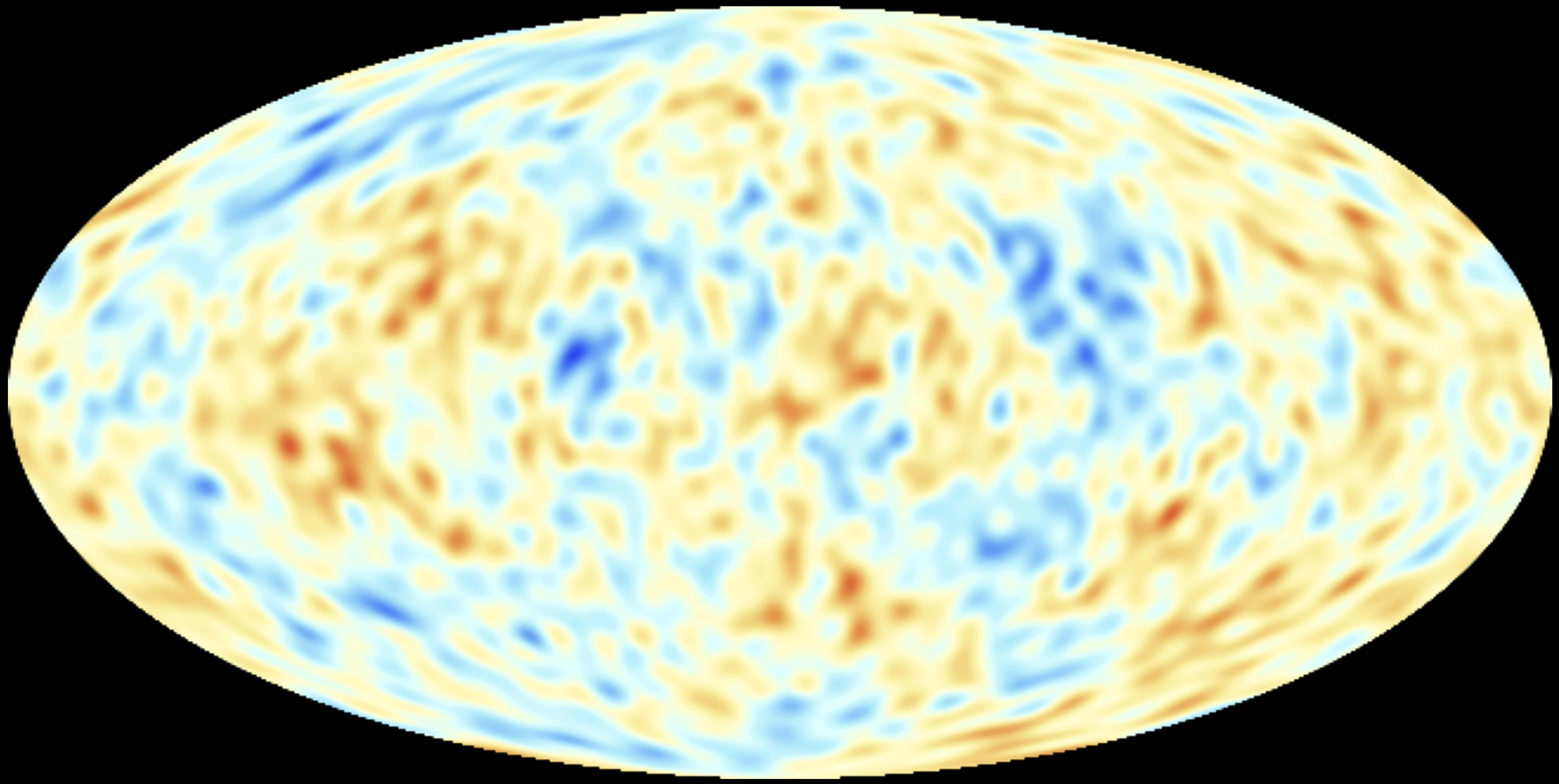
$$A_s(k_*) = 2.47 \times 10^{-9}$$
$$n_s(k_*) = 0.96$$

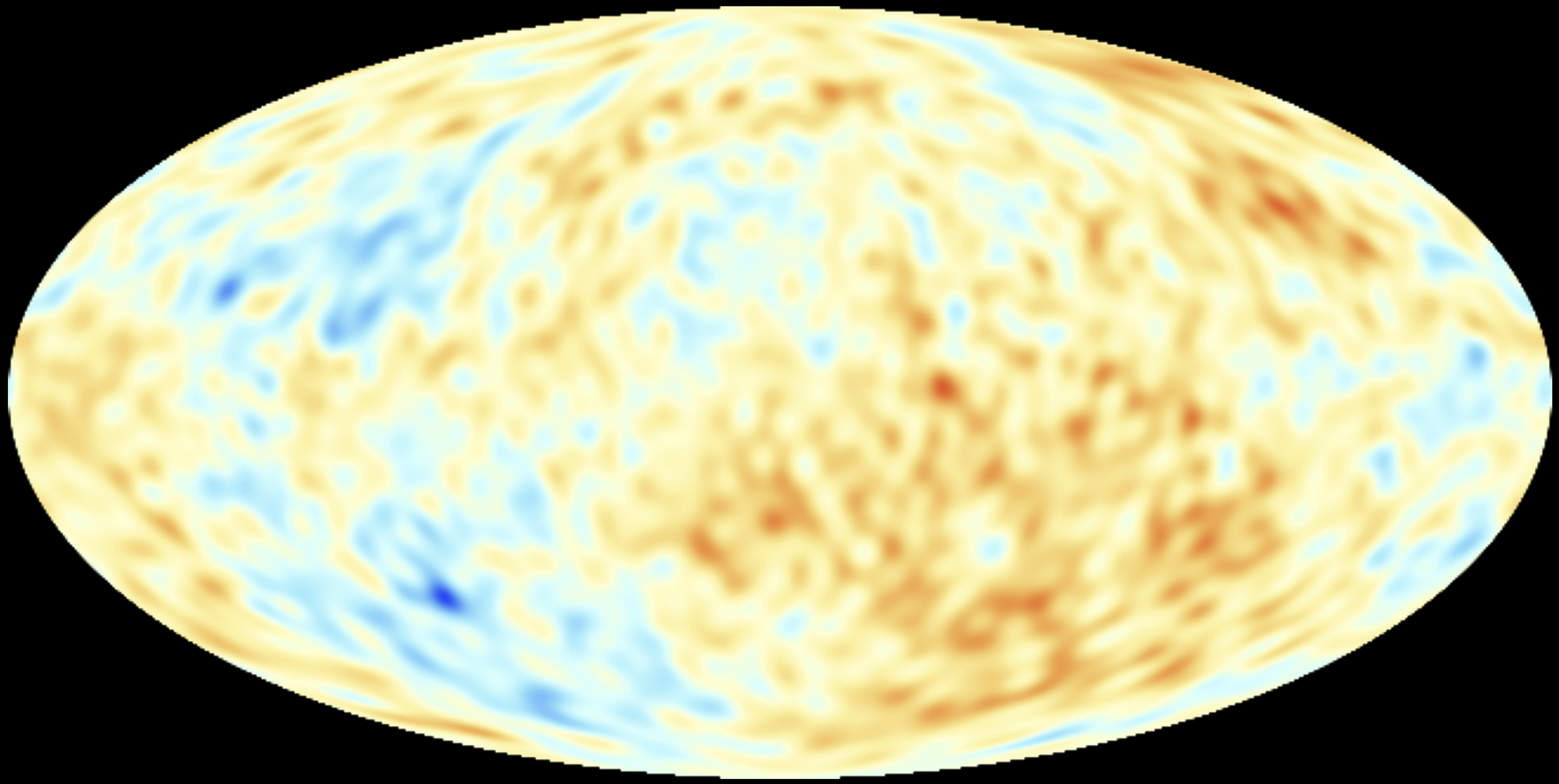
Mechanism: amplification of vacuum fluctuations by instabilities

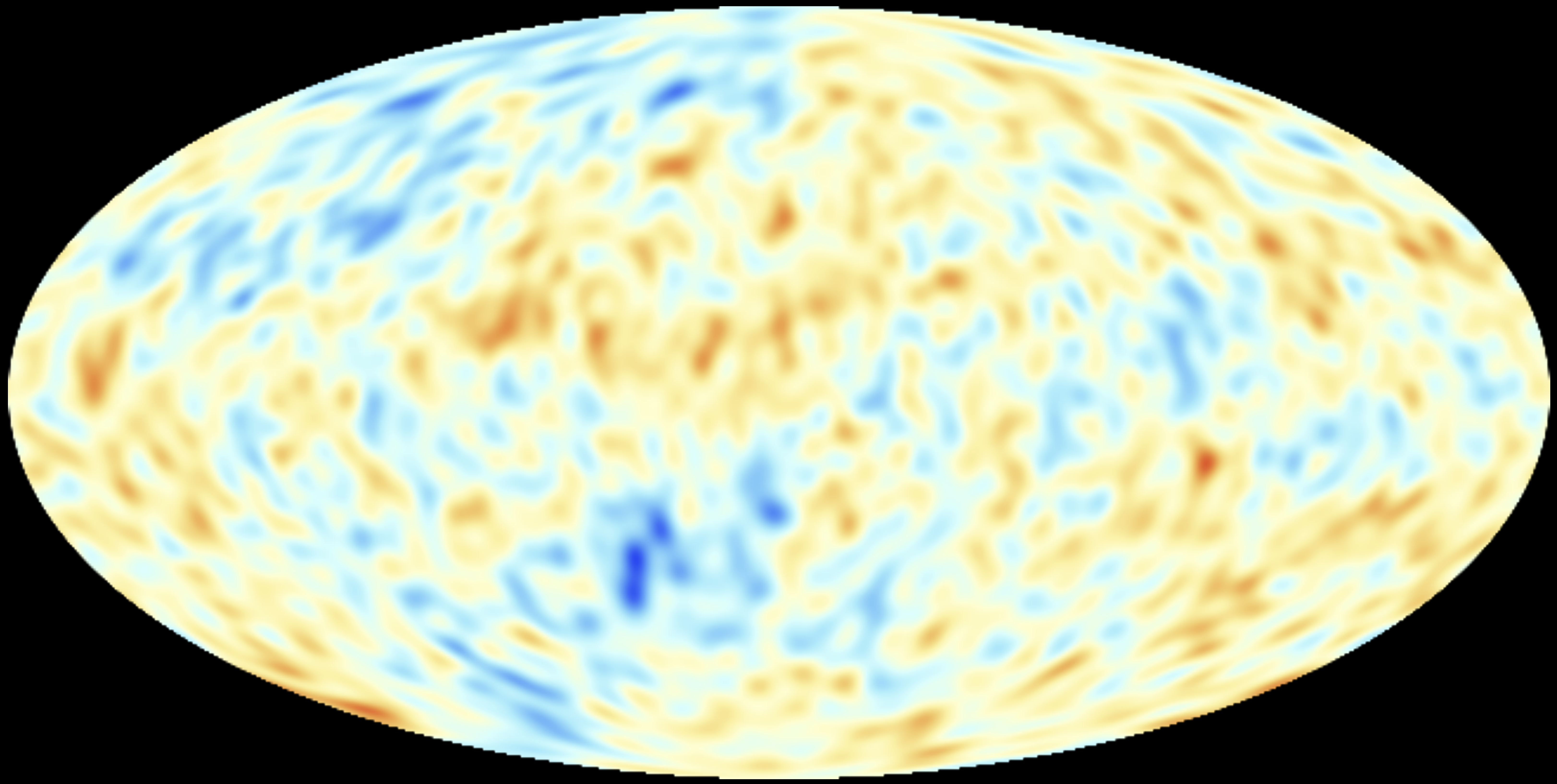


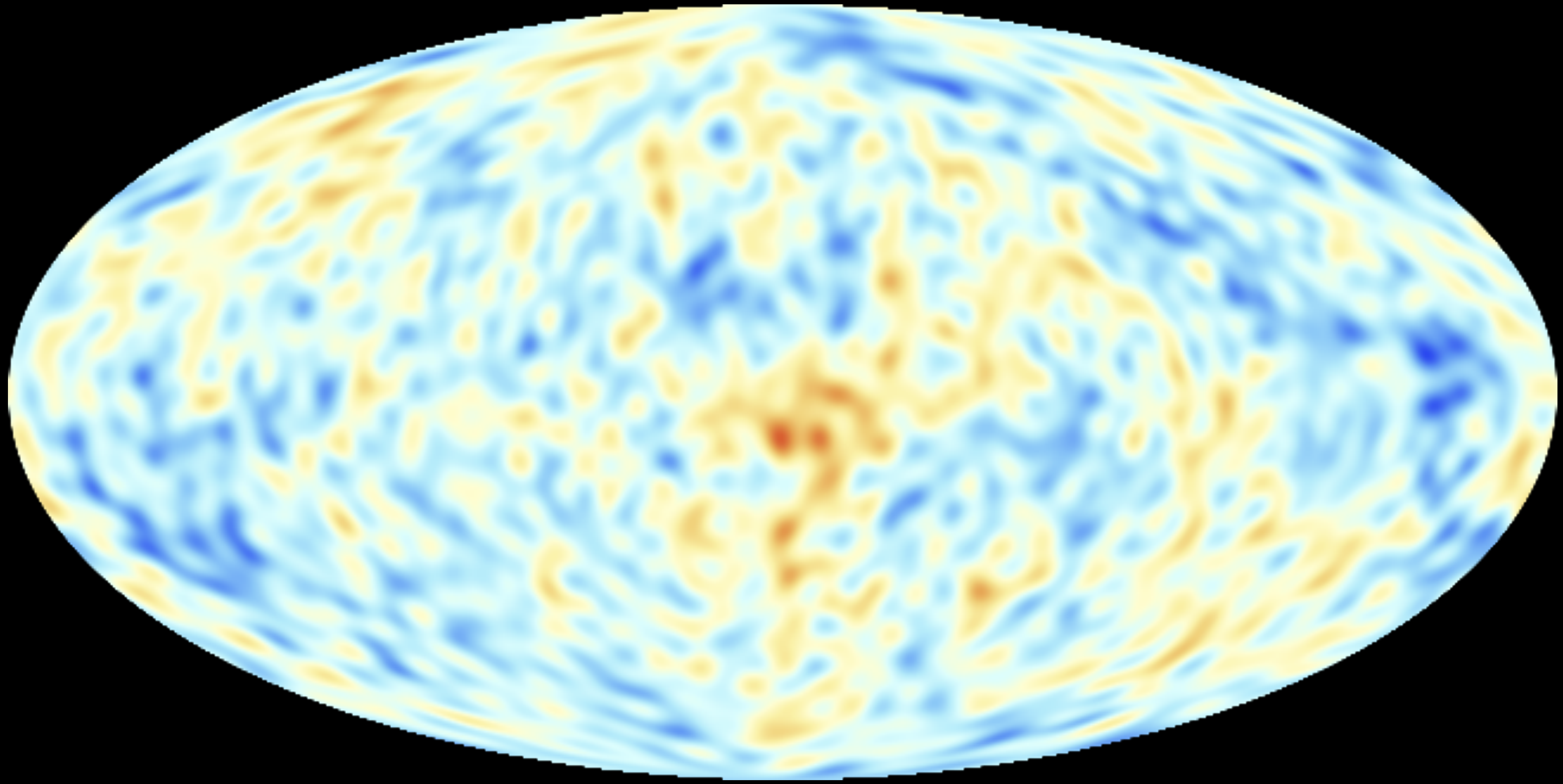
Harmonic oscillator with
time-dependent frequency

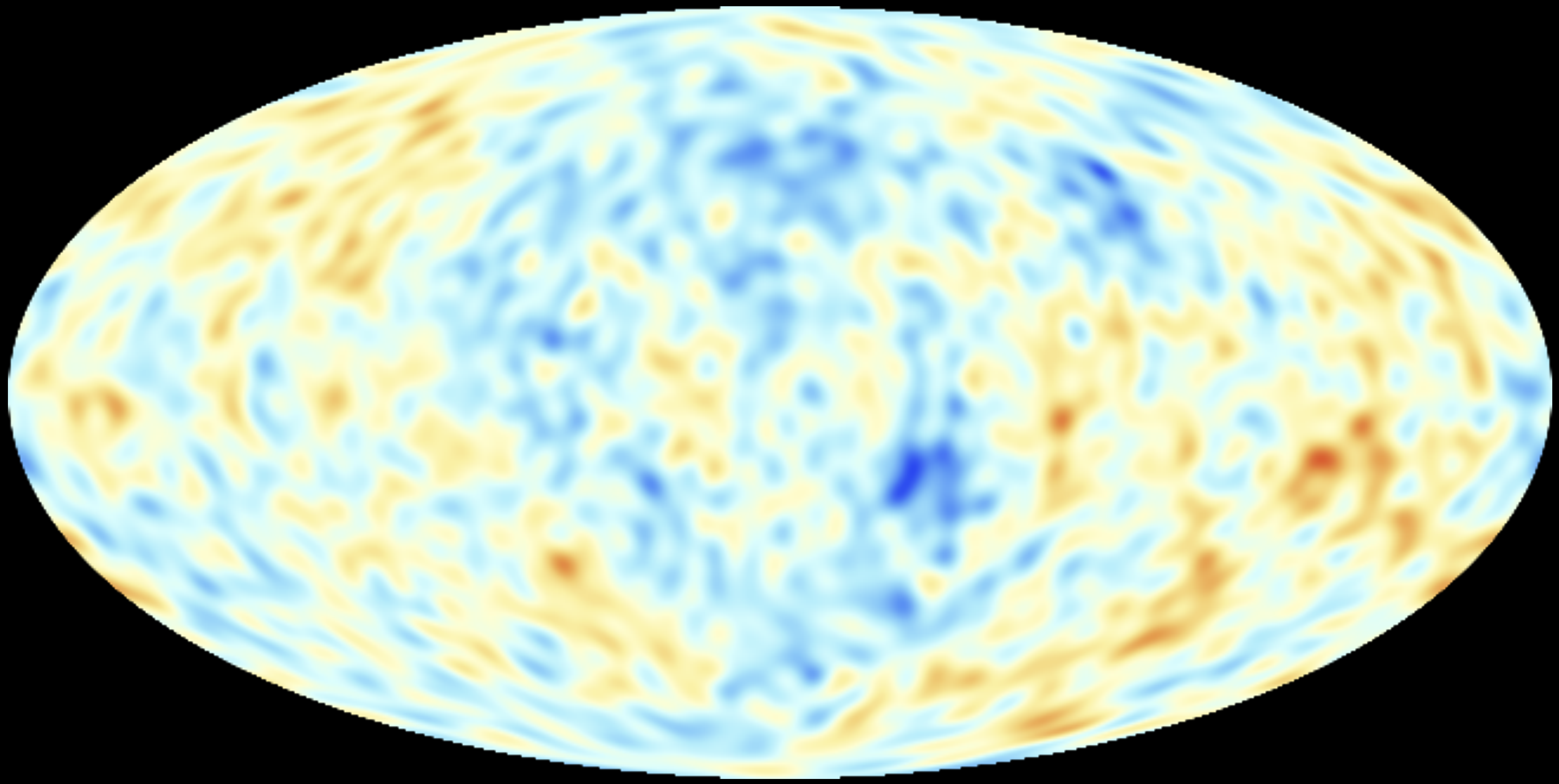
$$H(t) = \frac{1}{2}p^2 + \frac{1}{2}\big(k^2 - f(t)\big)\, q^2$$

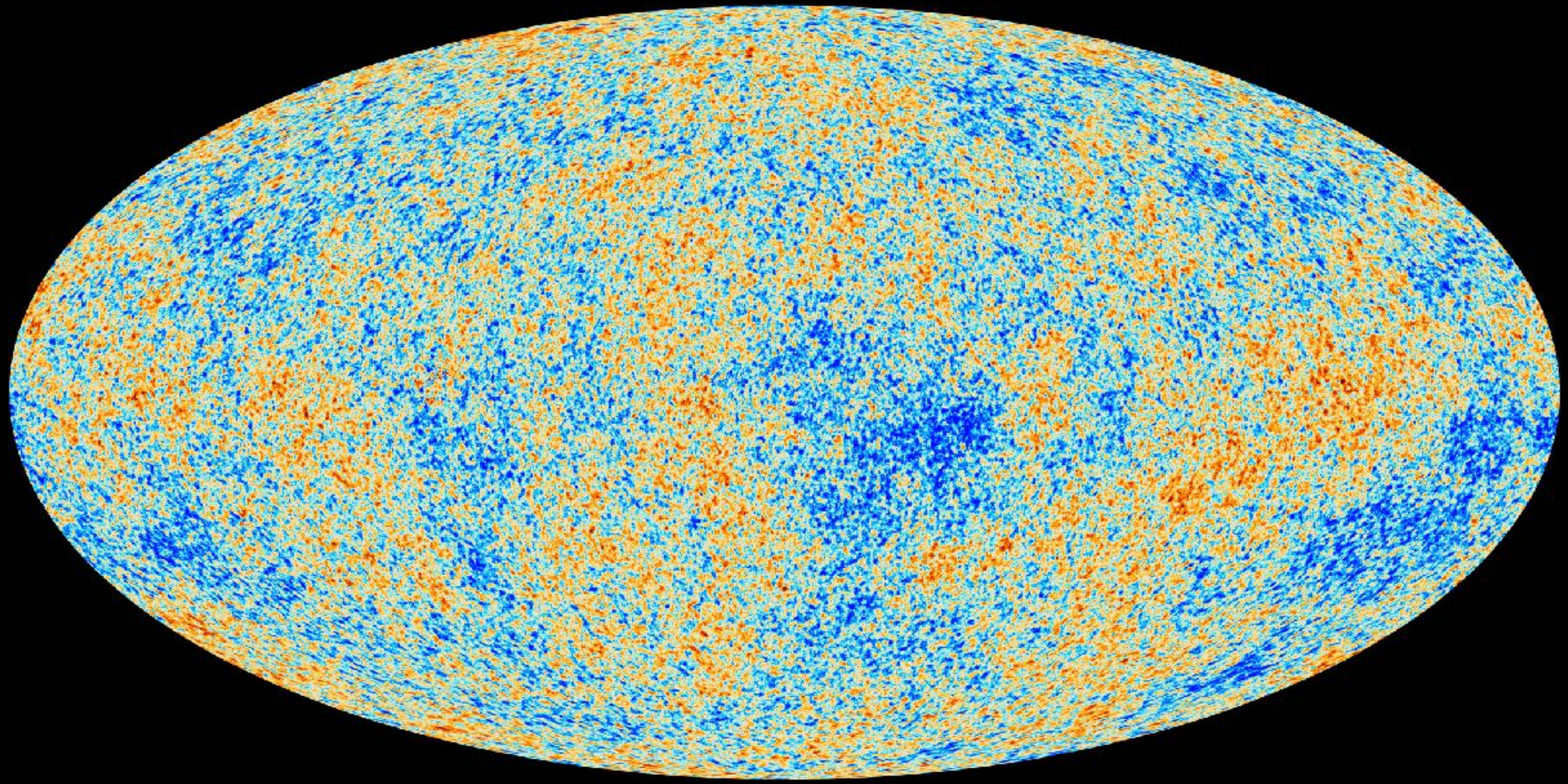




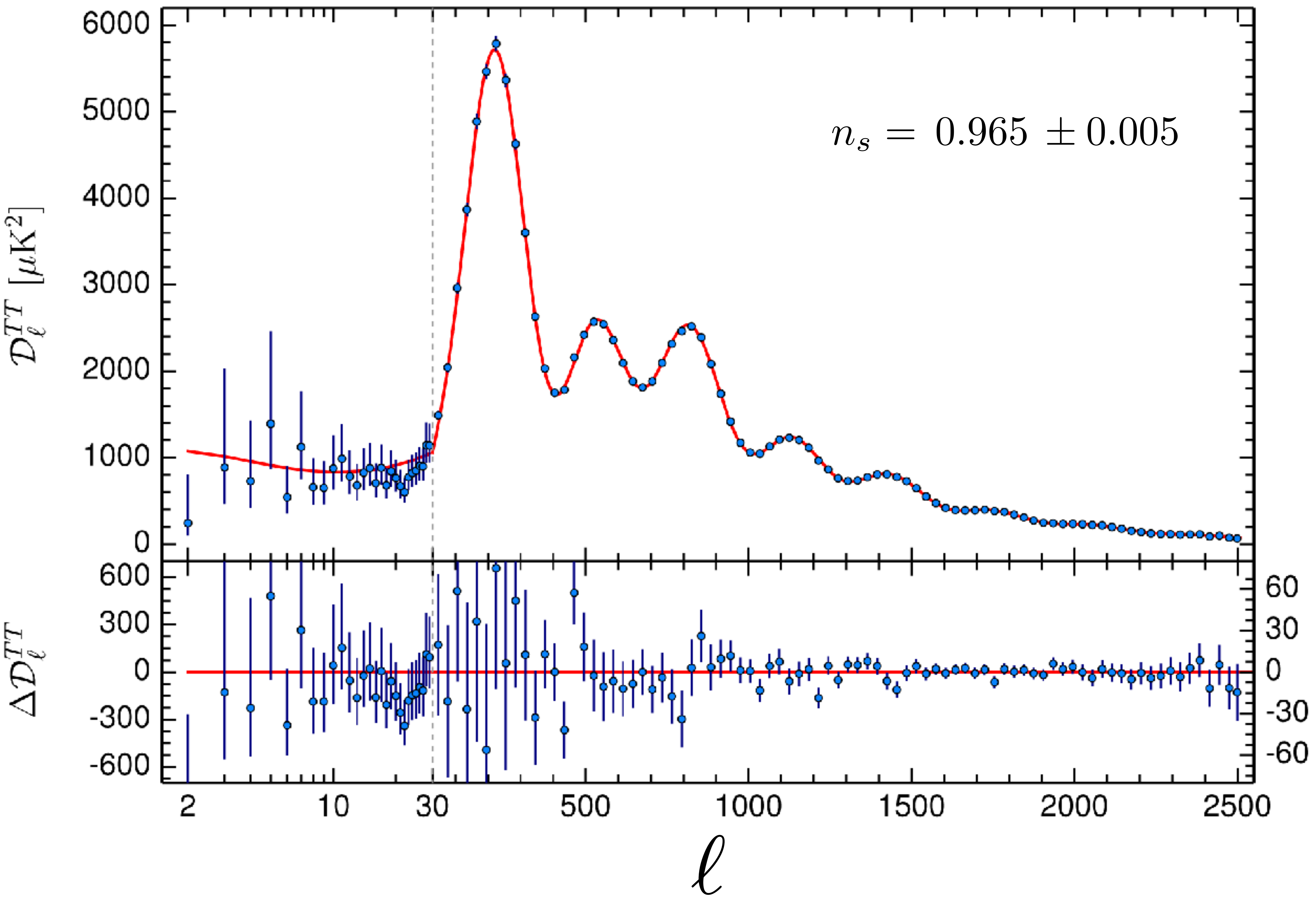




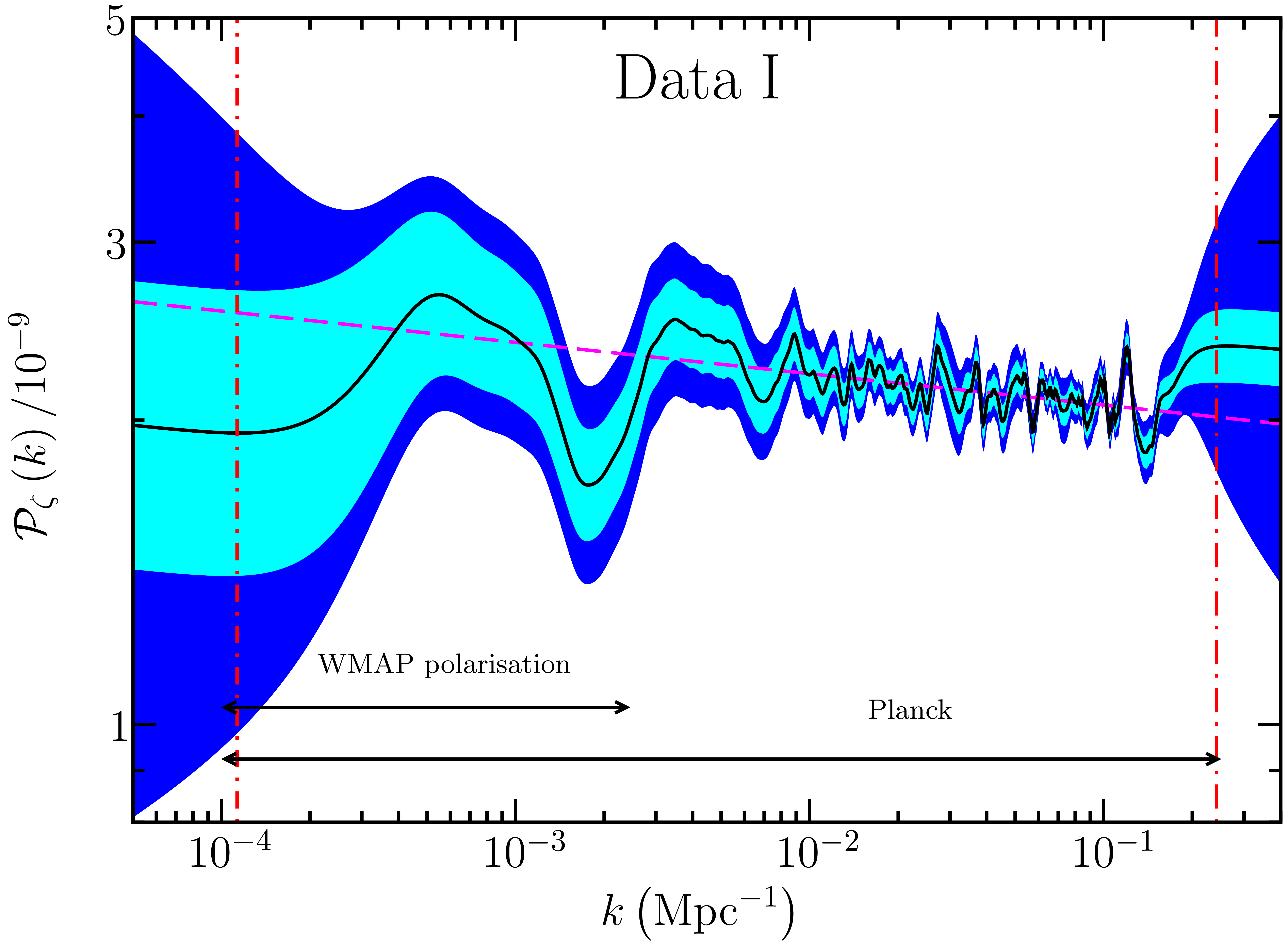




The anisotropies of the Cosmic Microwave Background
as observed by Planck



Planck Collaboration, arxiv.org/abs/1502.02114
``Planck 2015 results. XX. Constraints on inflation"



Reconstructed primordial power spectrum of curvature perturbations
[Hunt & Sarkar, JCAP 2015]

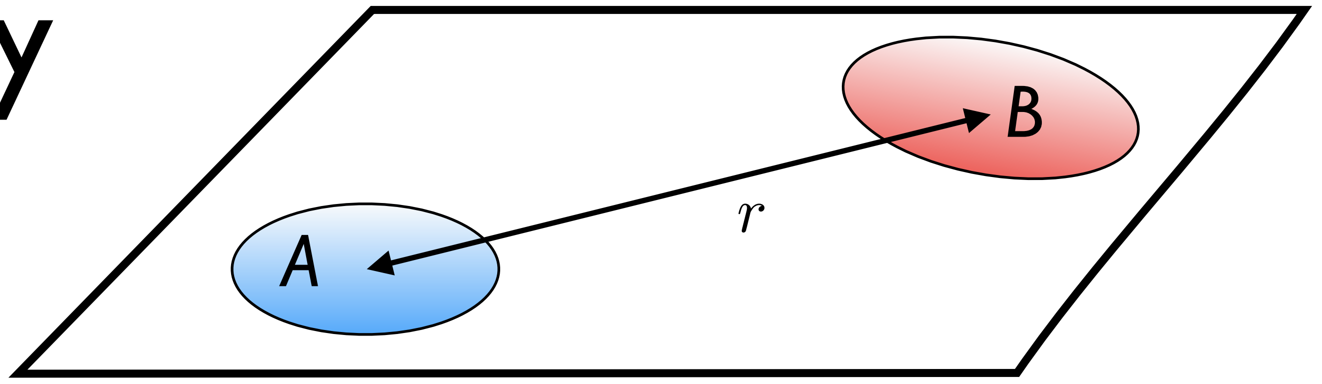
A distinguishing feature of loop quantum gravity:

→ existence of states with no correlation at space-like separation

Scenario

→ uncorrelated initial state and its phenomenological imprints

Emergence of space-like correlations in loop quantum gravity



States with *no* space-like correlations: allowed in quantum gravity

BKL conjecture (Belinsky-Khalatnikov-Lifshitz 1970)

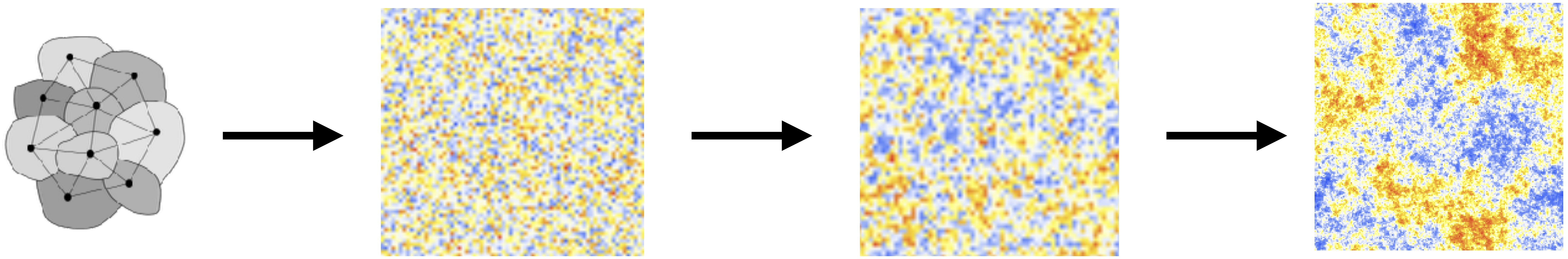
In classical General Relativity, the spatial coupling of degrees of freedom is suppressed in the approach to a space-like singularity

Quantum BKL conjecture (E.B.-Hackl-Yokomizo 2015)

In quantum gravity, correlations between spatially separated degrees of freedom are suppressed in the approach to a Planck curvature phase

$$\begin{cases} \hat{H}\,\Psi[g_{ij}(x),\varphi(x)] = 0 \\ \lim_{a\to 0} \Psi[a,\phi,\delta g_{ij}(x),\delta\varphi(x)] = \prod_{\vec{x}} \psi(\phi,\delta g_{ij}(x),\delta\varphi(x)) \end{cases}$$

Scenario: the correlations present at the beginning of slow-roll inflation are produced in a pre-inflationary phase when the LQG-to-QFT transition takes place



Inflation and spinfoams

- Effective spinfoam action

$$S[e^I, \omega^{IJ}, r, \lambda^{IJ}] = \int \Big((1 + 2\alpha\, r) B_{IJ} \wedge F^{IJ} - \frac{\alpha\, r^2}{1+\gamma^2}\, \frac{1}{4!} \epsilon_{IJKL} B^{IJ} \wedge B^{KL} + B_{IJ} \wedge \nabla \lambda^{IJ} \Big)$$

where $$B_{IJ} = \frac{1}{8\pi G} \Big(\frac{1}{2}\, \epsilon_{IJKL}\, e^K \wedge e^L - \frac{1}{\gamma}\, e_I \wedge e_J \Big)$$

γ = Barbero-Immirzi parameter

r = 0-form, effective Ricci scalar at a coarse-graining scale

α = coupling constant dimensions of *Area*

- It provides an embedding in spinfoams of the Starobinsky model (1979)

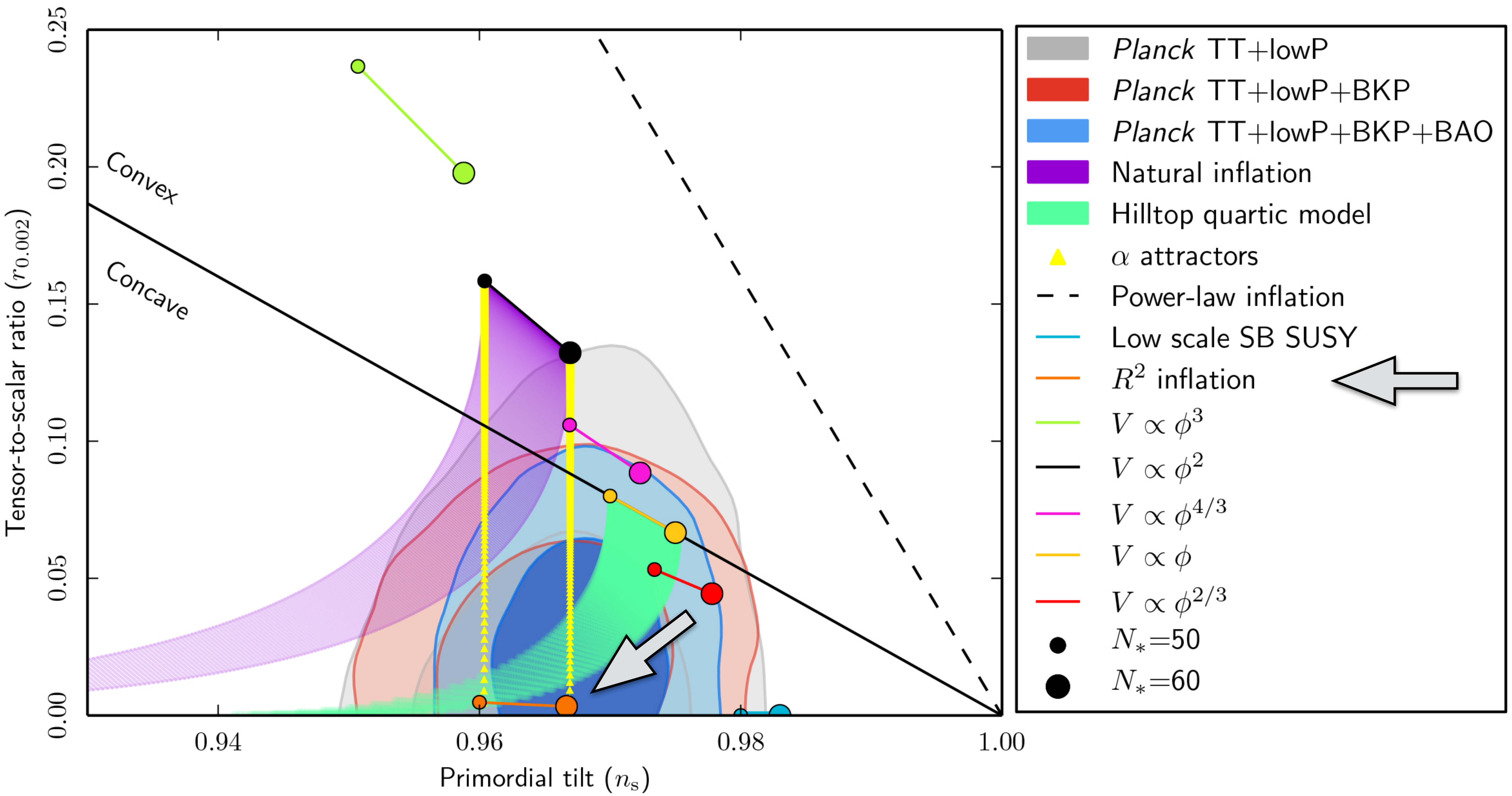
$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, \big(R + \alpha\, R^2\big) \quad \Rightarrow \quad \mathcal{G}_{\mu\nu} + \alpha\, \mathcal{H}_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Friedman eq: $$H^2 + 6\alpha \left(6 H^2 \dot{H} - \dot{H}^2 + 2 H \ddot{H}\right) = 0$$

gravity-driven inflation

E.B.-Fernandez-Rincon 2017

PLANCK 2015



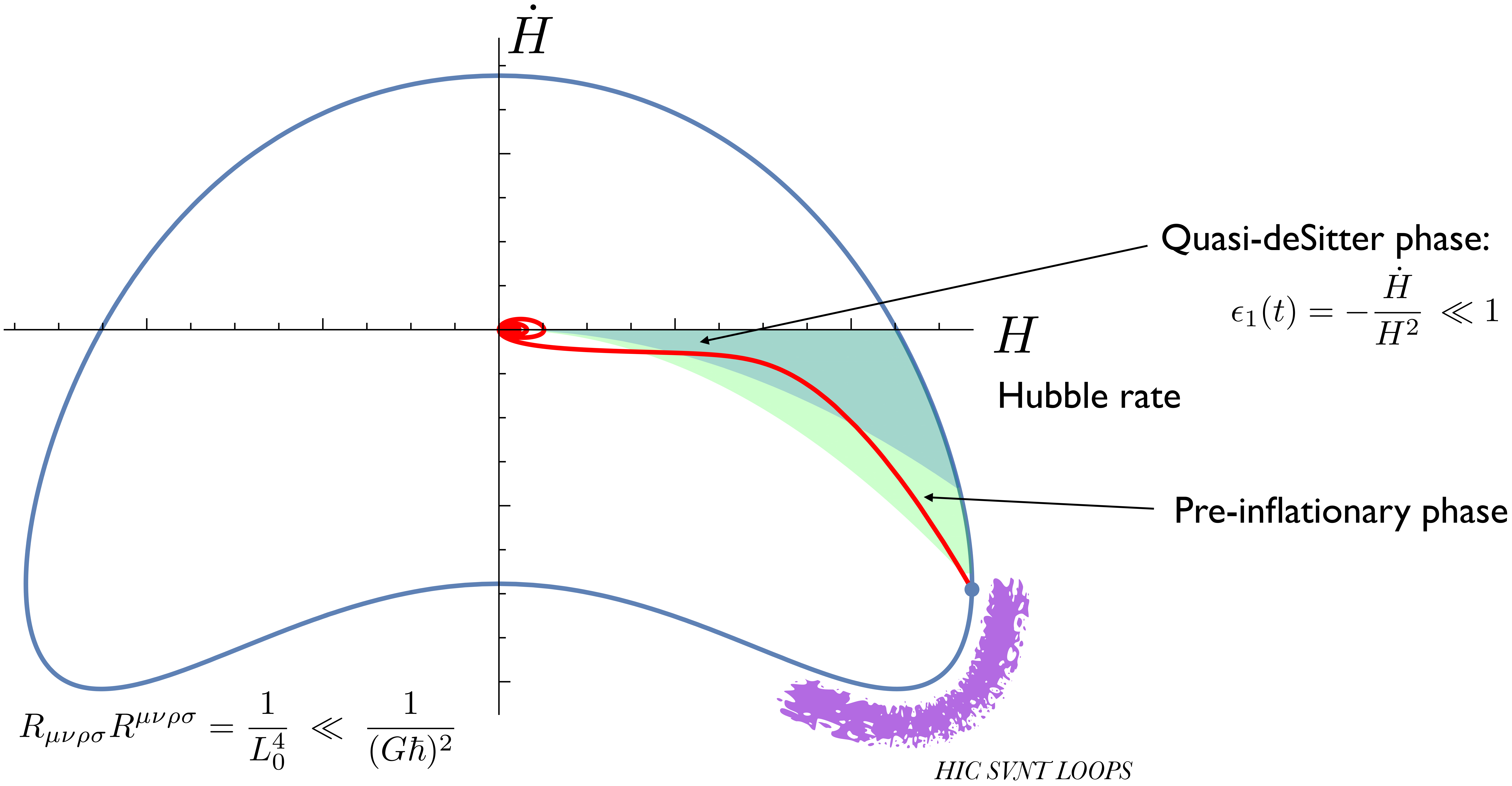
at the scale $k_* = 0.002\ \mathrm{Mpc}^{-1}$

$$A_s = (2.474 \pm 0.116) \times 10^{-9}$$
$$n_s = 0.9645 \pm 0.0062$$
$$r < 0.11$$

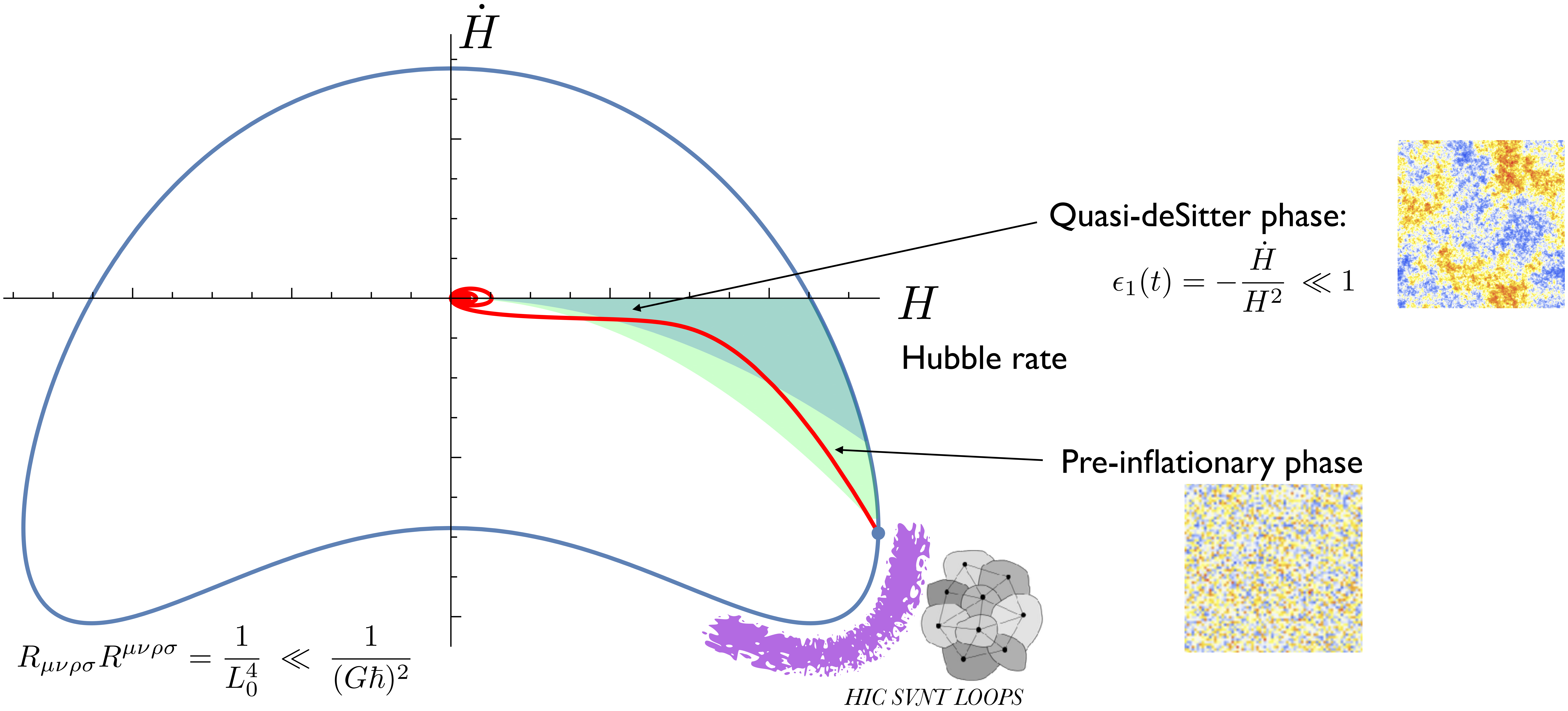
Primordial spectra from adiabatic vacuum in the quasi de-Sitter phase of the $R + \alpha R^2$ model

$$\begin{cases} \alpha \approx 3.54 \times 10^{10}\ G\hbar \\ H_* \approx 1.05 \times 10^{-5} \dfrac{1}{\sqrt{G\hbar}} \end{cases}$$

Background dynamics and pre-inflationary initial conditions



Perturbations and pre-inflationary initial conditions

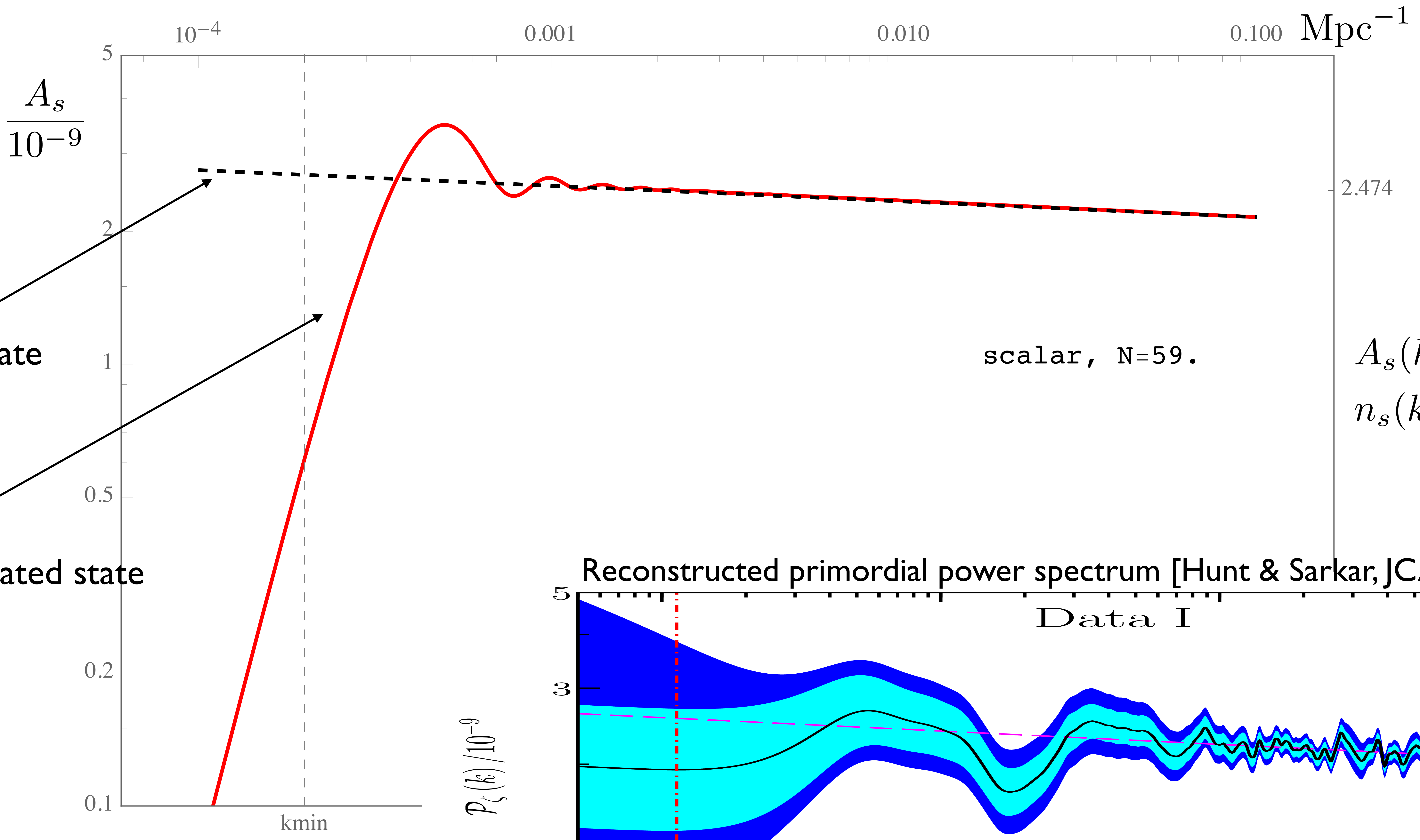


Scalar power spectrum with LQG-to-QFT initial conditions

E.B.-Fernandez 2017

Power suppression at large angular scales

Scalar power spectrum with LQG-to-QFT initial conditions



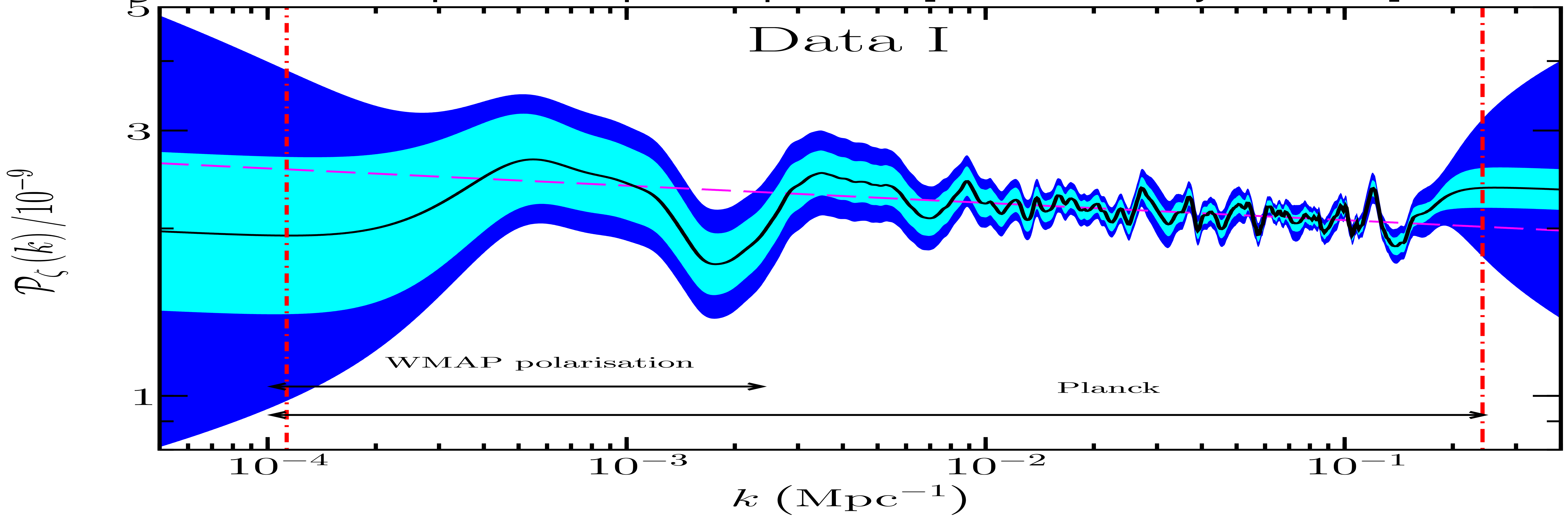
Bunch-Davies state

Initially uncorrelated state

$A_s(k_*) = 2.47 \times 10^{-9}$

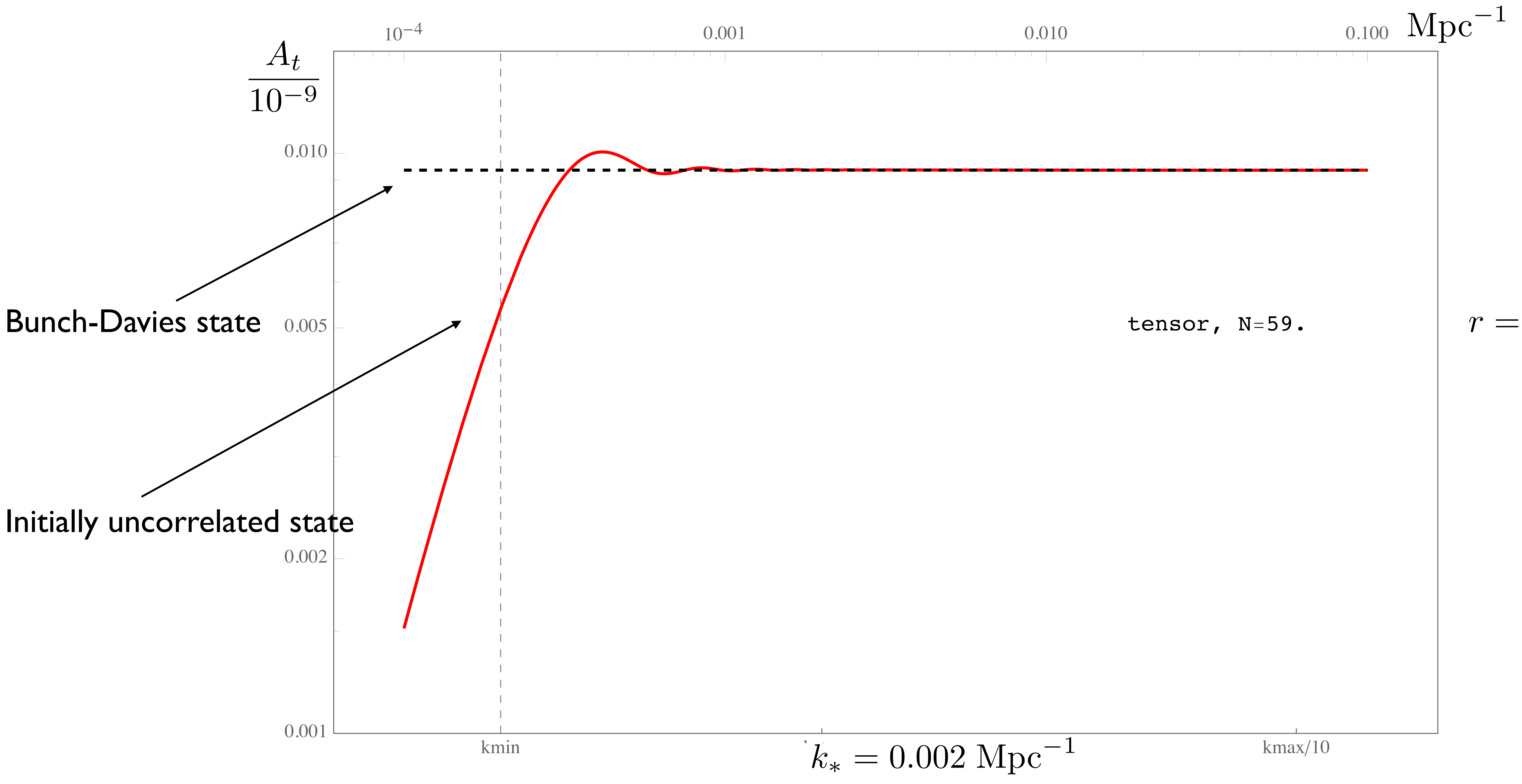
$n_s(k_*) = 0.96$

Reconstructed primordial power spectrum [Hunt & Sarkar, JCAP 2015]



Power suppression at large angular scales

Tensor power spectrum with LQG-to-QFT initial conditions

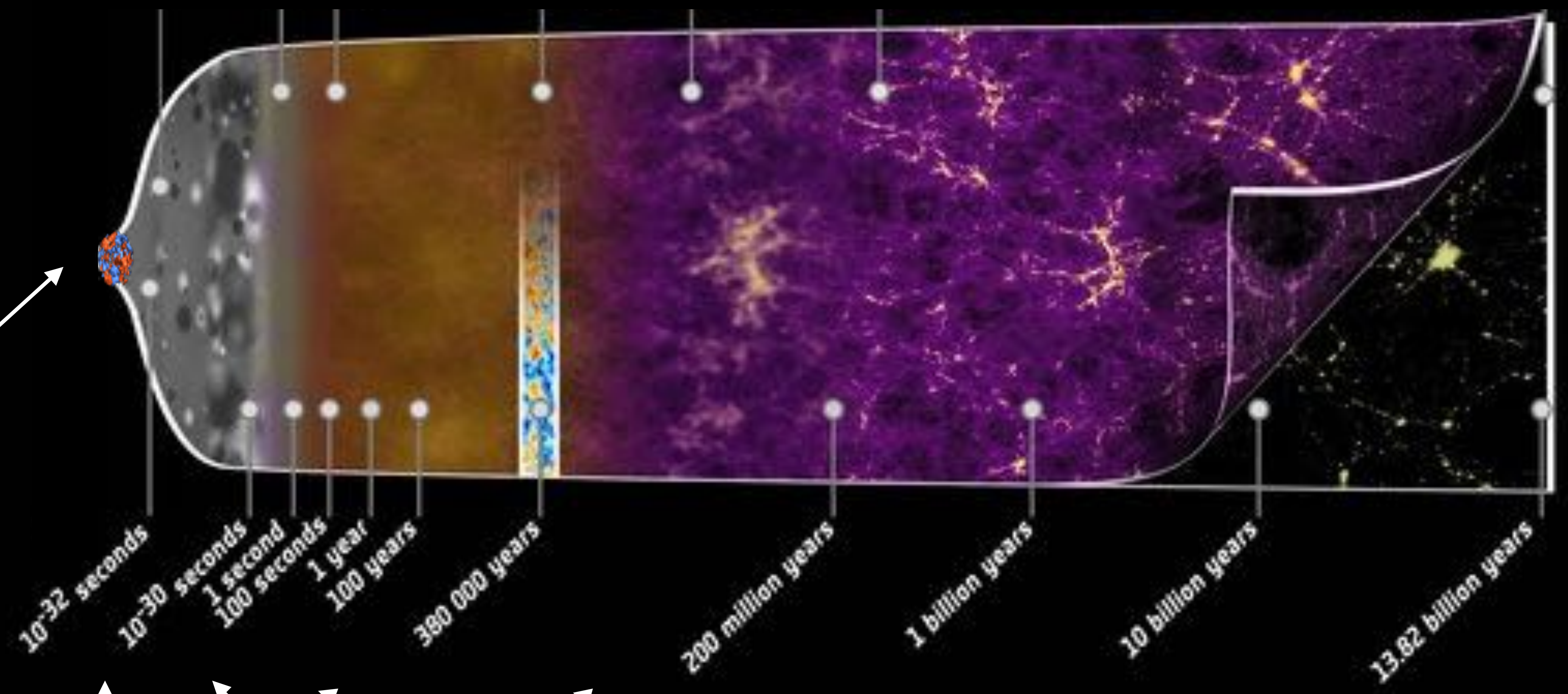


$$r = \frac{A_t(k_*)}{A_s(k_*)} = 0.0038$$

E.B.-Fernandez 2017

Power suppression at large angular scales

Scenario for the emergence of primordial entanglement in loop quantum gravity



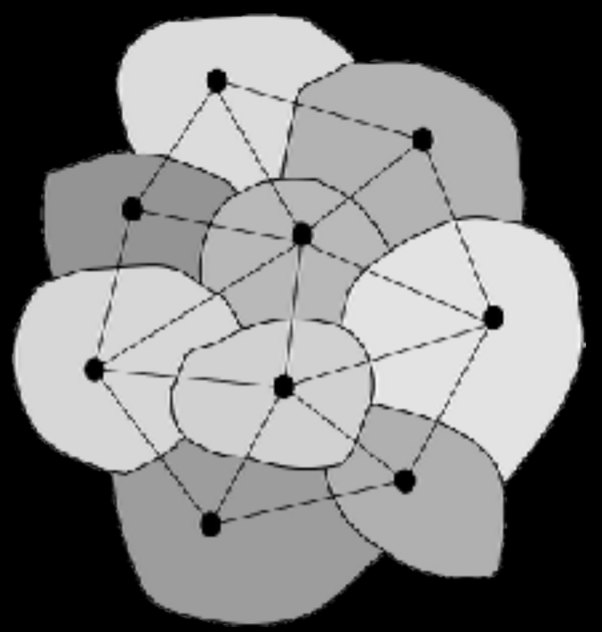
Planck Scale

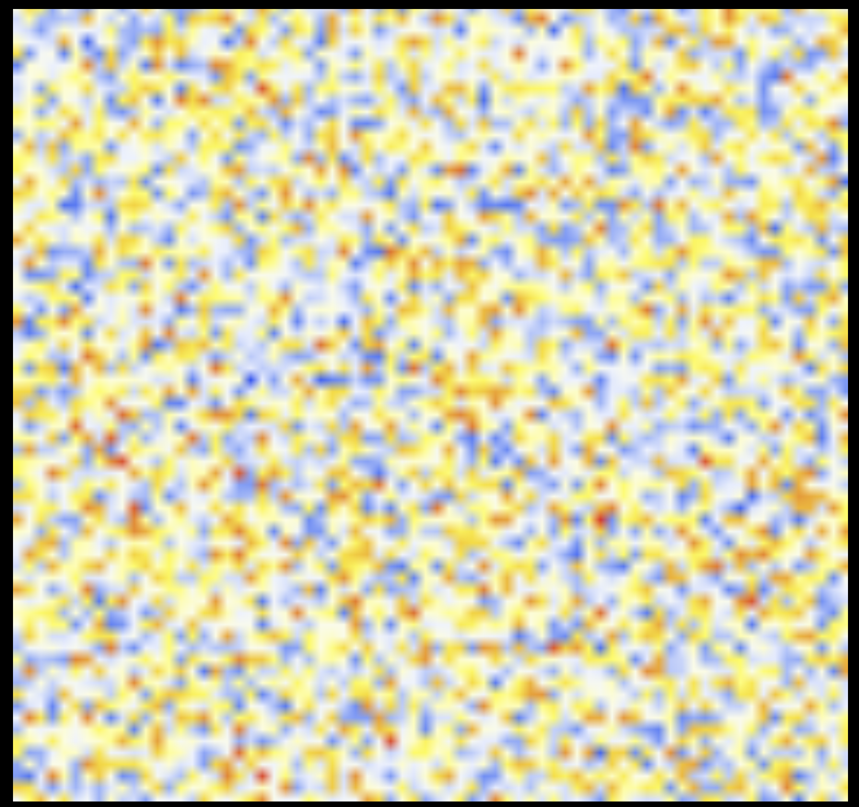
pre-infl. phase

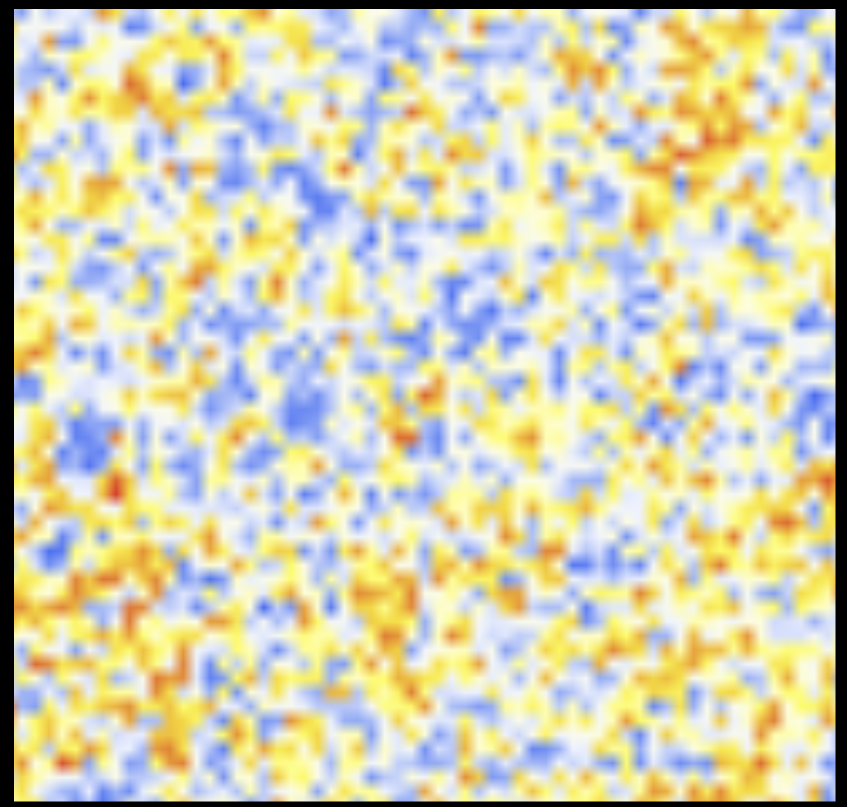
Inflation

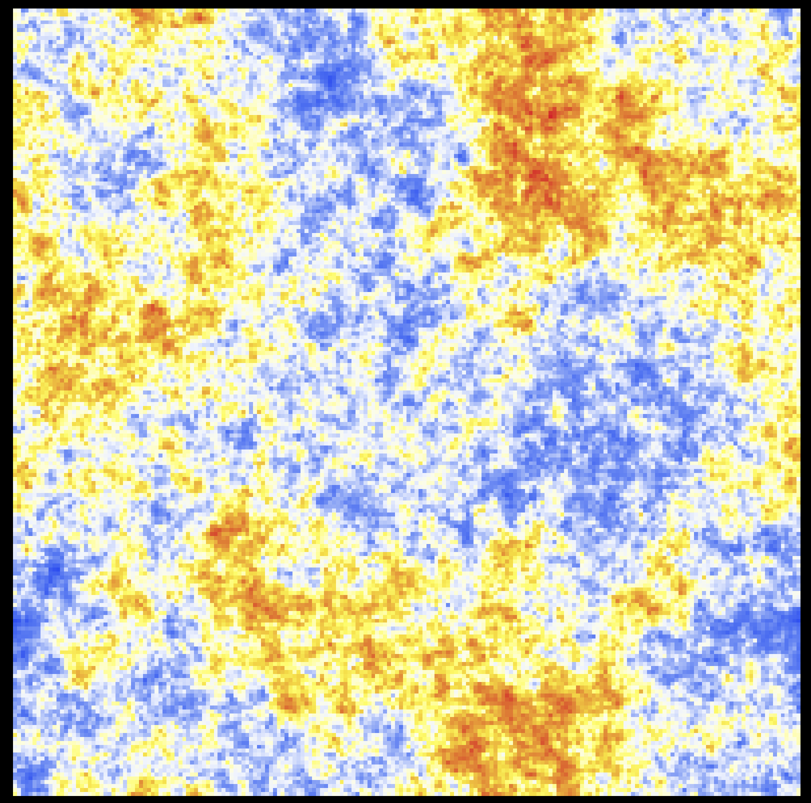
Hot Big Bang

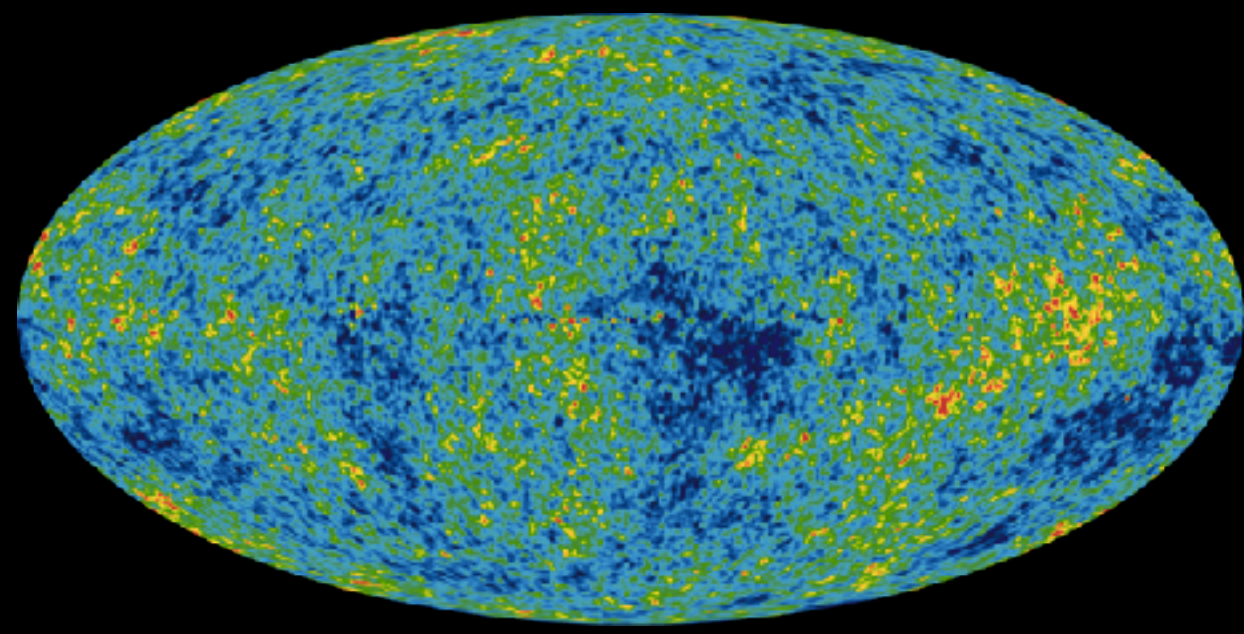
CMB











Inflation and spinfoams

- Effective spinfoam action

$$S[e^I, \omega^{IJ}, r, \lambda^{IJ}] = \int \left((1 + 2\alpha\, r) B_{IJ} \wedge F^{IJ} - \frac{\alpha\, r^2}{1+\gamma^2} \frac{1}{4!} \epsilon_{IJKL} B^{IJ} \wedge B^{KL} + B_{IJ} \wedge \nabla \lambda^{IJ} \right)$$

where $$B_{IJ} = \frac{1}{8\pi G} \left(\frac{1}{2} \epsilon_{IJKL}\, e^K \wedge e^L - \frac{1}{\gamma}\, e_I \wedge e_J \right)$$

γ = Barbero-Immirzi parameter

r = 0-form, effective Ricci scalar at a coarse-graining scale

α = coupling constant
dimensions of *Area*

- It provides an embedding in spinfoams of the Starobinsky model (1979)

$$S[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, \left(R + \alpha\, R^2 \right) \quad \Rightarrow \quad \mathcal{G}_{\mu\nu} + \alpha\, \mathcal{H}_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

Friedman eq: $$H^2 + 6\alpha \left(6H^2 \dot{H} - \dot{H}^2 + 2H\ddot{H} \right) = 0$$

gravity-driven inflation

E.B.-Fernandez-Rincon 2017

Primordial spectra from adiabatic vacuum in the quasi de-Sitter phase

$$36\,\epsilon_1\,\alpha H^2 \;=\; 1$$

- Scalar perturbations

$$A_s \equiv \frac{k_*^3\,P_s(k_*,t_*)}{2\pi^2} \approx \frac{G\hbar\,H_*^2}{2\pi\ \epsilon_{1*}}$$

$$n_s \equiv 1 + k\frac{d}{dk}\log\left(k^3 P_s(k,t_*)\right)\Big|_{k=k_*} \approx 1 - 2\epsilon_{1*} - \epsilon_{2*} \approx 1 - 4\epsilon_{1*}$$

- Tensor perturbations

$$A_t \equiv \frac{k_*^3\,P_t(k_*,t_*)}{2\pi^2} \approx \frac{G\hbar\,H_*^2}{2\pi}\,48\,\epsilon_{1*}$$

$$n_t \equiv k\frac{d}{dk}\log\left(k^3 P_t(k,t_*)\right)\Big|_{k=k_*} \approx -2\epsilon_{1*} + \epsilon_{2*}$$

$$r \equiv \frac{A_t}{A_s} \approx 48\epsilon_{1*}^2$$

$$N_* = \int_{t_*}^{t_{\rm end}} H(t)dt \;=\; 18\,H_*^2\,\alpha - \frac{1}{2}$$

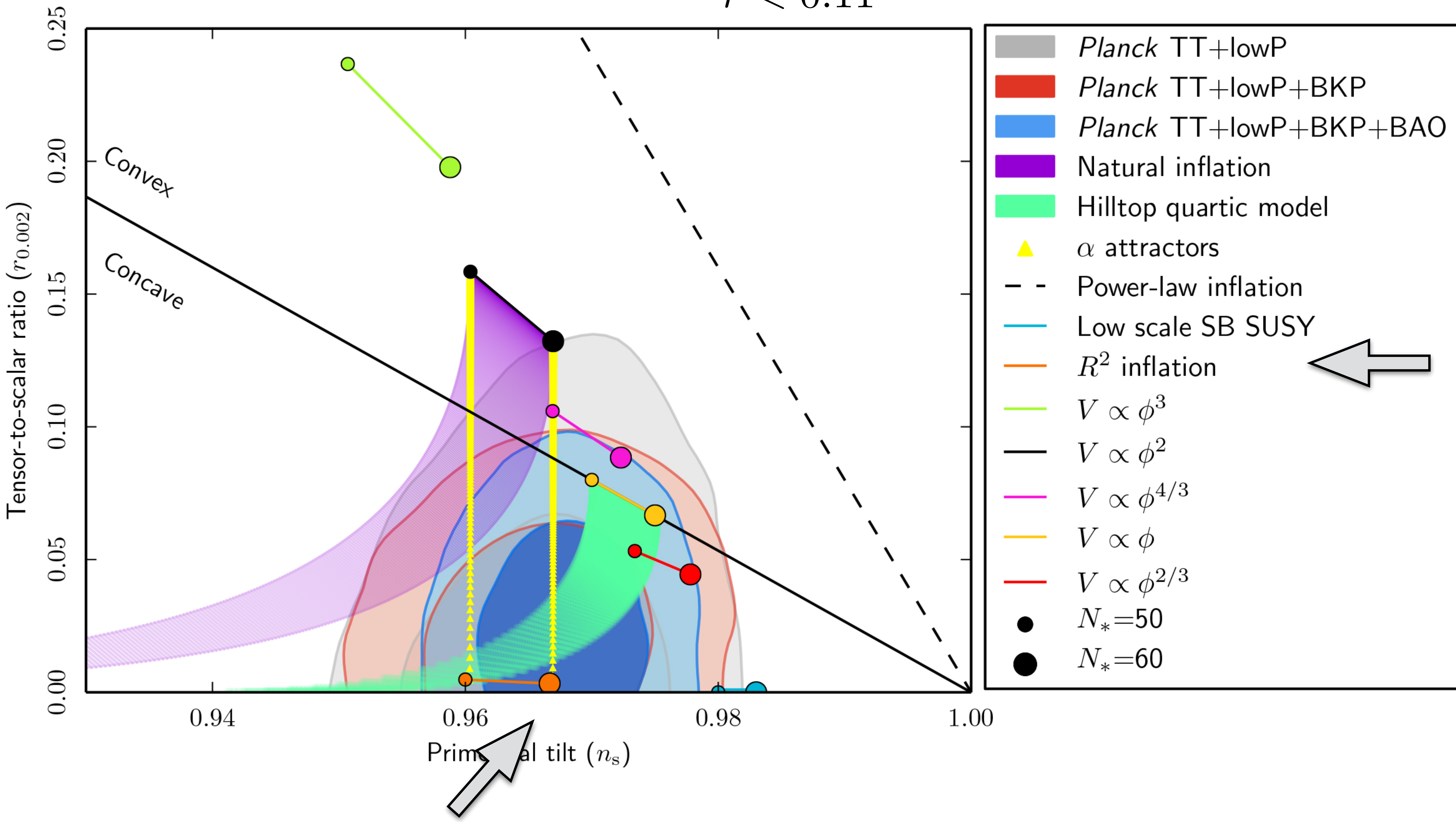
$$\begin{cases} \alpha \approx 3.54 \times 10^{10}\ G\hbar \\ H_* \approx\ 1.05 \times 10^{-5}\ \dfrac{1}{\sqrt{G\hbar}} \\ r \approx 2.4 \times 10^{-3} \\ N_* \approx\ 70 \end{cases}$$

PLANCK 2015 $\quad (k_* = 0.002\ \mathrm{Mpc}^{-1})$

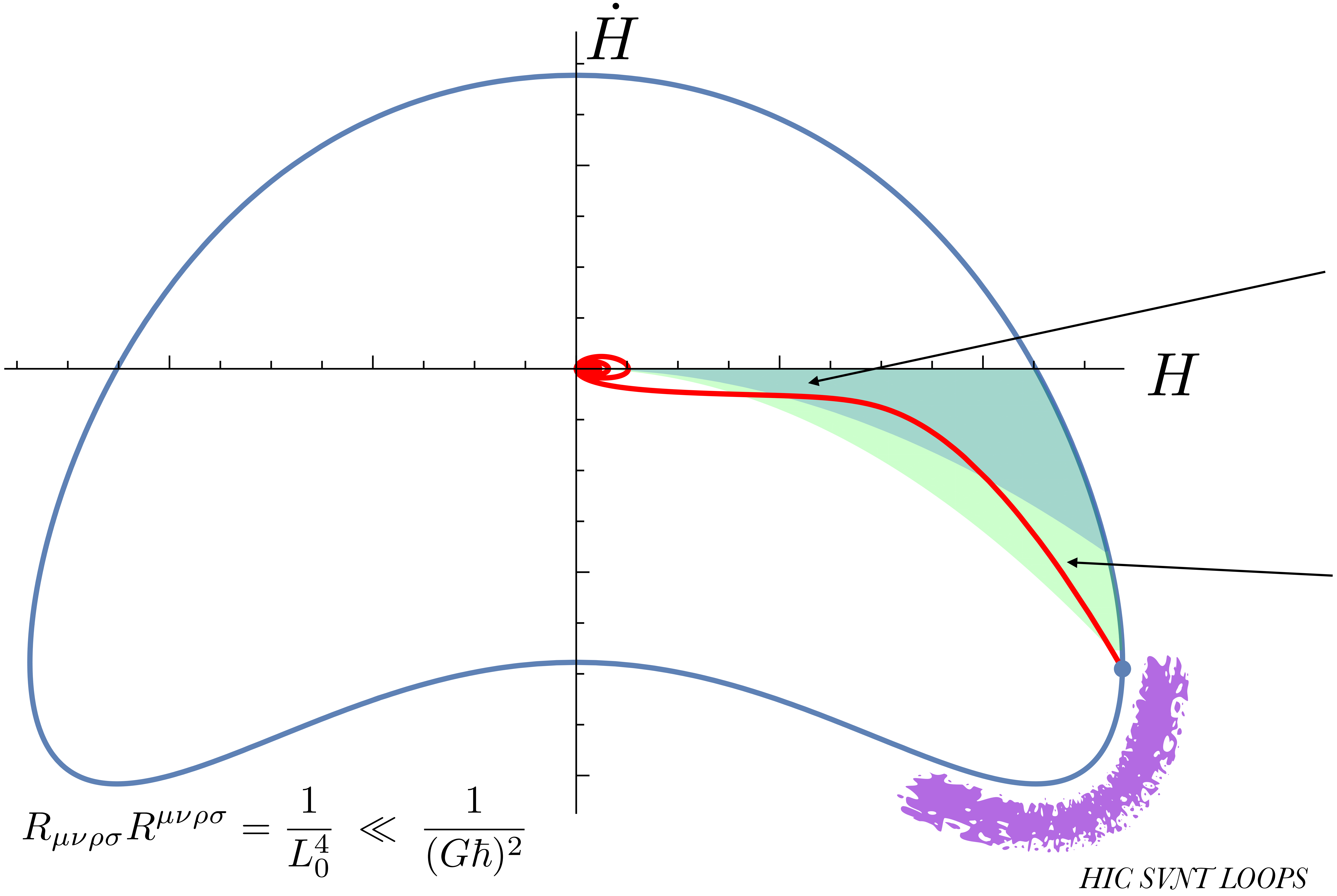
$$A_s = (2.474 \pm 0.116) \times 10^{-9}$$

$$n_s = 0.9645 \pm 0.0062$$

$$r < 0.11$$



Background dynamics and pre-inflationary initial conditions



$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = \frac{1}{L_0^4} \ll \frac{1}{(G\hbar)^2}$$

Friedman eq:

$$H^2 + 6\alpha\left(6H^2\dot{H} - \dot{H}^2 + 2H\ddot{H}\right) = 0$$

Quasi-deSitter phase:

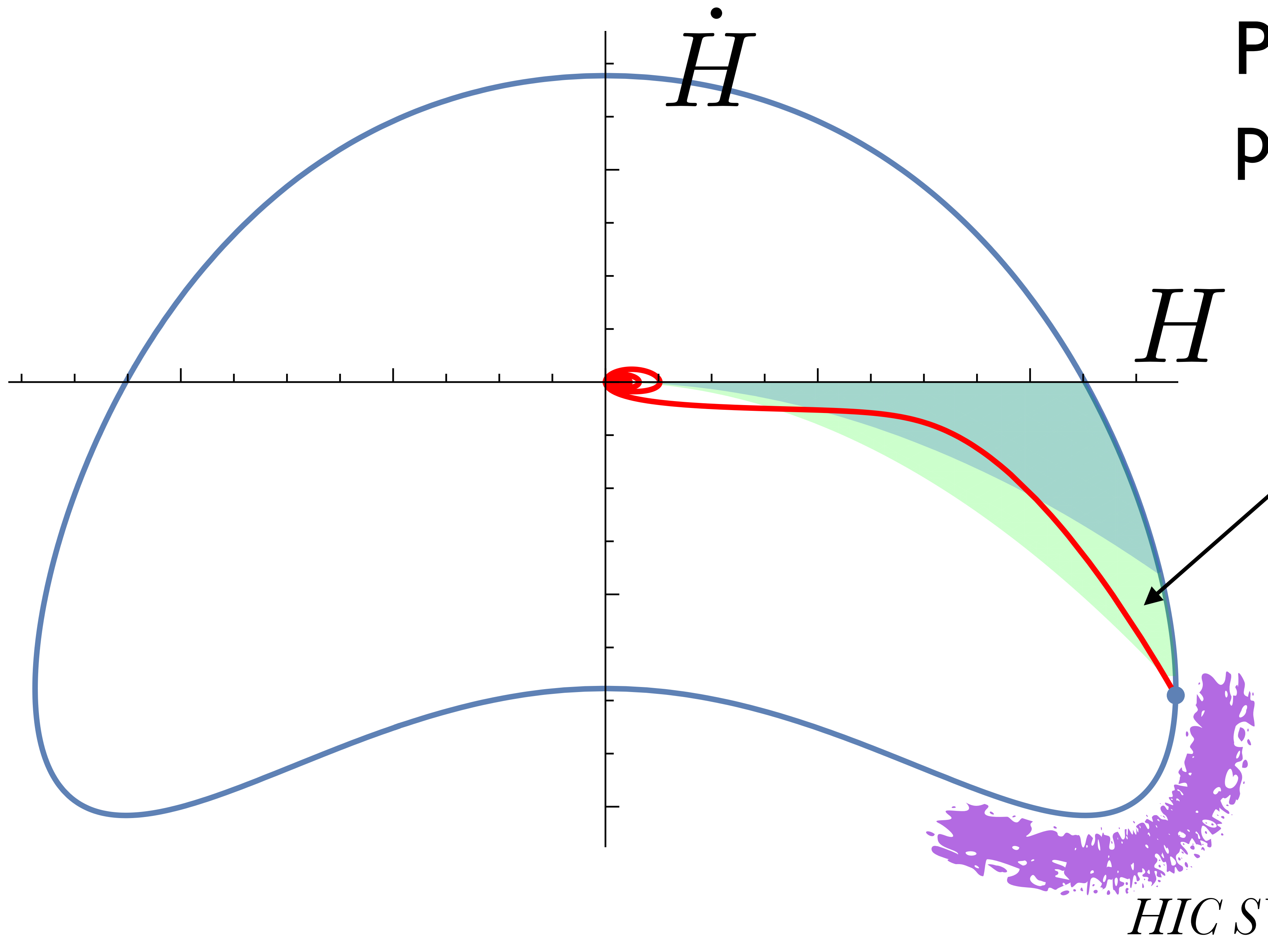
$$\epsilon_1(t) = -\frac{\dot{H}}{H^2} \ll 1$$

$$\dot{H} = -\frac{1}{36\alpha}$$

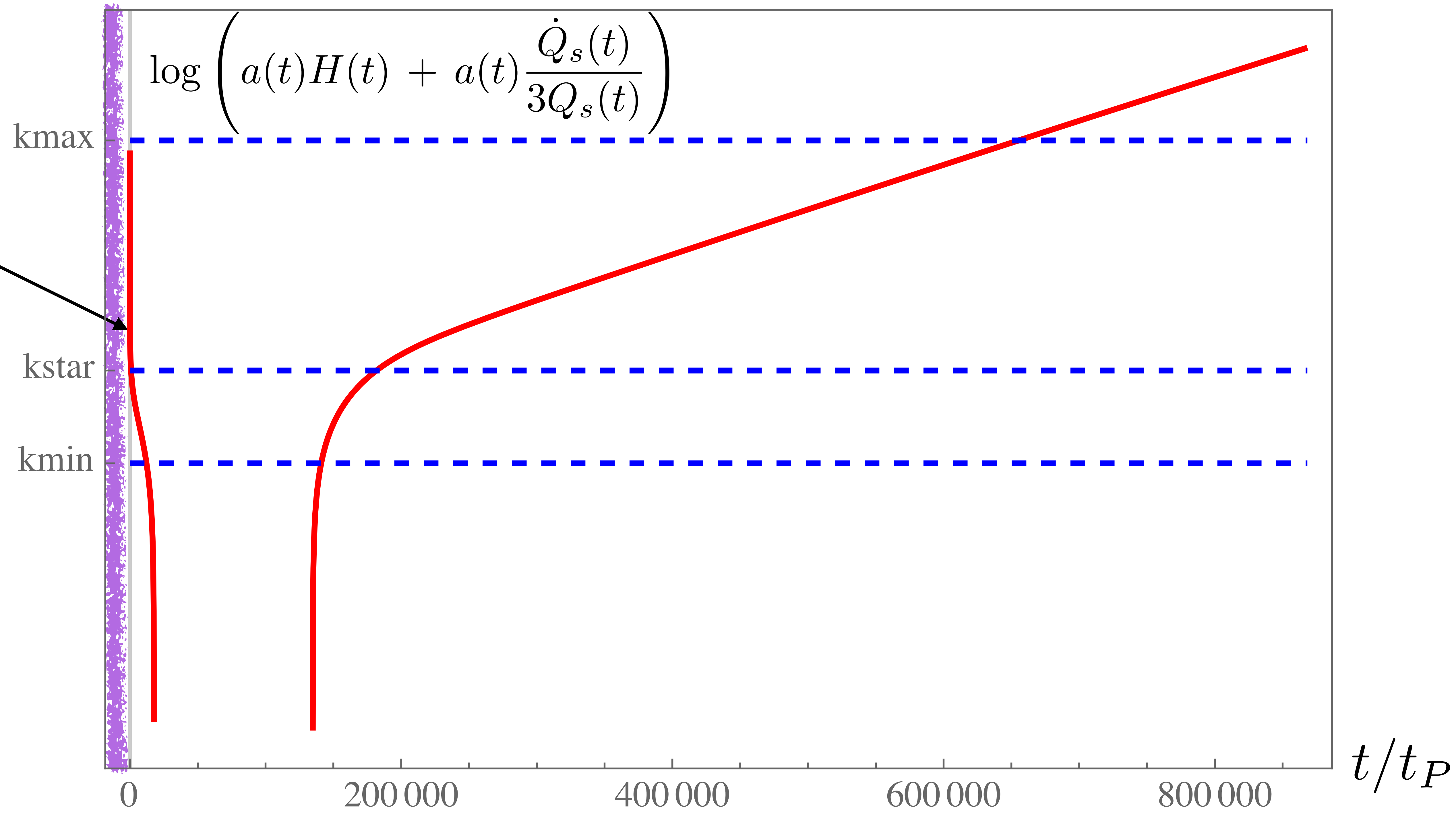
Pre-inflationary phase

$$H(t) \approx \frac{1}{2t}$$

Pre-inflationary initial conditions: scalar and tensor modes



Pre-inflationary phase $H(t) \approx \frac{1}{2t}$



In the pre-inflationary phase
both scalar and tensor perturbations satisfy

$$\ddot{u}(k,t) + \frac{1}{t}\,\dot{u}(k,t) + \frac{k^2}{H_c\,t}\,u(k,t) = 0$$

adiabatic vacuum $\quad u_0(k,t) = \sqrt{\frac{\pi}{2}}\left(J_0\left(2k\sqrt{t/H_c}\right) - i\,Y_0\left(2k\sqrt{t/H_c}\right)\right)$

vanishing correlations in the limit $t \to 0$, Bunch-Davies like correlations produced before the quasi-de Sitter phase

Plan:

I) Entanglement in simple systems

II) Building space from entanglement

→ a) Entanglement, mutual information and bosonic correlators

b) Gluing quantum polyhedra with entanglement

c) Entanglement and Lorentz invariance

III) Entanglement in the sky

Defining entanglement entropy in loop quantum gravity

Entanglement entropy $S_R(|\psi\rangle) = -\mathrm{Tr}(\rho \log \rho)$ [Ohya-Petz book 1993]

characterizes the statistical fluctuations in a sub-algebra of observables $\mathcal{A}_R \subset \mathcal{A}$

Two extreme choices of subalgebra:

a) Determine the algebra of Dirac observables of LQG, then consider a subalgebra ⟵ *difficult to use*

b) Enlarge the Hilbert space of LQG to a bosonic Fock space, then consider a bosonic subalgebra [EB-Hackl-Yokomizo 2015] ⟵ *useful for building space*

Other choices:

- In lattice gauge theory, trivial center sub-algebra [Casini-Huerta-Rosalba 2013]
- Adding d.o.f. (on the boundary), electric center subalgebra [Donnelly 2012] [Donnelly-Freidel 2016] [Anza-Chirco 2016][Han et al 2017] [Chirco-Mele-Oriti-Vitale et al 2017] [Delcalp-Dittrich-Riello 2017]
- Intertwiner subalgebra (at fixed spin) [Livine-Feller 2017]
- …

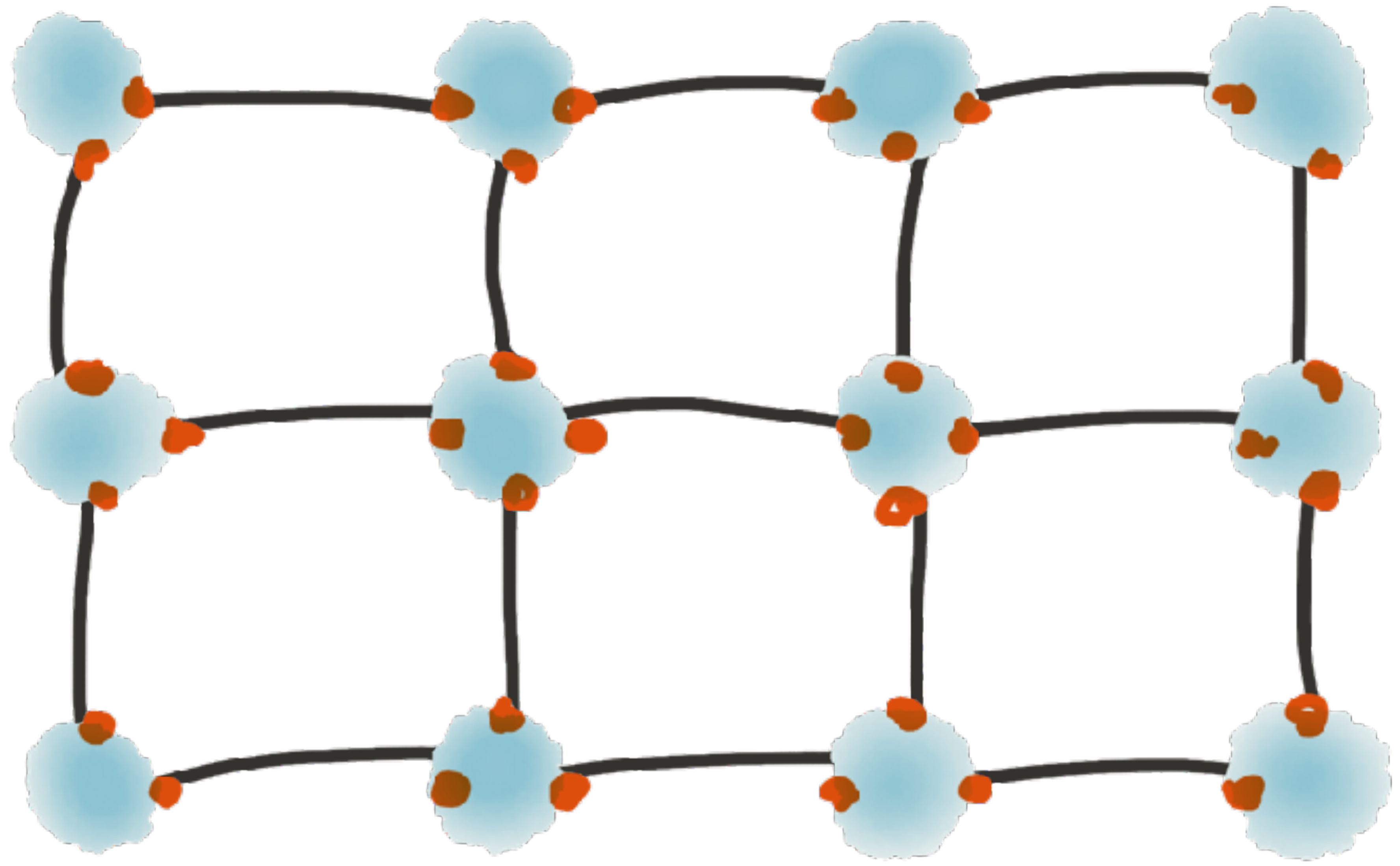
Bosonic formulation of LQG on a graph

[also known as the *twistorial* formulation]

- Two oscillators per end-point of a link

spin from oscillators $$|j,m\rangle = \frac{(a^{0\dagger})^{j+m}}{\sqrt{(j+m)!}}\frac{(a^{1\dagger})^{j-m}}{\sqrt{(j-m)!}}|0\rangle$$

[Schwinger 1952]



- Hilbert space of LQG and the bosonic Hilbert space

$$L^2(SU(2)^L/SU(2)^N) \subset \mathcal{H}_{\text{bosonic}}$$

$$|\psi\rangle = \sum_{n_i=1}^{\infty} c_{n_1\cdots n_{4L}}\,|n_1,\ldots,n_{4L}\rangle$$

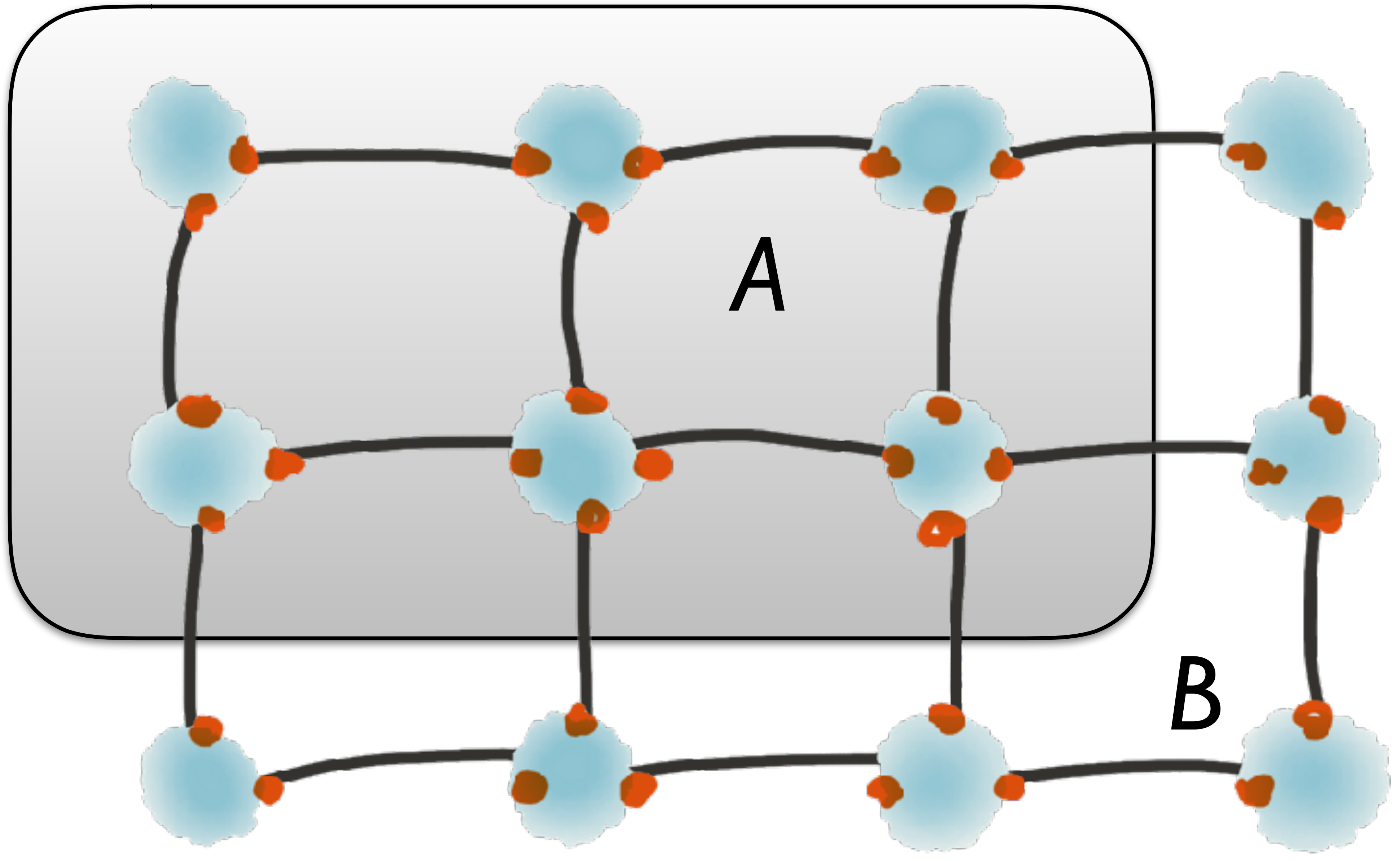
[Girelli-Livine 2005] [Freidel-Speziale 2010]
[Livine-Tambornino 2011] [Wieland 2011]

[EB-Guglielmon-Hackl-Yokomizo 2016]

- The bosonic Hilbert space factorizes over nodes: easy to define and compute the entanglement entropy

- Geometric operators in a region *R* of the graph generate a subalgebra $\mathcal{A}_R^{\text{LQG}} \subset \mathcal{A}_R^{\text{bosonic}}$

Entanglement entropy of a bosonic subalgebra *A*



- Spin-network state $|\Gamma, j_l, i_n\rangle$

  factorized over nodes

  no correlations, zero entanglement entropy in *A*

- Coherent states $P|z\rangle = P\, e^{z^i_A\, a^{A\dagger}_i}|0\rangle$

  not factorized over nodes *only* because of the projector *P*

  exponential fall off of correlations

  area law from Planckian correlations only

- Squeezed states $P|\gamma\rangle = P\, e^{\gamma^{ij}_{AB}\, a^{A\dagger}_i a^{B\dagger}_j}|0\rangle$

  not factorized over nodes because of the projector *P and* because of off-diag. terms in γ^{ij}_{AB}

  long-range correlations from γ^{ij}_{AB}

  efficient parametrization of a corner of the Hilbert space characterized by correlations

  zero-law, area-law, volume-law entanglement entropy depending on γ^{ij}_{AB}

Long-range correlations and the bosonic mutual information

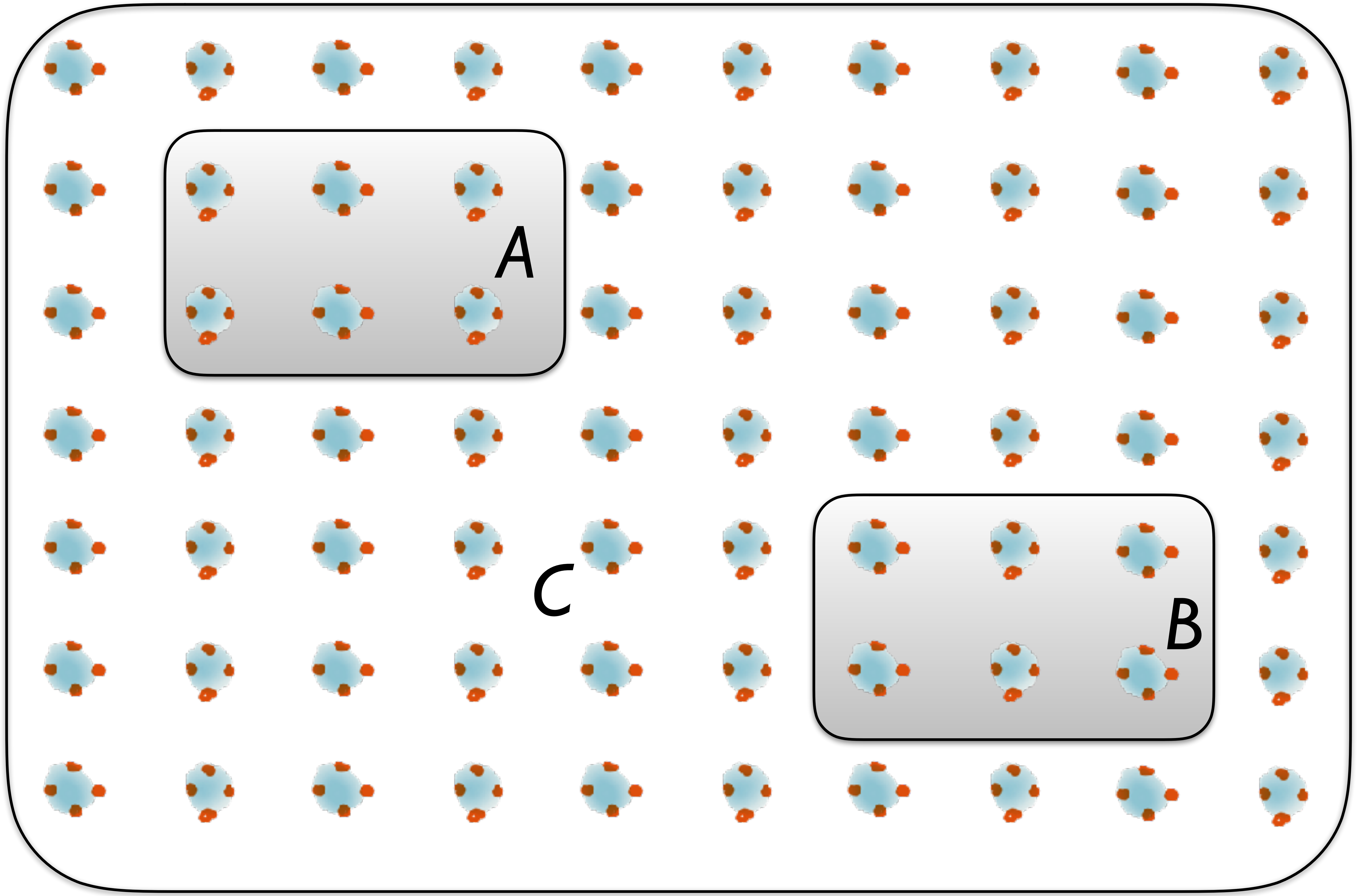
- Macroscopic observables in region *A* and *B*

- Correlations bounded by relative entropy of *A*, *B*

$$\frac{\Big(\langle \mathcal{O}_A\,\mathcal{O}_B\rangle - \langle \mathcal{O}_A\rangle\langle \mathcal{O}_B\rangle\Big)^2}{2\,\|\mathcal{O}_A\|^2\,\|\mathcal{O}_B\|^2} \leq I(A,B)$$

where

$$I(A,B) \equiv S(\rho_{AB}|\rho_A \otimes \rho_B) \;\; = S_A + S_B - S_{AB}$$



The bosonic formulation is useful because it allows us to define and compute the mutual information $I(A,B)$

This quantity bounds from above the correlations of all *LQG*-geometric observables in *A* and *B*

Plan:

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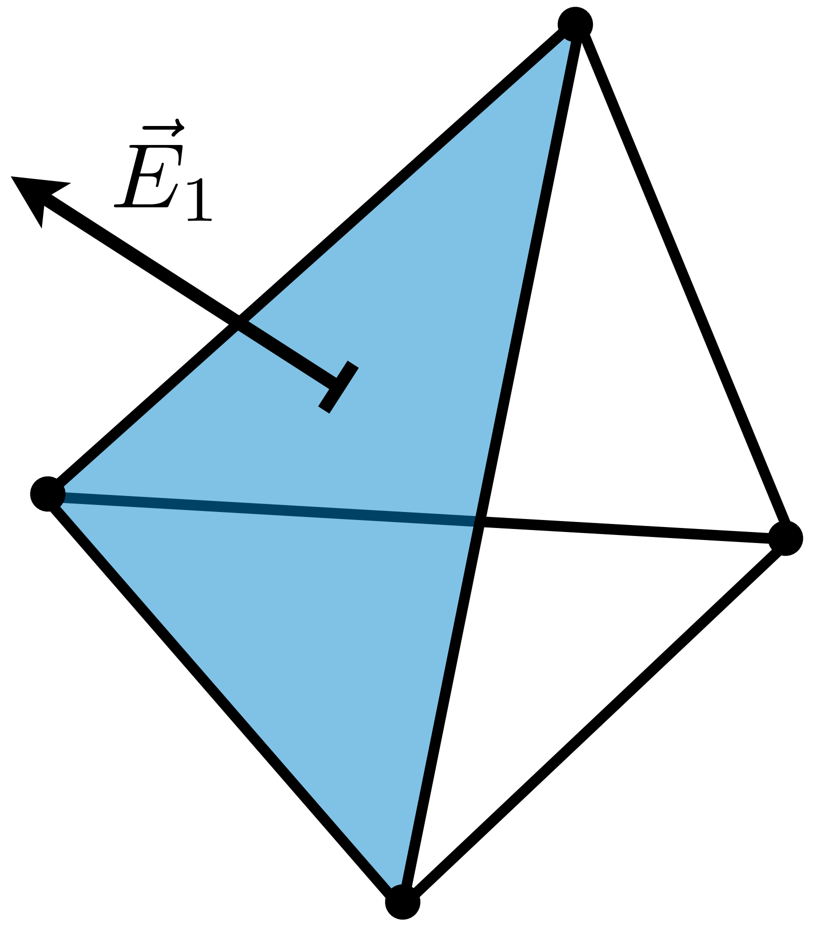
Classical geometry of a tetrahedron in $\mathbb{R}^3$ - area vectors

Area vectors $\vec{E}_a \qquad a = 1, 2, 3, 4$

Closure $\vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4 = 0$



- area of a face $A_a = |\vec{E}_a|$

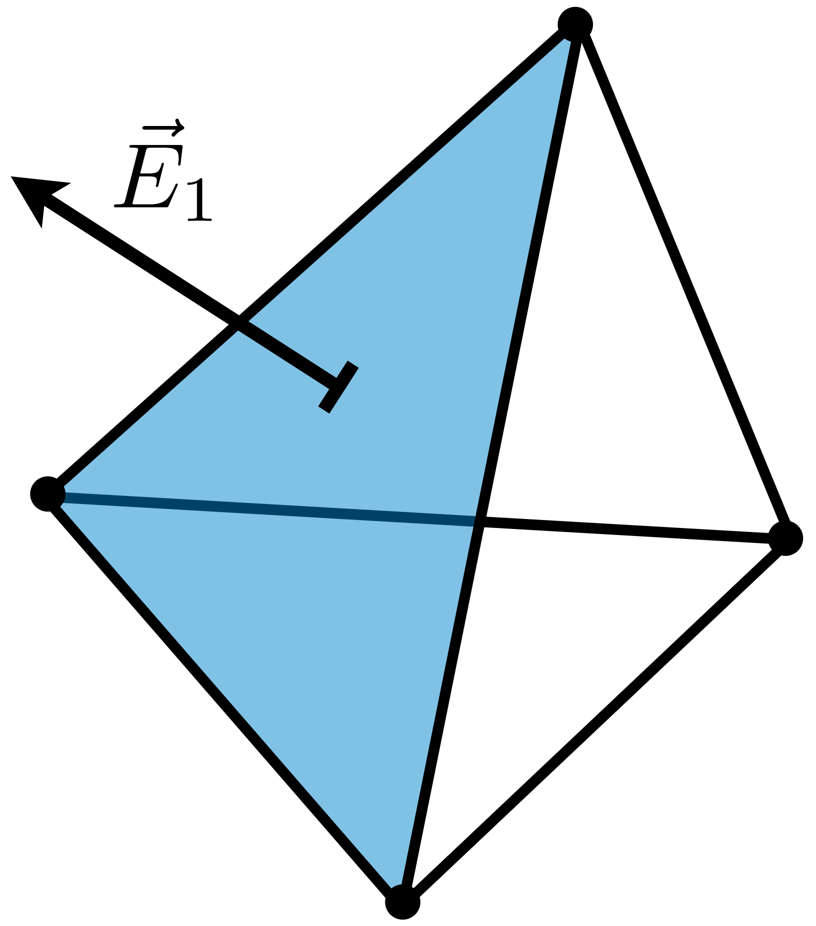
- angle between two faces $\vec{E}_a \cdot \vec{E}_b = A_a A_b \cos\theta_{ab}$

- volume of the tetrahedron

$$V = \frac{\sqrt{2}}{3}\sqrt{|\vec{E}_1 \cdot (\vec{E}_2 \times \vec{E}_3)|}$$

The phase space of a tetrahedron (face-areas A_a fixed)

$$\vec{E}_a = A_a\,\vec{n}_a \qquad\qquad a = 1, 2, 3, 4$$



Function $\quad f:\ S^2 \times S^2 \times S^2 \times S^2 \ \to \mathbb{R}$

Poisson brackets

$$\{f(\vec{E}_a),\, g(\vec{E}_a)\} \ = \sum_{a=1}^{4} \vec{E}_a \cdot \Big(\frac{\partial f}{\partial \vec{E}_a} \times \frac{\partial g}{\partial \vec{E}_a}\Big)$$

Fuctions invariant under rotations

q = angle between $\vec{E}_1 \times \vec{E}_2$ and $\vec{E}_3 \times \vec{E}_4$

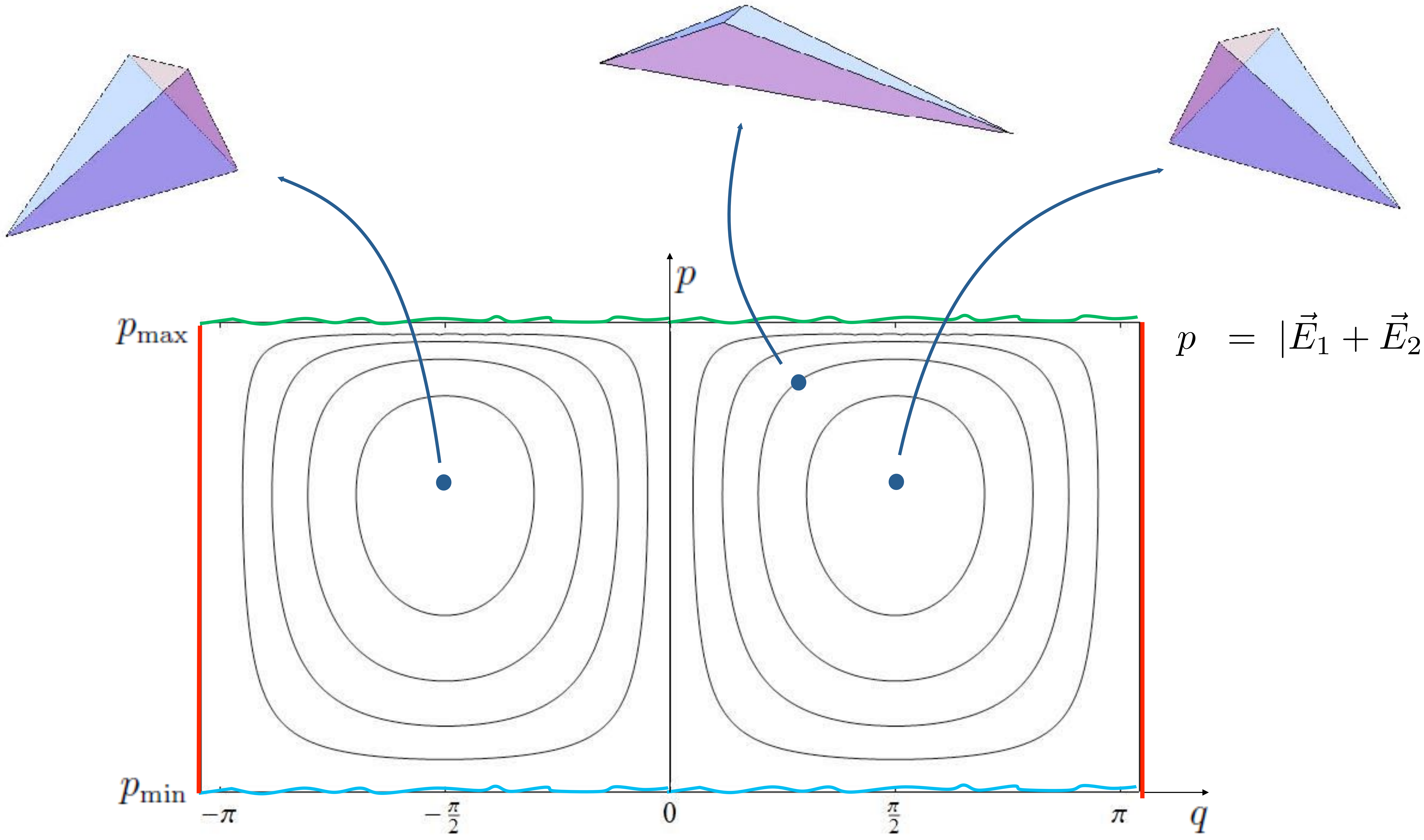
$p = |\vec{E}_1 + \vec{E}_2|$

Canonical variables $\quad \{q, p\} = 1$

Volume as a function of *q* and *p* (equal areas)

$$V = \frac{\sqrt{2}}{3}\sqrt{|\vec{E}_1 \cdot (\vec{E}_2 \times \vec{E}_3)|} \ = \ \frac{1}{3\sqrt{2}}\sqrt{p\,(p^2 - 4A^2)|\sin q|}$$

The phase space of a tetrahedron (face-areas A_a fixed)



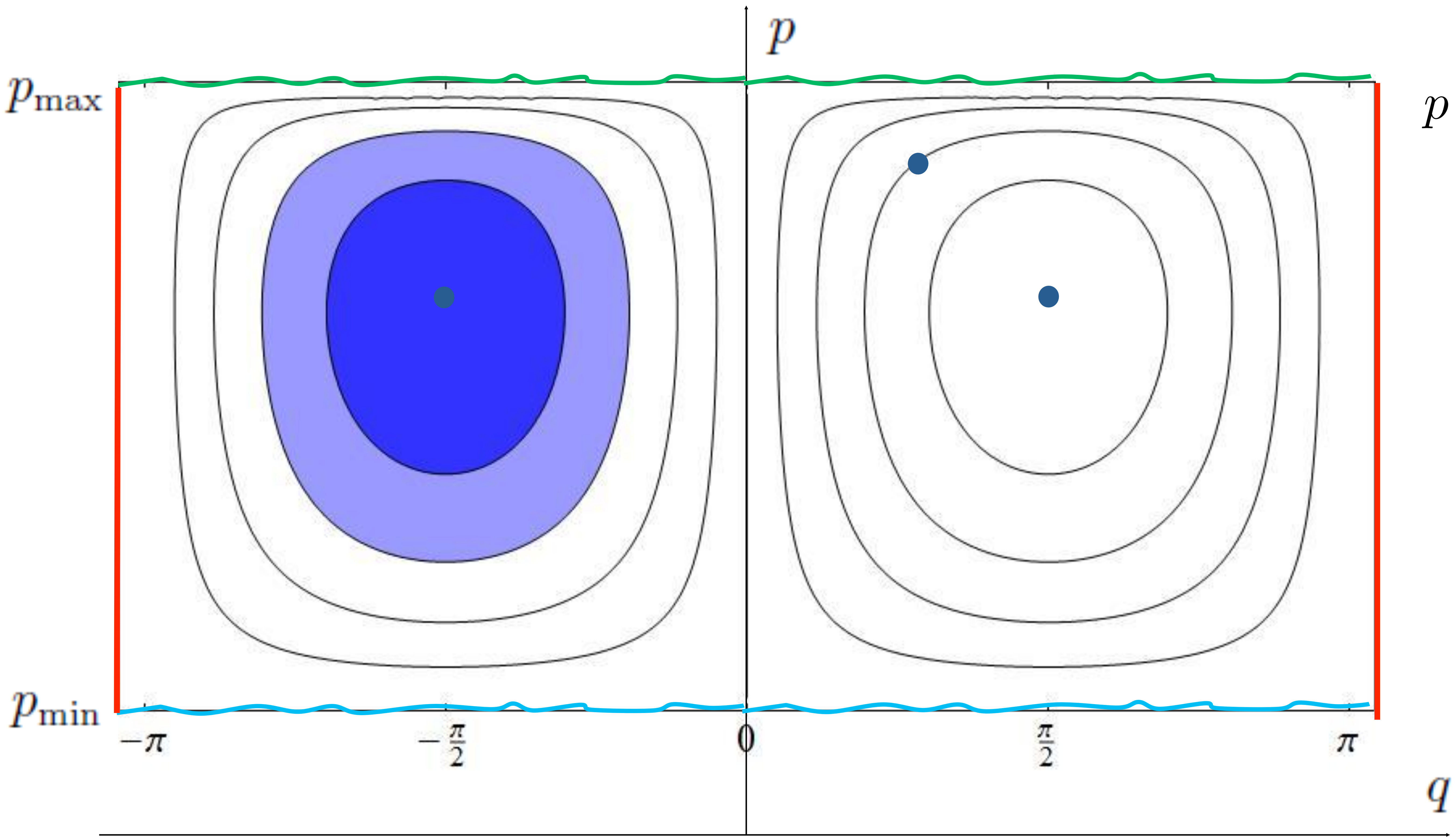
$$p = |\vec{E}_1 + \vec{E}_2|$$

Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

Quantization condition:

orbits of constant volume enclose an integer number
of phase-space cells of area $2\pi\hbar$



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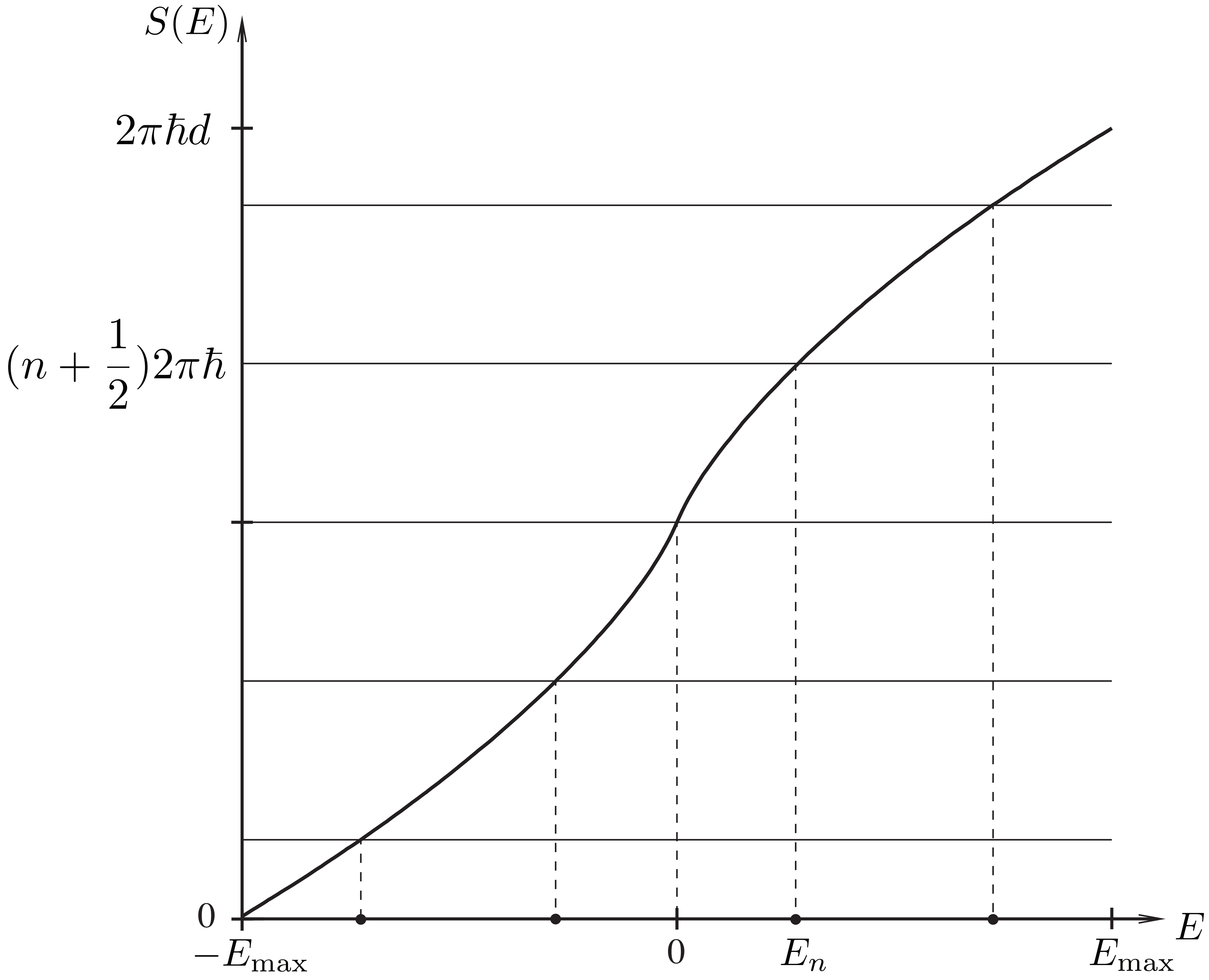


Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

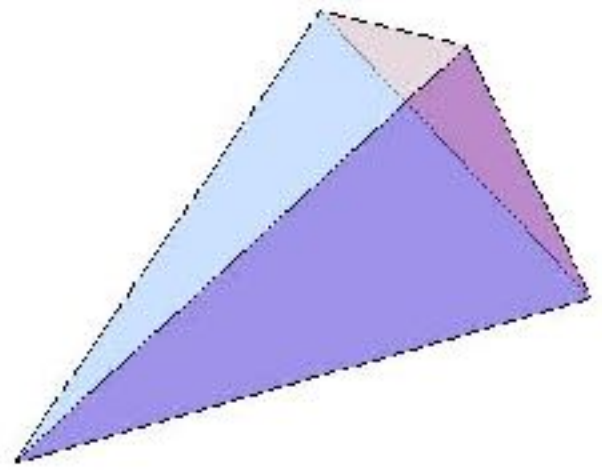
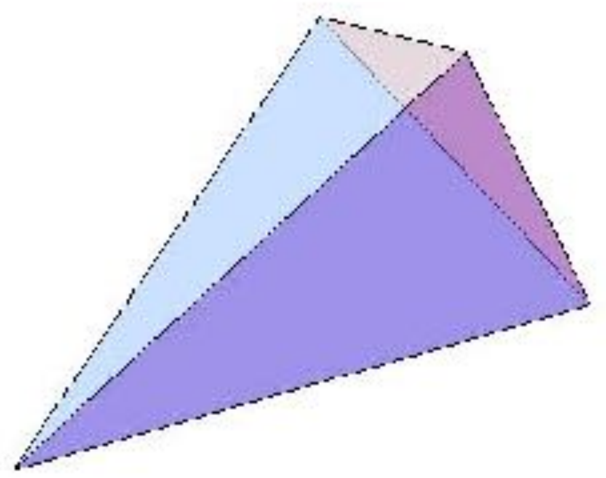


Table: Volume spectrum

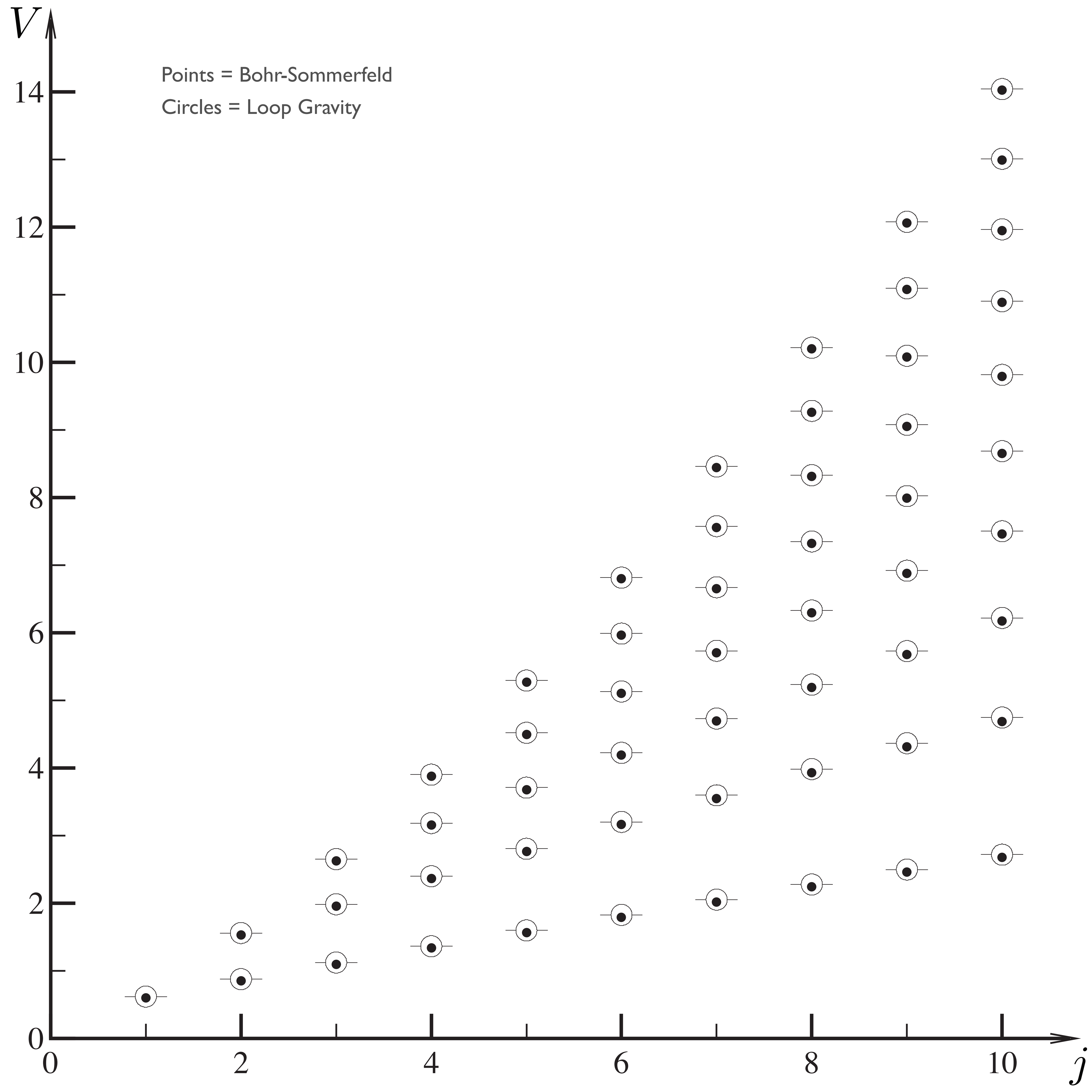
$j_1\ j_2\ j_3\ j_4$	Loop gravity	Bohr-Sommerfeld	Accuracy
$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$	0.310	0.252	19%
$\frac{1}{2}\ \frac{1}{2}\ 1\ 1$	0.396	0.344	13%
$\frac{1}{2}\ \frac{1}{2}\ \frac{3}{2}\ \frac{3}{2}$	0.464	0.406	12%
$\frac{1}{2}\ 1\ 1\ \frac{3}{2}$	0.498	0.458	8%
$1\ 1\ 1\ 1$	0	0	exact
	0.620	0.566	9%
$\frac{1}{2}\ \frac{1}{2}\ 2\ 2$	0.522	0.458	12%
$\frac{1}{2}\ 1\ \frac{3}{2}\ 2$	0.577	0.535	7%
$1\ 1\ 1\ 2$	0.620	0.598	4%
$\frac{1}{2}\ \frac{3}{2}\ \frac{3}{2}\ \frac{3}{2}$	0.620	0.598	4%
$1\ 1\ \frac{3}{2}\ \frac{3}{2}$	0	0	exact
	0.753	0.707	6%
	...		
$6\ 6\ 6\ 7$	1.828	1.795	1.8%
	3.204	3.162	1.3%
	4.225	4.190	0.8%
	5.133	5.105	0.5%
	5.989	5.967	0.4%
	6.817	6.799	0.3%

Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

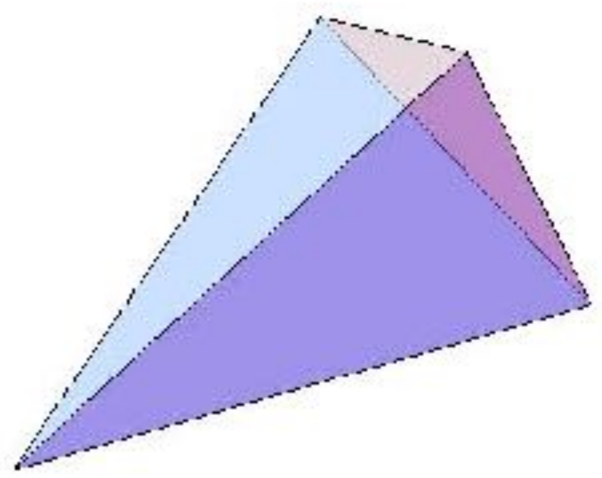


$$A_1 = j + 1/2$$
$$A_2 = j + 1/2$$
$$A_3 = j + 1/2$$
$$A_4 = j + 3/2$$



Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]

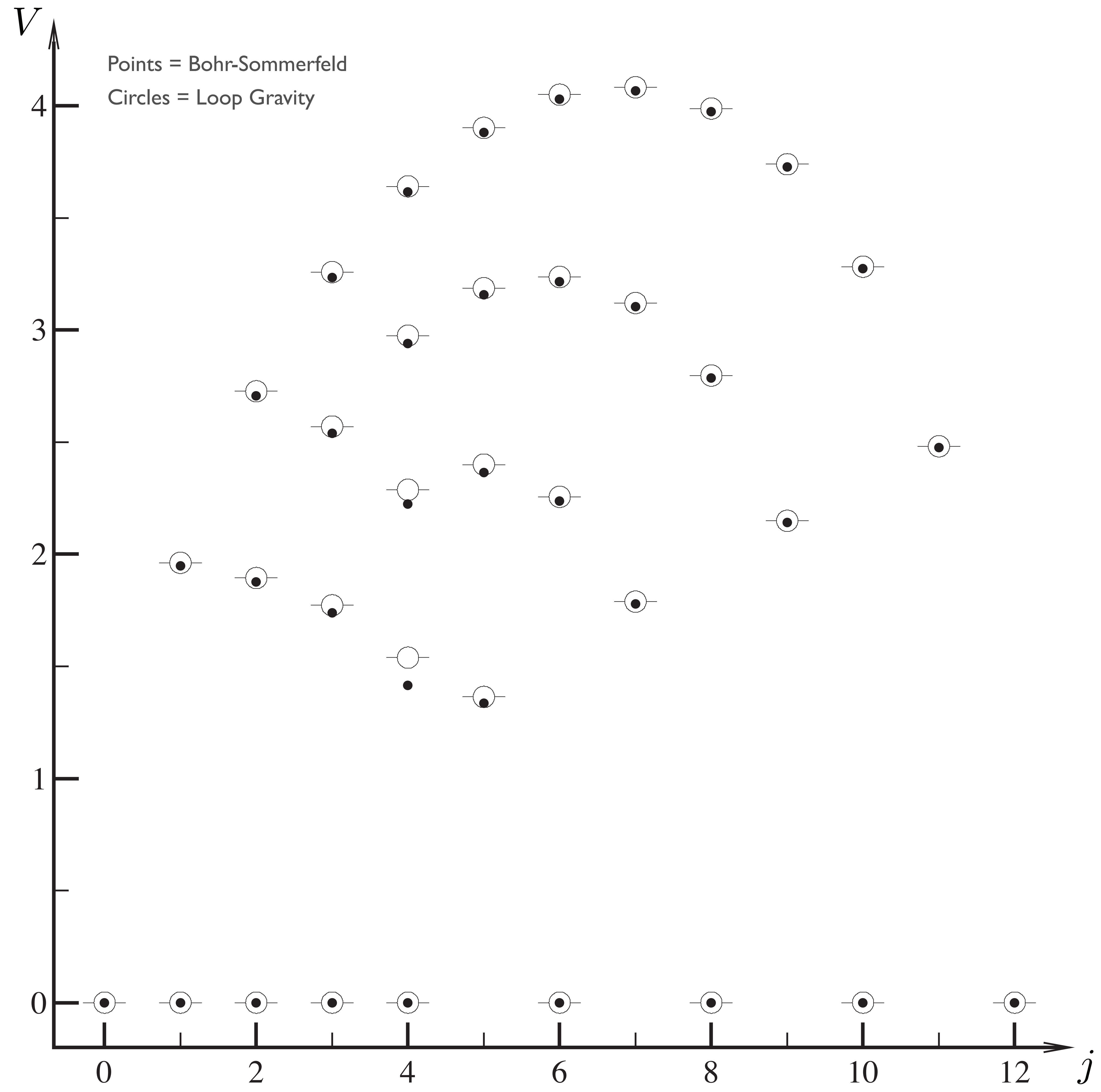


$A_1 = 9/2$

$A_2 = 9/2$

$A_3 = 9/2$

$A_4 = j + 1/2$



Quantum Geometry in intertwiner space

[Barbieri, *NPB* '97]

Spin: irreps of SU(2) $\quad |j, m\rangle \in \mathcal{H}_j$

Intertwiner: invariant tensor $\quad |i\rangle \in \mathrm{Inv}_{SU(2)}\,(\mathcal{H}_{j_1} \otimes \mathcal{H}_{j_2} \otimes \mathcal{H}_{j_3} \otimes \mathcal{H}_{j_4})$

$$|i\rangle = \sum_{m_1 m_2 m_3 m_4} i_{m_1 m_2 m_3 m_4}\, |j_1, m_1\rangle |j_2, m_2\rangle |j_3, m_3\rangle |j_4, m_4\rangle$$

Rovelli-Smolin '95
Ashtekar-Lewandowski '95

Quantum Geometry

- area normals $\quad \vec{E}_a \;=\; 8\pi G\hbar\gamma\, \vec{L}_a \qquad a = 1, 2, 3, 4$
- area operator $\quad A_a \;=\; |\vec{E}_a|$

  spectrum $\quad A_a\, |i\rangle \;=\; 8\pi G\hbar\gamma\, \sqrt{j_a(j_a + 1)}\; |i\rangle$
- angle operator $\quad \vec{E}_a \cdot \vec{E}_b$
  (*Penrose metric*)
- Volume operator $\quad V = \dfrac{\sqrt{2}}{3} \sqrt{|\vec{E}_1 \cdot (\vec{E}_2 \times \vec{E}_3)|}$

Exercise: Volume spectrum in $\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}$

Basis of intertwiner space $\quad |0\rangle, \quad |1\rangle$

Matrix elements of $\quad Q = \vec{L}_1 \cdot (\vec{L}_2 \times \vec{L}_3)$

$$Q_i{}^j = \langle i|\vec{L}_1 \cdot (\vec{L}_2 \times \vec{L}_3)|j\rangle = \begin{pmatrix} 0 & \mathrm{i}\frac{\sqrt{3}}{4} \\ -\mathrm{i}\frac{\sqrt{3}}{4} & 0 \end{pmatrix}$$

Eigenvectors and Eigenvalues

$$Q|q_\pm\rangle = q_\pm|q_\pm\rangle \qquad |q_\pm\rangle = \frac{|0\rangle \pm \mathrm{i}|0\rangle}{\sqrt{2}} \qquad q_\pm = \pm\frac{\sqrt{3}}{4}$$

Volume spectrum

$$V = (8\pi G\hbar\gamma)^{3/2}\,\frac{\sqrt{2}}{3}\sqrt{|Q|}$$

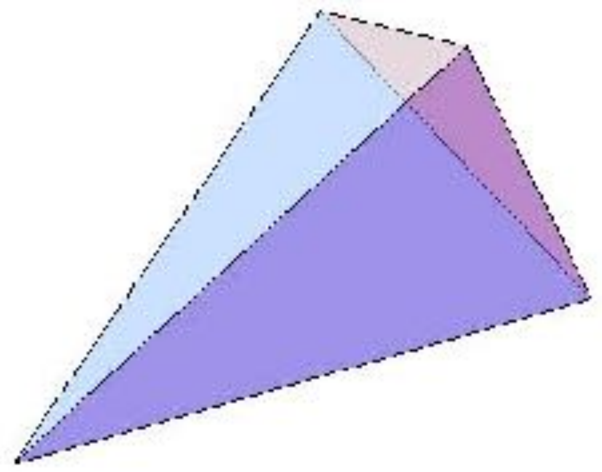
$$V|q_\pm\rangle = v_\pm|q_\pm\rangle$$

$$v_\pm = (8\pi G\hbar\gamma)^{3/2}\,\frac{\sqrt{2}}{3}\sqrt{\frac{\sqrt{3}}{4}}$$

$$\approx (8\pi G\hbar\gamma)^{3/2} \times 0.310$$

Bohr-Sommerfeld quantization of the Volume

[Bianchi-Haggard, *PRL* '11]



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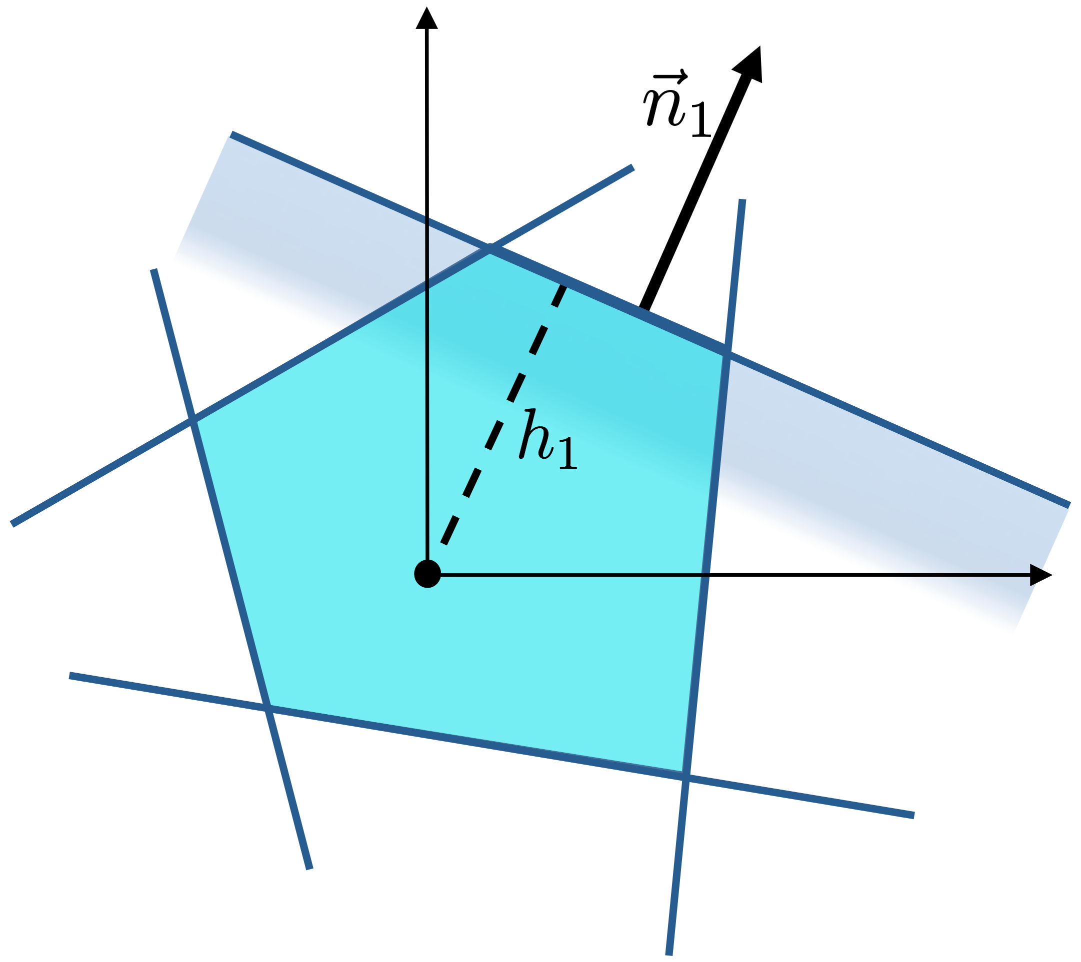
Beyond tetrahedra: number of faces $N > 4$

Bianchi-Dona-Speziale *PRD*'10

- Minkowski theorem [1897]

up to rotations, there is a unique convex polyhedron in 3d Euclidean space having faces with normals $\vec{E}_a = A_a \, \vec{n}_a$



A_a = areas

$\vec{n}_a$ = unit vectors

$$\sum_a A_a \vec{n}_a = 0$$

$$\mathcal{P}_N = \{\vec{E}_a, \, a = 1..N \,|\, \sum_a \vec{E}_a = 0 \,, \, \|\vec{E}_a\| = A_a\}/SO(3)$$

- Kapovich-Millson theorem [1996]

$\mathcal{P}_N$ has naturally the structure of a phase space

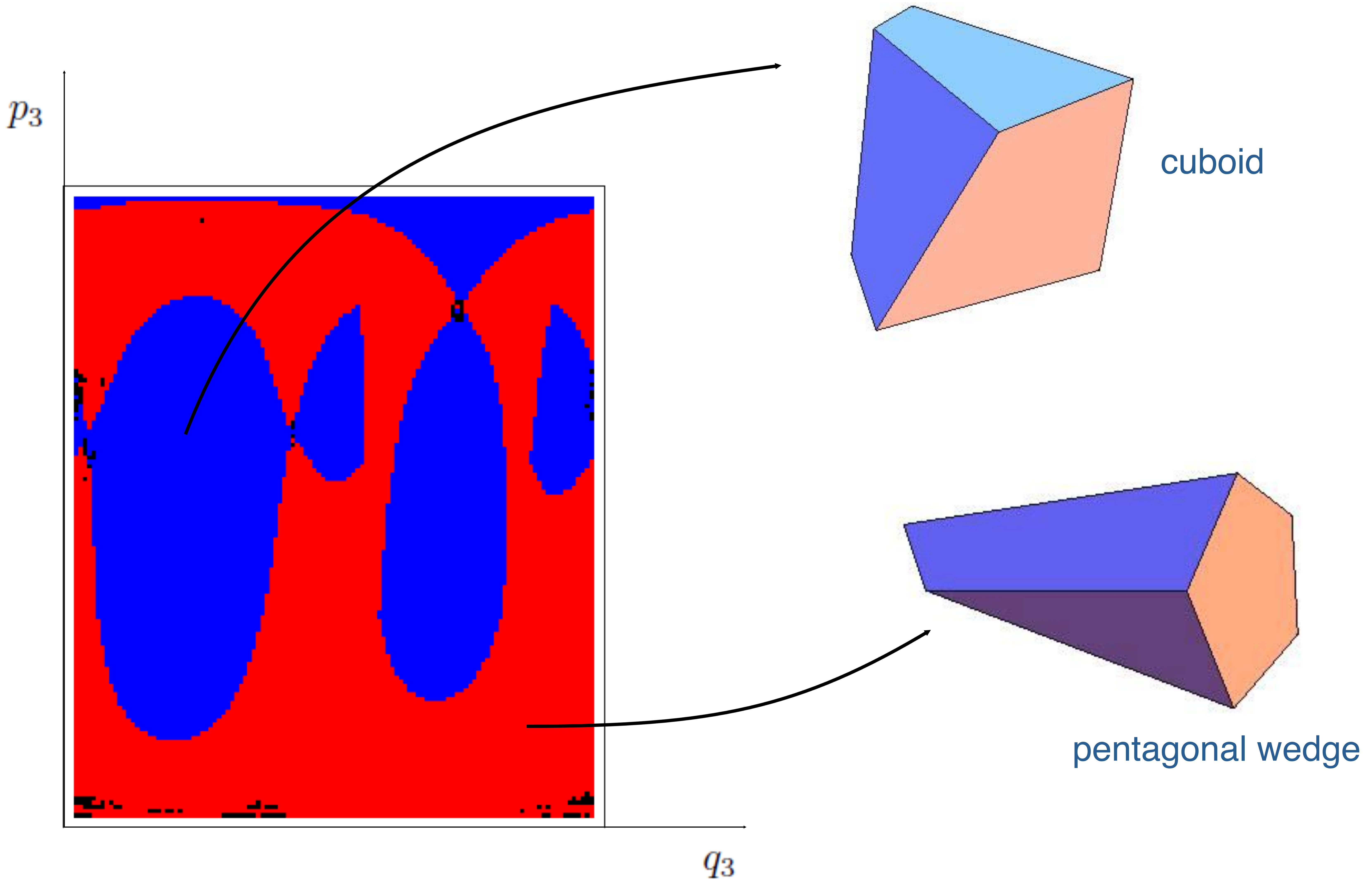
Poisson brackets $$\{f(\vec{E}_a), \, g(\vec{E}_a)\} = \sum_{a=1}^{N} \vec{E}_a \cdot \left(\frac{\partial f}{\partial \vec{E}_a} \times \frac{\partial g}{\partial \vec{E}_a}\right)$$

⇨ **Convex Euclidean polyhedra form a phase space**

Quantization ⇨ Hilbert space of intertwiners = nodes of a spin-network graph

Beyond tetrahedra: $F = 6$, the space of shapes of ...

Bianchi-Dona-Speziale *PRD*'10



Volume spectrum with Quantum Chaos behavior

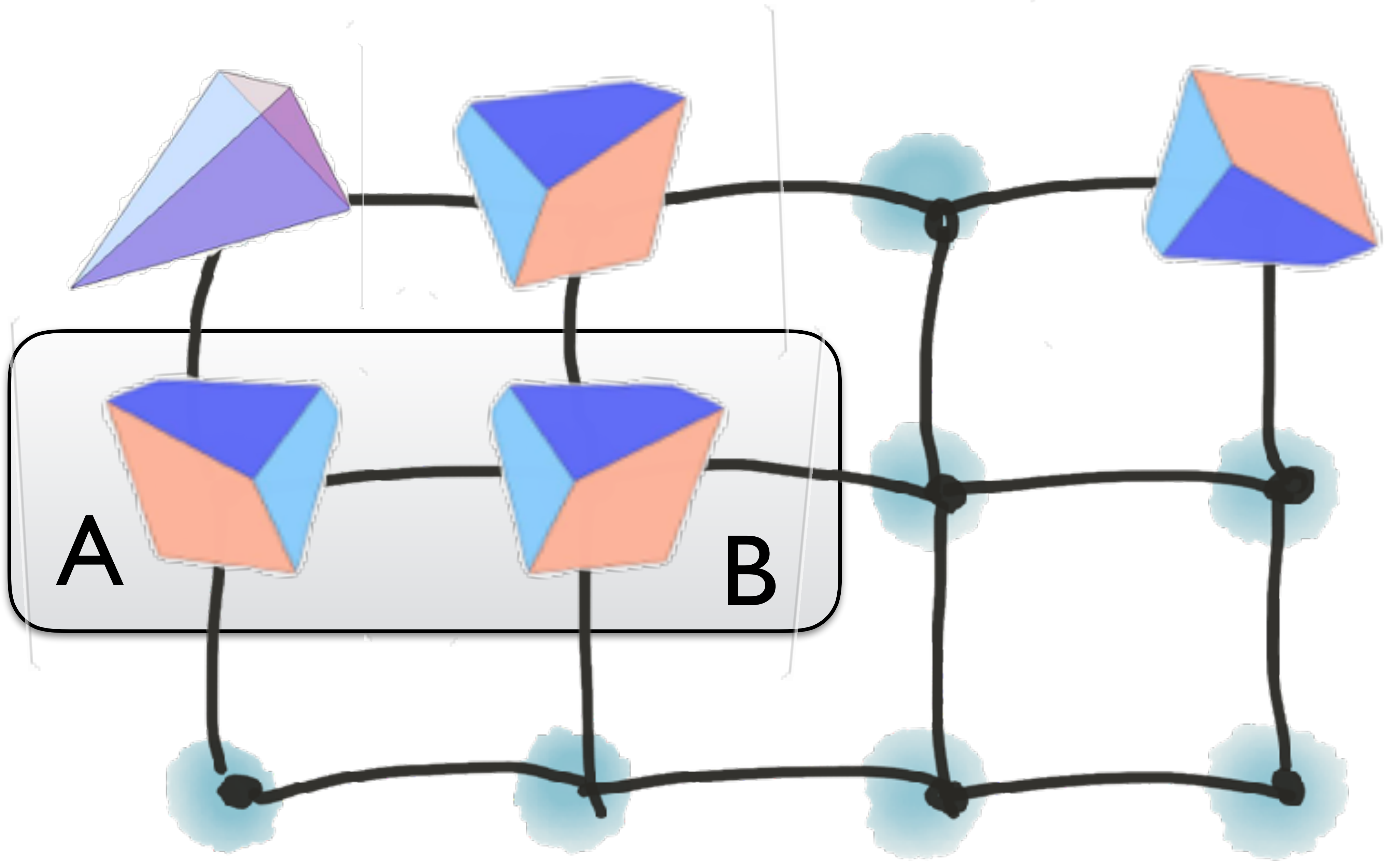
Haggard *PRD*'13

ColemanSmith-Muller *PRD*'13

Gluing quantum polyhedra with entanglement

- Fluctuations of nearby quantum shapes are in general uncorrelated: twisted geometry

[Dittrich-Speziale 2008] [EB 2008]
[Freidel-Speziale 2010]
[EB-Dona-Speziale 2010]
[Dona-Fanizza-Sarno-Speziale 2017]



- Saturating uniformly the short-ranged relative entropy

$$\frac{\Big(\langle \mathcal{O}_A\, \mathcal{O}_B\rangle - \langle \mathcal{O}_A\rangle\langle \mathcal{O}_B\rangle\Big)^2}{2\, \|\mathcal{O}_A\|^2\, \|\mathcal{O}_B\|^2} \leq I(A,B)$$

where

$$I(A,B) \equiv S(\rho_{AB}|\rho_A \otimes \rho_B) \;= S_A + S_B - S_{AB}$$

correlates fluctuations of the quantum geometry

State with $\displaystyle \max \sum_{\langle A,B\rangle} I(A,B)$ Glued geometry from entanglement [EB-Baytas-Yokomizo, to appear]

Plan:

I) Entanglement in simple systems

II) Building space from entanglement

a) Entanglement, mutual information and bosonic correlators

b) Gluing quantum polyhedra with entanglement

→ c) Entanglement and Lorentz invariance

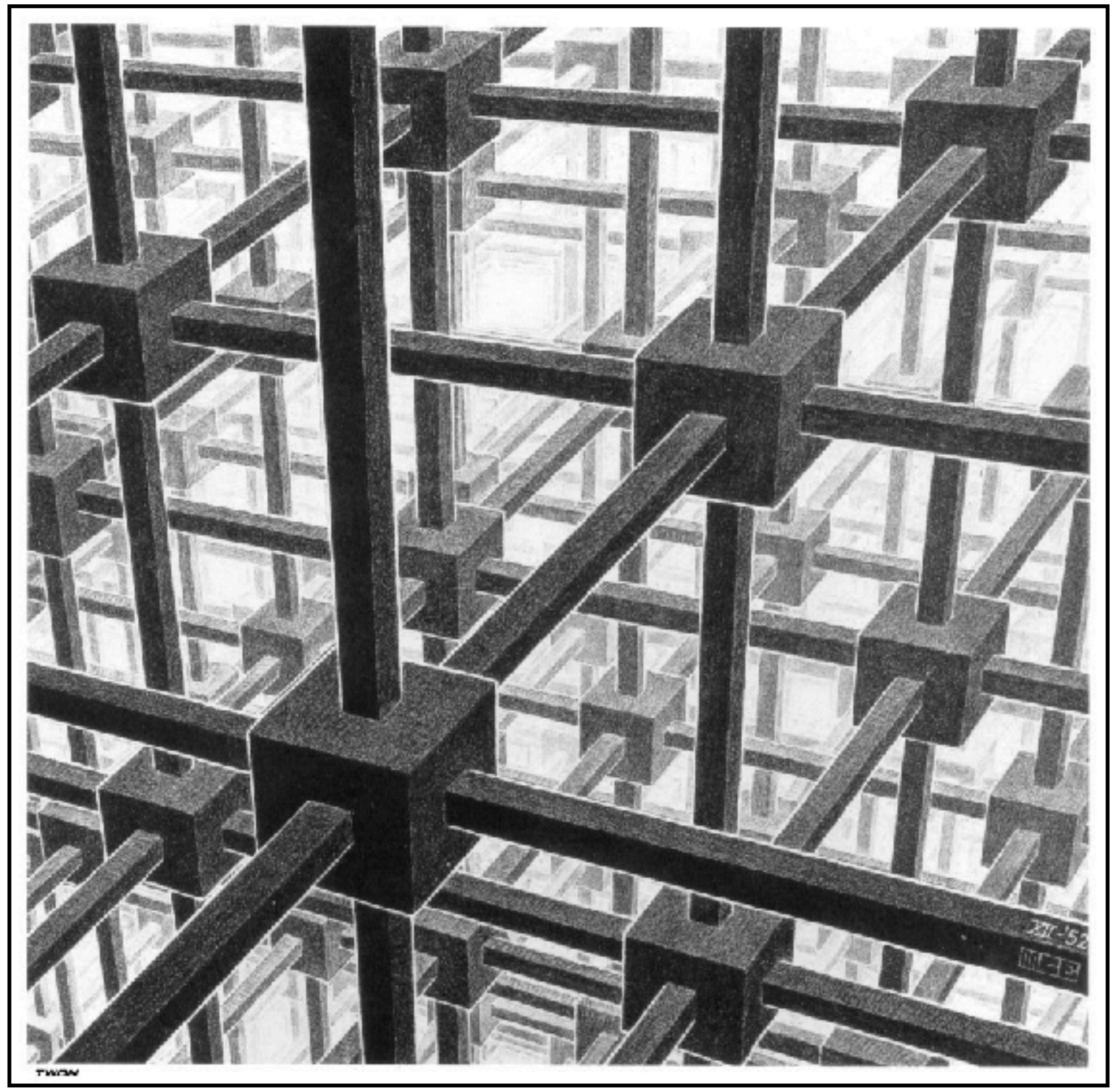
III) Entanglement in the sky

Lorentz invariance in LQG

- Discrete spectra are Lorentz covariant [Rovelli-Speziale 2002]

- Lorentz invariant state in LQG ?
  1) Minkowski geometry as expectation value
  2) Lorentz-invariant 2-point correlation functions, 3-point…

- Homogeneous and isotropic states in LQG ? similarly (1), (2)



Strategy: double-scaling encoded in the state

- use squeezed states defined in terms of 1- and 2-point correlations
- graph, e.g. cubic lattice with *N* nodes
- choose the diagonal entries of the squeezing matrix γ^{ij}_{AB} to fix the expectation value of the spin $\langle j \rangle$
- choose the off-diagonal entries of γ^{ij}_{AB} to fix the correlation function $\mathcal{C} = \langle \mathcal{O}_A\, \mathcal{O}_B \rangle - \langle \mathcal{O}_A \rangle \langle \mathcal{O}_B \rangle$ at a lattice distance n_0
- the correlation function can be expressed in terms of the physical length $\ell \sim n_0 \sqrt{\langle j \rangle}$
- take the limit of the squeezed state $|\gamma\rangle$ such that $\langle j \rangle \to 0$, $n_0 \to \infty$ with $\mathcal{C}(\ell)$ fixed
- the limit can be studied at fixed physical volume $V \sim N\, (\sqrt{\langle j \rangle}\,)^3$, with symmetries imposed on $\mathcal{C}(\ell)$

Toy model: 1d chain of quantum cubes with long-range entanglement and translational invariance [EB-Dona]