Quantum Gravity, or: Give me more Observables!

Frascati, 19 Dec 2017

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What is quantum gravity?

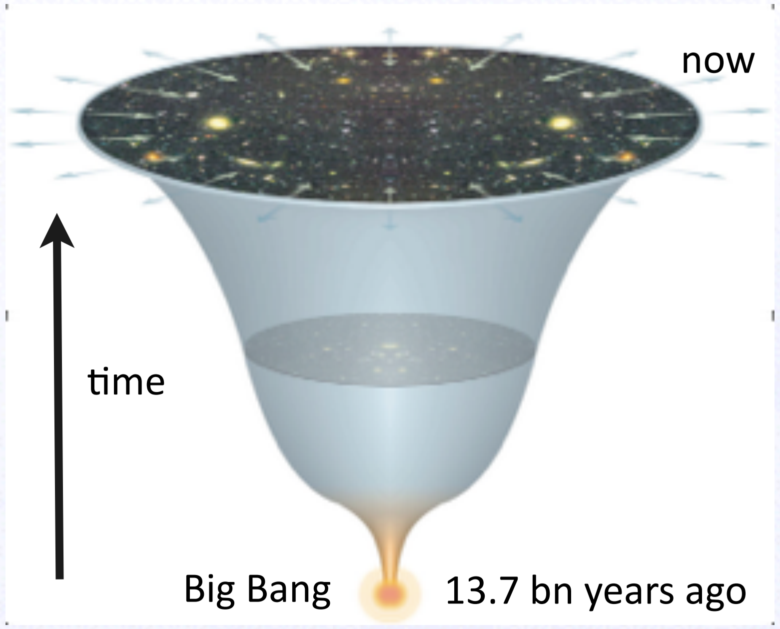
- *Quantum gravity* is the putative fundamental quantum theory underlying the classical field theory of General Relativity.
- It is *the* missing piece in our theoretical understanding of the four fundamental interactions.
- We do not know whether quantum gravity can/must be understood as part of a grand unifying dynamical principle.
- Applying the logic of Einstein's General Relativity, quantum gravity should also describe the dynamics of *spacetime* on all scales.
- The length scale at which quantum properties of the gravitational field must be taken into account is the *Planck length*

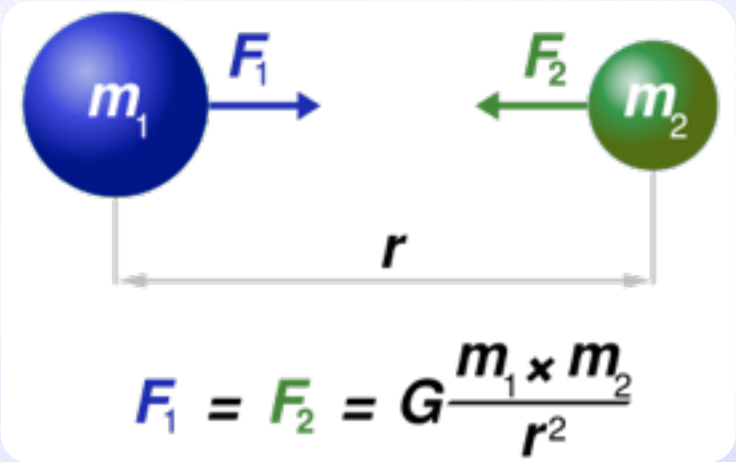
$$\ell_{\mathrm{Pl}} = \sqrt{\frac{G_{\mathrm{N}}\hbar}{c^3}} \approx 1.6 \times 10^{-35} m$$

(yes, this is *really* small; gravity is special!)

Questions that quantum gravity should answer

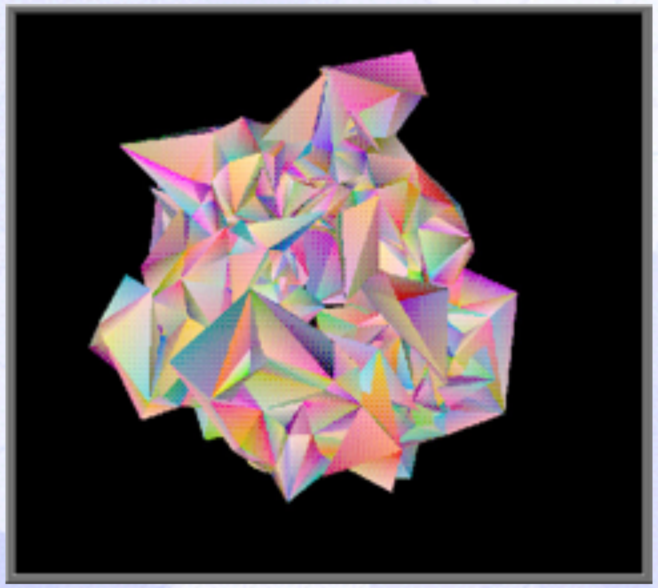
- What was the quantum behaviour of the very early universe?
- Are space and time fundamental or merely emergent on macroscopic scales?





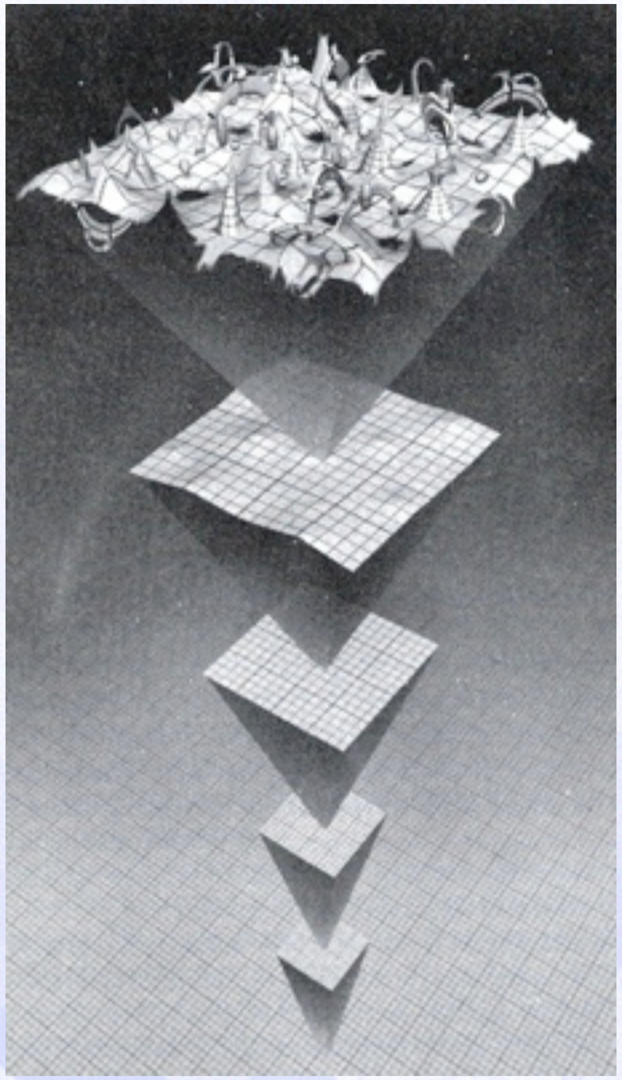
- Can we *derive* gravitational attraction from first-principles quantum dynamics @ ℓ_{Pl}?

- What is the quantum microstructure of spacetime? Can we use it to *explain* the observed large-scale de Sitter nature of our universe? Can we make de Sitter space "emerge"?



triangulated model of quantum space

Going beyond classical geometry/GR



zooming in on a piece of empty spacetime

What becomes of spacetime and the degrees of freedom of gravity at the Planck scale ℓ_{Pl} ?

- spacetime foam? wormholes?
- noncommutative space(time)?
- is there a shortest length scale?
- is there fundamental discreteness, "atoms of spacetime"? (there is little evidence, but plenty of 'intuition' it should be so)

However, the world is *quantum*, and physics on small scales is highly counterintuitive!

Which aspects of classical geometry and gravity will survive?
For example, what happens to the key classical notion of "curvature"?

Quantum gravity: where do we stand?

- perturbative quantum gravity “does not work” (nonrenormalizable)
  - perturbative ansatz $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ is misguided?
  - Minkowski metric $\eta_{\mu\nu}$ is the “wrong vacuum”?
  - smooth $g_{\mu\nu}$ ($h_{\mu\nu}$) not appropriate in quantum-fluctuating regime?
  - need a completely different “UV (ultraviolet) completion” of gravity (grand unified theory, stringy, …)?
- we have several nonperturbative candidate theories, working from different premises (some are more promising than others …)
- they are too incomplete and/or have too many free parameters to make any solid predictions; comparing them is also (still) difficult
- there is little if any quantum gravity phenomenology to speak of

How should we proceed?

We can postulate some Planckian degrees of freedom, with an associated quantum dynamics. Two extreme examples:

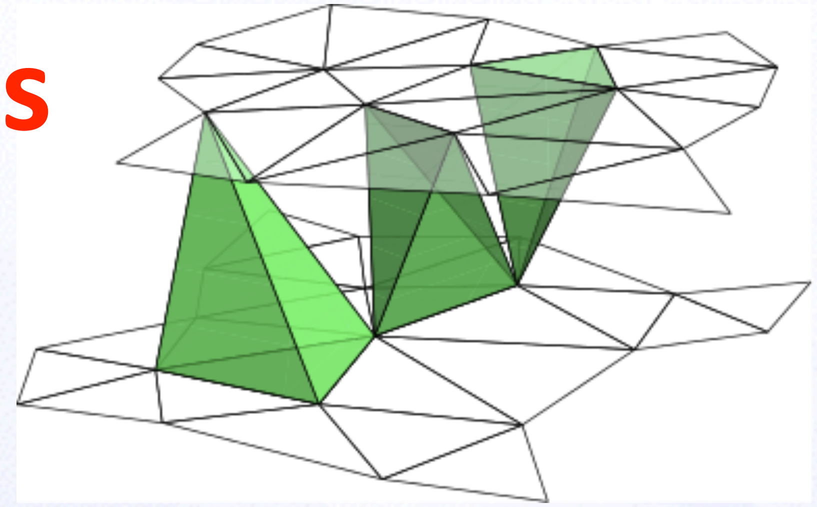
1.) **superstring theory** (lots of “exotic” ingredients, nonperturbative dynamics unknown)

Problem: “embarassment of riches”, no predictive power

2.) **causal set theory** (“bare bones” ingredients, quantum dynamical principle unknown)

Problem: what does this have to do with gravity? issue of the “classical limit”

Quantum Gravity, back to basics



part of a (piecewise flat) causal triangulation

Causal Dynamical Triangulations (CDT) is an attempt to bring quantum gravity back into the fold of ordinary quantum field theory, without appealing to a grand unified dynamical principle.

Analogous to QCD on the lattice, CDT uses a lattice regularization to define a theory of quantum gravity nonperturbatively.* However, the spacetime lattices are dynamical and no lattice/background is distinguished.

As expected, the theory has divergences in the continuum limit as the UV regulator is removed. They must be renormalized appropriately.

Quantitative results so far are in a highly quantum fluctuating regime, <u>far away from (semi-)classicality</u>, apart from a few global observables.

*nonperturbative ≈ large fluctuations on small scales

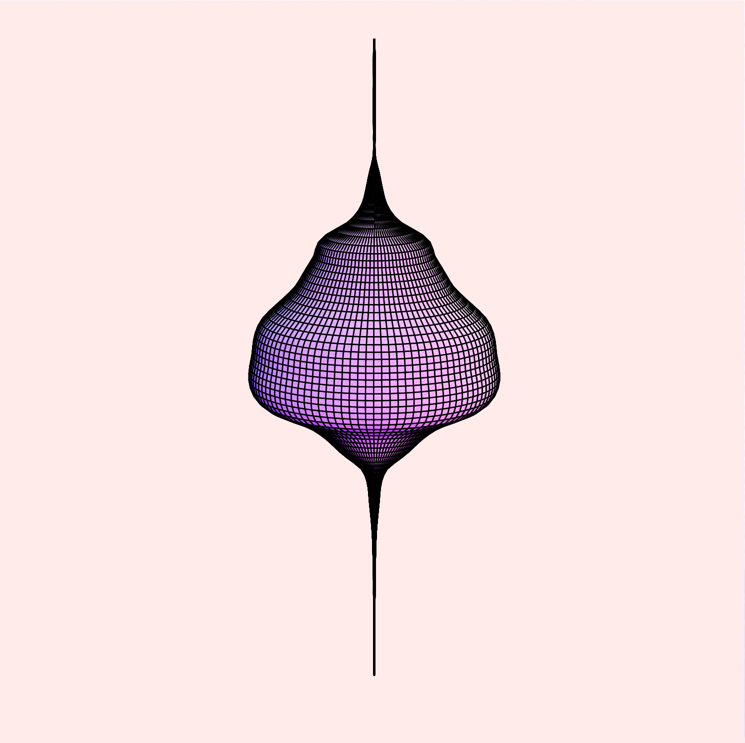
What is the overall outlook of CDT quantum gravity?

- CDT quantum gravity depends on a minimalist set of ingredients and just two(!) free parameters, and is conceptually simple.
- Nevertheless, understanding its nonperturbative dynamics is *not* easy.
- We have been able to extract <u>*new and exciting results*</u> from evaluating a handful of **nonperturbative quantum observables**. These results are robust and quantitative, and therefore potentially falsifiable.
- causal structure plays a crucial role ("Euclidean QG" not good enough)
- "discrete" aspects of CDT are merely an intermediate feature, before a continuum limit is taken (no evidence of "fundamental discreteness")

(J. Ambjørn, A. Görlich, J. Jurkiewicz & RL, "Nonperturbative Quantum Gravity", Physics Report 519 (2012) 127 [arXiv: 1203.3591])

The Emergence of Classical Spacetime from Causal Dynamical Triangulations (CDT)

CDT is currently the only candidate quantum theory of gravity which can generate *dynamically* a spacetime with semiclassical properties from pure quantum excitations, without using a background metric.



Other key results:

- crucial role of causal structure
- nonstandard treatment of symmetries and observables
- scale-dependent spacetime dimension (2 → 4)
- nontrivial phase structure, with "classical" phases
- second-order phase transitions (unique!)
- applicability of renormalization group methods

Quantum Gravity from CDT

The formal, ill-defined continuum gravitational path integral

Newton's constant

$$Z(G_N,\Lambda) = \int_{\substack{\text{spacetimes}\\ g\in\mathcal{G}}} \mathcal{D}g\; \mathrm{e}^{iS^{\rm EH}_{\rm G_N,\Lambda}[g]}$$

cosmological constant

Einstein-Hilbert action

("sum over histories")

is turned into a finite regularized sum over triangulated spacetimes,

$$Z(G_N,\Lambda) := \lim_{\substack{a\to 0\\ N\to\infty}} \sum_{\substack{\text{inequiv.}\\ \text{triangul.s}\\ T\in\mathcal{G}_{a,N}}} \frac{1}{C(T)}\, \mathrm{e}^{iS^{\rm Regge}_{\rm G_N,\Lambda}[T]}$$

UV cutoff

building blocks

|Aut(T)|

whose continuum limits are investigated after an analytic continuation.

(N.B.: the inclusion of matter is straightforward)

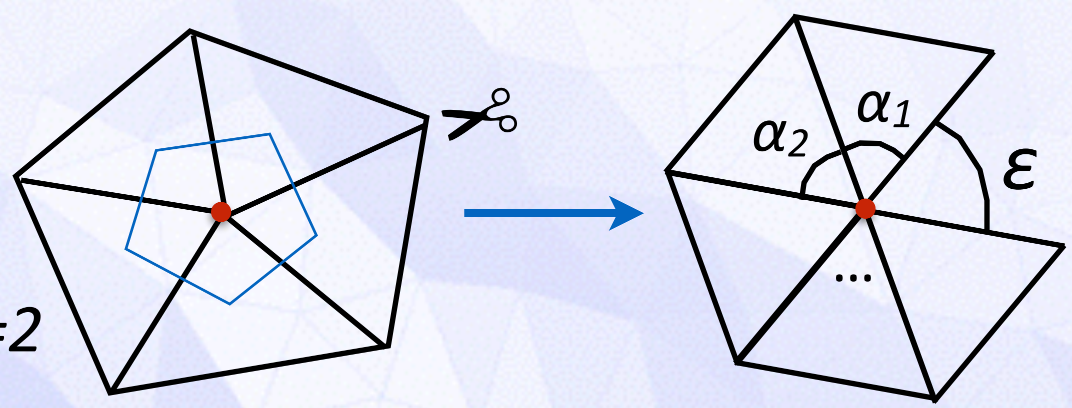
gravity action: $S^{\rm EH} = \dfrac{1}{G_{\rm N}}\displaystyle\int d^4x\sqrt{-\det g}\,(R[g,\partial g,\partial^2 g]-2\Lambda)$

Crucial: "General Relativity without coordinates"

A curved *d*-dimensional spacetime in CDT is described by a piecewise flat manifold, made from flat, triangular *d*-dimensional building blocks.

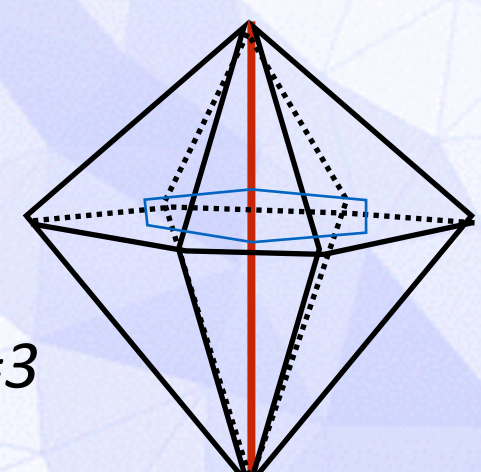
Geometry is described uniquely by the edge lengths of the *d*-simplices and how simplices are 'glued' together. **No coordinates are needed** (Regge, 1961) and **the CDT path integral has no coordinate redundancies**.

Curvature is located at "hinges" τ (subsimplices of dim. *d-2*), in the form of deficit angles ε, measured by parallel-transport around minimal loops $L(\tau)$.

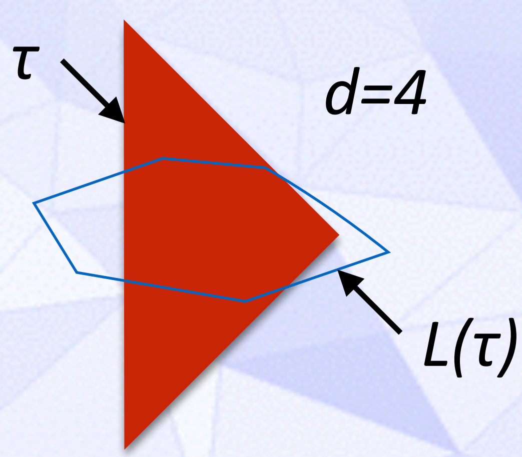


d=2

$$\varepsilon = 2\pi - \sum_i \alpha_i$$



d=3

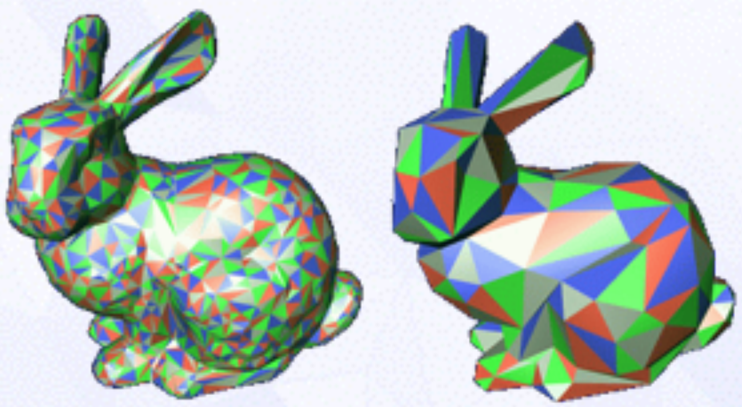


In *d=4*, each hinge τ is shared by a ring of four-simplices (not shown).

red: hinge τ, blue: minimal loop $L(\tau)$

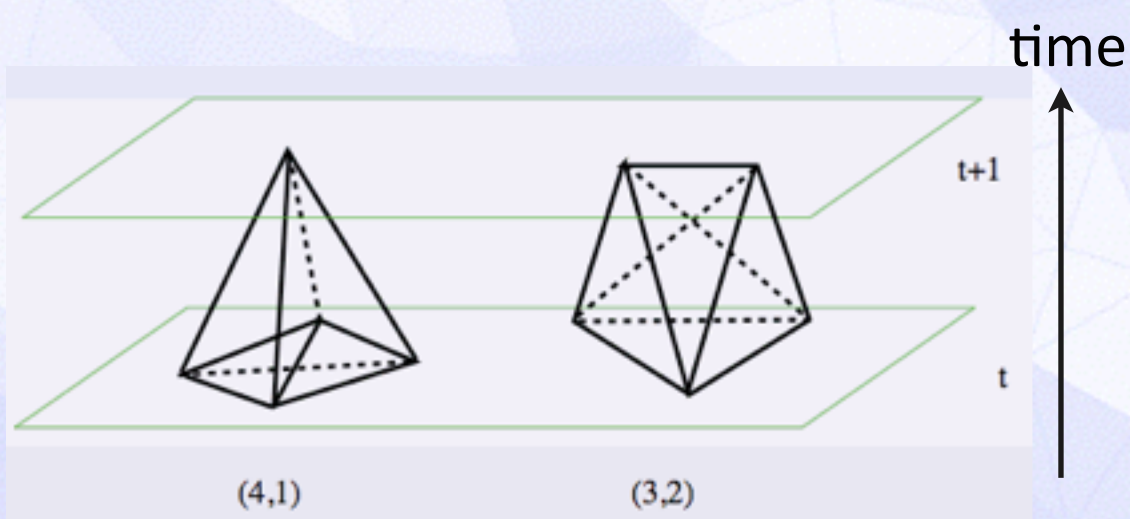
Key ingredients of the CDT approach:

- representing curved spacetimes by piecewise flat triangulations makes the path integral well defined at an intermediate ("regularized") stage



approximating a given *classical* curved surface through triangulation

- crucial to obtain a semiclassical limit: spacetimes must have causal structure
- crucial in d = 4: nonperturbative comput. tools (Monte Carlo simulations) to extract quantitative results



simplicial 4d building blocks of CDT

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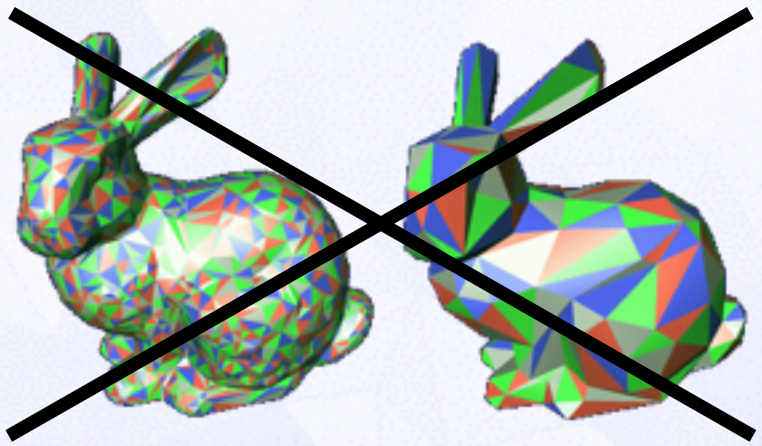

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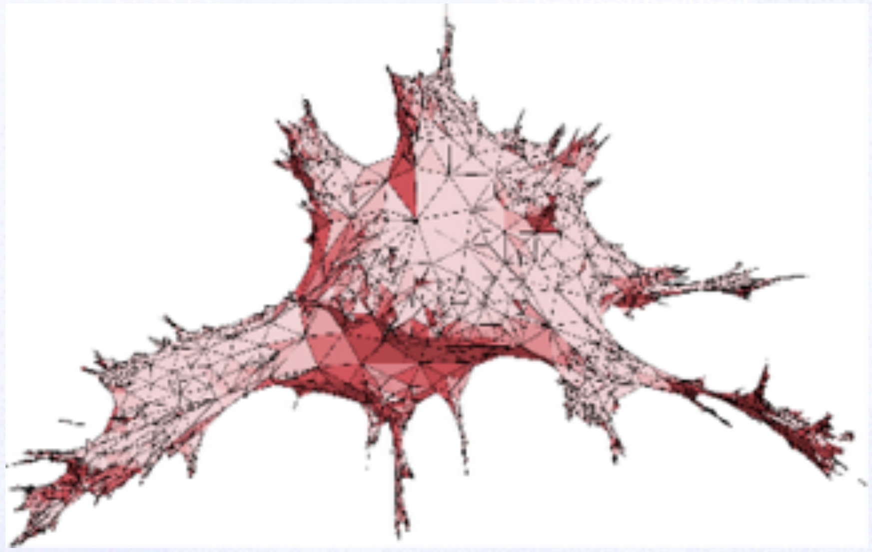
instead:



Quantum Theory: approximating the space of *all* curved geometries by a space of triangulations

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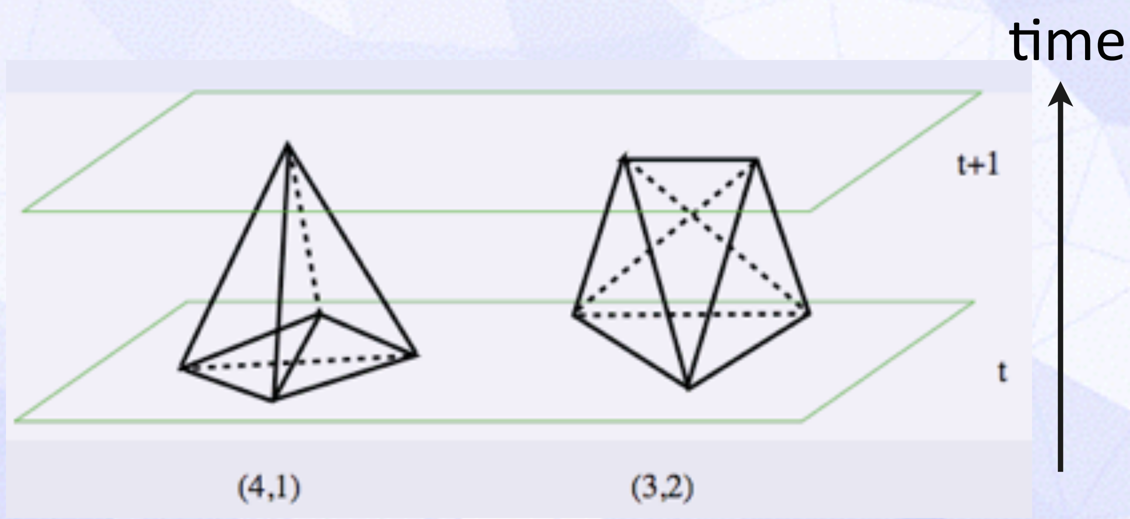

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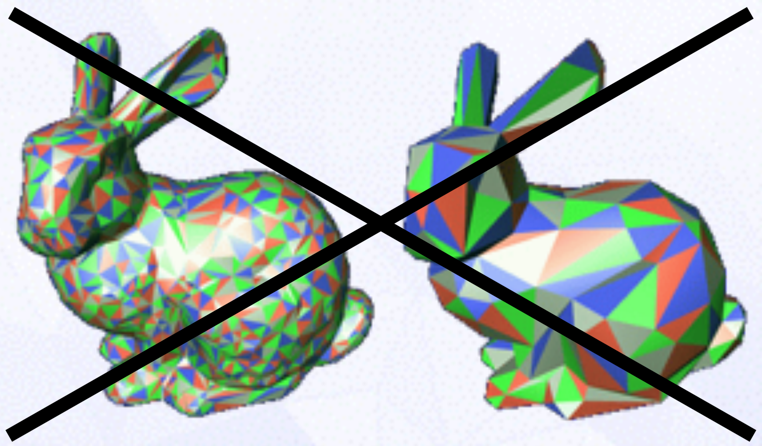

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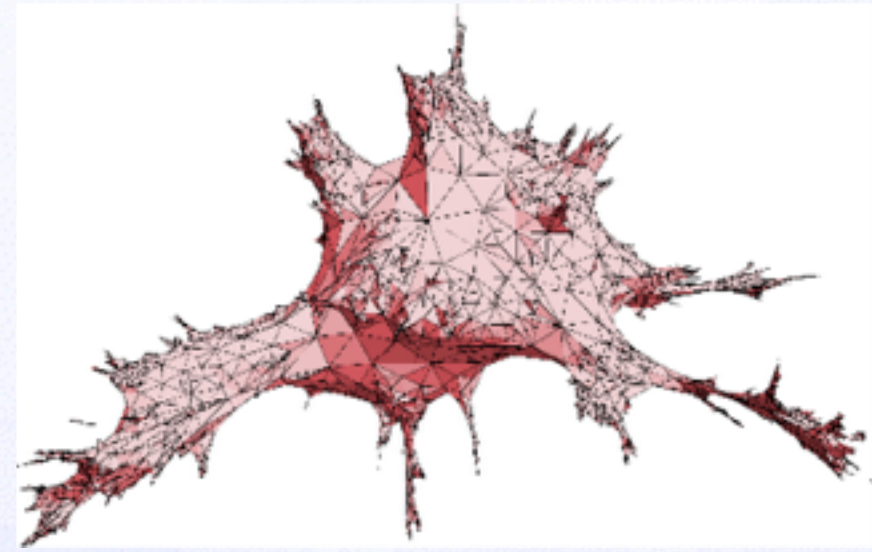

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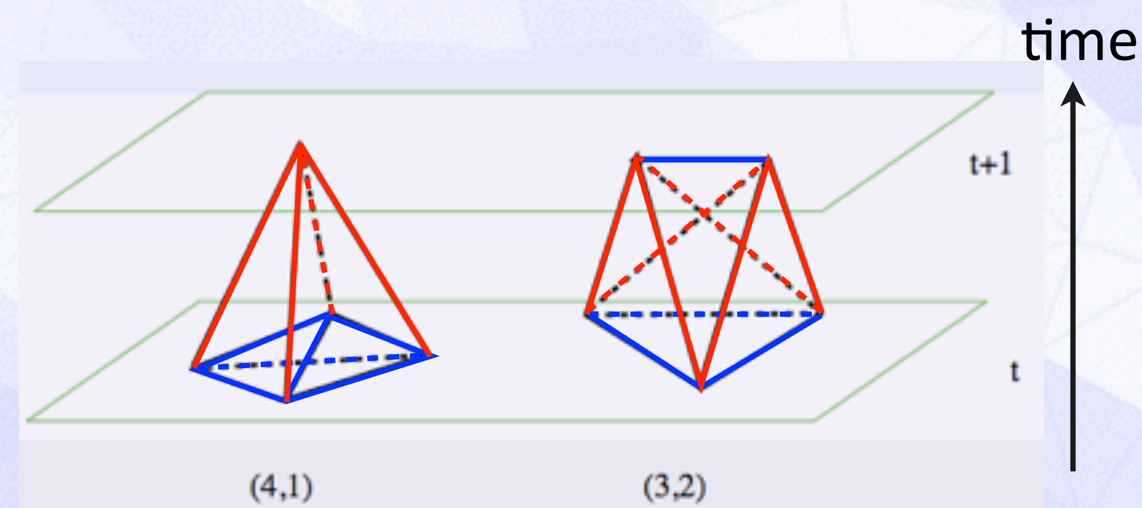


simplicial 4d building blocks of CDT

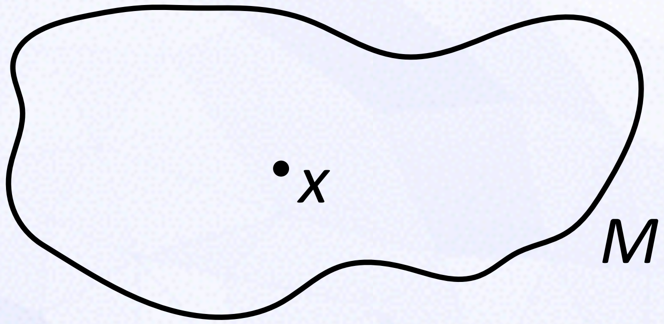
spacelike edge, squared length a^2

timelike edge, squared length $-\alpha a^2$, $\alpha > 0$

Everything we have learned about "quantum spacetime" in CDT comes from measuring <u>quantum observables</u>.

The story of observables: classical

- Recall that four-dimensional spacetime diffeomorphisms (coordinate transformations) are the invariance group of general relativity.
- Classical gravitational observables are diffeomorphism-invariant (and therefore nonlocal) quantities.
  For example, $g_{\mu\nu}(x)$ is not an observable,
  while $\int_M d^4x \sqrt{g}\, R(x)$ is.



- In continuum approaches to quantum gravity, implementing diffeomorphism invariance is a source of endless problems, e.g.
  - *path integral*: must gauge-fix à la Faddeev-Popov
  - *canonical quantum gravity*: must solve the quantum constraints
- In causal dynamical triangulations, what are quantum-gravitational **observables** in the absence of diffeomorphisms?

[health warning: "observable" does not imply a direct link to phenomenology]

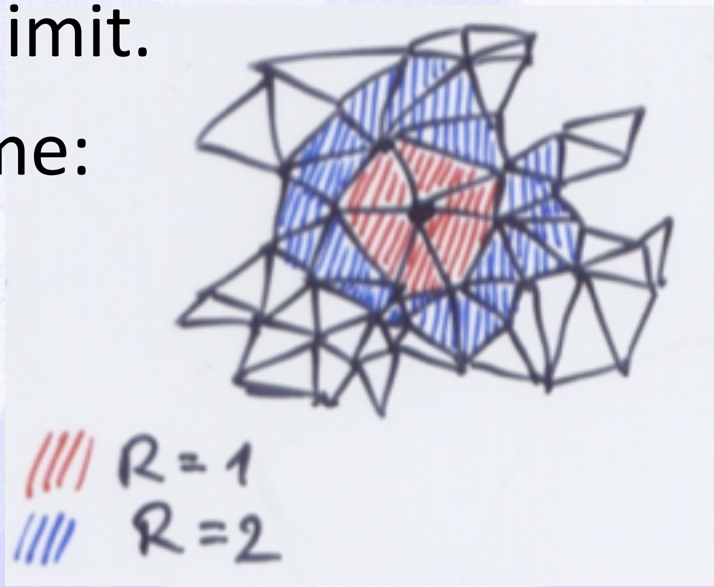
The story of observables: quantum

- Observables are (still) formulated in terms of geometric notions, like geodesic distances and volumes, which do not rely on smoothness.
- A quantity "at a point" is still not a meaningful concept, and must be averaged over spacetime, in addition to 'summing over geometries'.
- We are interested in how (the expectation values of) observables depend on scale, because we want to quantify their quantum behaviour and their classical limit.
- → **good quantum observables** are (i) geometric, (ii) scalable, (iii) finite, (iv) computable, and (v) have a classical limit.
- prototype: Hausdorff dimension d_H of spacetime:

  measuring the volume of geodesic balls,

$$\langle \mathrm{Vol}(B_R) \rangle \propto R^{d_H},$$

  d_H in general depends on the scale R.



geodesic balls B_R of radius R in 2D

Nonperturbative "geometry" behaves strangely

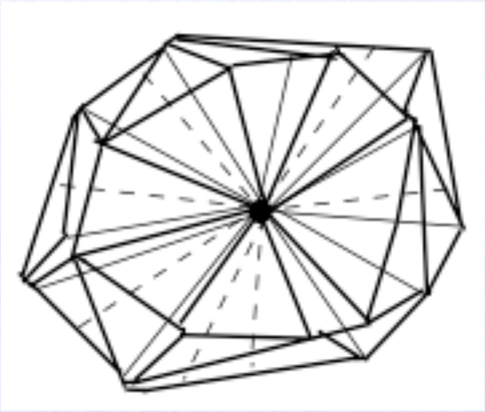
Isn't it obvious that by gluing together four-dimensional building blocks, one will obtain a (quantum) spacetime of dimension 4?

No. Generically it does not happen when quantum fluctuations are large.

This was only gradually understood, using computer "experiments". In DT models prior to CDT, one of two things happened to "quantum geometry":

Hausdorff dimension

it polymerized (small G_N^{bare}), $d_H = 2$



it crumpled (large G_N^{bare}), $d_H = \infty$

This degenerate behaviour is generic for (Euclidean) DT in dimension d > 2. Branched polymers are a generic finding of stat mech models of QG.

Causal DT was invented to cure this problem and appears to do so!

Dimension is not what it used to be …

Totally unexpected: spacetime dimension, a “pregeometric” property, becomes dynamical in the presence of large curvature fluctuations.

The absence of any regime where the dimension at large scales is equal to 4 is enough to rule out a candidate theory of quantum gravity!

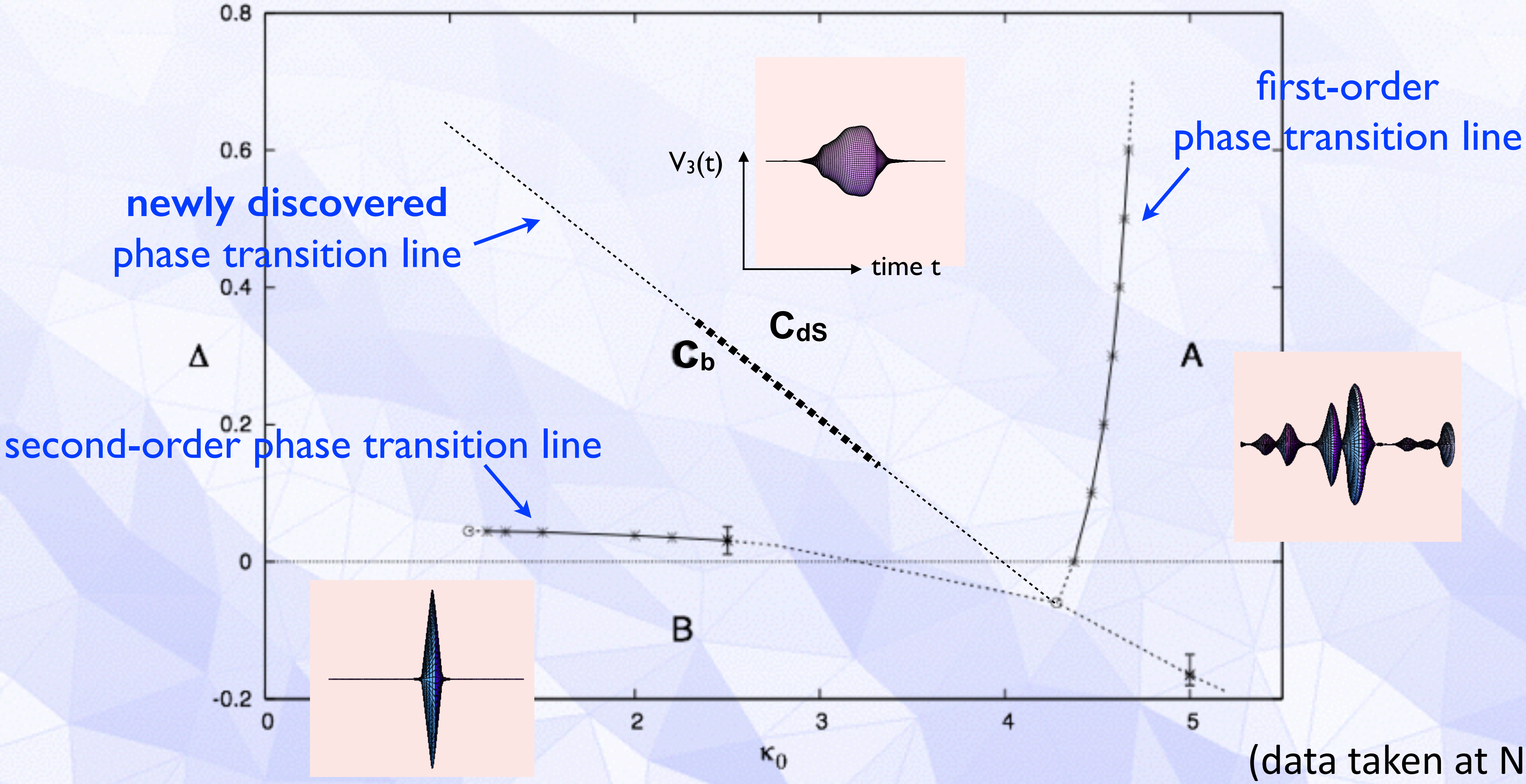
Besides the Hausdorff dimension, one can also measure the quantum geometry’s spectral dimension (by setting up a diffusion process).

“Dimension” in nonperturbative quantum gravity is no longer fixed a priori, but reflects a particular quantum dynamics. It is *not* pre-determined by the dimensionality of the triangular building blocks used.

Also in CDT quantum gravity, the dimension is dynamical, but in a part of phase space it is now equal to 4, within measuring accuracy!

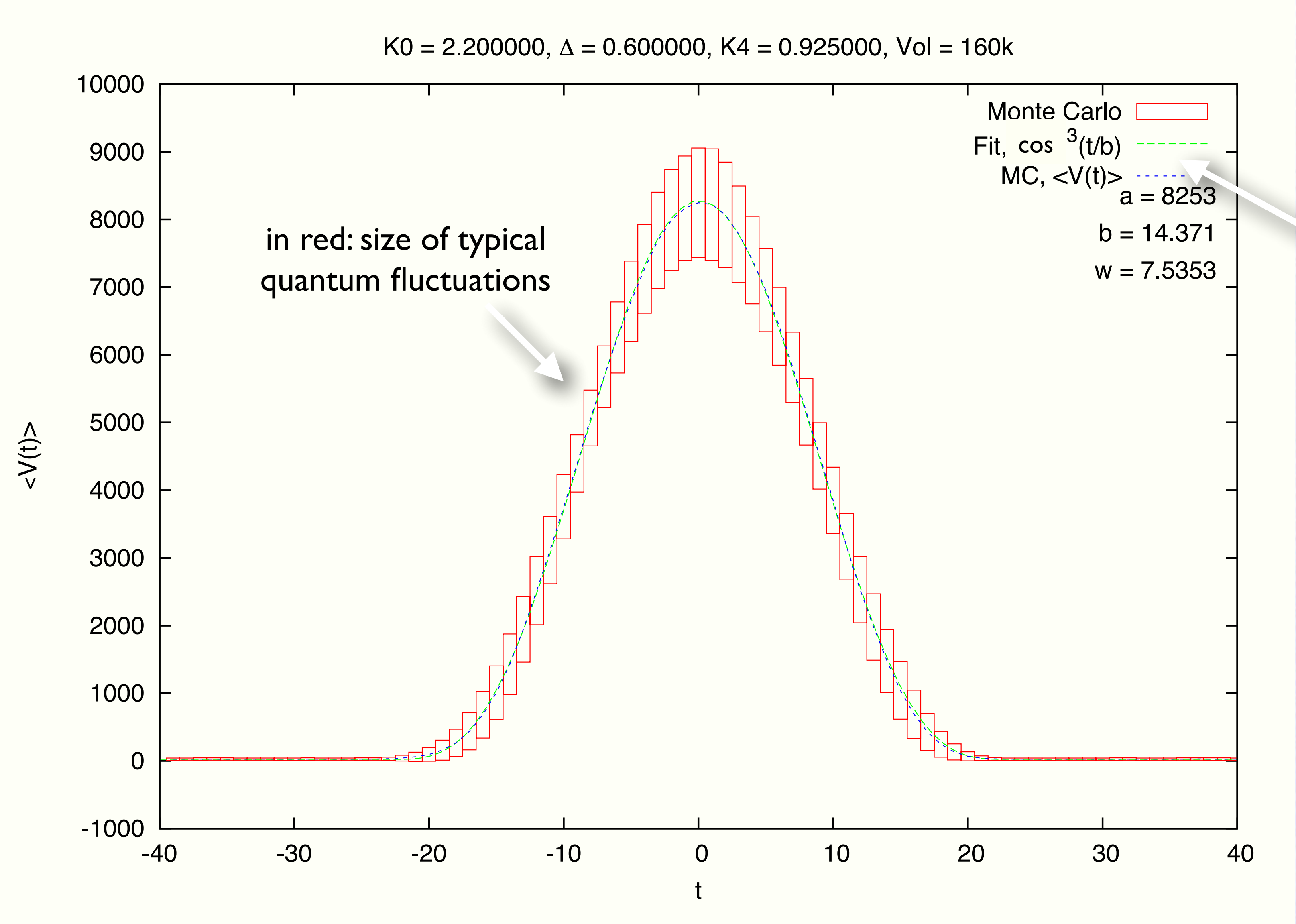
Phase diagram of CDT quantum gravity

Phases are characterized by their "volume profiles" $\langle V_3(t) \rangle$. In the de Sitter phase C_{dS} and the bifurcation phase C_b the large-scale dimension of the dynamically generated "quantum spacetime" appears to be 4 and therefore compatible with General Relativity.



(data taken at N_4=80.000)

"CDT Classic": universal de Sitter-like volume profile in phases C



fit to $\cos^3(t/b)$

The measured average volume profile $<V_3(t)>$ of the universe, as function of Euclidean proper time t, matches to great accuracy a corresponding minisuperspace calculation derived from GR.

The classical line element of Euclidean de Sitter space, derived by *assuming homogeneity and isotropy a priori*, as function of Euclidean proper time *t=iτ*, is

$$ds^2 = dt^2 + a(t)^2 d\Omega^2_{(3)} = dt^2 + c^2 \cos^2\left(\frac{t}{c}\right) d\Omega^2_{(3)}$$

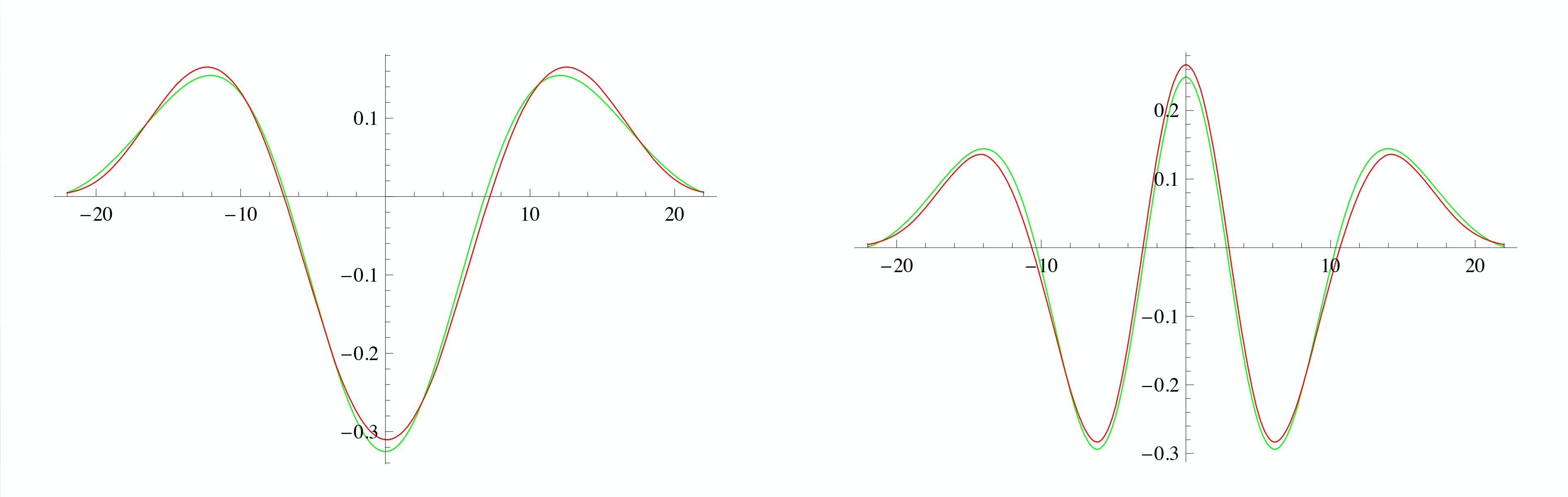
scale factor

volume el. S^3

In addition, expanding the minisuperspace action around the de Sitter solution,

$$S_{\rm eu}(V_3) = S(V_3^{\rm dS}) + \kappa \int dt\ \delta V_3(t) \hat{H} \delta V_3(t)$$

the eigenmodes of $\hat{H}$ match well with those extracted from the simulations:

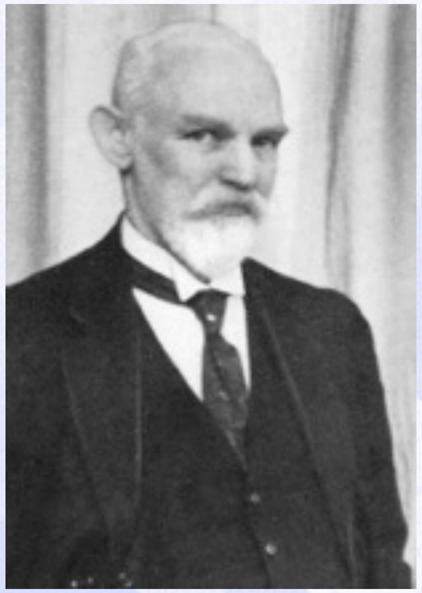


(N.B.: no further fitting necessary)

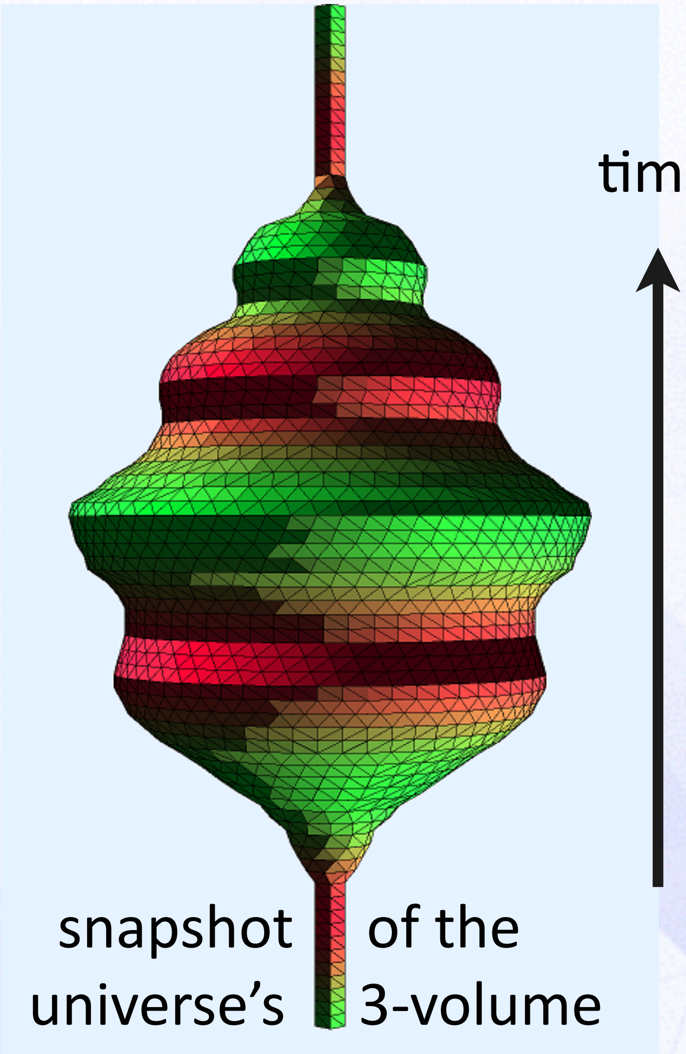
(J. Ambjørn, A. Görlich, J. Jurkiewicz, RL, PRL 100 (2008) 091304, PRD 78 (2008) 063544, NPB 849 (2011) 144 (with J. Gizbert-Studnicki, T. Trzesniewski))

Dynamical emergence of spacetime (out of "quantum foam")

For suitable choice of couplings, CDT quantum gravity dynamically produces a "<u>quantum spacetime</u>", that is, a ground state ("vacuum"), whose macroscopic scaling properties are ***four-dimensional*** and whose macroscopic shape is that of a well known classical cosmology, ***de Sitter space***. The matching of <u>shapes</u> (incl. their quantum fluctuations) is excellent. In nonperturbative, background-independent gravity, this is unprecedented.



Willem de Sitter



(ensemble average gives the "shape")

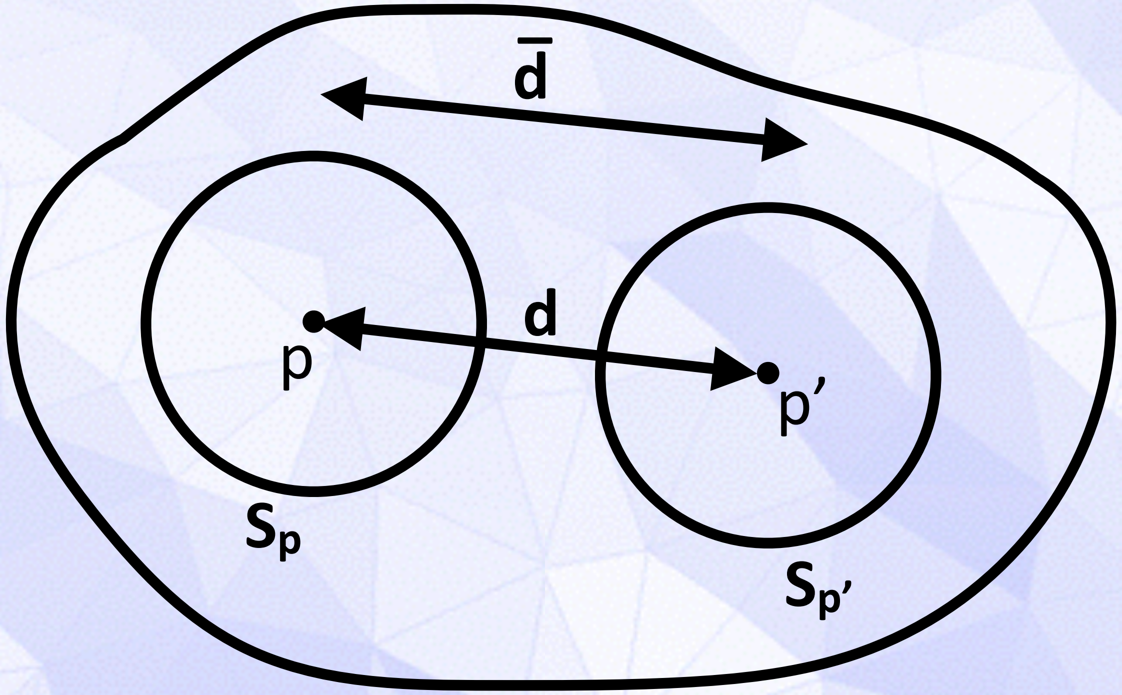
The region in phase space where we see interesting physics is far away from the perturbative regime. Quantum fluctuations are large and local geometry is highly nonclassical (N.B.: these fluctuations are <u>*not*</u> shown in the snapshot).

Can we probe more local geometric features, by using suitable observables?

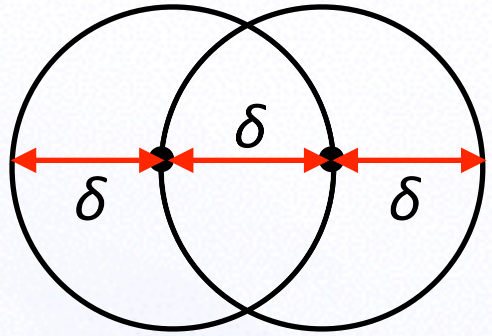
A new observable: quantum Ricci curvature

- Curvature is a crucial property of classical spacetime, but computing the Riemann tensor $R^{\kappa}{}_{\lambda\mu\nu}[g,\partial g,\partial^2 g;x)$ requires a smooth metric *g*.
- Finding a meaningful "quantum curvature" observable, applicable in nonperturbative quantum gravity, has so far received little attention.
- We adapt a classical characterization of curvature to construct a scalable, robust and computable notion of quantum Ricci curvature.

- key is the sphere-distance criterion: *"On a metric space with positive (negative) Ricci curvature, the distance $\bar{d}$ of two nearby spheres S_p and $S_{p'}$ is smaller (bigger) than the distance d of their centres."*



Our variant uses the ***average sphere distance*** of two spheres of radius δ whose centres are a distance δ apart,



$$\bar{d}(S_p^\delta, S_{p'}^\delta) := \frac{1}{vol(S_p^\delta)} \frac{1}{vol(S_{p'}^\delta)} \int_{S_p^\delta} d^{D-1}q\,\sqrt{h} \int_{S_{p'}^\delta} d^{D-1}q'\,\sqrt{h'}\; d(q,q'),$$

which is defined purely in terms of volume and distance measurements and therefore can be implemented on general metric spaces.

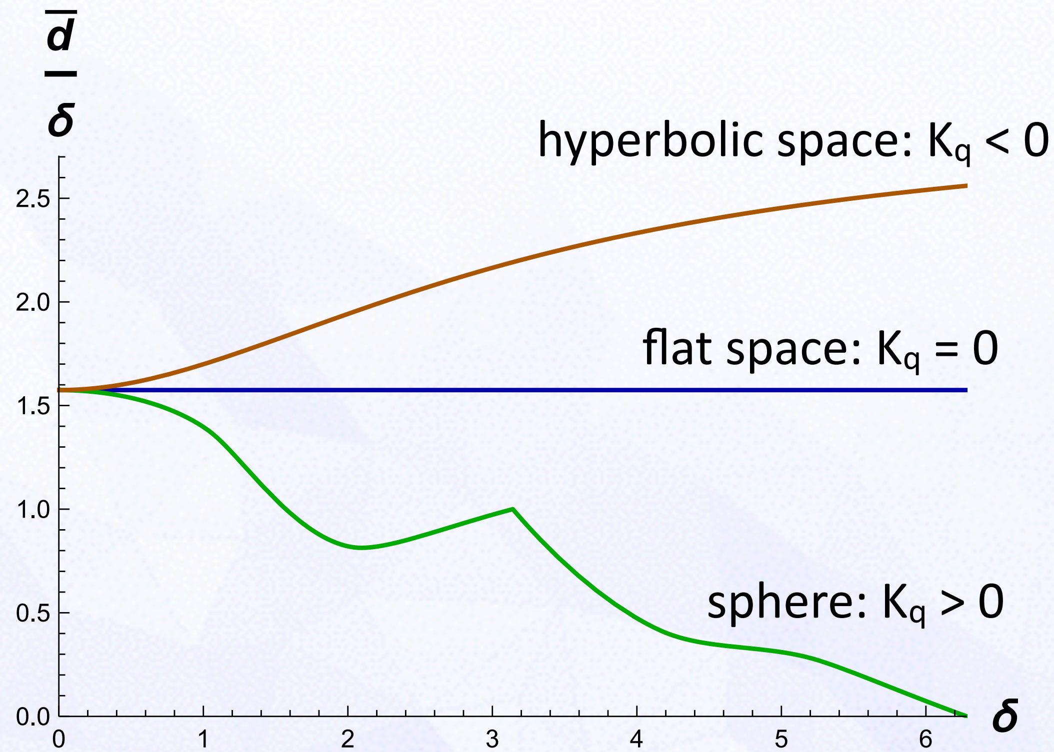
The **"quantum Ricci curvature K_q at scale δ"** is then

$$\frac{\bar{d}(S_p^\delta, S_{p'}^\delta)}{\delta} = c_q(1 - K_q(p,p')), \quad \delta = d(p,p'), \quad c_q > 0,$$

where c_q is a non-universal constant depending on the space under consideration. We have evaluated K_q on classical model spaces, and tested it on a variety of mainly 2D PL spaces (= triangulations), and computed it in nonperturbative 2D Euclidean quantum gravity.

(N. Klitgaard and RL, "Introducing Quantum Ricci curvature", on arXiv, "Quantizing Quantum Ricci curvature", to appear)

Quantum Ricci curvature in action



reference curves for the normalized average sphere distance $\bar{d}/\delta$ on 2D constantly curved continuum spaces

$$\text{recall} \quad \frac{\bar{d}}{\delta} = c_q(1 - K_q)$$

First quantum measurements of the expectation value

$$\langle \bar{d}(S_p^\delta, S_{p'}^\delta)/\delta \rangle$$

in 2D DT quantum gravity show that its curvature is best matched by that of a 4D(!) continuum sphere.

Summary

CDT quantum gravity comes with a simple set-up, only two tunable couplings and advanced computational tools. So far, this approach has been very fruitful. By studying a handful of observables, one has obtained remarkable and unique nonperturbative results, including

- the emergence of semiclassical 4D de Sitter space from Planckian "quantum foam" (without a priori background), and
- the presence of second-order phase transitions, which provide natural candidates for taking a scaling limit.

More quantum observables are needed to understand the nature of the newly found phase transition and of the (Planckian) physics near the 2nd-order phase transitions, and to improve the existing RG analysis (looking for evidence of asymptotic safety).

In a way, this is all good old quantum field theory, but with a twist …

Out-of-the-box Idea

Classical gravity has an "unusual" invariance group, which is intimately tied to the representation of spacetime as a smooth continuum, and which we do not expect to be relevant at the Planck scale.

To understand quantum gravity at a fundamental level, we must in more radical ways "let go of $g_{\mu\nu}(x)$ and associated smooth structures" and think more in terms of geometric observables.

The lessons learned so far from CDT quantum gravity, a **continuum, non-smooth approach**, are encouraging and show that this possible!

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Frascati, 19 Dec 2017

Thank you!