

# Hyperbolic Higgs, Clockworking/Linear Dilaton.

Rome  
Dec 18<sup>th</sup> 2017

Matthew McCullough



Superheroic Higgs,  
Clockwork and Dilaton,  
and The Renormalization

*Weird ideas?*

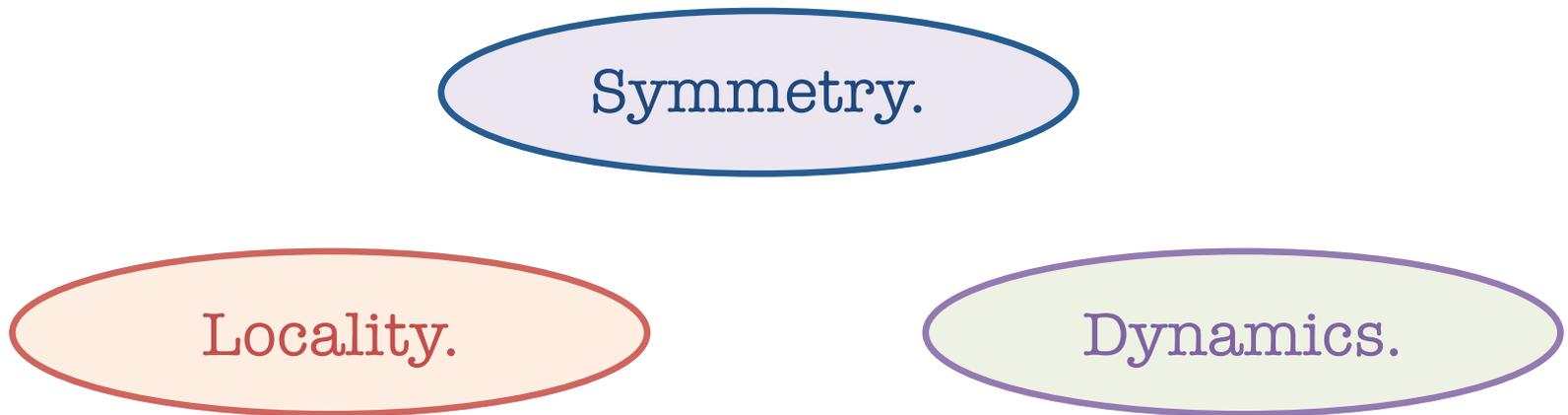
Rome  
Dec 18<sup>th</sup> 2017

Matthew McCullough



# Hierarchy Problem

Many\* approaches follow three basic paradigms...



This talk will cover/review three recent variations on these themes. Only the first two contain my own work.

# Hierarchy Problem

Many\* approaches follow three basic paradigms...

*In some sense, this talk will summarise mainstream model building. Weird enough? Open to interpretation...*

This talk will cover/review three recent variations on these themes. Only the first two contain my own work.

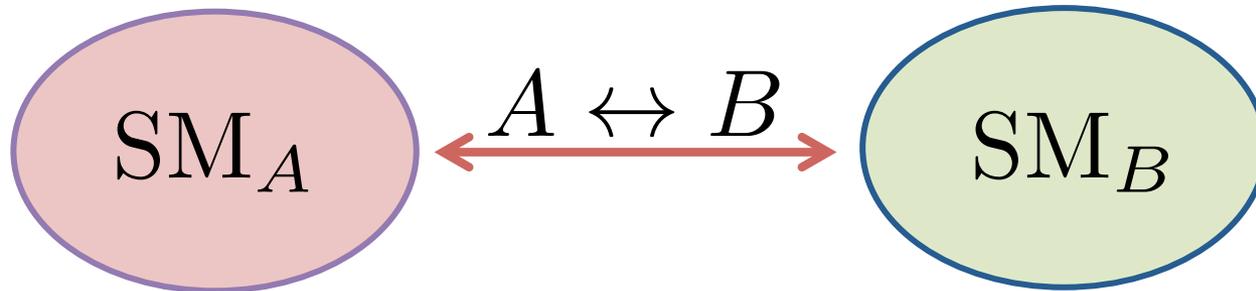
Part I

Symmetry

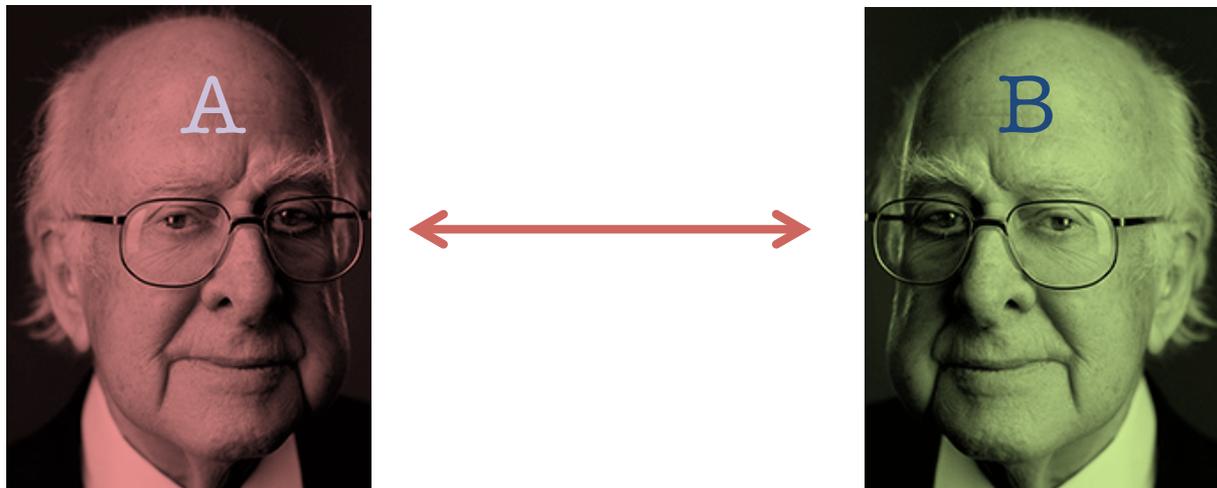
(This talk: Neutral Naturalness)

# Twin Higgs

- Take two identical copies of the Standard Model:

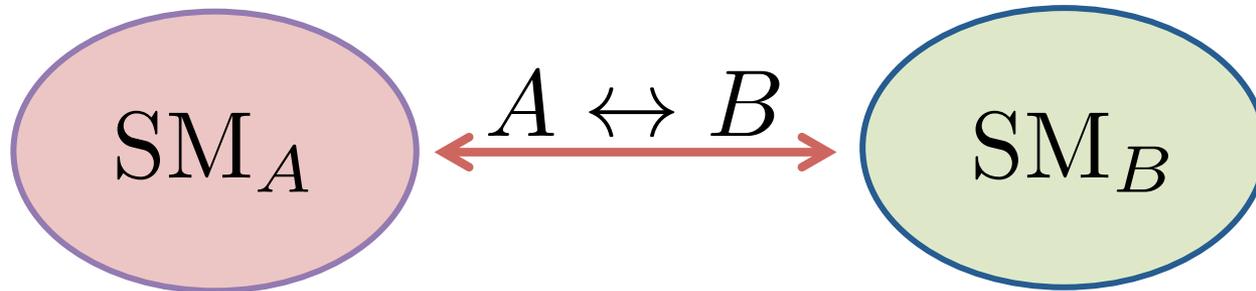


- Why would you want to do this?



# Twin Higgs

- Take two identical copies of the Standard Model:



- Enhance symmetry structure to global  $SU(4)$ :

Desired quartic dictated by accidental symmetry:

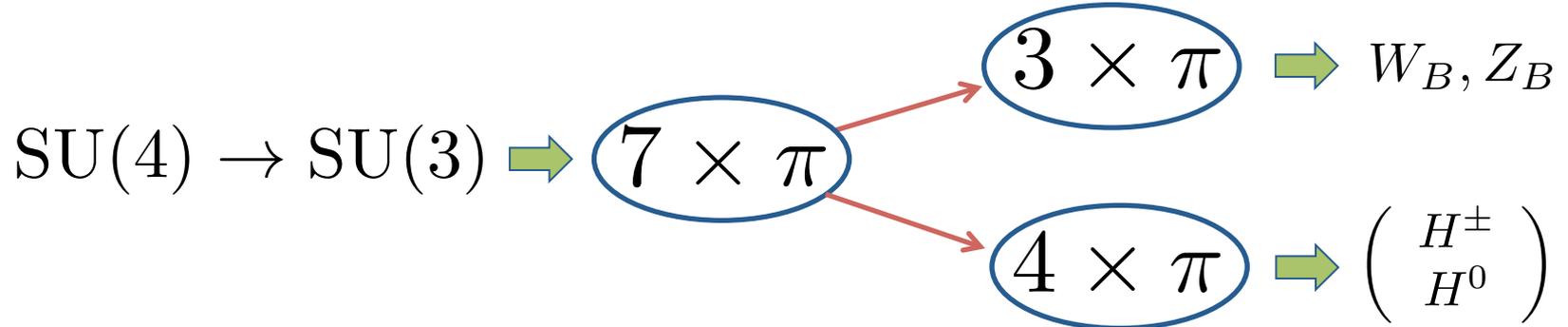
$$V_{\text{Higgs}} = \lambda (|H_A|^2 + |H_B|^2)^2 - \Lambda^2 (|H_A|^2 + |H_B|^2)$$

Exchange enforces equal quadratic corrections for each Higgs. Thus masses still respect  $SU(4)$  symmetry.

# Twin Higgs

- Total symmetry-breaking pattern is:  $SU(4) \rightarrow SU(3)$

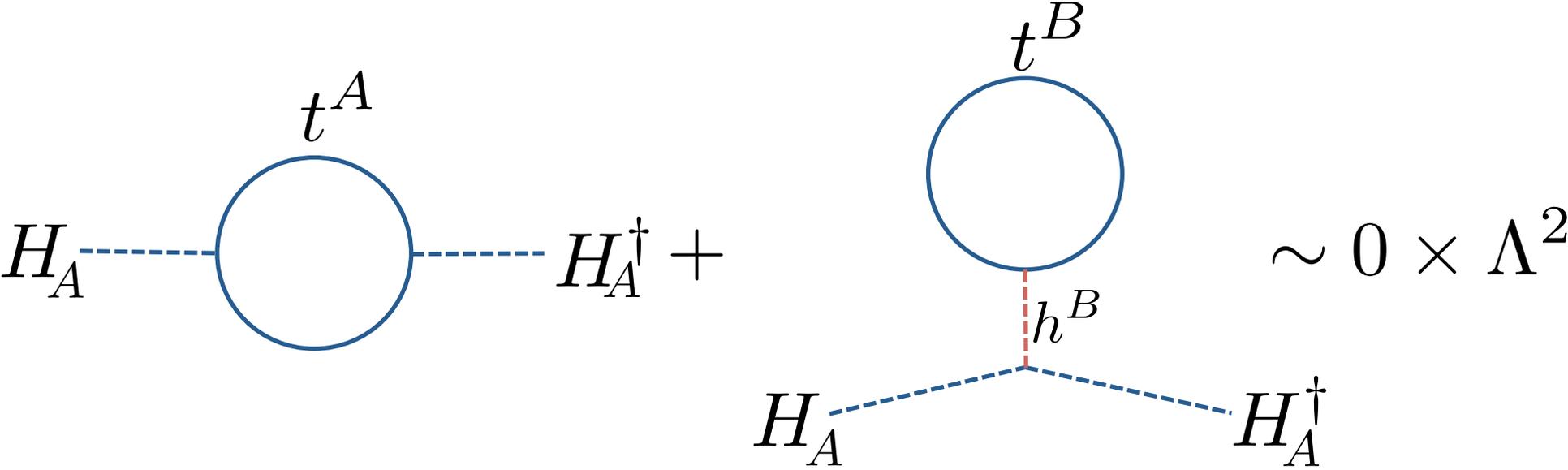
- Thus 7 pseudo-Goldstone bosons:



- The SM Higgs light because of the symmetry-breaking pattern!
- Hierarchy problem solved all the way up to the scale:  $\Lambda$

# Twin Higgs

- In usual “quadratic divergences” parlay:

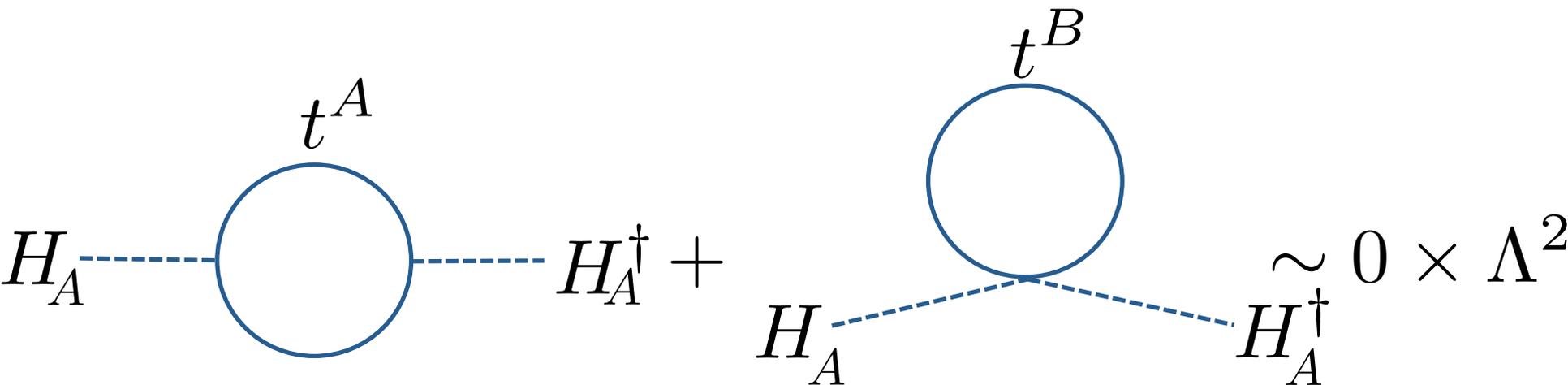


Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

# Twin Higgs

- In usual “quadratic divergences” parlay:

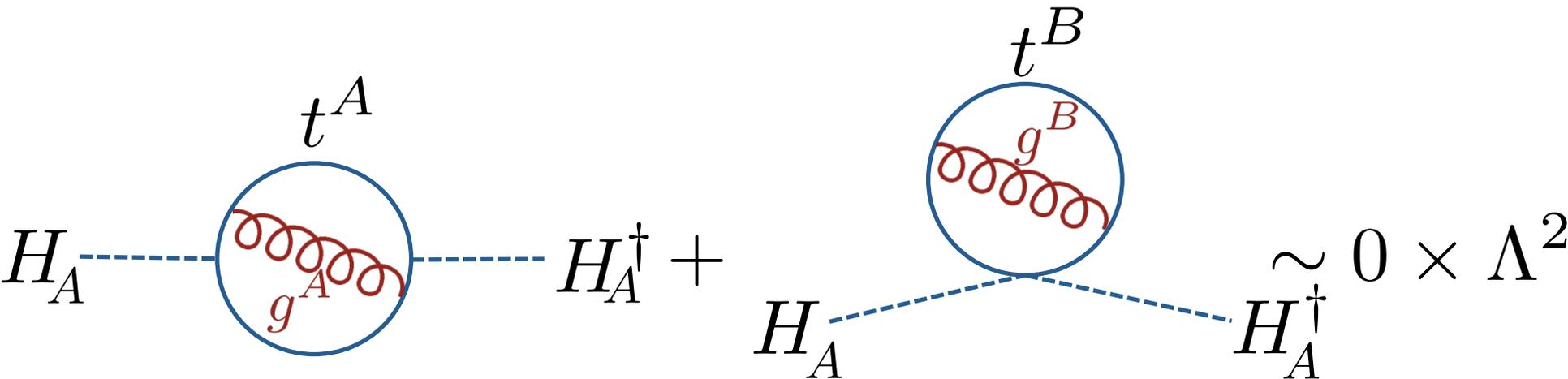


Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

# Twin Higgs

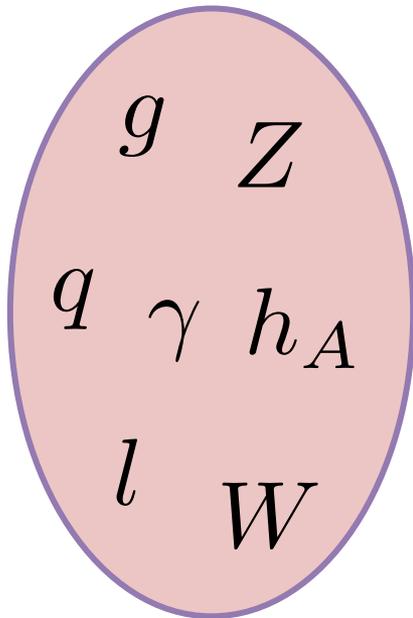
- In usual “quadratic divergences” parlay:



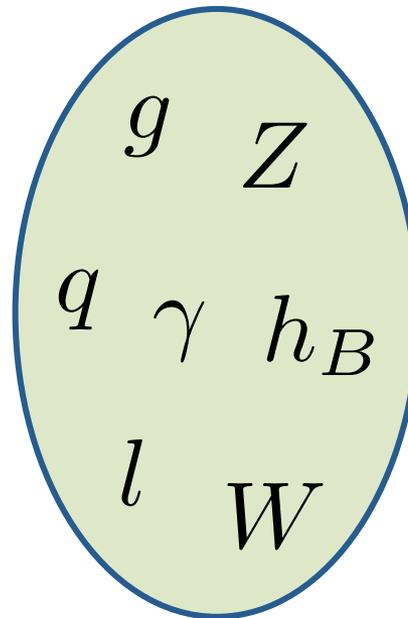
Quadratic divergences from SM top quark loops cancelled by loops of “Twin” top quarks.

- Cancellation persists for all Twin particles: Twin W-bosons, Twin gluons, etc.

Standard  
Model



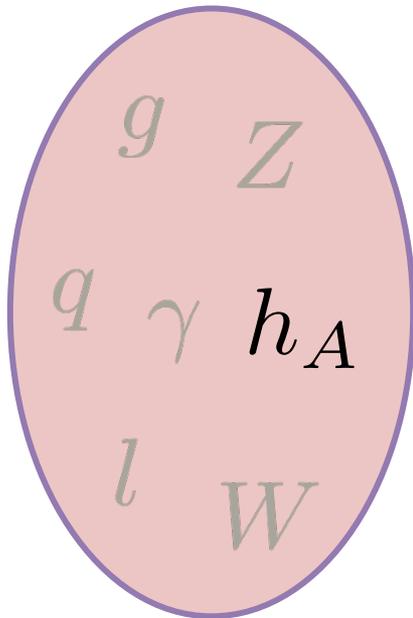
“Twin”  
Standard  
Model



These fields  
completely  
neutral:  
“Neutral  
Naturalness”

Predictions for Twin sector most robust for the Twins  
of the SM fields that couple most strongly to Higgs.

Standard  
Model

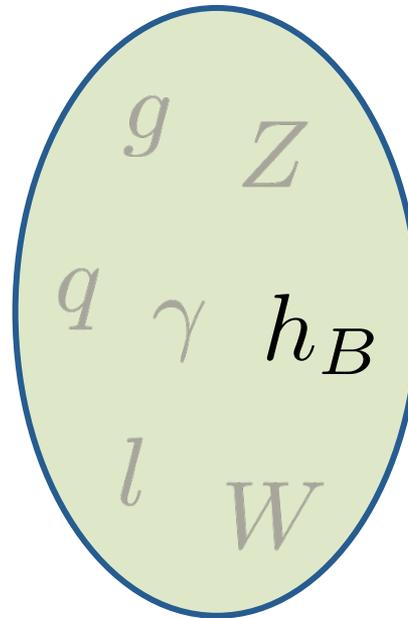


$\sim m^2 h_A h_B$

A double-headed orange arrow pointing between the two model ovals.

Only  
communication  
through small  
“Higgs Portal”  
mixing

“Twin”  
Standard  
Model



These fields  
completely  
neutral:  
“Neutral  
Naturalness”



# Hyperbolic Higgs

In progress...  
Cohen, Craig,  
Giudice, MM.

- The landscape of top partners:

		<i>scalar</i>	<i>fermion</i>
<i>strong direct production</i> {	<i>QCD</i>	SUSY	Composite Higgs/ RS
<i>DY direct production</i> {	<i>EW</i>	folded SUSY	Quirky Little Higgs
<i>Higgs portal direct production</i> {	<i>singlet</i>	?	Twin Higgs

Table from Curtin and Verhaaren.

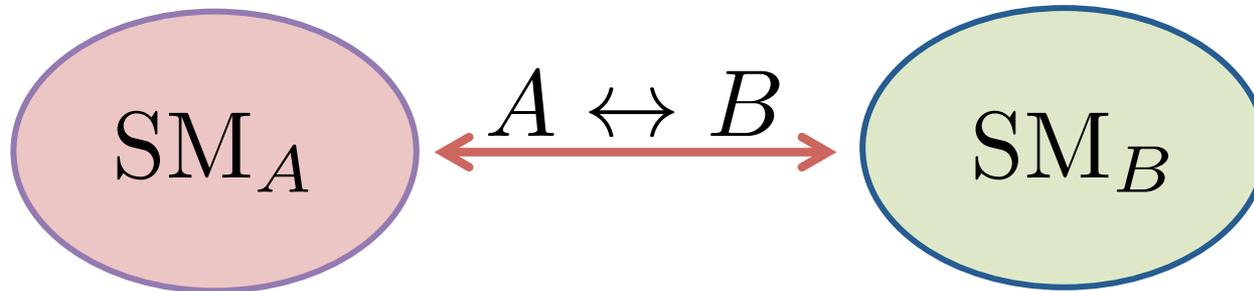
Mirror Glueballs      Higgs coupling shifts  
Higgs portal observables      ~ tuning

- This section: The last box.

# Hyperbolic Higgs

In progress...  
Cohen, Craig,  
Giudice, MM.

- Take two identical copies of the MSSM:



- Take a large D-term with equal and opposite charges for Higgses:

$$V_{\mathcal{H}} = \frac{g_{\mathcal{H}}^2}{2} (|H|^2 - |H_{\mathcal{H}}|^2)^2$$

This enforces that the scalar potential respects an accidental  $SU(2,2)$  symmetry. Not symmetry of theory.

# Hyperbolic Higgs

In progress...  
Cohen, Craig,  
Giudice, MM.

- Remove **scalar matter in A**, and **fermions in B**:

$$\mathcal{L} = \lambda_t H \psi_Q \psi_{U^c} + \text{h.c.}$$

$$+ \lambda_t^2 \left( |H_{\mathcal{H}} \cdot \tilde{Q}_{\mathcal{H}}|^2 + |H_{\mathcal{H}}|^2 |\tilde{U}_{\mathcal{H}}^c|^2 \right)$$

- Quadratic corrections respect the accidental SU(2,2) symmetry:

$$V_{\mathcal{H}} = -\Lambda^2 \left( |H|^2 - |H_{\mathcal{H}}|^2 \right) + \frac{g_{\mathcal{H}}^2}{2} \left( |H|^2 - |H_{\mathcal{H}}|^2 \right)^2$$

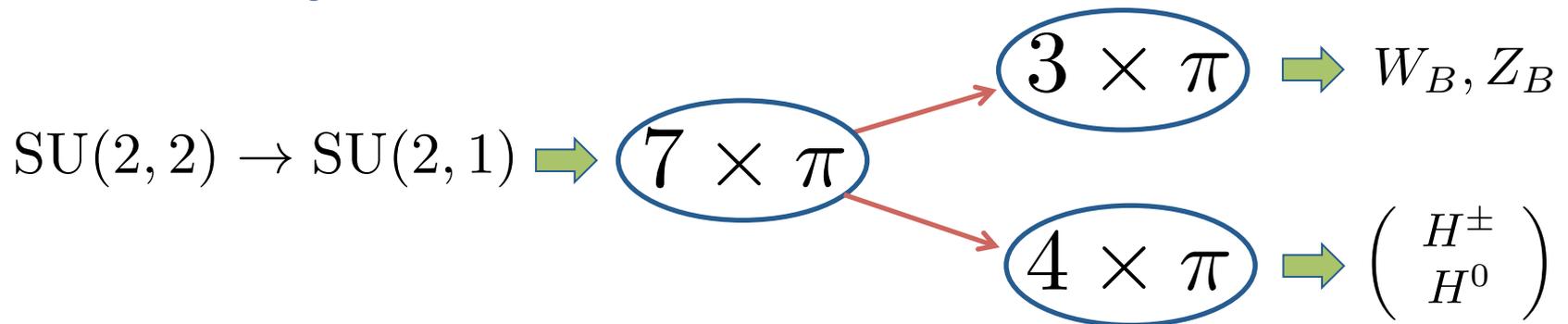
Thus, at level of one-loop corrections, scalar potential respects an accidental SU(2,2) symmetry.

# Hyperbolic Higgs

- Total symmetry-breaking pattern is:

$$SU(2, 2) \rightarrow SU(2, 1)$$

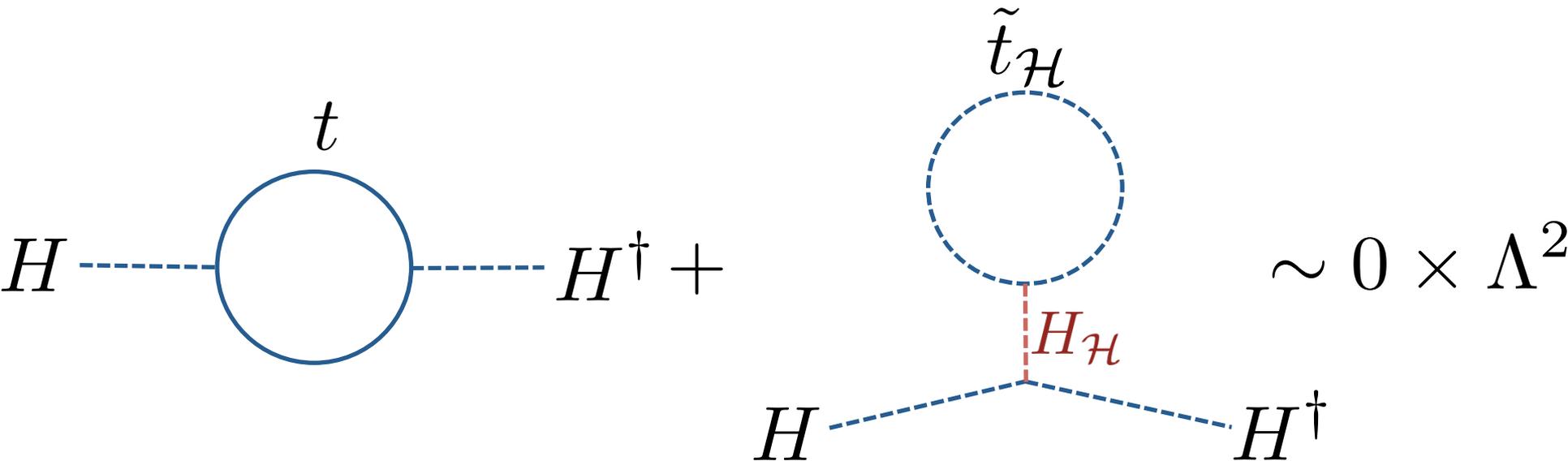
- Thus 7 Quasi-Goldstone bosons:



- The SM Higgs light because of the symmetry-breaking pattern!
- Higgs not really a Goldstone. More like an accidental flat direction...

# Hyperbolic Higgs

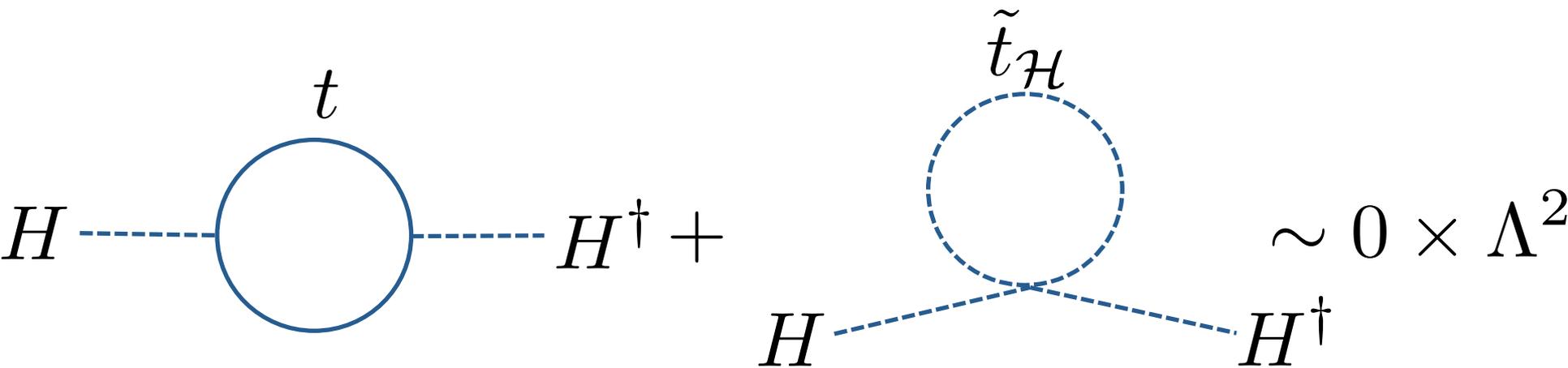
- In usual “quadratic divergences” parlay:



Quadratic divergences from SM top quark loops cancelled by loops of “Hyperbolic” stop squarks.

# Hyperbolic Higgs

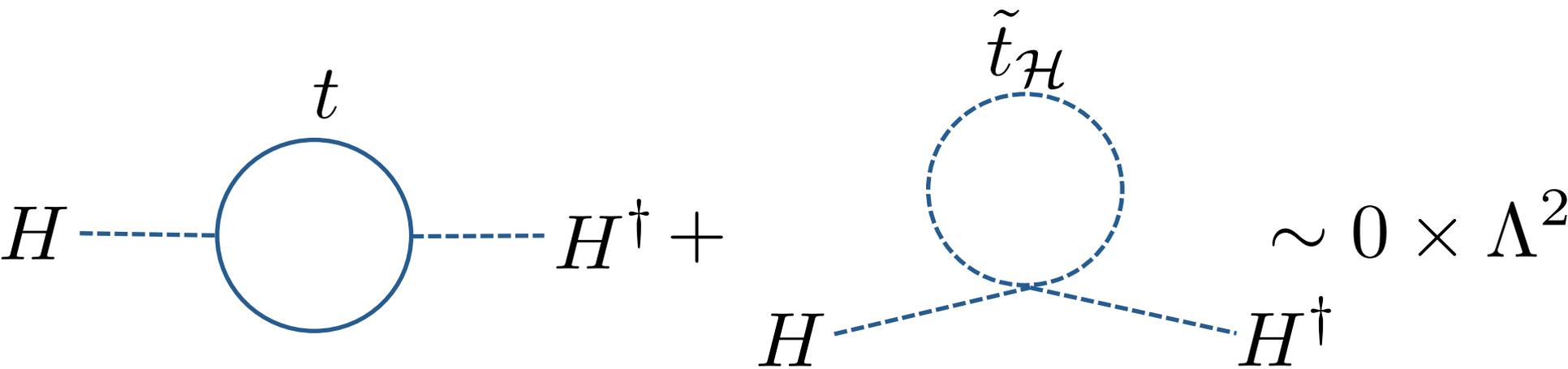
- In usual “quadratic divergences” parlay:



Quadratic divergences from SM top quark loops cancelled by loops of “Hyperbolic” stop squarks.

# Hyperbolic Higgs

- In usual “quadratic divergences” parlay:



Quadratic divergences from SM top quark loops cancelled by loops of “Hyperbolic” stop squarks.

$$\mathcal{L} \sim \lambda_t H \psi_Q \psi_{U^c} + \text{h.c.} + \lambda_t^2 |H|^2 \left( |\tilde{t}_{\mathcal{H}}^L|^2 + |\tilde{t}_{\mathcal{H}}^R|^2 \right)$$

# UV-Completion

- Scherk-Schwarz provides a natural home for the top sector. Take a flat extra dimension:

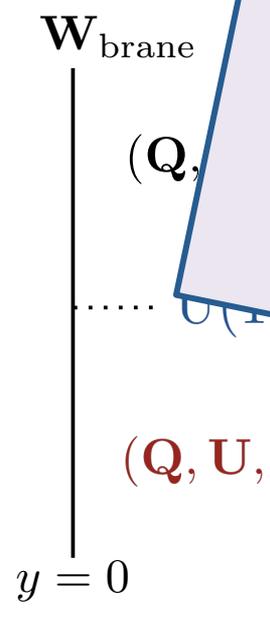
$V_{\text{CW}}(H) = \frac{1}{2} \text{Tr} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4}$   
 $\times \log \frac{p^2 + (n + q_B)^2/R^2 + M^2(H)}{p^2 + (n + q_F)^2/R^2 + M^2(H)}$

$V_{\text{CW}}(H_{\mathcal{H}}) = \frac{1}{2} \text{Tr} \sum_{n=-\infty}^{\infty} \int \frac{d^4 p}{(2\pi)^4}$   
 $\times \log \frac{p^2 + (n + q_F)^2/R^2 + M^2(H_{\mathcal{H}})}{p^2 + (n + q_B)^2/R^2 + M^2(H_{\mathcal{H}})}$

- Scherk-Schwarz: “project out” modes and automatically give opposite sign corrections!

# UV-Completion

- Scherk-Schwarz provides a natural home for the top sector in extra dimension:



One-loop corrections:

$$V_{CW} \ni -\frac{7 \zeta(3) \lambda_t^2}{32 \pi^2 (\pi R)^2} \left\{ 3 |H|^2 - 3 |H_{\mathcal{H}}|^2 - |Q_{\mathcal{H}}|^2 - 2 |U_{\mathcal{H}}^c|^2 \right\} \frac{M^2(H)}{M^2(H)}$$

$$V_{CW}(H_{\mathcal{H}}) = \frac{1}{2} \sum_{n=-\infty}^{\infty} \log \frac{p^2 + (n + q_F)^2 / R^2 + M^2(H_{\mathcal{H}})}{p^2 + (n + q_B)^2 / R^2 + M^2(H_{\mathcal{H}})}$$

- Scherk-Schwarz: “project out” modes and automatically give opposite sign corrections.

# A Shallow Grave.

- We also need the Hyperbolic quartic. Use gauge D-term, but haven't seen a new gauge force...

$$V_{U(1)_{\mathcal{H}}} \ni \frac{g_{\mathcal{H}}^2}{2} \xi \left( |H_{\mathcal{H}}|^2 - |H|^2 - f_{\mathcal{H}}^2 \right)^2$$

- Supersymmetric breaking: D-term vanishes. Must have SUSY breaking, parameterised by

$$\xi = \left( 1 - \frac{M_V^2}{M_S^2} \right)$$

- But this feeds into U(2,2) violating soft masses!

$$V_{U(1)_{\mathcal{H}}} \ni -\frac{g_{\mathcal{H}}^2 M_V^2}{16 \pi^2} \log (1 - \xi) \left( |H_{\mathcal{H}}|^2 + |H|^2 \right)$$

# A Shallow Grave.

- We also need the Hyperbolic quartic. Use gauge D-term, ... a new gauge force...

No free lunch....

Internal tension between LEP, SUSY-breaking, and desired  $U(2,2)$  symmetry.

Fine-tuning? May not be much better than a Scherk-Schwarz MSSM, such as 1404.7554.

Must

- But this feeds into  $U(2,2)$  violating ...

$$V_{U(1)_{\mathcal{H}}} \ni -\frac{g_{\mathcal{H}}^2 M_V^2}{16 \pi^2} \log(1 - \xi) \left( |H_{\mathcal{H}}|^2 + |H|^2 \right)$$

# Phenomenology

Phenomenology has not been studied, however one aspect could be radically different to Twin. If...

$$\langle \tilde{t}_{\mathcal{H}} \rangle \neq 0$$

Then:

- Hyperbolic QCD is broken, so no glueball signatures, no hidden sector hadronisation.
- Longitudinal modes of Hyperbolic Gluons are Top Partners!
- Radial modes of Hyperbolic Stops mix with Higgs, so Higgs becomes, partially, its own top partner!

# Part II

## Locality

(This talk: Linear Dilaton/Clockworking)

# A Clockwork Scalar

Choi & Im,  
Kaplan &  
Rattazzi. See  
also Dvali.

Take  $N+1$  copies of spontaneously broken global  $U(1)$ .  
At low energies only have Goldstones:

$$\phi_j \sim \frac{f}{\sqrt{2}} e^{i\pi_j/f} \quad , \quad j = 0, \dots, N$$

Now explicitly break  $N$  of the  $U(1)$  symmetries with  
spurions,

$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

This action is justified by symmetry assignments for  
spurions.

# A Clockwork Scalar

Take  $N+1$  copies of original story, assume  $\lambda \approx 1$ , such that at low energies only have Goldstones:

$$\phi_j \sim \frac{f}{\sqrt{2}} e^{i\pi_j/f} \quad , \quad j = 0, \dots, N$$

Now explicitly break  $N$  of the  $U(1)$  symmetries with spurions,

$$\mathcal{L} = \mathcal{L}(\phi_j) - \sum_{j=0}^{N-1} \epsilon \phi_j^* \phi_{j+1}^3 + h.c.$$

Can take other “q”

This action is justified by symmetry assignments for spurions.

# A Clockwork Scalar

Action given by

$$\mathcal{L} = \frac{1}{2} \sum_{j=0}^N (\partial_\mu \pi_j)^2 - \frac{m^2 f^2}{2} \sum_{j=0}^{N-1} \left( e^{\frac{i}{f} (q\pi_{j+1} - \pi_j)} + h.c. \right)$$


**Spontaneous** symmetry breaking pattern:

$$U(1)^{N+1} \rightarrow \emptyset$$

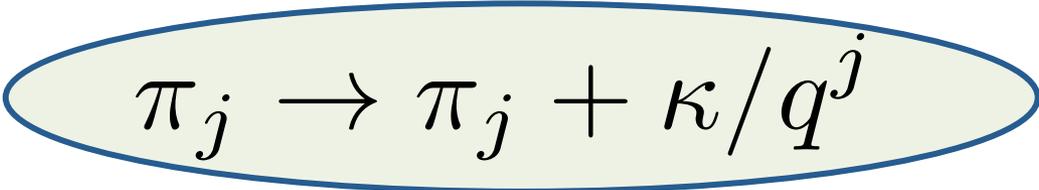
So expect  $N + 1$  Goldstones.

**Explicit** symmetry breaking:

$$U(1)^{N+1} \rightarrow U(1)$$

So expect  $N$  pseudo-Goldstones and one true Goldstone.

Can identify true Goldstone direction from remaining shift symmetry


$$\pi_j \rightarrow \pi_j + \kappa / q^j$$

# A Clockwork Scalar

Identify Goldstone couplings by promoting shift parameter to a field:

$$\pi_j \rightarrow \pi_j + a(x)/q^j$$

Now, imagine we had some fields charged under last  $U(1)_N$ , thus coupled to  $\pi_N$ . Coupling to massless Goldstone becomes:

$$\frac{\pi_N}{f} \rightarrow \frac{a_0}{q^N f}$$

Exponentially small coupling has been generated from a theory with no exponential parameters!

# A Clockwork Scalar

Peculiar spectrum, reminiscent of Condensed Matter...

## Mass matrix

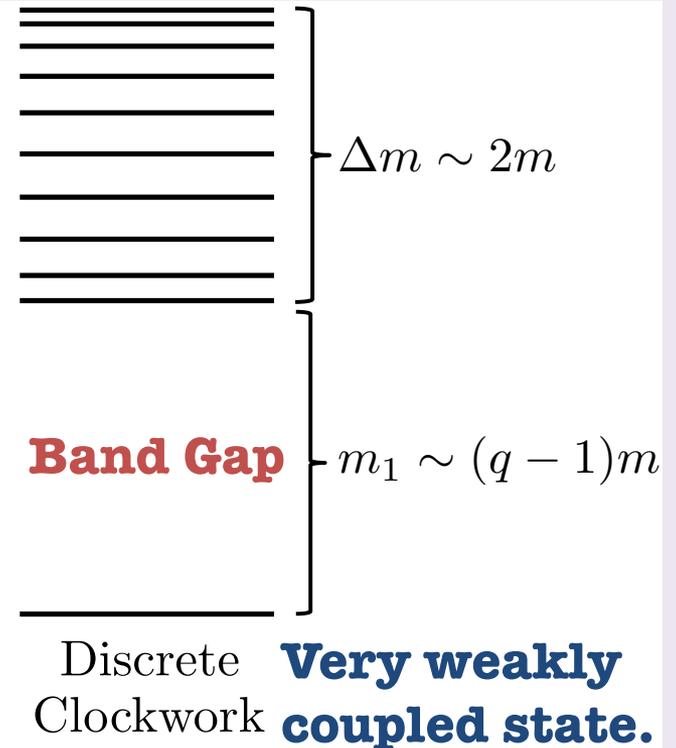
$$M_\pi^2 = m^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}.$$

## Eigenvalues for “Clockwork Gears”

$$m_{a_k}^2 = \left( q^2 + 1 - 2q \cos \frac{k\pi}{N+1} \right) m^2$$

$k = 1, \dots, N$

## Mass spectrum



How might this be useful in practise?

# Continuum Clockworking / Linear Dilaton Model

For details, ask me afterwards. Short story:  
There is a solution to Einstein's equations for  
gravity + dilaton with the metric

$$ds^2 = e^{\frac{4k|y|}{3}} (dx^2 + dy^2)$$

that offers an extra-dimensional approach to the  
hierarchy problem with a very different  
phenomenology to RS or LED. Proposed by  
Antoniadis, Arvanitaki, Dimopoulos, Giveon.

# Continuum Clockworking / Linear Dilaton Model

For more details, ask me afterwards. Short story:  
This theory shows up when you take the  
continuum limit of the clockwork model.

$$ds^2 = e^{\frac{4\kappa|y|}{3}} (dx^2 + \dots)$$

that offers an extra-dimensional approach to the  
hierarchy problem with a very different  
phenomenology to RS or LED. Proposed by  
Antoniadis, Arvanitaki, Dimopoulos, Giveon.

# The Clockwork Metric

Put a massless scalar in this background and decompose to find 5D eigenstates (KK):

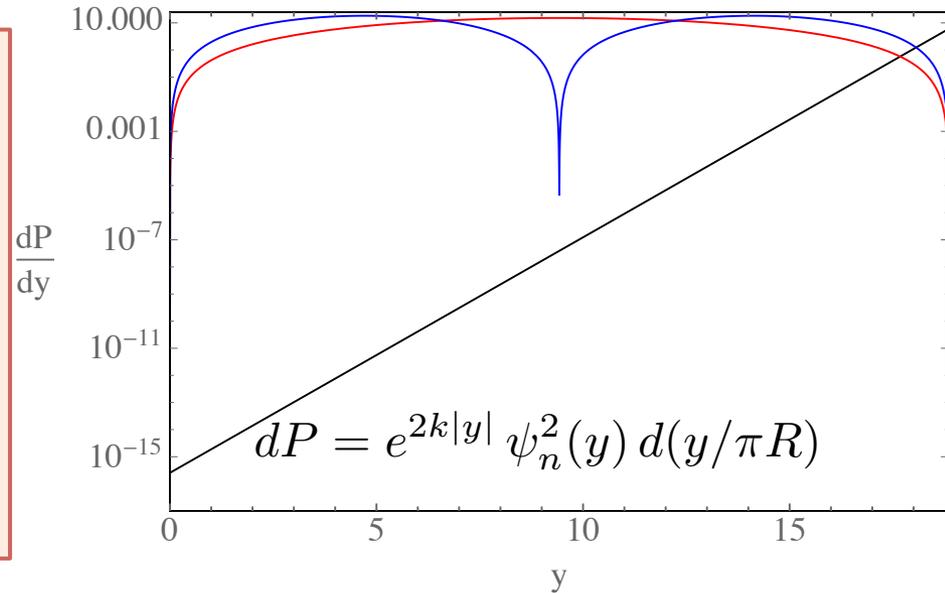
$$\phi(x, y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x) \psi_n(y)}{\sqrt{\pi R}} \quad \longrightarrow \quad \text{SM?} \left| \begin{array}{c} \text{Gravity} \\ y = 0 \qquad \qquad \qquad y = \pi R \end{array} \right.$$

Find a zero-mode:

Mass:  $m_0^2 = 0$

Wavefunction:

$$\psi_0(y) = \sqrt{\frac{k\pi R}{e^{2k\pi R} - 1}}$$



# The Clockwork Metric

Put a massless scalar in this background and decompose to find 5D eigenstates (KK):

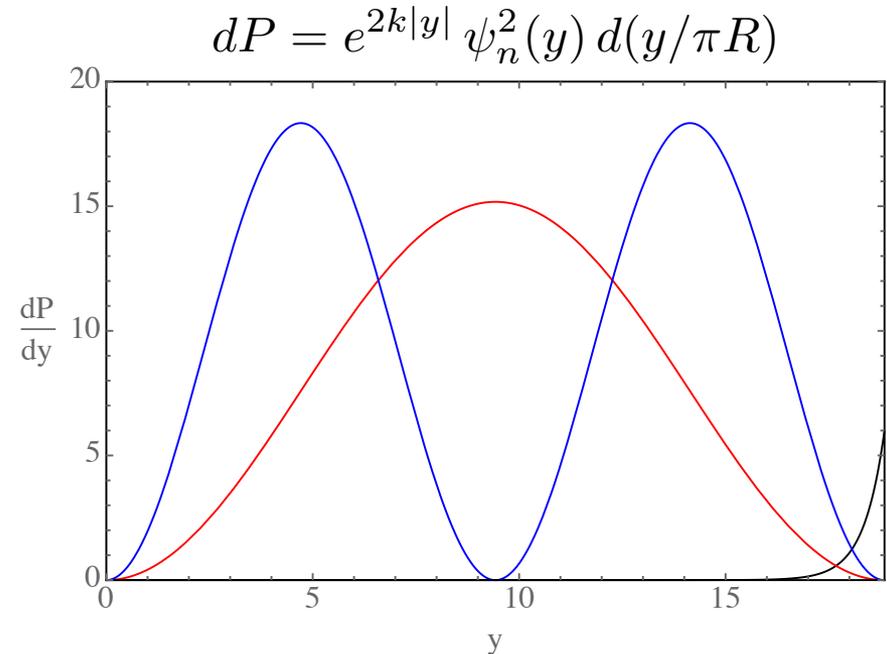
$$\phi(x, y) = \sum_{n=0}^{\infty} \frac{\tilde{\phi}_n(x) \psi_n(y)}{\sqrt{\pi R}} \quad \longrightarrow \quad \text{SM?} \left| \begin{array}{c} \text{Gravity} \\ y = 0 \qquad \qquad \qquad y = \pi R \end{array} \right.$$

Find excited modes:

Mass:  $m_n^2 = k^2 + \frac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R} e^{-k|y|} \left( \frac{kR}{n} \sin \frac{n|y|}{R} + \cos \frac{ny}{R} \right)$$



# The Clockwork Metric

Put a massless scalar in this background and decompose to find 5D eigenstates (KK):

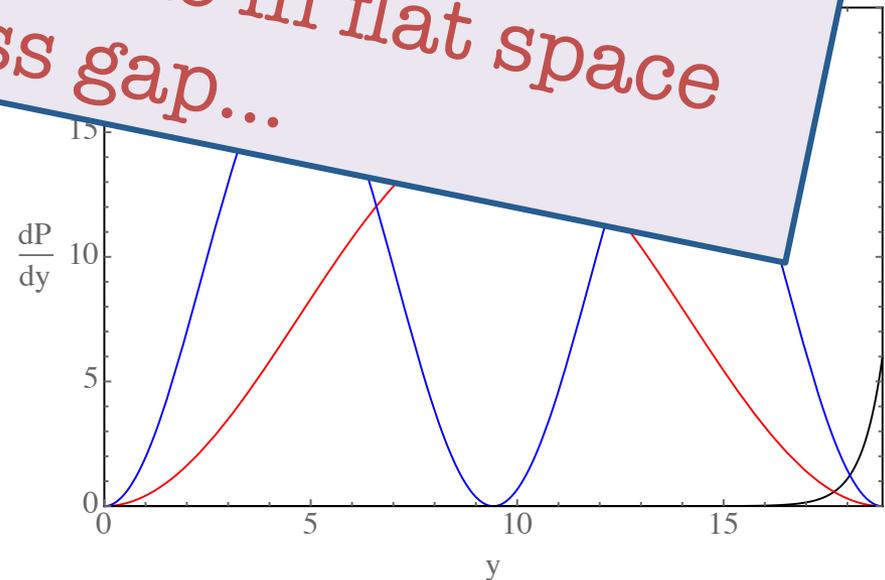
*Zero mode density warped, like in RS...*

*KK mode density just like in flat space with a mass gap...*

Mass:  $m_n^2 = k^2 + \frac{n^2}{R^2}$

Wavefunction:

$$\psi_n(y) = \frac{n}{m_n R} e^{-k|y|} \left( \frac{kR}{n} \sin \frac{n|y|}{R} + \cos \frac{ny}{R} \right)$$



# An Analogy

Is there a physical picture for what is going on?

When modes are decomposed as KK states:

$$h_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} \frac{\tilde{h}_{\mu\nu}^{(n)}(x) \psi_n(y)}{\sqrt{\pi R}}$$

they must satisfy the following equation of motion:

$$\left( \partial_y^2 + 2k\partial_y + \partial_x^2 \right) \tilde{h}_{\mu\nu}^{(n)}(x) \psi_n(y) = 0$$

Remind you of anything?

# An Analogy

When modes are decomposed as KK states:

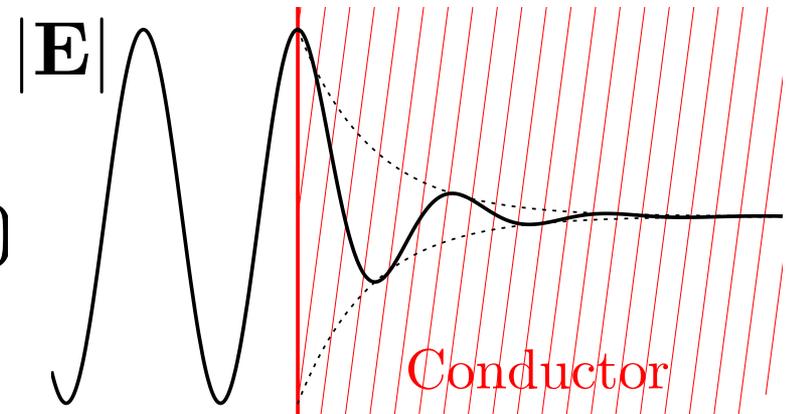
$$h_{\mu\nu}(x, y) = \sum_{n=0}^{\infty} \frac{\tilde{h}_{\mu\nu}^{(n)}(x) \psi_n(y)}{\sqrt{\pi R}}$$

they must satisfy the following equation of motion:

$$(\partial_y^2 + 2k\partial_y + \partial_x^2) \tilde{h}_{\mu\nu}^{(n)}(x) \psi_n(y) = 0$$

Maxwell's equations for EM wave in a conductor:

$$(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2) \mathbf{E} = 0$$



# An Analogy

W can be decomposed as KK states:

General solution for stationary 4D particle:

$$\sim e^{-ky} e^{i(m_n t + \sqrt{m_n^2 - k^2} y)}$$

they must satisfy

$$(\partial_y^2 + 2k\partial_y + \partial_x^2) h_{\mu\nu}^{(n)}(x)$$

EM wave in a conductor:

General solution for EM wave in conductor:

$$\sim e^{-\delta x} e^{i(\omega t + kx)}$$

$$(\nabla^2 - \mu\sigma\partial_t - \mu\epsilon\partial_t^2)$$

# The Hierarchy Problem

Graviton 0-mode and KK states have same decomposition. If SM fields on brane at end:

$$\mathcal{L} = -\frac{h_{\mu\nu}(x, 0) T_{\mu\nu}^{SM}(x)}{M_5^{3/2}} = -\sum_{n=0}^{\infty} \frac{\tilde{h}_{\mu\nu}^{(n)}(x) T_{\mu\nu}^{SM}(x)}{\Lambda_n}$$

Interaction scale

Excited graviton modes:

$$\Lambda_n = \sqrt{M_5^3 \pi R \left(1 + \frac{k^2 R^2}{n^2}\right)}$$

True massless graviton:

$$\Lambda_0 = M_P = \sqrt{\frac{M_5^3}{k}} \sqrt{e^{2k\pi R} - 1}$$

Exponentially enhanced

# Phenomenology

Things get really interesting when looking to the phenomenology...

This talk: Recent paper with Giudice, Kats, Torre, Urbano.

Previous related studies:

- Antoniadis, Arvanitaki, Dimopoulos, Giveon, 2011. (Large-k)
- Baryakhtar, 2012. (All-k)
- Cox, Gherghetta, 2012. (Dilatons)
- Giudice, Plehn, Strumia, 2004. Franceschini, Giardino, Giudice, Lodone, Strumia, 2011. (Large extra dimensions, pheno similar.)

# Phenomenology

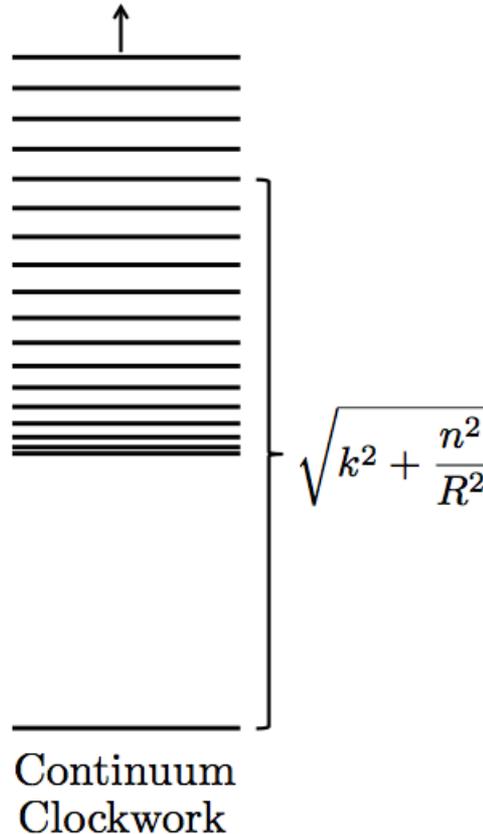
Irreducible prediction:

In this theory  
Planck scale is:

$$M_P \sim \sqrt{\frac{M_5^3}{k}} e^{k\pi R}$$

So if all other  
parameters at the  
weak scale, require:

$$kR \sim 11$$



But the mass  
spectrum is given by:

$$m_n \sim k \left( 1 + \frac{n^2}{2(kR)^2} \right)$$

Thus the first few  
states will always be  
split by %'s, with the  
relative splitting  
decreasing for  
heavier modes.

This splitting is thus a key prediction of the theory.

# Phenomenology

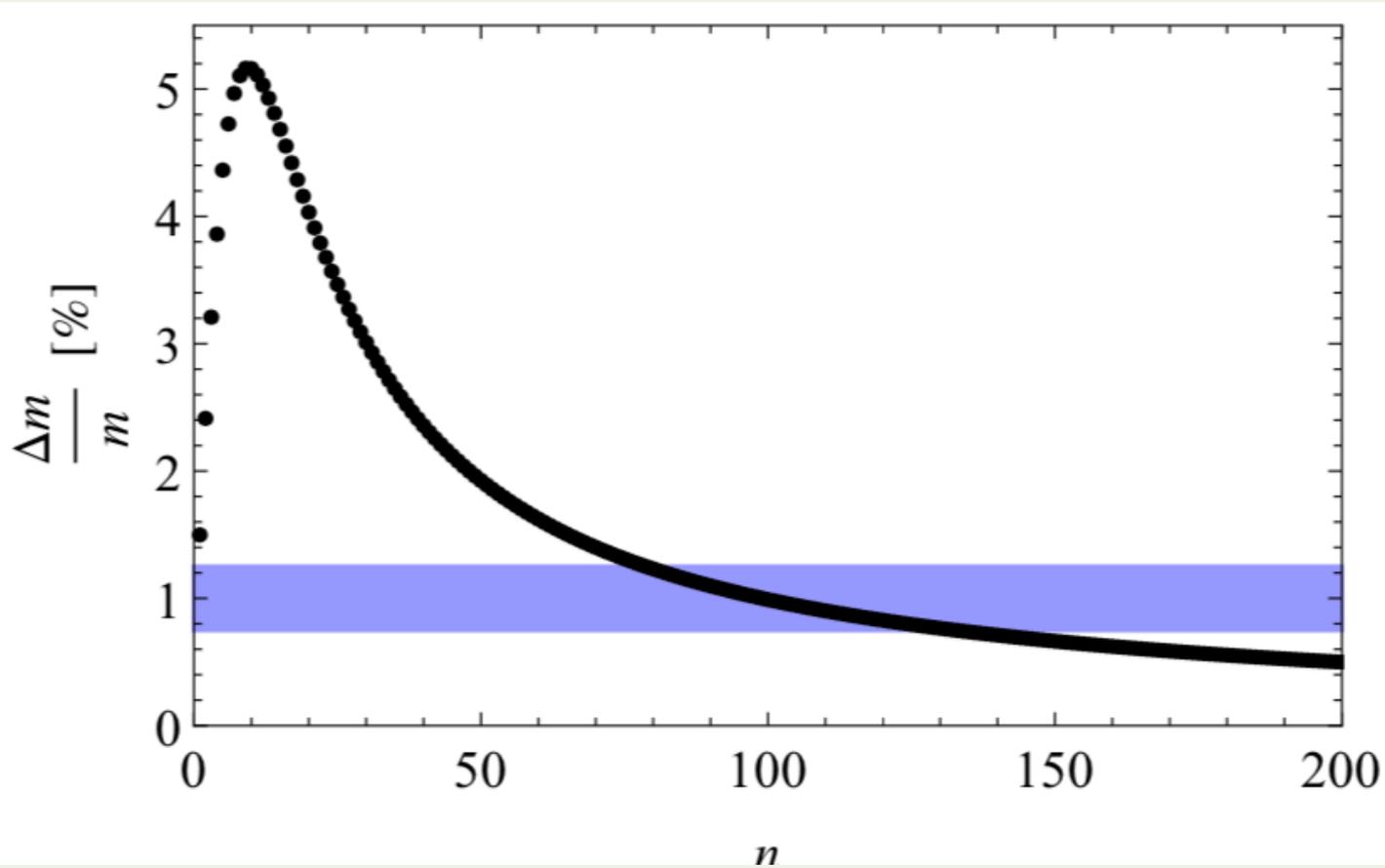
Irre

Mass splitting:

In t  
Plan

$M_I$

So i  
par  
wea



en by:

$$\left( \frac{n^2}{(kR)^2} \right)$$

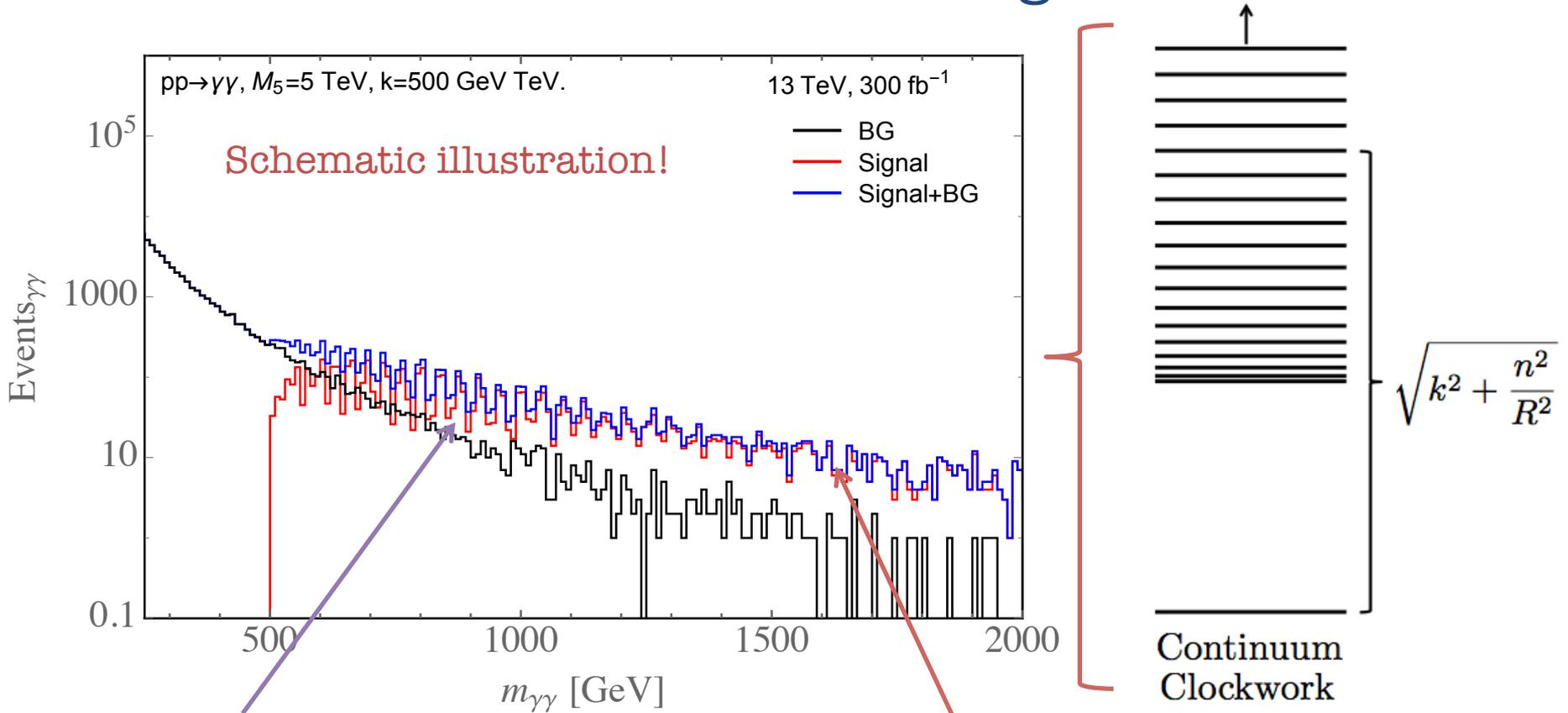
ew  
ys be  
h the  
g

This

theory.

# Phenomenology

At colliders would look something like:

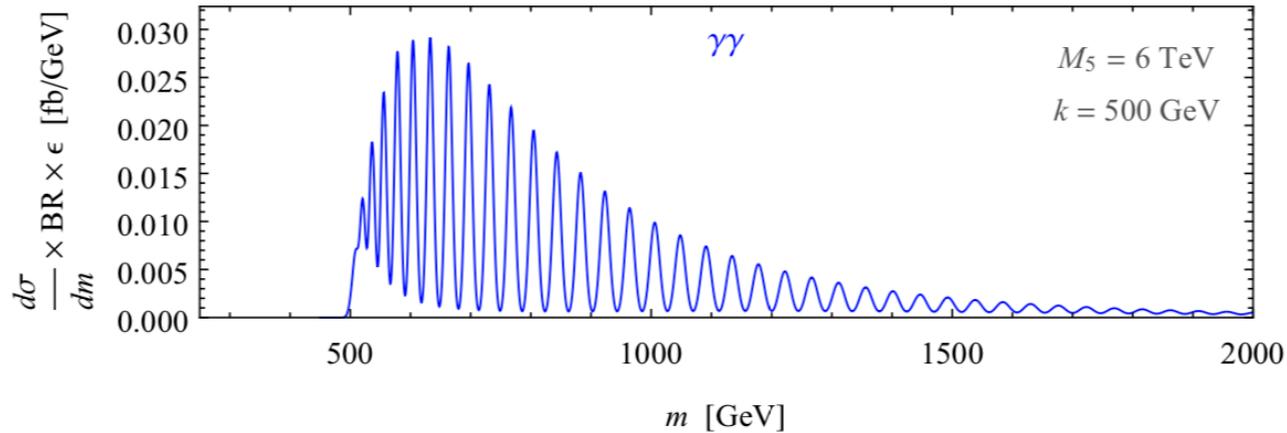


Most interestingly, due to splittings, signal appears to “oscillate”. Thus get extra sensitivity by doing spectral analysis... The “power spectrum” of LHC data!

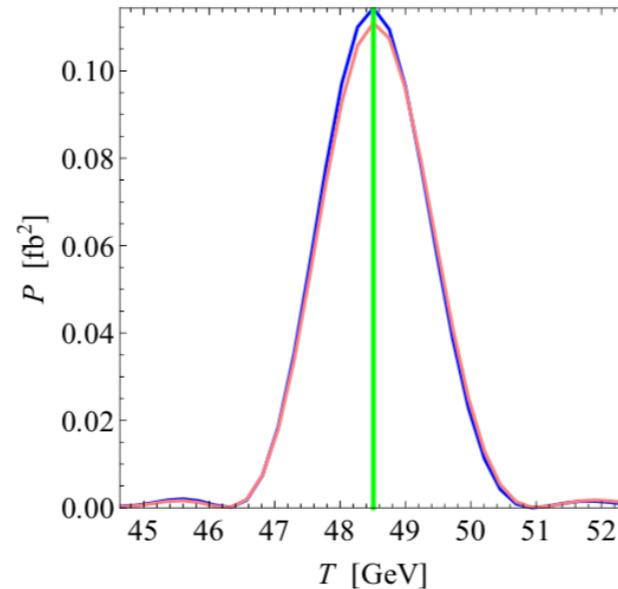
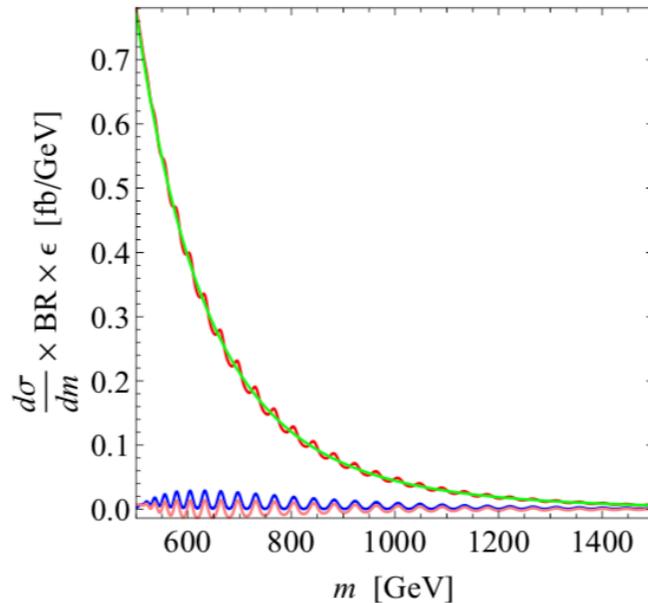
Can search for continuum spectrum at high energies. BG modelling essential...

# Phenomenology

Extract the oscillations, subtract off background:



And then Fourier-transform what's left over!

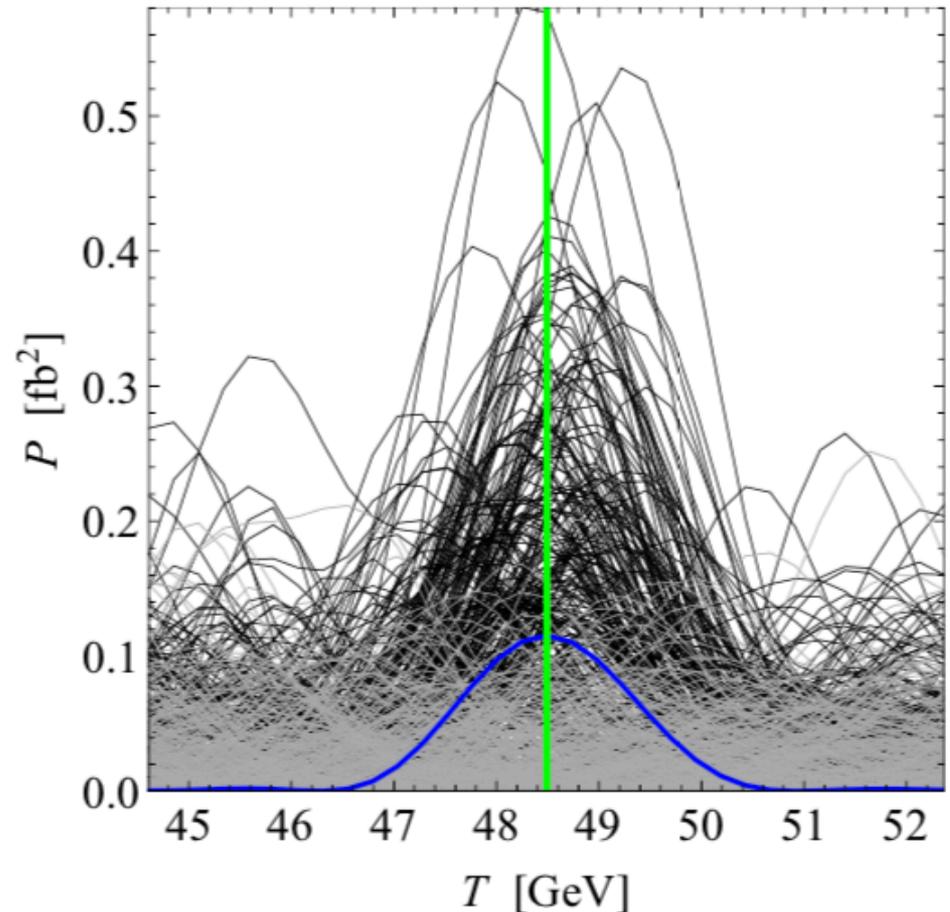


# Phenomenology

Even when statistical fluctuations and experimental resolution are included, such that reality is a bit more messy:

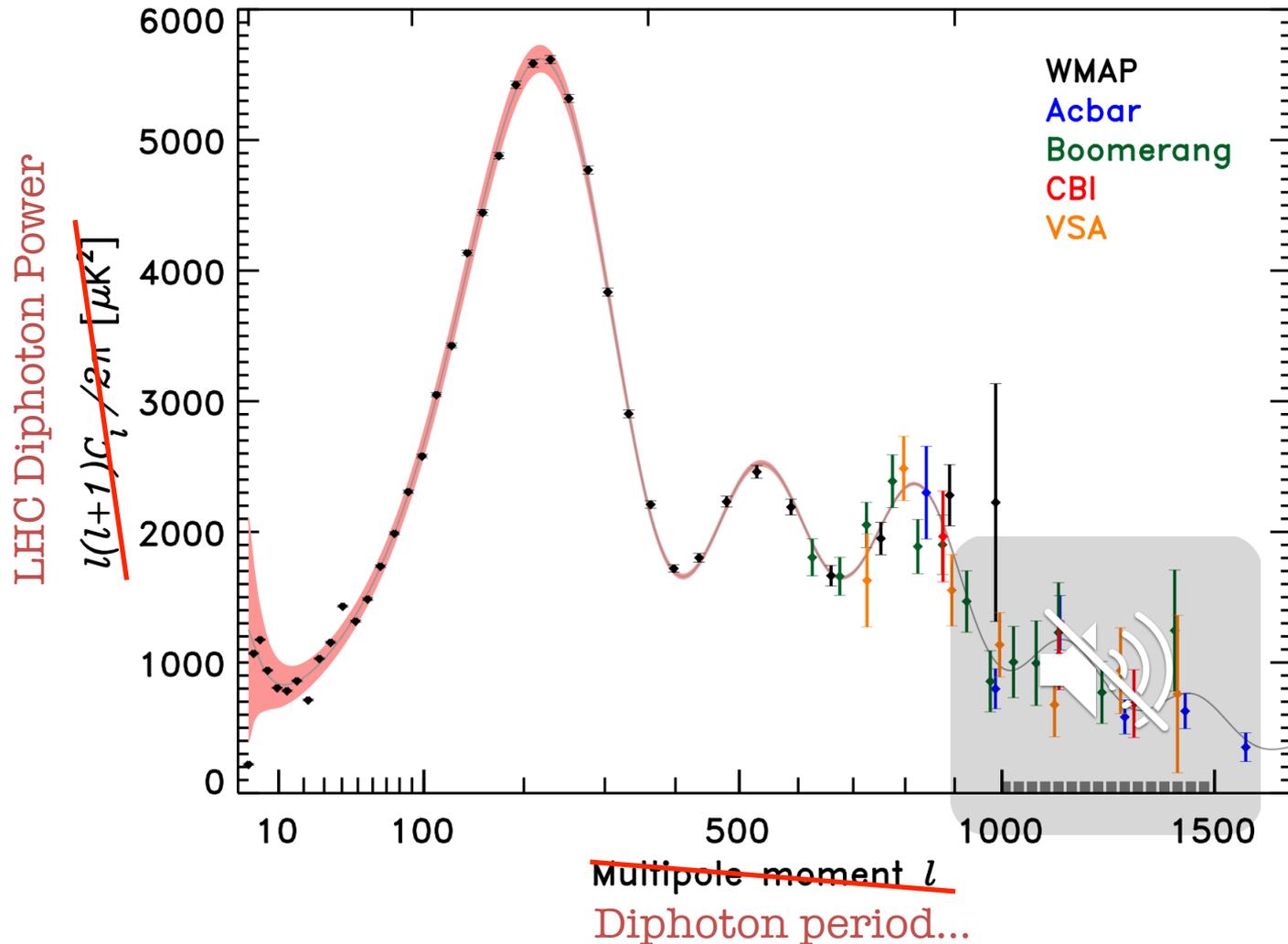
The residual power spectrum of signal+background.

The peak is at the frequency of the oscillations, which correspond to the inverse radius of the extra dimension.



# Phenomenology

Irrespective of the clockwork, it would be a very cool thing to know the LHC diphoton power spectrum!!

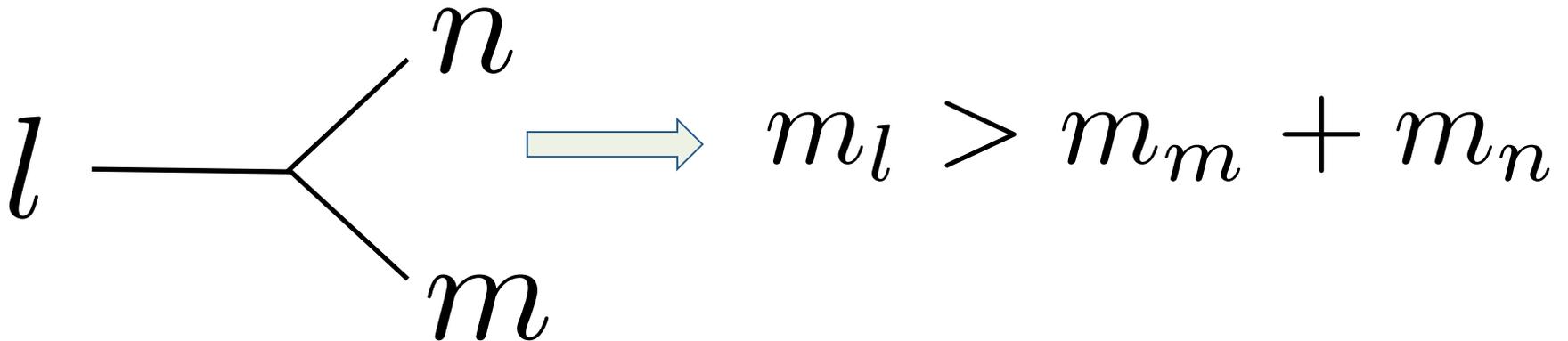


# Phenomenology

In the linear dilaton theory we have broken translation invariance by “k”, resulting in the modification

$$p_0^2 - \underline{p}_3^2 = k^2 + \frac{n^2}{R^2}$$

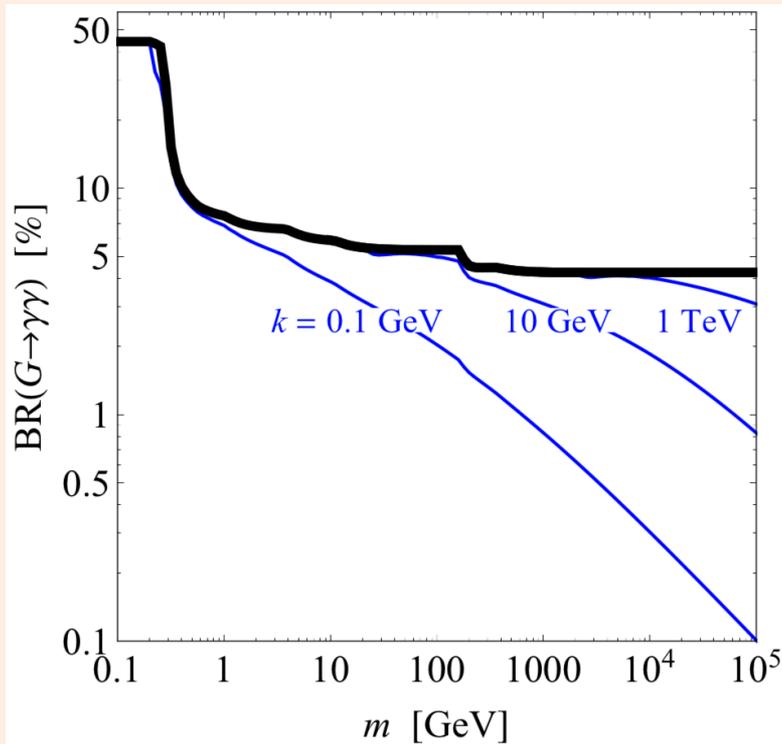
where the latter can be interpreted as the extra-dimensional momentum, and we now have


$$l \begin{array}{l} \nearrow n \\ \searrow m \end{array} \longrightarrow m_l > m_m + m_n$$

However, still a preference for nearby states.

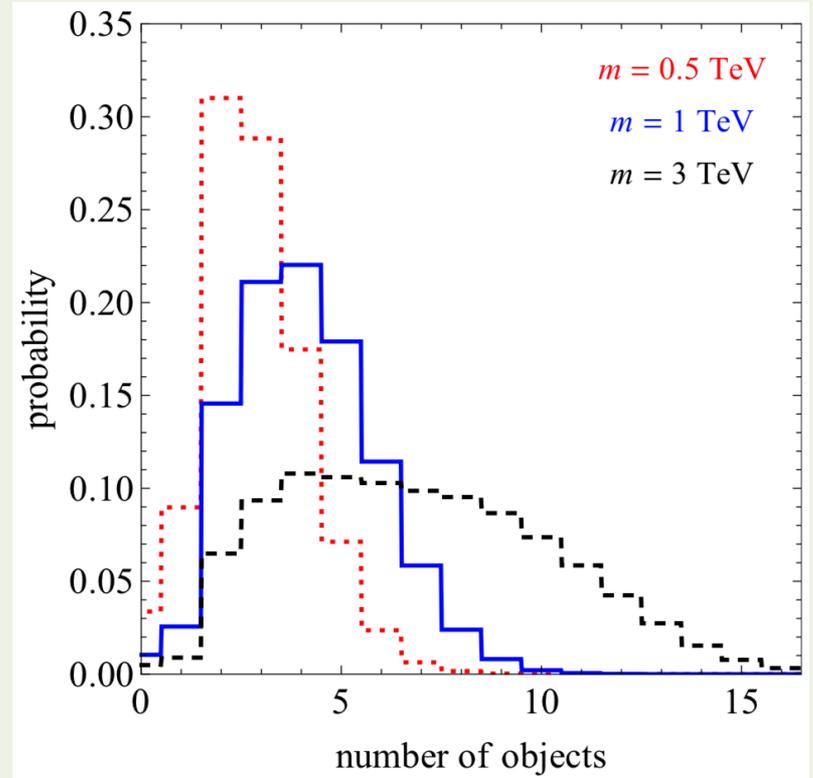
# Phenomenology

## Suppressed SM Signatures:



Black line ignores graviton cascades.

## Graviton Cascades:

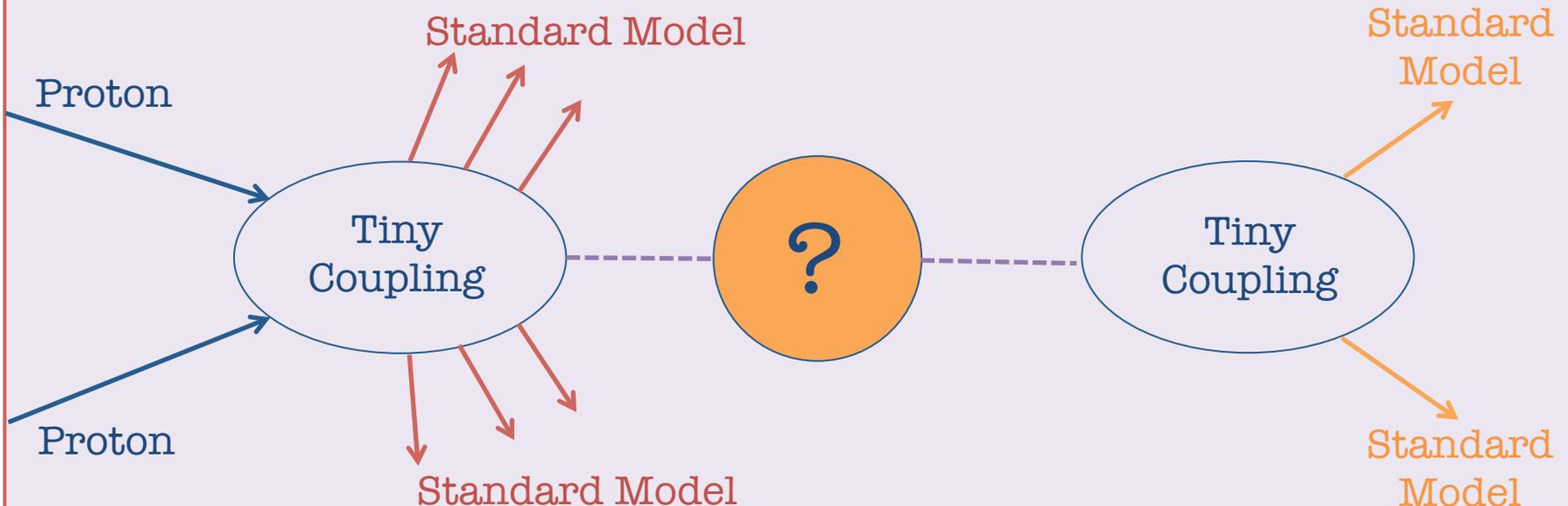


Large multiplicity final states predicted and calculable!

# Phenomenology

## Resonant Displaced Particles!

Typically particles with displaced decays are assumed not to be singly-produced

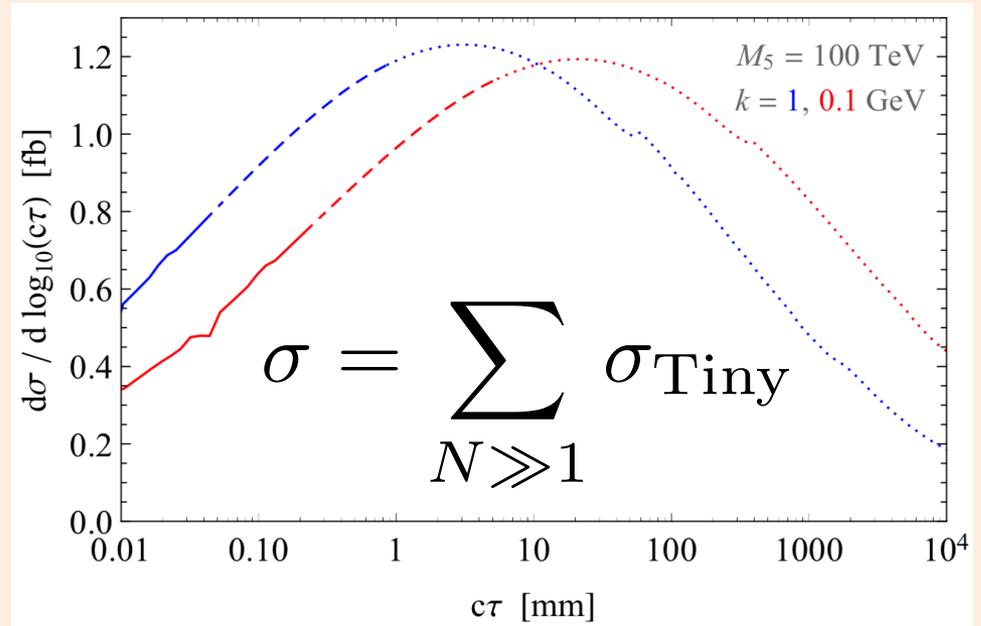
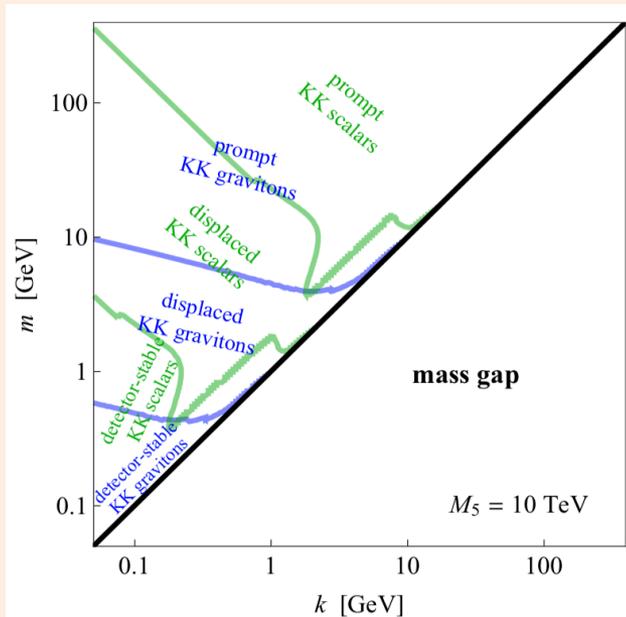


Unless there is some extreme phase-space suppression, displaced decays requires small couplings, which predicts tiny production rates.

# Phenomenology

## Resonant Displaced Particles!

Loophole: Can overcome tiny couplings if there are many particles:



The couplings here are miniscule, leading to large displacements, but the number of states is enormous. S-channel production of long-lived particles!

# Phenomenology

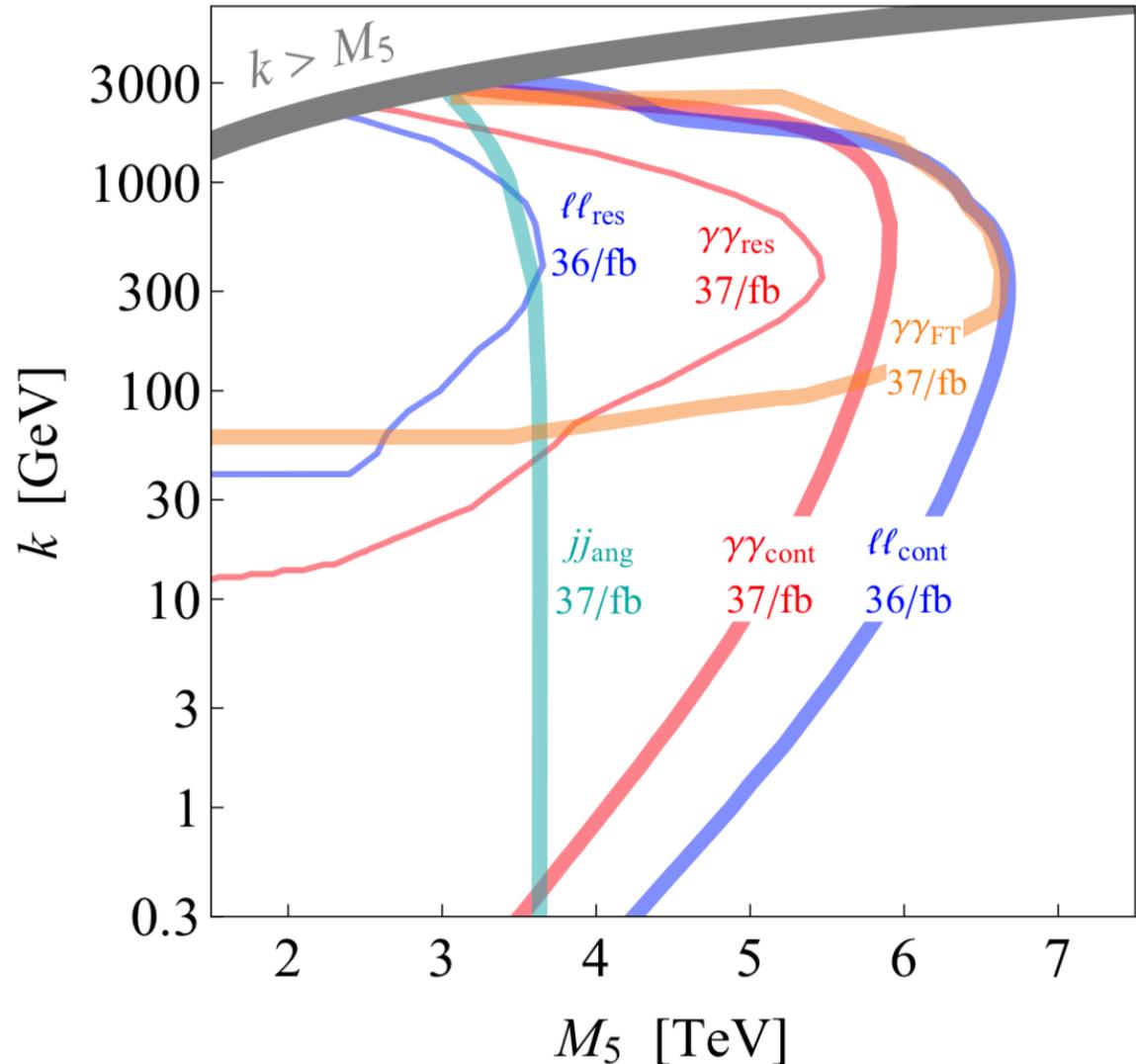
## Summary of constraints:

Included in here are:

- Single bump-hunts
- Dijet angular correlations.
- High  $p_T$  continuum excesses.

The estimate for the Fourier-space search is also shown.

The weakening of SM limits due to graviton decays is clear, as well as the strength of the FT search.



# Part III

Dynamics

*Ran out of time...*

(Cosmological Relaxation)

# Summary

## Conference email/website:

In spite of its consolidated experimental success, the standard model of particle physics falls short of describing all observed phenomena. Elegant and well motivated **theoretical ideas like Supersymmetry, Technicolor, Gand Unification, have so far found no support from experimental results**, and the longed-for discovery of some kind of physics beyond the standard model that could guide us to replace these ideas with new theoretical paradigms, has so far escaped all experimental efforts. Given this situation, any serious attempt to approach the incompleteness of the standard model from originally different and unconventional perspectives should receive proper consideration. **Fearless exploration outside the box might provide more insights than lengthy struggles trough standard thinking.**

**Suggested addition: Fearless experimental and theoretical exploration outside the box ...**

# Summary

Many\* approaches follow three basic paradigms...

Symmetry.

Locality.

Dynamics.

Some of these ideas are, in the current context, more radical than others. In any case, LHC results have catalyzed plenty of weird theory ideas...

# Summary

Many\* approaches follow three basic paradigms...

*We are lucky to have a broad experimental program. The weirder the theory, the weirder the signature, the better!*

Locality.

Some of these ideas are, in the current context, more radical than others. In any case, LHC results have catalyzed plenty of weird theory ideas...