Asymptotic safety

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Historical perspective

Old point of view: Only renormalizable theories are consistent and useful. All others should be discarded.
“G ’t Hooft turned the Weinberg-Salam frog into a beautiful prince”

Modern point of view: renormalizability not necessary.
EFT framework dominant in current particle physics.
Even WS theory is seen as EFT.

Proposal: use AS as criterion to select viable theories.
Gain predictivity and range of applicability.

CRITERIA

We judge theories on the basis of their predictivity and range of validity.

In a renormalizable theory only a finite number of parameters have to be determined from experiment, the others can be calculated. Renormalizable theories are highly predictive.

This does not mean that non-renormalizable theories have no predictive power.
EFT based on an expansion in E/M. Predictive for $E \ll M$.

Energy expansion

Typical situation: M mass of unreachable heavy states.
Effective action functionals for low mass fields at energy E contains non-renormalizable terms suppressed by inverse powers of M:

$$S(\phi) = \sum_i g_i \mathcal{O}_i(\phi)$$

If $[\mathcal{O}_i] = -2, 0, 2, 4 \ldots$, then $d_i \equiv [g_i] = 2, 0, -2, -4 \ldots$
Generically $g_i = \tilde{g}_i M^{d_i}$ where $\tilde{g}_i$ are dimensionless.

At a given order there can be more observable quantities than undetermined parameters so the theory is predictive within its low energy domain of validity.

UV completeness

In order to narrow down class of theories demand predictivity at all energies.

Beyond EFT: Asymptotic freedom (e.g. QCD).

Generalizes to asymptotic safety

WILSON'S RG

Rename $M \to k$.

Think of S_k as the result of having integrated out all modes with energy $> k$.
k is the UV cutoff for the EFT describing the low momentum modes.

Dependence of S_k on k is such that the PI on modes from k to 0 is fixed.

S_0 is $-\log Z$.

[Could also use a 1PI effective action.]

Theory space

$$S_k(\phi) = \sum_i g_i(k)\mathcal{O}_i(\phi)$$

Beta functions

$$\beta_i(g_j, k) = k\frac{dg_i}{dk}$$

Dimensionless combinations $\tilde{g}_i = k^{-d_i} g_i$
are coordinates in theory space.

$$\tilde{\beta}_i(\tilde{g}_j) \equiv k\frac{d\tilde{g}_i}{dk} = -d_i\tilde{g}_i + k^{-d_i}\beta_i$$

and from dimensional analysis $\tilde{\beta}_i(\tilde{g}_j)$.
Well-defined flow on theory space.

Theory = RG trajectory

ASYMPTOTIC SAFETY

Qualitative behavior of the flow is determined by fixed points.

Perturbation theory is based on Gaussian fixed point.
In PT all but a finite number of $\tilde{g}_i$ blow up for $k \to \infty$.
Sometimes they even blow up at finite k (Landau poles).
Generically leads to divergences in physical observables.

Perhaps instead they reach another fixed point.

UV safe RG trajectories = *renormalizable* RG trajectories
= RG trajectories that reach a FP in the UV
= UV complete theories

PREDICTIVITY

Define S_{UV} the basin of attraction of the fixed point.

Linearized flow

$$k\frac{dy_i}{dk} = M_{ij}y_j \; ; \qquad M_{ij} = \frac{\partial\tilde{\beta}_i}{\partial\tilde{g}_j} \qquad y_i = \tilde{g}_i - \tilde{g}_{i*}$$

Diagonalize: $z_i = S^{-1}_{ij}y_j$, $S^{-1}MS = \mathrm{diag}(\lambda_1, \lambda_2 \ldots)$

$$k\frac{dz_i}{dk} = \lambda_i z_i$$

- $\lambda_i < 0 \implies z_i$ **relevant**
- $\lambda_i > 0 \implies z_i$ **irrelevant**

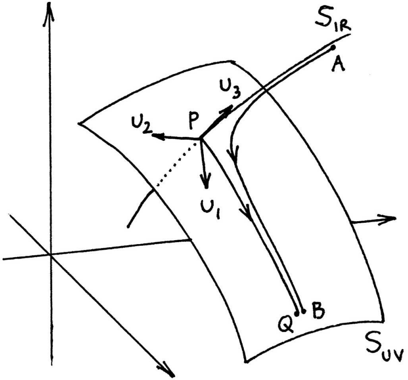
PREDICTIVITY

dim(S_{UV})=# of negative eigenvalues of M.

If the space of renormalizable trajectories is finite dimensional,

AS $\implies$ all irrelevant $z_i = 0$
$\implies$ *at all energy scales* only finitely many parameters are free.

GENERAL PICTURE



Examples

QCD:

- Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- $M_{ij}\Big|_* = -d_i \delta_{ij}$
- relevant couplings=renormalizable couplings

Non-AF examples?

General NLSM

$$\frac{1}{2}Z\int d^dx\,\partial_\mu\varphi^\alpha\partial^\mu\varphi^\beta h_{\alpha\beta}(\varphi)$$

$Z=\frac{1}{g^2}$

$Z\approx \mathrm{mass}^{d-2}$, $g\approx \mathrm{mass}^{\frac{2-d}{2}}$

nonrenormalizable in $d>2$

Ricci flow:

$$\frac{d}{dt}(Zh_{\alpha\beta})=2c_dk^{d-2}R_{\alpha\beta}$$

$c_d=\frac{1}{(4\pi)^{d/2}\Gamma(d/2+1)}$

$O(N)$ NLSM (Zinn-Justin)

In $d = 2$:

$$\beta_{g^2} = -2c_2(N-2)\tilde{g}^4$$

In $d = 2+\epsilon$: $\tilde{g}^2 = k^{d-2}g^2$ and

$$\beta_{g^2} = \epsilon\tilde{g}^2 - 2c_d(N-2)\tilde{g}^4$$

non gaussian FP at

$$\tilde{g}^2_* = \frac{d-2}{2}\frac{1}{c_d(N-2)} \; .$$

$\left.\frac{d\beta}{d\tilde{g}}\right|_* = 2-d.$
Confirmed by Monte-Carlo calculations in $d = 3$.

Other old examples

Gross-Neveu model in $d = 3$

Gauge theories in $d = 4$: one loop

$$L_{YM} = -\frac{1}{4}\mathrm{tr}F_{\mu\nu}F^{\mu\nu}$$
$$L_F = \bar{\psi} i \not{D} \psi$$

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$\beta_g = -B\alpha_g^2$$

$$B = -\frac{4}{3}\epsilon\ ; \qquad \epsilon = \frac{N_F}{N_c} - \frac{11}{2}$$

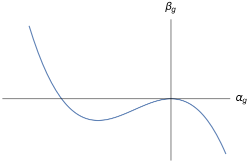
$$N_F < \tfrac{11}{2}N_c \Longrightarrow \epsilon < 0 \Longrightarrow B > 0 \Longrightarrow \mathrm{AF}$$

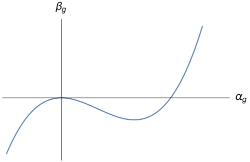
Gauge theories in $d = 4$: two loops

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$$\alpha_{g*} = B/C \; ; \qquad \beta'(\alpha_{g*}) = B^2/C$$

UV FP if $C < 0$, IR FP if $C > 0$.





[W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244]

found C=>0

[T. Banks and A. Zaks, Nucl. Phys. B 196 (1982)]

From AF to AS: Yukawa couplings

$$
\begin{aligned}
L_H &= \mathrm{tr}(\partial^\mu H)^\dagger(\partial_\mu H) \\
L_Y &= y\,\mathrm{tr}(\bar\psi_L H \psi_R + \bar\psi_R H \psi_L)
\end{aligned}
$$

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2}$$

$$
\begin{aligned}
\beta_g &= \alpha_g^2\left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{3} + \epsilon\right)\alpha_y\right] \\
\beta_y &= \alpha_y\left[(13 + 2\epsilon)\alpha_y - 6\alpha_g\right]
\end{aligned}
$$

Fixed points

$$(\alpha_{g*}, \alpha_{y*}) = \left(-\frac{4\epsilon}{75+26\epsilon}, 0\right)$$

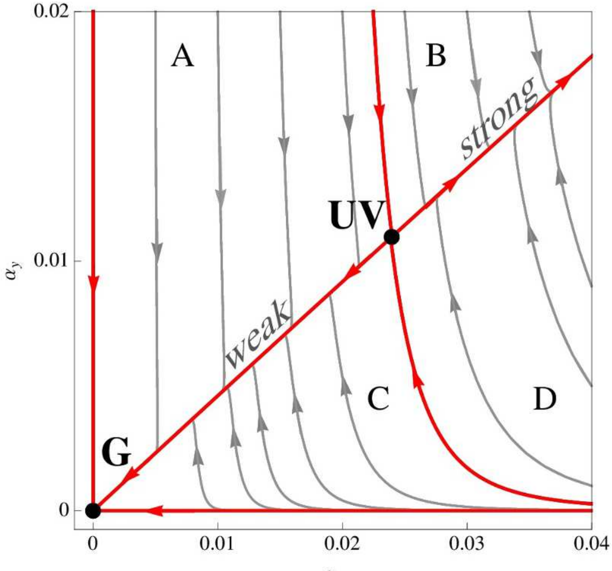
for $\epsilon < 0$, Banks-Zaks

$$\begin{aligned}(\alpha_{g*}, \alpha_{y*}) &= \left(\frac{2\left(13\epsilon+2\epsilon^2\right)}{57-46\epsilon-8\epsilon^2}, \frac{12\epsilon}{57-46\epsilon-8\epsilon^2}\right) \\ &\approx (0.456\epsilon + O(\epsilon^2), 0.211\epsilon + O(\epsilon^2))\end{aligned}$$

for $\epsilon > 0$, Litim and Sannino

[D.F. Litim and F. Sannino, JHEP 1412 (2014) 178]

Phase diagram



WITH SCALAR INTERACTIONS

$$V = -u\,\mathrm{tr}((H^\dagger H)^2) - u\,(\mathrm{tr}(H^\dagger H))^2$$

Fixed point persists

Applications to BSM physics

[A. Bond, G. Hiller, K. Kowalska, D. Litim, Directions for model building from asymptotic safety. arXiv:1702.01727 [hep-ph]]
[R.B. Mann, J.R. Meffe, F. Sannino, T.G. Steele, Z.W. Wang and C. Zhang, Asymptotically safe Standard Model via vector-like fermions arXiv:1707.02942 [hep-th]]

THE GOSPEL ACCORDING TO DE WITT

Expand

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Gauge fix using a background gauge fixing condition e.g.

$$S_{GF}(\bar{g}, h) = \frac{1}{2}\int dx \sqrt{-\bar{g}}\, \bar{g}^{\mu\nu}\chi_\mu\chi_\nu \; ; \qquad \chi_\mu = \bar{\nabla}^\nu h_{\nu\mu} - \frac{1}{2}\bar{\nabla}_\mu h$$

Add ghost Lagrangian

$$S_{ghost}(\bar{g}, \bar{c}, c) = \int dx \sqrt{-\bar{g}}\, \bar{c}^\mu(-\delta^\nu_\mu \bar{\nabla}^2 - \bar{R}^\nu{}_\mu)c_\nu$$

Compute $\Gamma(\bar{g}, h)$.
Formalism preserves background gauge invariance:
$\delta_\epsilon \bar{g}_{\mu\nu} = \mathcal{L}_\epsilon \bar{g}_{\mu\nu}$, $\delta_\epsilon h_{\mu\nu} = \mathcal{L}_\epsilon h_{\mu\nu}$.

ISSUES

Non-renormalizable (Goroff and Sagnotti 1985)

- interaction strength grows like $\tilde{G} = Gk^2$
- violation of unitarity
- lack of predictivity

Gravity

$$S = \int dx \sqrt{g} \left[2m_P^2 \Lambda - m_P^2 R + \ell_1 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \ell_2 R_{\mu\nu} R^{\mu\nu} + \ell_3 R^2 + O(\partial^6) \right]$$

$R \sim \Gamma\Gamma \sim (g^{-1}\partial g)^2$

m_P similar to f_π.
(Analogy even better in unimodular case $\sqrt{g} = 1$.)

Concrete problem: quantum corrections to Newtonian potential

A prediction of quantum gravity

$$V(r) = -\frac{Gm_1m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2c^3} + \dots\right]$$

Comes from non-local terms in the effective action, e.g.

$$\int dx\sqrt{g}\,[RF_1(\Box)R + R_{\mu\nu}F_2(\Box)R^{\mu\nu} + \dots]$$

Local terms cannot be predicted

Lessons

- this is a quantum theory of gravity
- it agrees with all experimental data
- has vast range of applicability

but

- open issues in the UV, IR, strong field...
- not a quantum theory of spacetime

Before giving up QFT...

Try to extend beyond Planck scale

- higher derivative gravity (renormalizable and AF)
- special combinations of gravity and matter (SUGRA)
- give up Lorentz invariance (Hořava)
- non-perturbative framework

One Loop Corrections in Einstein's Theory

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = ck^{d-2}$$

$$k\frac{dG}{dk} = -16\pi c G^2 k^{d-2}$$

$$\tilde{G} = Gk^{d-2}$$

$$k\frac{d\tilde{G}}{dk} = (d-2)\tilde{G} - 16\pi c\tilde{G}^2$$

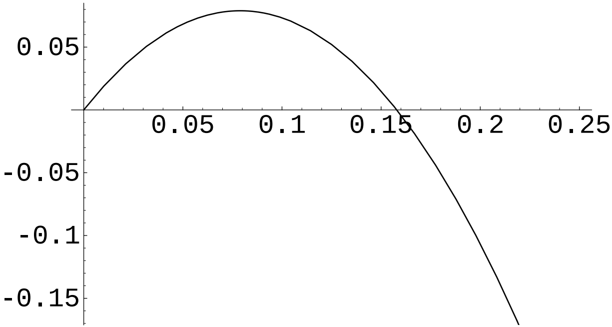
fixed point at $\tilde{G} = (d-2)/16\pi c$

$c = \frac{11}{3\pi}, \frac{35}{8\pi}, \frac{23}{3\pi}, \ldots$

GRAVITY IN $d = 2 + \epsilon$

$d = 2 + \epsilon$

$$\tilde{G} = G k^{\epsilon}$$
$$\beta_{\tilde{G}} = \epsilon \tilde{G} - \frac{38}{3} \tilde{G}^2$$



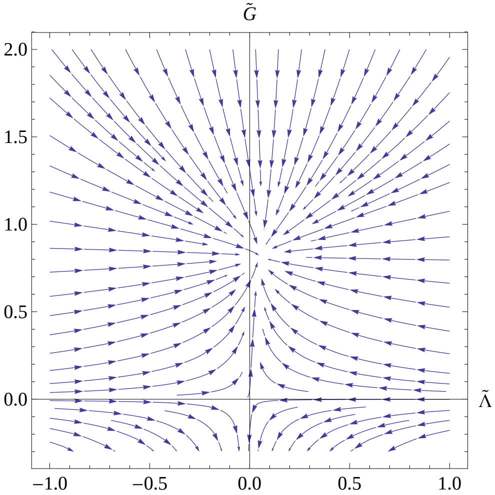
ONE LOOP BETA FUNCTIONS

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{46\tilde{G}^2}{6\pi},$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{4\pi} - \frac{16\tilde{G}\tilde{\Lambda}}{6\pi}$$

$$\tilde{\Lambda}_* = \frac{3}{62} \qquad \tilde{G}_* = \frac{12\pi}{46}$$

One loop flow



Topologically massive gravity

Action

$$S(g) = \frac{1}{16\pi G}\int d^3x\sqrt{g}\left(2\Lambda - R + \frac{1}{2\mu}\,\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}_{\lambda\sigma}\left(\partial_\mu\Gamma^{\sigma}_{\nu\rho} + \frac{2}{3}\Gamma^{\sigma}_{\mu\tau}\Gamma^{\tau}_{\nu\rho}\right)\right)$$

Dimensionless combinations of couplings

$$\nu = \mu G\ ; \qquad \tau = \Lambda G^2\ ; \qquad \phi = \mu/\sqrt{|\Lambda|}$$

R.P., E. Sezgin, Class.Quant.Grav. 27 (2010) 155009, arXiv:1002.2640 [hep-th]

Recently extended to TM SUGRA: R.P., M. Perry, C. Pope, E. Sezgin, arXiv 1302.0868

Beta functions of

$$\begin{aligned}
\beta_\nu &= 0\,, \\
\beta_{\tilde{G}} &= \tilde{G} + B(\tilde{\mu})\tilde{G}^2\,, \\
\beta_{\tilde{\Lambda}} &= -2\tilde{\Lambda} + \frac{1}{2}\tilde{G}\left(A(\tilde{\mu},\tilde{\Lambda}) + 2B(\tilde{\mu})\tilde{\Lambda}\right)
\end{aligned} \tag{1}$$

Since $\nu = \mu G = \tilde{\mu}\tilde{G}$ is constant

can replace $\tilde{\mu}$ by $\nu/\tilde{G}$

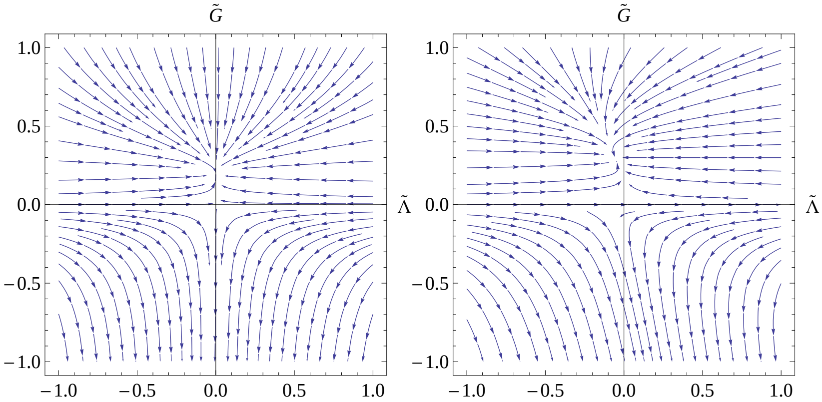


Figure: The flow in the $\tilde{\Lambda}$-$\tilde{G}$ plane for $\nu = 5$ (left) and $\nu = 0.1$ (right).

HIGHER DERIVATIVE GRAVITY

$$\Gamma_k = \int d^4x \sqrt{g} \left[2Z\Lambda - ZR + \frac{1}{2\lambda} \left(C^2 - \frac{2\omega}{3} R^2 + 2\theta E \right) \right]$$

$$Z = \frac{1}{16\pi G}$$

[K.S. Stelle, Phys. Rev. **D16**, 953 (1977).]

[J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).]

[E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).]

[I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).]

[G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).]

[A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006).]

[M. Niedermaier, Nucl. Phys. B833, 226-270 (2010).]

[N. Ohta and R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398]

BETA FUNCTIONS I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25+1098\omega+200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56-171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1+\lambda_0\frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

BETA FUNCTIONS II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right] - \frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83+70\omega+8\omega^2)/18\pi$

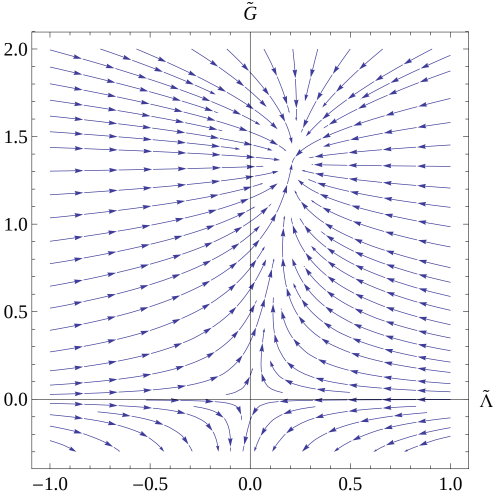
Flow in $\tilde\Lambda$–$\tilde G$ plane I

$$\beta_{\tilde\Lambda} = -2\tilde\Lambda + \frac{2\tilde G}{\pi} - q_*\tilde G\tilde\Lambda$$
$$\beta_{\tilde G} = 2\tilde G - q_*\tilde G^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde\Lambda_* = \frac{1}{\pi q_*} \approx 0.221 \,, \qquad \tilde G_* = \frac{2}{q_*} \approx 1.389 \,.$$

Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



Functional renormalization

Define the EAA $\Gamma_k(\phi)$. It satisfies

$$k\frac{d\Gamma_k}{dk} = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2\Gamma_k}{\delta\phi\delta\phi} + R_k\right)^{-1} k\frac{dR_k}{dk}$$

$$\Gamma_k(\phi) = \sum_i g_i(k)\mathcal{O}_i(\phi)$$

extract β_i.

Since $\lim_{k\to 0}\Gamma_k = \Gamma$, can use FRGE to calculate the EA.

For gravity $\Gamma_k(h_{\mu\nu}, \bar{g}_{\mu\nu})$

Most work deals with

$$\Gamma_k(g_{\mu\nu}) = \Gamma_k(0, g_{\mu\nu})$$

[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. Percacci, Class. and Quantum Grav. **15** 3449 (1998)]

Einstein–Hilbert truncation I

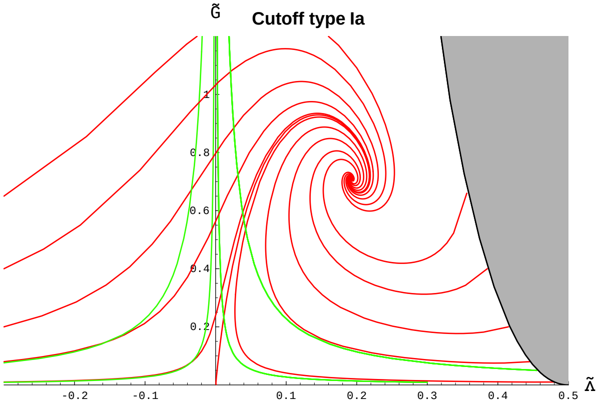
$$\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu}) = S_{EH}(\bar{g}_{\mu\nu}+h_{\mu\nu})+S_{GF}(\bar{g}_{\mu\nu}, h_{\mu\nu})+S_{ghost}(\bar{g}_{\mu\nu}, \bar{C}^{\mu}, C_{\nu})$$

$$S_{EH}(g_{\mu\nu}) = \int dx \sqrt{g} Z (2\Lambda - R) \; ; \quad Z = \frac{1}{16\pi G}$$

$$\beta_{\tilde{\Lambda}} = \frac{-2(1-2\tilde{\Lambda})^2\tilde{\Lambda} + \frac{36-41\tilde{\Lambda}+42\tilde{\Lambda}^2-600\tilde{\Lambda}^3}{72\pi}\tilde{G} + \frac{467-572\tilde{\Lambda}}{288\pi^2}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

$$\beta_{\tilde{G}} = \frac{2(1-2\tilde{\Lambda})^2\tilde{G} - \frac{373-654\tilde{\Lambda}+600\tilde{\Lambda}^2}{72\pi}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

EINSTEIN–HILBERT TRUNCATION III



$f(R)$ GRAVITY

$$\Gamma_k(g_{\mu\nu}) = \int d^4x \sqrt{g} f(R)$$

$$f(R) = \sum_{i=0}^{n} g_i(k) R^i$$

n=6
A. Codello, R.P. and C. Rahmede Int.J.Mod.Phys.A23:143-150 arXiv:0705.1769 [hep-th];
n=8
A. Codello, R.P. and C. Rahmede Annals Phys. 324 414-469 (2009) arXiv: arXiv:0805.2909;
P.F. Machado, F. Saueressig, Phys. Rev. **D** arXiv: arXiv:0712.0445 [hep-th]
n=35
K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]
n=∞
Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]
Dario Benedetti, arXiv:1301.4422 [hep-th]

$f(R)$ GRAVITY $n = 8$

Position of FixedPoint ($\times 10^{-3}$)

n	$\tilde{g}_{0*}$	$\tilde{g}_{1*}$	$\tilde{g}_{2*}$	$\tilde{g}_{3*}$	$\tilde{g}_{4*}$	$\tilde{g}_{5*}$	$\tilde{g}_{6*}$	$\tilde{g}_{7*}$	$\tilde{g}_{8*}$
1	5.23	-20.1							
2	3.29	-12.7	1.51						
3	5.18	-19.6	0.70	-9.7					
4	5.06	-20.6	0.27	-11.0	-8.65				
5	5.07	-20.5	0.27	-9.7	-8.03	-3.35			
6	5.05	-20.8	0.14	-10.2	-9.57	-3.59	2.46		
7	5.04	-20.8	0.03	-9.78	-10.5	-6.05	3.42	5.91	
8	5.07	-20.7	0.09	-8.58	-8.93	-6.81	1.17	6.20	4.70

Critical exponents

n	$Re\vartheta_1$	$Im\vartheta_1$	ϑ_2	ϑ_3	$Re\vartheta_4$	$Im\vartheta_4$	ϑ_6	ϑ_7	ϑ_8
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.3

$f(R)$ GRAVITY $n = 8$ PREDICTIONS

Critical surface:

$$\tilde{g}_3 = 0.00061243 + 0.06817374\,\tilde{g}_0 + 0.46351960\,\tilde{g}_1 + 0.89500872\,\tilde{g}_2$$
$$\tilde{g}_4 = -0.00916502 - 0.83651466\,\tilde{g}_0 - 0.20894019\,\tilde{g}_1 + 1.62075130\,\tilde{g}_2$$
$$\tilde{g}_5 = -0.01569175 - 1.23487788\,\tilde{g}_0 - 0.72544946\,\tilde{g}_1 + 1.01749695\,\tilde{g}_2$$
$$\tilde{g}_6 = -0.01271954 - 0.62264827\,\tilde{g}_0 - 0.82401181\,\tilde{g}_1 - 0.64680416\,\tilde{g}_2$$
$$\tilde{g}_7 = -0.00083040 + 0.81387198\,\tilde{g}_0 - 0.14843134\,\tilde{g}_1 - 2.01811163\,\tilde{g}_2$$
$$\tilde{g}_8 = 0.00905830 + 1.25429854\,\tilde{g}_0 + 0.50854002\,\tilde{g}_1 - 1.90116584\,\tilde{g}_2$$

$f(R)$ GRAVITY $n = 35$

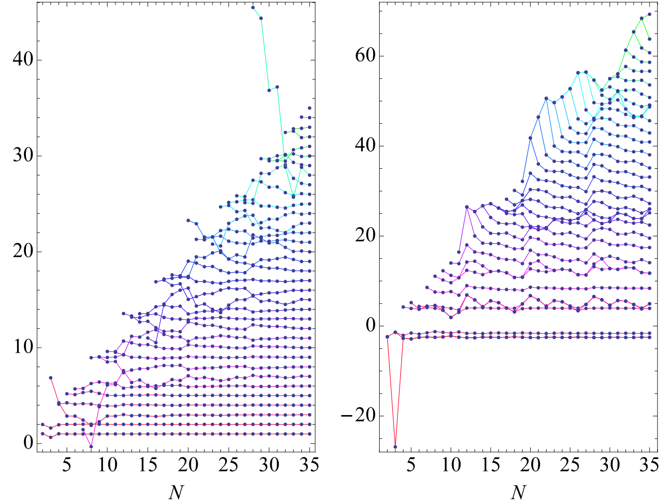
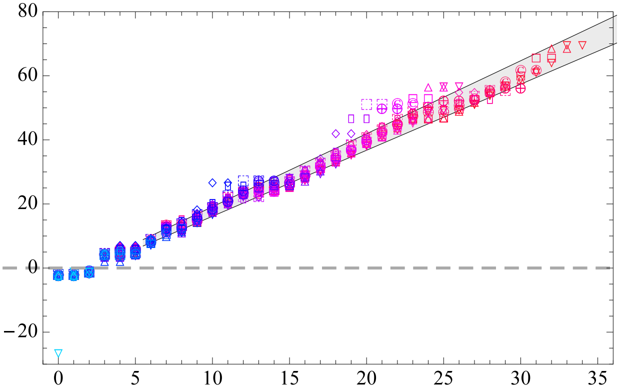


Figure: Left: couplings Right: scaling exponents

$f(R)$ GRAVITY $n = 35$



CURRENT FRONTIER

- functional truncations
- split transformations

Functional truncations

ERGE well suited to study flow of potential

$$\Gamma_k[\phi] = \int d^d x \left(V(\phi^2) + \frac{1}{2}(\partial\phi)^2 \right)$$

Successfully reproduces properties of many critical models.

$f(R)$ GRAVITY, FUNCTIONAL TREATMENT

Do not expand $f(R)$ but write flow equation for f

$$\partial_t \tilde{f}(\tilde{R}) = \beta(\tilde{f}, \tilde{f}', \tilde{f}'', \tilde{f}''')$$

where $\tilde{R} = R/k^2$, $\tilde{f} = f/k^4$.
For large $\tilde{R}$

$$\tilde{f}(\tilde{R}) = A\tilde{R}^2 \left(1 + \sum_{n>0} d_n \tilde{R}^{-n}\right)$$

Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]

$f(R)$ GRAVITY, FUNCTIONAL TREATMENT

Theorem 1: $\Gamma_*(g_{\mu\nu}) = A_* \int d^4x \sqrt{g} R^2$, $A_* \neq 0$

Theorem 2: if $\tilde{f}_*$ exists, the spectrum of perturbations is discrete, real, and there are at most finitely many relevant direction.

D. Benedetti, arXiv:1301.4422 [hep-th]

Analytic and numerical solutions have been found.
Situation not yet completely clarified.

[N. Ohta, R.P., G.P. Vacca, Eur. Phys. J. C (2016) 76:46 arXiv:1511.09393 [hep-th]]

Alternative approach

Expand

$$\begin{aligned}\Gamma_k(h;\bar g) &= \Gamma_k(0;\bar g)+\int\Gamma'_k(0;\bar g)h+\int\Gamma''_k(0;\bar g)h^2\\ &\quad+\int\Gamma'''_k(0;\bar g)h^3+\int\Gamma''''_k(0;\bar g)h^4+\dots\end{aligned}$$

from FRGE obtain

$$\begin{aligned}\dot\Gamma_k(h;\bar g) &= \dot\Gamma_k(0;\bar g)+\int\dot\Gamma'_k(0;\bar g)h+\int\dot\Gamma''_k(0;\bar g)h^2\\ &\quad+\int\dot\Gamma'''_k(0;\bar g)h^3+\int\dot\Gamma''''_k(0;\bar g)h^4+\dots\end{aligned}$$

can read off beta functions.

VERTEX EXPANSION

Flat space expansion.
Flow extracted from 3- and 4-point functions.
FP again present.

[N. Christiansen, J. Pawlowski, A. Rodigast, Phys.Rev. D93 (2016) no.4, 044036 arXiv:1403.1232 [hep-th]]
[N. Christiansen, B. Knorr, J. Meibohm, J. Pawlowski, M. Reichert, Phys.Rev. D92 (2015) no.12, 121501 arXiv:1506.07016 [hep-th]]

Split symmetry

Because

$$S = S(g_{\mu\nu}) = S(\bar{g}_{\mu\nu} + h_{\mu\nu})$$

the bare action is invariant under

$$\begin{aligned} \delta\bar{g}_{\mu\nu} &= \epsilon_{\mu\nu} \ , \\ \delta h_{\mu\nu} &= -\epsilon_{\mu\nu} \ . \end{aligned}$$

but the EAA $\Gamma_k(\mathbf{h}; \bar{\mathbf{g}})$ is not.

$$\frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta h^n} \neq \frac{\delta^{(n)}\Gamma_k(h;\bar{g})}{\delta \bar{g}^n}$$

Split symmetry

Expanding the Hilbert action

$$S(g) = S(\bar{g}) + \int S'(\bar{g})h + \int S''(\bar{g})h^2 + \int S'''(\bar{g})h^3 + \int S''''(\bar{g})h^4 + \ldots$$

all contain the same Newton constant.

Due to violation of split symmetry each term of the expansion gives a different “beta function”

[T. Denz, J. Pawlowski and M. Reichert, Towards apparent convergence in asymptotically safe quantum gravity arXiv:1612.07315 [hep-th]]

Dealing with the split symmetry violation

- Write the anomalous Ward identity for the split symmetry or a subgroup thereof
- Solve it to eliminate from the EAA a number of fields equal to the number of parameters of the transformation
- Write the flow equation for the EAA depending on the remaining variables

Carried through for $\epsilon_{\mu\nu} = \epsilon \bar{g}_{\mu\nu}$

[P. Labus, T.R. Morris, Z.H. Slade Phys.Rev. D94 (2016) no.2, 024007 arXiv:1603.04772 [hep-th]]
[T.R. Morris, JHEP 1611 (2016) 160 arXiv:1610.03081 [hep-th]]

[R.P., G.P. Vacca, Eur.Phys.J. C77 (2017) no.1, 52 arXiv:1611.07005 [hep-th]]

SUMMARY

- Fixed point appears in all approximations tried so far
- Canonical dimension seems to be a reasonably good guide
- Some uncertainty on the number of relevant deformations
- Truly functional truncations can be studied, but are hard. (Less tolerant of bad approximations.)

but

- Use of background field method makes effective action depend on two fields.
- Pure gravity has no local observables.
- No reliable calculation of physical observable effect.

ENTER MATTER

- because it's there
- gravitational scattering of matter less exotic than graviton scattering
- possible experimental constraints from known physics
- because it may help (large N limit)

Non-interacting matter:

[P. Donà, A. Eichhorn, R.P. arXiv:1311.2898 [hep-th](2013)]

PERTURBATIVE BETA FUNCTIONS WITH MATTER

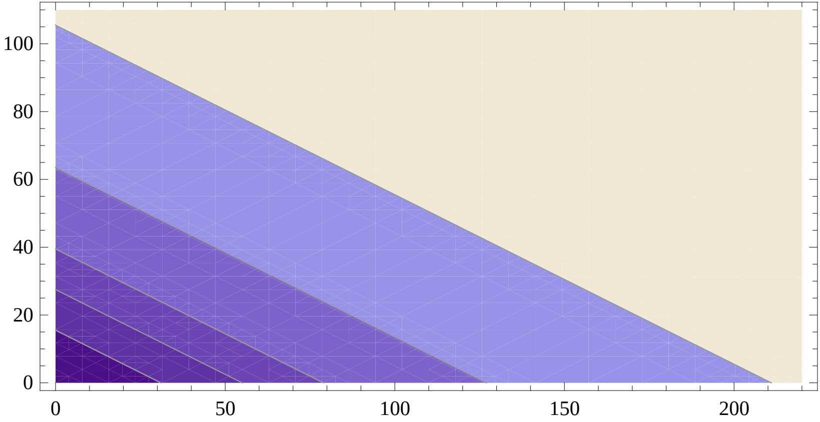
$$\beta_{\tilde{G}} = 2\tilde{G} + \frac{\tilde{G}^2}{6\pi}\left(N_S + 2N_D - 4N_V - 46\right),$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{\tilde{G}}{4\pi}\left(N_S - 4N_D + 2N_V + 2\right) + \frac{\tilde{G}\tilde{\Lambda}}{6\pi}\left(N_S + 2N_D - 4N_V - 16\right)$$

$$\tilde{\Lambda}_* = -\frac{3}{4}\frac{N_S - 4N_D + 2N_V + 2}{N_S + 2N_D - 4N_V - 31},$$

$$\tilde{G}_* = -\frac{12\pi}{N_S + 2N_D - 4N_V - 46}.$$

EXCLUSION PLOTS $N_V = 0, 6, 12, 24, 45$



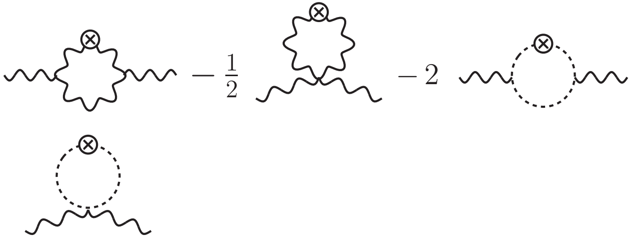
Truncated FRGE, bimetric formalism

$$
\begin{aligned}
\Gamma_k(\bar{g},h) &= \frac{1}{16\pi G}\int d^dx\sqrt{\bar{g}}\,(-\bar{R}+2\Lambda)\\
&+\frac{Z_h}{2}\int d^dx\sqrt{\bar{g}}\,h_{\mu\nu}K^{\mu\nu\alpha\beta}((-\bar{D}^2-2\Lambda)\mathbf{1}^{\rho\sigma}_{\alpha\beta}+W^{\rho\sigma}_{\alpha\beta})h_{\rho\sigma}\\
&-\sqrt{2}Z_c\int d^dx\sqrt{\bar{g}}\;\bar{c}_\mu\Big(\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\kappa\nu}D_\rho+\bar{D}^\rho\bar{g}^{\mu\kappa}g_{\rho\nu}D_\kappa-\bar{D}^\mu\bar{g}^{\rho\sigma}g_{\rho\nu}D_\sigma\Big)c^\nu
\end{aligned}
$$

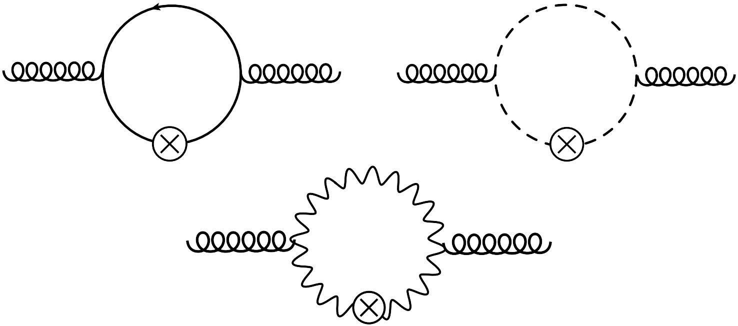
$$
\begin{aligned}
S_S &= \frac{Z_S}{2}\int d^dx\sqrt{g}\,g^{\mu\nu}\sum_{i=1}^{N_S}\partial_\mu\phi^i\partial_\nu\phi^i\\
S_D &= iZ_D\int d^dx\sqrt{g}\sum_{i=1}^{N_D}\bar{\psi}^i\nabla\!\!\!/\,\psi^i,\\
S_V &= \frac{Z_V}{4}\int d^dx\sqrt{g}\sum_{i=1}^{N_V}g^{\mu\nu}g^{\kappa\lambda}F^i_{\mu\kappa}F^i_{\nu\lambda}+\dots
\end{aligned}
$$

Graviton+ghost contributions to η_h

$$\partial_t \gamma_k^{(2,0,0;0)} =$$

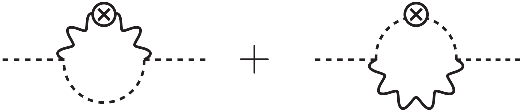


MATTER CONTRIBUTION TO η_h

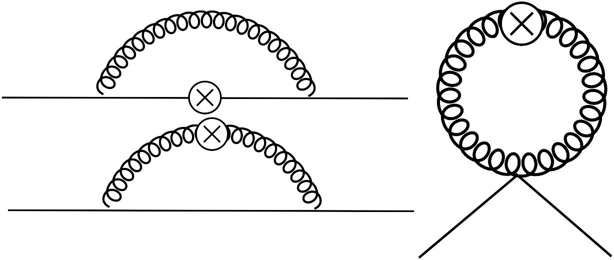


GRAVITON+GHOST CONTRIBUTIONS TO η_C

$$\partial_t \gamma_k^{(0,1,1;0)} =$$



GRAVITON CONTRIBUTION TO MATTER η



General structure of anomalous dimensions

For $\Psi = h, c, S, D, V$,

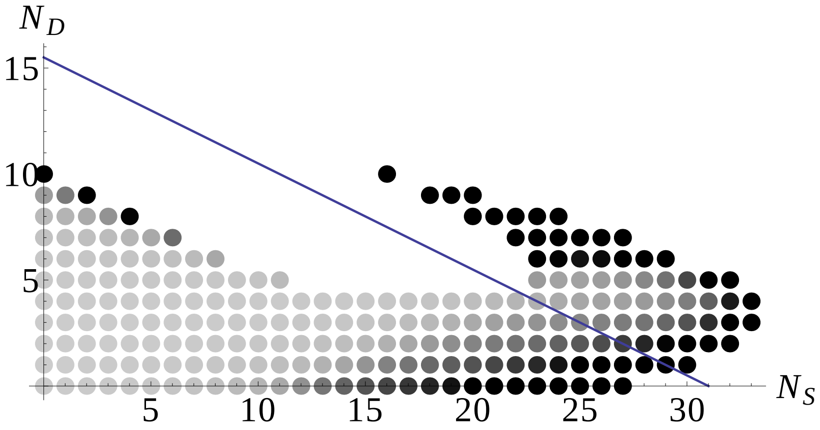
$$\eta_\Psi = -\frac{1}{Z_\Psi} k \frac{dZ_\Psi}{dk}$$

$$\vec{\eta} = (\eta_h, \eta_c, \eta_S, \eta_D, \eta_V)$$

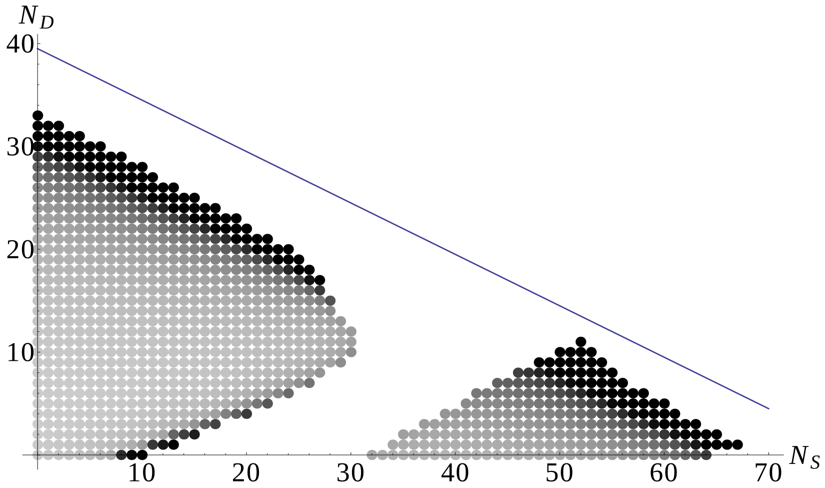
$$\vec{\eta} = \vec{\eta}_1(\tilde{G}, \tilde{\Lambda}) + \mathbf{A}(\tilde{G}, \tilde{\Lambda})\vec{\eta}$$

- one loop anomalous dimensions $\vec{\eta} = \vec{\eta}_1$
- RG improved anomalous dimensions $\vec{\eta} = (\mathbf{1} - \mathbf{A})^{-1}\vec{\eta}_1$

EXCLUSION PLOT $N_V = 0$



Exclusion plot $N_V = 12$



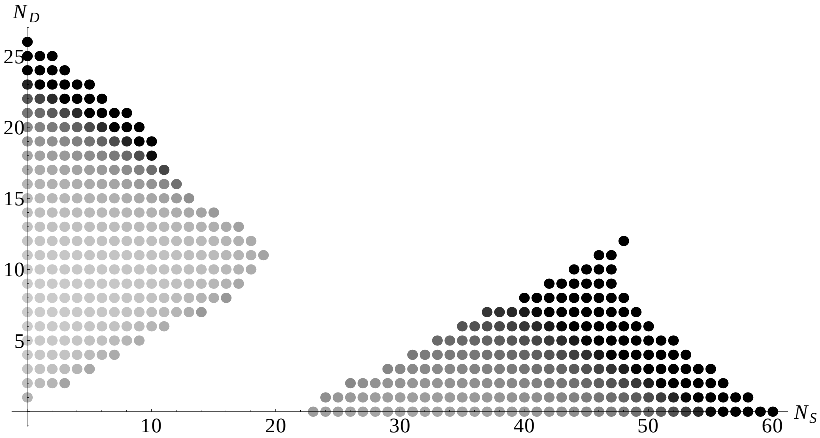
STANDARD MODEL MATTER

	1L-II	full-II	1L-Ia	full-Ia
$\tilde{\Lambda}_*$	-2.399	-2.348	-3.591	-3.504
$\tilde{G}_*$	1.762	1.735	2.627	2.580
θ_1	3.961	3.922	3.964	3.919
θ_2	1.644	1.651	2.178	2.187
η_h	2.983	2.914	4.434	4.319
η_c	-0.139	-0.129	-0.137	-0.125
η_S	-0.076	-0.072	-0.076	-0.073
η_D	-0.015	0.004	-0.004	0.016
η_V	-0.133	-0.145	-0.144	-0.158

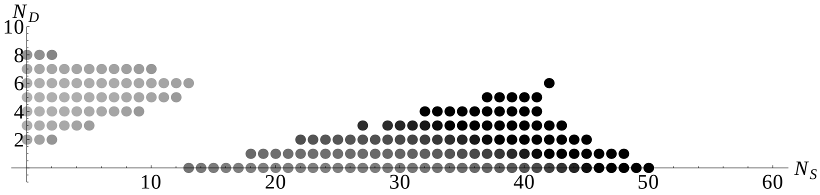
SPECIFIC MODELS

model	N_S	N_D	N_V	$\tilde{G}_*$	$\tilde{\Lambda}_*$	θ_1	θ_2	η_h
no matter	0	0	0	0.77	0.01	3.30	1.95	0.27
SM	4	45/2	12	1.76	-2.40	3.96	1.64	2.98
SM +dm scalar	5	45/2	12	1.87	-2.50	3.96	1.63	3.15
SM+ 3 ν's	4	24	12	2.15	-3.20	3.97	1.65	3.71
SM+3ν's + axion+dm	6	24	12	2.50	-3.62	3.96	1.63	4.28
MSSM	49	61/2	12	-	-	-	-	-
SU(5) GUT	124	24	24	-	-	-	-	-
SO(10) GUT	97	24	45	-	-	-	-	-

EXCLUSION PLOT $N_V = 12$, $d = 5$



EXCLUSION PLOT $N_V = 12$, $d = 6$



TURN ON MATTER INTERACTIONS

Can AF matter coexist with AS gravity?

Conjecture:

- Interactions respecting the global symmetries of the kinetic term will have non-zero couplings at a FP.
- Interactions that violate the symmetries of the kinetic term could have a fixed point at zero coupling.

[A. Eichhorn and A. Held, (2017), arXiv:1705.02342 [gr-qc].]

Confirmed in several cases of scalar, fermion and vector (F^4) interactions.

GRAVITY+SCALAR

$$\Gamma_k[g,\phi] = \int d^d x \sqrt{g} \left(V(\phi^2) - F(\phi^2) R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$$

[G. Narain, R.P., Class. and Quantum Grav. 27, 075001 (2010)]

[T. Henz, J. Pawlowski, A. Rodigast, C. Wetterich, Phys. Lett. B727 (2013) 298]

[D. Benedetti and F. Guarnieri, New J. of Phys. (2014) 053051]

[R.P., G.P. Vacca, Eur.Phys.J. C75 (2015) 5, 188, arXiv:1501.00888 [hep-th]]

Polynomial truncations

$$
\begin{aligned}
\tilde{V}(\tilde{\phi}^2) &= \tilde{\lambda}_0 + \tilde{\lambda}_2\tilde{\phi}^2 + \tilde{\lambda}_4\tilde{\phi}^4 + \dots \\
\tilde{F}(\tilde{\phi}^2) &= \tilde{\xi}_0 + \tilde{\xi}_2\tilde{\phi}^2 + \dots
\end{aligned}
$$

Only “Gaussian matter fixed point”, in accordance with general expectation.

FLOW EQUATIONS $d = 3$

R.P., G.P. Vacca, Eur.Phys.J. C75 (2015) 5, 188, arXiv:1501.00888 [hep-th]

$$\dot{v} = -3\,v + \frac{1}{2}\phi\, v' + \frac{f + 4f'^2}{6\pi^2\left(4f'^2 + f(1+v'')\right)} + O(\dot{f})$$

$$\dot{f} = -f + \frac{1}{2}\phi\, f' + \frac{25}{36\pi^2} + f\,\frac{(f+4f'^2)(1+3v''-2f'') + 2fv''^2}{12\pi^2(4f'^2 + f(1+v''))^2} + O(\dot{f})$$

Compare with equation for pure scalar in LPA

$$\dot{v} = -3\,v + \frac{1}{2}\phi\, v' + \frac{1}{6\pi^2(1+v'')}$$

Gravitationally dressed Wilson-Fisher fixed point

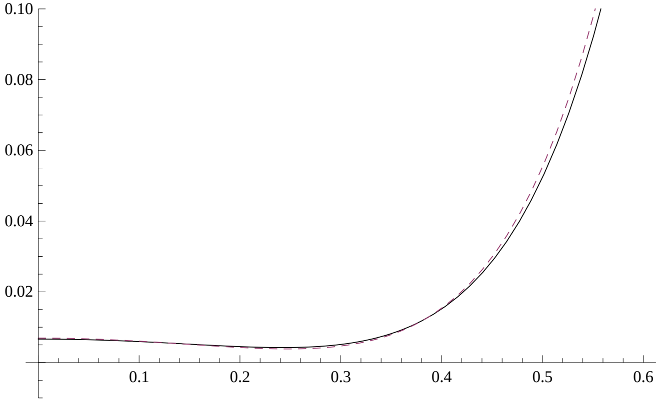


Figure: Solid curve: potential with gravity; dashed curve: LPA approximation of potential of Wilson-Fisher fixed point without gravity.

FLOW EQUATIONS $d = 4$

$$\dot{v} = -4v + \varphi v' + \frac{1}{16\pi^2} + \frac{f + 3f'^2}{32\pi^2\left(3f'^2 + f(1 + v'')\right)} + O(\dot{f})$$

$$\dot{f} = -2f + \varphi f' + \frac{37}{384\pi^2} + f\frac{(f + 3f'^2)(1 - 3f'' + 3v'') + 2fv''^2}{96\pi^2(3f'^2 + f(1 + v''))^2} + O(\dot{f})$$

ANALYTIC SOLUTIONS $d = 4$ $N = 1$

FP1

$$v_* = \frac{3}{128\pi^2} \approx 0.00237 \; ; \qquad f_* = \frac{41}{768\pi^2} \approx 0.00541$$

FP2

$$v_* = \frac{3}{128\pi^2} \approx 0.00237 \; ; \qquad f_* = \frac{37}{768\pi^2} + \frac{1}{6}\varphi^2 \approx 0.0049 + 0.167\varphi^2$$

FP3

$$v_* = \frac{3}{128\pi^2} \approx 0.002374 \; ; \qquad f_* = -\frac{41}{420\pi^2}\varphi^2 \approx -0.0976\varphi^2$$

$O(N)$-INVARIANT SCALARS

Analytic solutions of functional equations known for $d < d_{max}$ and $N < N_{max}$.

All quadratic in φ except candidate Wilson-Fisher FP for $N = 2$.

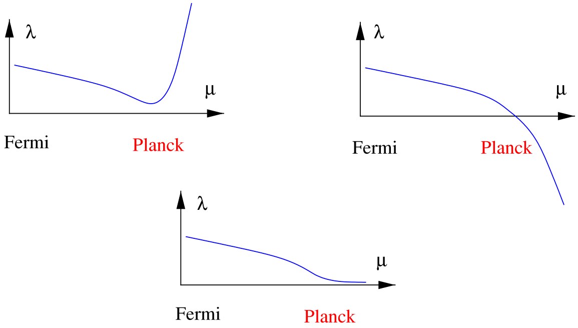
[P. Labus, R.P., G.P. Vacca, Phys.Lett. B753 (2016) 274-281 arXiv:1505.05393 [hep-th]]

CORRECTIONS TO RUNNING OF λ

$$\begin{aligned} V(\phi^2) &= \lambda_0 + \lambda_2\phi^2 + \lambda_4\phi^4 + \dots \\ F(\phi^2) &= \xi_0 + \xi_2\phi^2 + \dots \end{aligned}$$

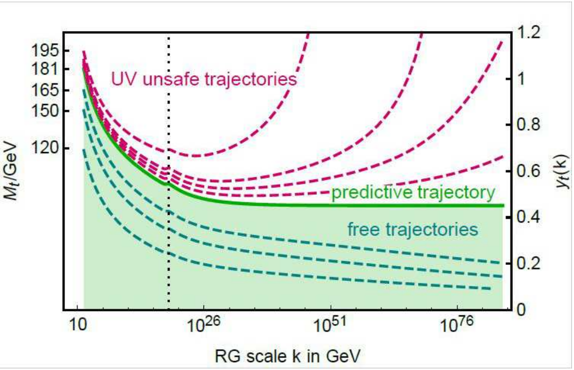
$$\partial_t\lambda_4 = \frac{9\lambda_4^2}{2\pi^2} + \frac{\tilde{G}\lambda_4}{\pi} + \dots$$

Prediction of the Higgs mass



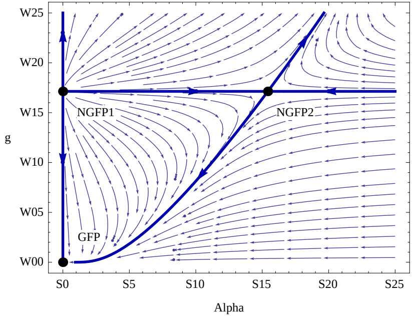
[M. Shaposhnikov and C. Wetterich, Phys.Lett. B683, 196 (2010)]

Calculation of top mass



[A. Eichhorn and A. Held, arXiv:1707.01107 [hep-th]]

POSSIBLE CALCULATION OF FINE STRUCTURE CONSTANT



$\alpha - \alpha_*$ irrelevant at NGFP2.

[U. Harst and M. Reuter, JHEP 1105, 119 (2011), arXiv:1101.6007 [hep-th]]

[A. Eichhorn and F. Versteegen, arXiv:1709.07252 [hep-th]]

[A. Eichhorn, A. Held and C. Wetterich, arXiv:1711.02949 [hep-th]]

CONTINUUM, COVARIANT APPROACH TO QG

Extrapolation to infinite energy used as a principle to select theories $\Longrightarrow$ predictivity.

AS: STRENGTHS AND WEAKNESSES

- use powerful QFT tools
- bottom up approach $\Longrightarrow$ guaranteed to give correct low energy limit
- highly predictive: only few parameters undetermined
- inclusion of matter relatively easy
- particle physics constrained

but

- strong coupling
- parametrization/gauge/scheme dependence
- no robust calculation of physical observable yet

Final comments

AS, QFT for gravity, FRGE all independent notions.

In gravity other techniques could play a role:

- ϵ expansion
- large N expansion
- two loop calculations
- generalization of CFT techniques

AS may play role in BSM physics aside from gravity.
New fermion fields required (perhaps many).

Gravity with matter may be the most promising avenue