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Vacuons and Quasiphonons: The Hidden side of Bogoliubov Collective Excitations



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The Interacting N Bosons Hamiltonian

$$H_{bos} = \sum_{\mathbf{k}} \overbrace{\left(\frac{\hbar^2 k^2}{2M} \right)}^{\mathcal{T}(k)} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k}_1, \mathbf{k}_2, \mathbf{q}} \hat{u}(q) b_{\mathbf{k}_2 - \mathbf{q}}^\dagger b_{\mathbf{k}_1 + \mathbf{q}}^\dagger b_{\mathbf{k}_1} b_{\mathbf{k}_2}$$

Bogoliubov:

Drop all couplings between excited free-particle states $| \vec{k} \neq 0 \rangle$
(Low Energy Approximation)



**The Truncated Hamiltonian
(Canonic)**

$$H_c = \overbrace{\frac{\hat{u}(0)N^2}{2}}^{E_{in}} + \sum_{\mathbf{k} \neq 0} \overbrace{\left[\mathcal{T}(k) + \tilde{N}_{in} \hat{u}(k) \right]}^{\tilde{\epsilon}_1(k)} b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{2} \sum_{\mathbf{k} \neq 0} \hat{u}(k) [b_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger (b_0^\dagger)^2 + b_{\mathbf{k}} b_{-\mathbf{k}} (b_0^\dagger)^2]$$

Bogoliubov Canonic Approximation (BCA):

$$(b_0^\dagger)^2 \approx b_0^2 \approx N_{\mathbf{k}=0}$$

+ transformation preserving canonicity (Adam & Bru, 2004)

$$\beta_{\mathbf{k}} = b_0^\dagger \left(\tilde{N}_{in} + 1 \right)^{-1/2} b_{\mathbf{k}} \quad , \quad \beta_{\mathbf{k}}^\dagger = b_{\mathbf{k}}^\dagger \left(\tilde{N}_{in} + 1 \right)^{-1/2} b_0$$



BCA Hamiltonian (Canonic)

$$H_{BCA} = E_{in} + \sum_{\mathbf{k} \neq 0} \overbrace{[\mathcal{T}(k) + N \hat{u}(k)]}^{\epsilon_1(k)} \beta_{\mathbf{k}}^\dagger \beta_{\mathbf{k}} + \frac{N}{2} \sum_{\mathbf{k} \neq 0} \hat{u}(k) [\beta_{\mathbf{k}}^\dagger \beta_{-\mathbf{k}}^\dagger + \beta_{\mathbf{k}} \beta_{-\mathbf{k}}]$$

Bogoliubov transformations:

$$B_{\mathbf{k}}^\dagger = w_+^* \beta_{\mathbf{k}}^\dagger - w_-^* \beta_{-\mathbf{k}} \quad ; \quad B_{\mathbf{k}} = w_+ \beta_{\mathbf{k}} - w_- \beta_{-\mathbf{k}}^\dagger$$



$$H_{BCA} = \sum_{\mathbf{k}} \left[\left(B_{\mathbf{k}}^\dagger B_{\mathbf{k}} + 1/2 \right) \epsilon(k) - \frac{\epsilon_1(k)}{2} \right]$$

$$E_{BCA}(\eta, k) = \left[\epsilon(k) \left(\eta + \frac{1}{2} \right) - \frac{\epsilon_1(k)}{2} \right] \quad (\eta = 0, 1, \dots)$$

$$\epsilon(k) = \sqrt{\epsilon_1^2(k) - N^2 \hat{u}^2(k)}$$

BCA Collective Excitations (CEs)

Equation for BCA vacuum:

$$B_{\mathbf{k}} | \emptyset \rangle_{BCA} = 0$$

**Equation for the eigenstate corresponding to η
CEs of moment \vec{k}**

$$| \mathbf{k}, \eta \rangle_{BCA} = \frac{(B_{\mathbf{k}}^\dagger)^\eta}{\sqrt{\eta!}} | \emptyset \rangle_{BCA}$$

Exact Diagonalization of H_c

in the TL ($N \rightarrow \infty$)

(L. Ferrari, 2016, 2017)

$$H_c = E_{in} + \sum_{\mathbf{k} \neq 0} \overbrace{\left[\tilde{\epsilon}_1(k) b_{\mathbf{k}}^\dagger b_{\mathbf{k}} + \frac{1}{2} \hat{u}(k) \left(b_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger (b_0) \right)^2 + b_{\mathbf{k}} b_{-\mathbf{k}} (b_0^\dagger)^2 \right]}^{h_c(\mathbf{k})}$$

Single-k Schrödinger equation:

$$[h_c(\mathbf{k}) + h_c(-\mathbf{k})] |E, \mathbf{k}\rangle = E |E, \mathbf{k}\rangle$$

Base: Fock states with j bosons in $|-\mathbf{k}\rangle$ and $j+\eta$ bosons in $|\mathbf{k}\rangle$

$$|j, \mathbf{k}\rangle_\eta = \frac{(b_0^\dagger)^{N-2j-\eta}}{\sqrt{(N-2j-\eta)!}} \frac{(b_{\mathbf{k}}^\dagger)^{j+\eta} (b_{-\mathbf{k}}^\dagger)^j}{\sqrt{j!(j+\eta)!}} |\emptyset\rangle_{true}$$

Warning! Total momentum = $\eta \hbar \mathbf{k}$



After some sweat ...



(no blood, no tears! ...)

The Solution!



$$|E, \mathbf{k}\rangle \equiv |S, \mathbf{k}, \eta\rangle = \sum_{j=0}^{\infty} \phi_{S,\eta}(j,k) |j, \mathbf{k}\rangle_{\eta} \quad (S = 0, 1, \dots)$$

exponential

$$\phi_{S,\eta}(j,k) = \overbrace{x^j(k)}^{\text{exponential}} \underbrace{\sqrt{\binom{j+\eta}{j}} \sum_{m=0}^S C_{S,\eta}(m,k) j^m}_{\text{polynomial } S\text{-degree}}$$



Schrödinger Equation



System of S+2 equations determining $E, x, C\dots(n)$

$$\frac{(\epsilon_1\eta - E)C(n) + 2\epsilon_1C(n-1)}{N\hat{u}(k)} + x \left[(1+\eta) \sum_{m=n}^S C(m) \binom{m}{n} + \sum_{m=n-1}^S C(m) \binom{m}{n-1} \right] + \\ + \frac{1}{x} \sum_{m=n-1}^S C(m) \binom{m}{n-1} (-1)^{n-m+1} = 0 \quad (n = 0, 1, \dots, S+1)$$



$$E \equiv E_S(\eta, k) = \epsilon(k)(\eta + 2S) - [\epsilon_1(k) - \epsilon(k)]$$

$$x(k) = \frac{\epsilon(k) - \epsilon_1(k)}{N\hat{u}(k)}$$

Differences and analogies with BCA results

$$E_S(\eta, k) = E_{BCA}(\eta, k) + E_{BCA}(2S, k)$$



NEW!

$$| 0, \mathbf{k}, \eta \rangle = | \mathbf{k}, \eta \rangle_{BCA} = \frac{(B_{\mathbf{k}}^\dagger)^\eta}{\sqrt{\eta!}} | \emptyset \rangle_{BCA} \quad (\eta = 0, 1, \dots)$$

The BCA eigenstates coincide with the exact eigenstates with S=0

$$| S \neq 0, \mathbf{k}, \eta \rangle \quad (S = 1, 2, \dots) \quad \text{**NEW!**}$$

Not exactly:

Dziarmaga and Sacha discovered something like $|S, \vec{k}, 0 \rangle$ in 2003

Free Bosons Limit

$$\xi = \frac{\hbar}{\sqrt{2MN\hat{u}(0)}}$$

$k \gg \xi^{-1} \Rightarrow \mathcal{T}(k) \gg N\hat{u}(k)$ (**Kinetic energy >> Interaction energy**)

$E \propto k^2$: particle-like CEs

$$E_S(\eta, k) \rightarrow \overbrace{\mathcal{T}(k)(\eta + 2S)}^{\text{but}}$$

$$\vec{P}_{tot} = \hbar\eta\vec{k}$$



S numerates **pairs** of bosons with **opposite** momentum $\vec{k}, -\vec{k}$
(bosonic Cooper pairs)

η numerates **unpaired bosons** with momentum \vec{k}

Strong coupling Limit $k \ll \xi^{-1}$



$E \propto k$: wave-like CEs

$$E_S(\eta, k) \rightarrow \underbrace{\frac{\hbar^2 k}{\sqrt{2M\xi}} \eta}_{\vec{P}_{tot} = \hbar \vec{k} \cdot \eta} + \underbrace{\frac{\hbar^2 k}{\sqrt{2M\xi}} 2S}_{\text{momentless}} - N\hat{u}(0)$$



η Quasiparticles (QPs)
with velocity

$$v_{QP} = \sqrt{\frac{N\hat{u}(0)}{M}}$$



S Vacuons

NEW!

Why “Vacuons” ?:

Each eigenstate $| S, \mathbf{k}, 0 \rangle$ is a momentless “vacuum” of QPs

Passing from a vacuum to the next neighbor:

$$| S, \mathbf{k}, 0 \rangle \longrightarrow \pm 2\epsilon(k) \longrightarrow | S \pm 1, \mathbf{k}, 0 \rangle$$

means creating/annihilating a Vacuon

Instead:

$$| S, \mathbf{k}, \eta \rangle \longrightarrow \pm \epsilon(k) \longrightarrow | S, \mathbf{k}, \eta \pm 1 \rangle$$

means creating/annihilating a QP
on the same S-vacuum

Differences and analogies with BCA results (continued)

$B_k^\dagger (B_k)$ do not create (annihilate) any **exact CE**, if $S \neq 0$

However:

$$B_k^\dagger B_k | S, \mathbf{k}, \eta \rangle = (S + \eta) | S, \mathbf{k}, \eta \rangle$$



The BCA number operator numerates the **CEs** in each **exact eigenstate** :

$| S, \mathbf{k}, \eta \rangle \rightarrow S$ **Vacuons** + η **Quasi-phonons**

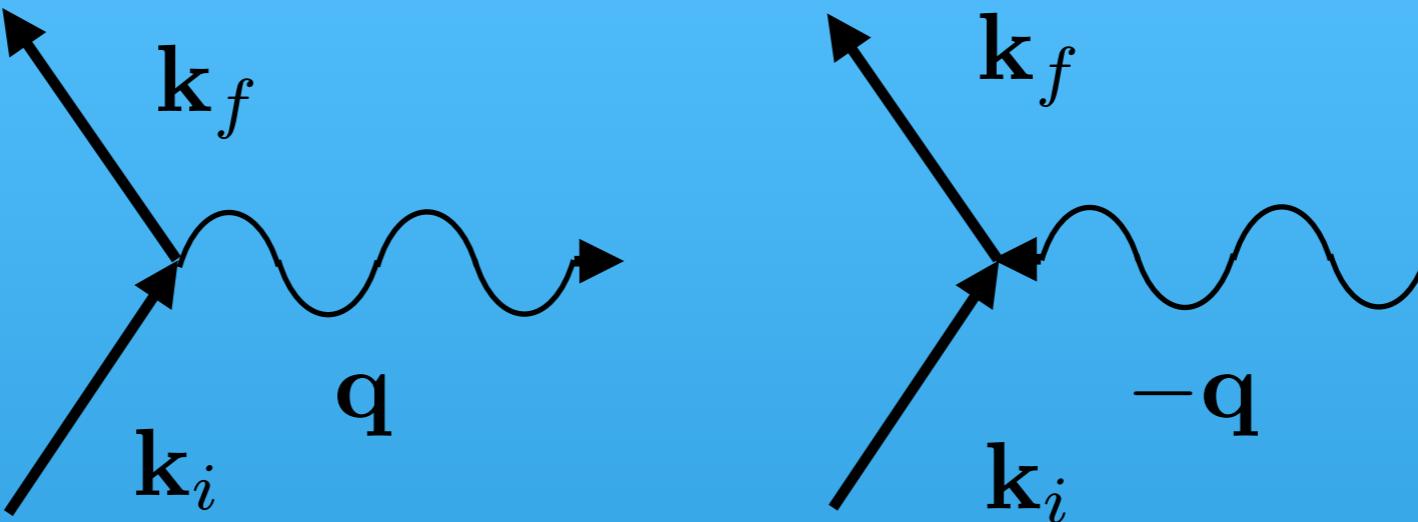
Mixed transitions like

$$| S, \mathbf{k}, \eta \rangle \rightarrow | S', \mathbf{k}, \eta' \rangle$$

are possible, e.g. by scattering of an external particle moving in the gas:

$$U_{int} = W_0 \delta(\vec{r}_{particle} - \vec{r}_{boson})$$

Emission/adsorption of a QP by a particle



$$\begin{aligned} & \langle \mathbf{k}_i | \langle \eta, \mathbf{q}, S | U | S', \mathbf{q}', \eta', \rangle | \mathbf{k}_f \rangle = \\ & = W_+(S, S', \eta) \delta_{\mathbf{k}_f, \mathbf{k}_i - \mathbf{q}} \delta_{\eta', \eta+1} + W_-(S, S', \eta) \delta_{\mathbf{k}_f, \mathbf{k}_i + \mathbf{q}} \delta_{\eta', \eta-1} \end{aligned}$$

The matrix element is $\neq 0$ even if $S \neq S'$!

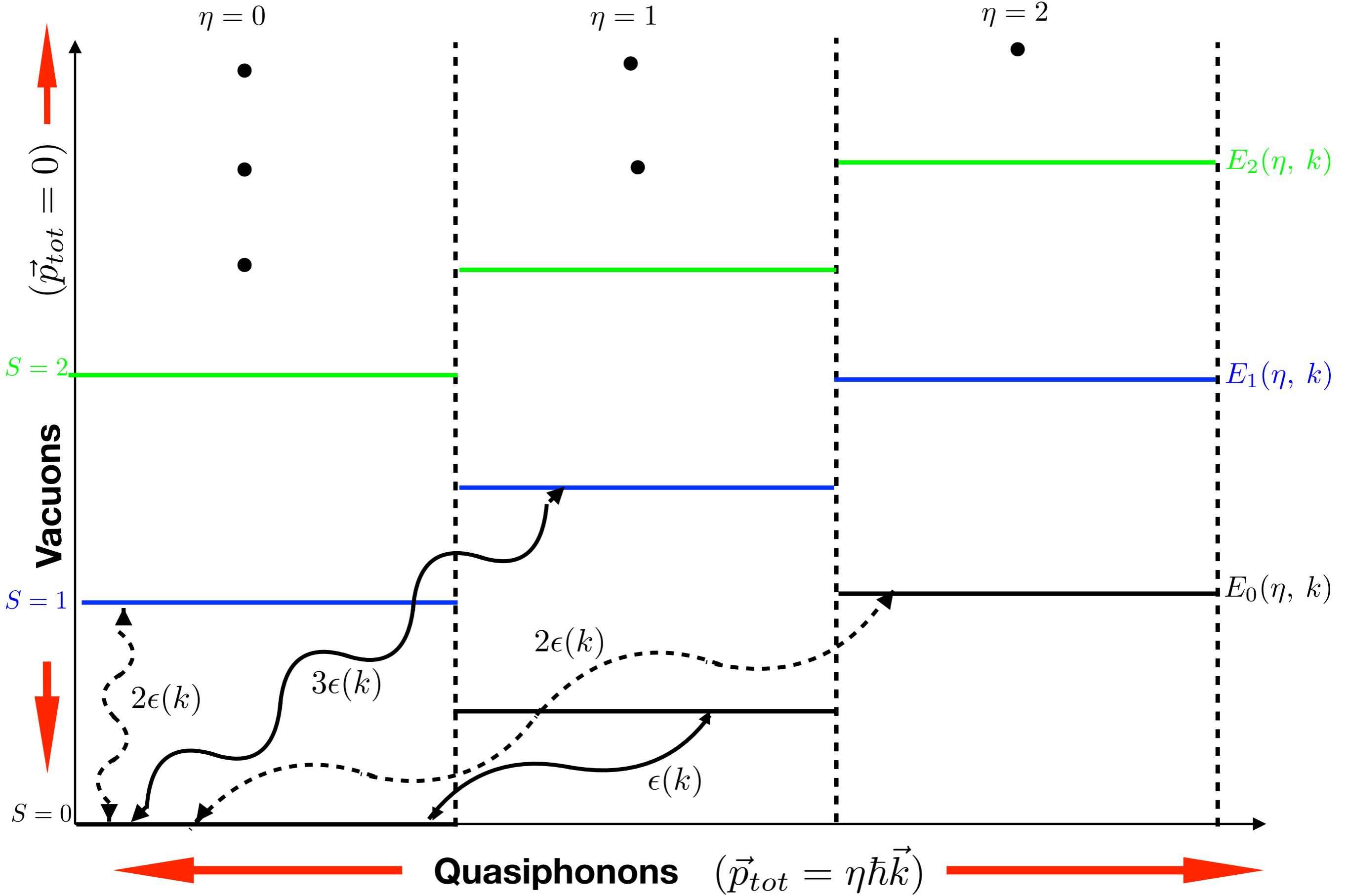
The creation/annihilation of a QP is a *single-particle* process even if $|S - S'|$ vacuons are created/annihilated simultaneously

The matrix element is 0 if $\eta' \neq \eta \pm 1$

The creation/annihilation of a vacuon is first-order forbidden unless a QP is created/annihilated simultaneously



Spectrum



In any scattering experiment revealing the eigenvalues,
transitions involving odd multiples of

$$\omega_{min}(k) = \epsilon(k)/\hbar$$

are first-order allowed

Transitions involving even multiples
are first-order forbidden

The frequency $2\omega_{min}(k)$ does not appear at first order!

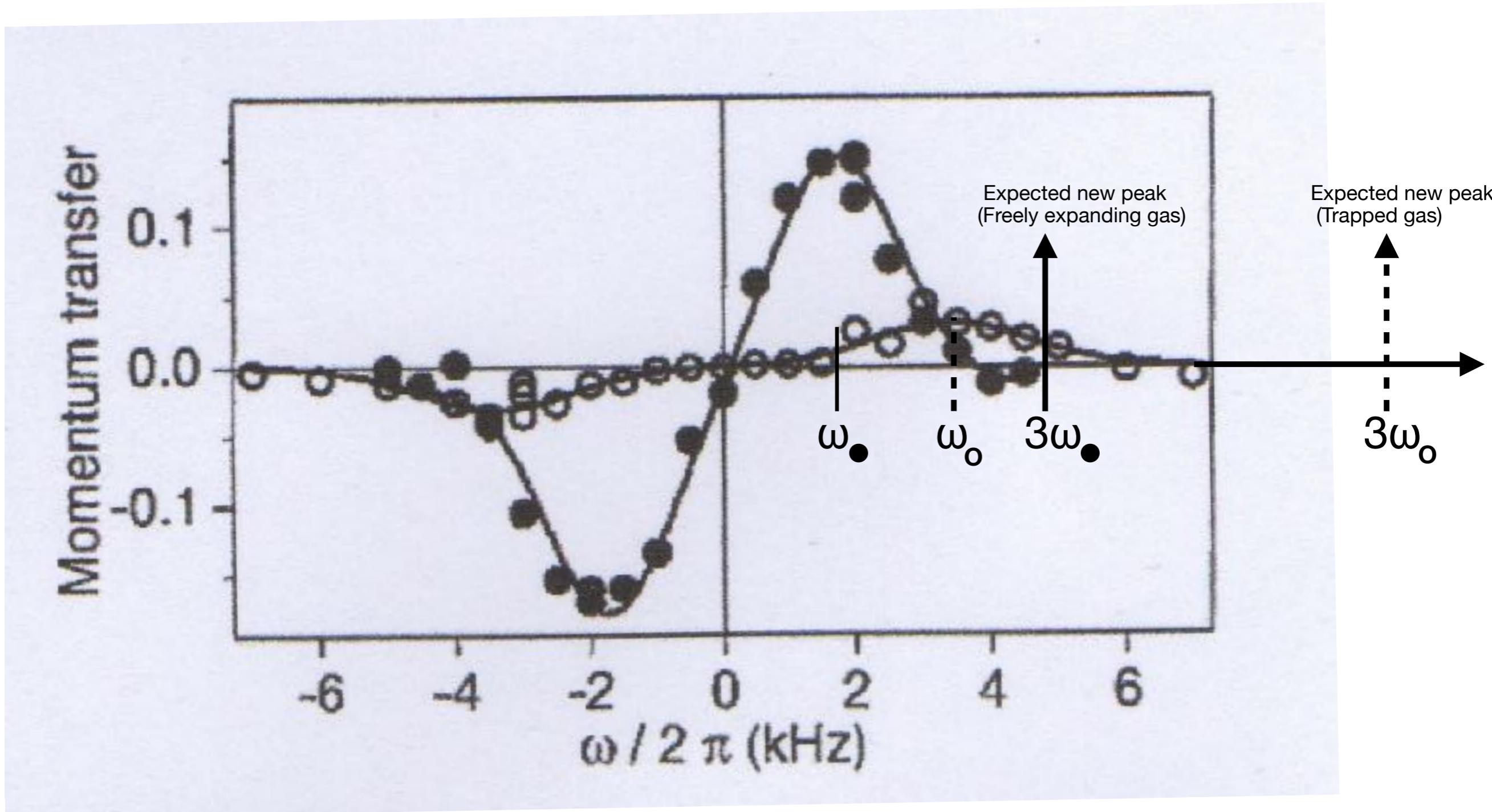
Vacuons are revealed by resonant effects at

$$\omega_{min}(k), 3\omega_{min}, \dots, (2S + 1)\omega_{min}, \dots$$

Any experimental evidence?



2 Photons Bragg spectroscopy: Stamper-Kurn et al , 1999



Expected positions of the peaks due to the activation of 1 QP
+ 1 Vacuon

Vacuons might have been observed experimentally, but:

a new version of Murphy's Law did apply:

Data stop right before the effect predicted!

Hope in future experiments (or in a future life)



Thank you for your attention



Dissipation: a new picture of Superfluidity

**Slowing down of a particle of mass M_p and initial velocity \mathbf{v}_i
by creation of 1 QP and S vacuons**

energy-moment conservation
$$\begin{cases} \frac{M_p v_i^2}{2} = \frac{M_p v_f^2}{2} + \epsilon(k)(1 + 2S) \\ M_p \mathbf{v}_i = M_p \mathbf{v}_f + \hbar \mathbf{k} \end{cases}$$



$$v_i > v_c(S) = v_{QP}(1 + 2S)$$

When v_i crosses each $v_c(S)$

from below (above), a new channel of energy dissipation is open (closed)

Figure 1(a)

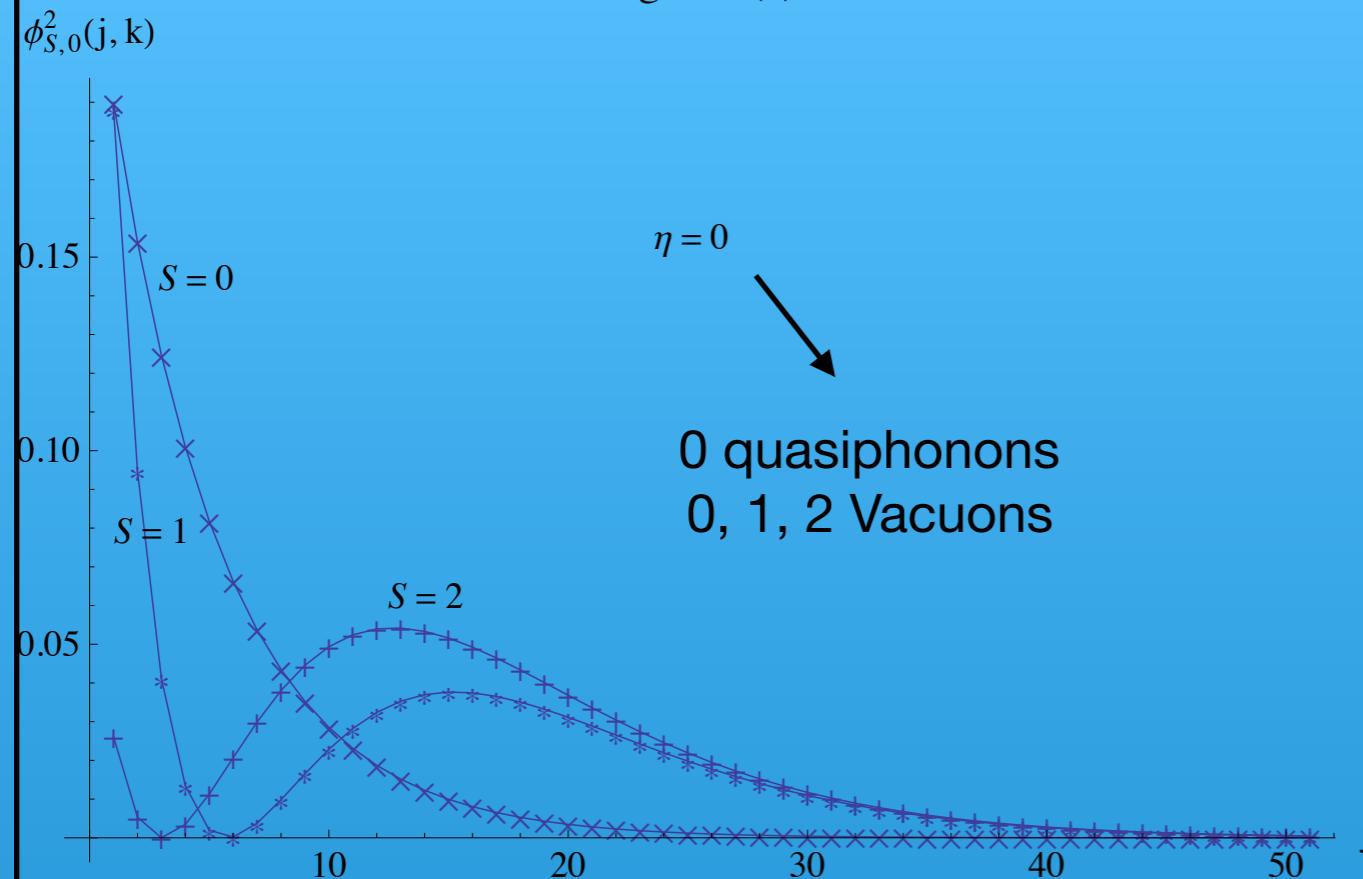
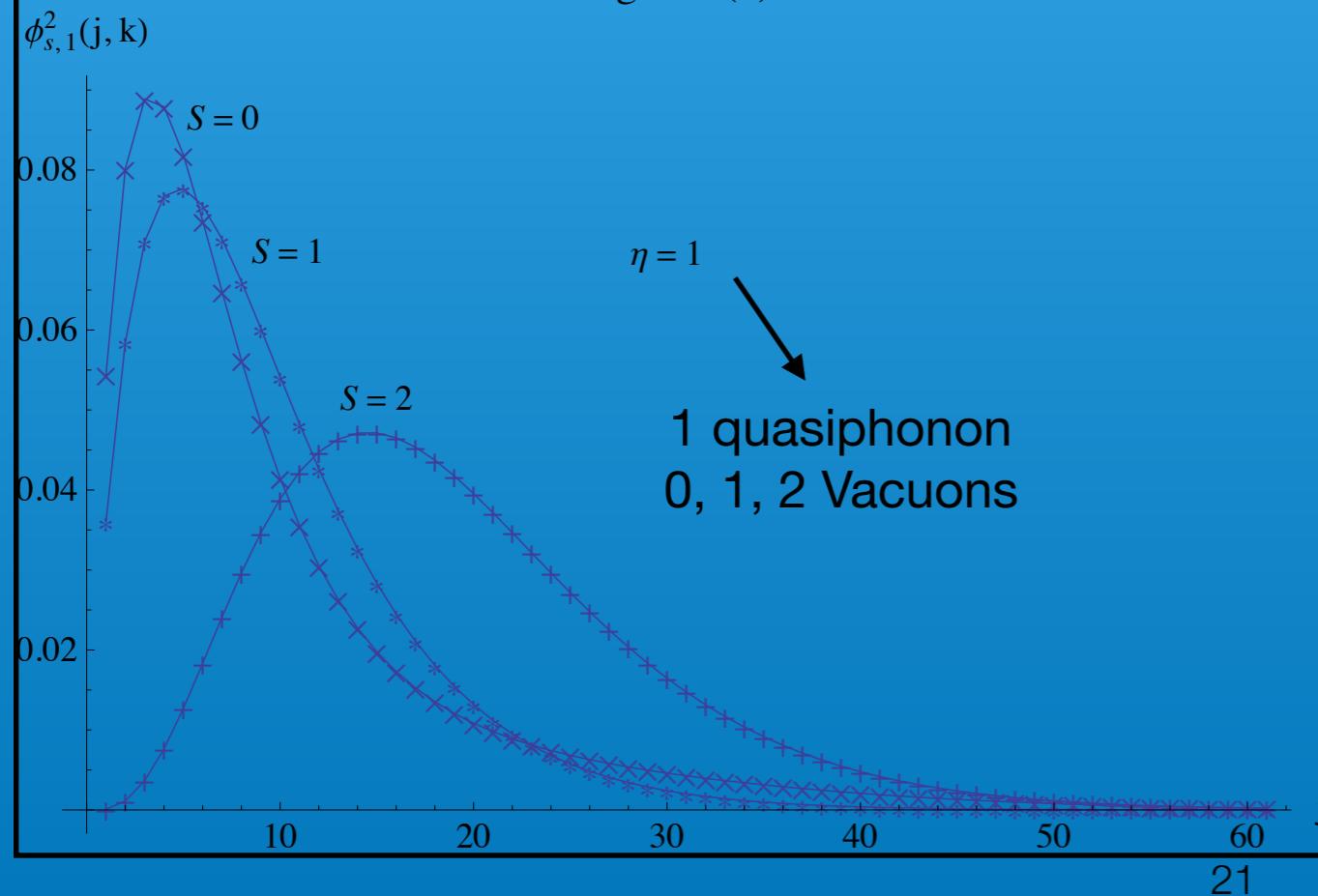


Figure 1(b)



The different shape and values of the square amplitudes

$$|\phi_{S,0}(j, k)|^2, |\phi_{S,1}(j, k)|^2$$

show that the creation of a QP changes the structure of the corresponding “vacuum” too:



Vacuons and QPs are not independent CEs, though the energy eigenvalue is a sum of a QP and a Vacuon contribution:

$$E_S(\eta, k) = \underbrace{\epsilon(k)\eta + \frac{\epsilon(k) - \epsilon_1(k)}{2}}_{\text{QP contribution} = E_{BCA}(k)} + \underbrace{2\epsilon(k)S + \frac{\epsilon(k) - \epsilon_1(k)}{2}}_{\text{Vacuon contribution}}$$