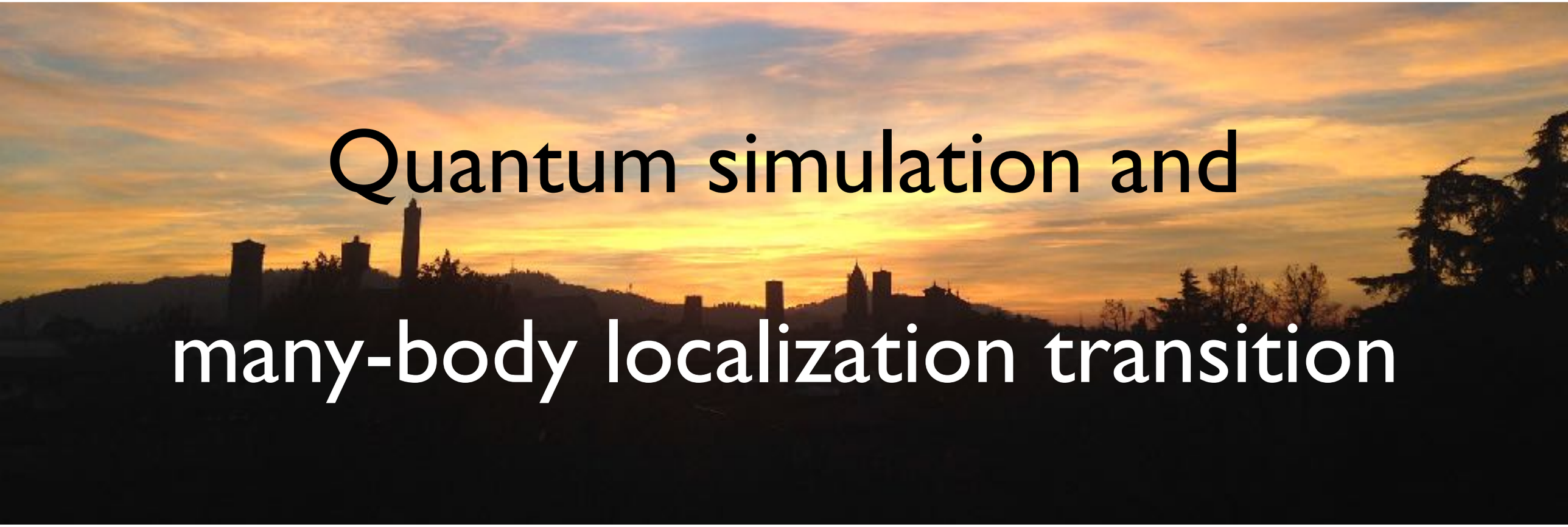


Quantum simulation and many-body localization transition

Piero Naldesi

LPMMC - Laboratoire de Physique et Modélisation des Milieux Condensés

Grenoble

MBL - Physics in the middle of the spectrum

Elisa Ercolessi

Dipartimento di Fisica e Astronomia,
Università di Bologna.

Tommaso Roscilde

Laboratoire de Physique,
Ecole Normale Supérieure de Lyon.

OUTLINE

- Introduction to Many-Body Localization (MBL)
- Detecting a Many-Body Mobility Edge with Quantum Quenches
- Decaying of correlation & localization length
- Conclusions & Perspectives

INTRODUCTION - Many-Body Localization

Many-Body Localization?

A stable dynamical phase of matter which *breaks ergodicity* and *doesn't thermalize.*

Key ingredients

Isolated interacting system

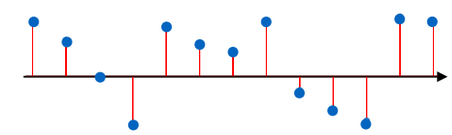
Spins

Bosons

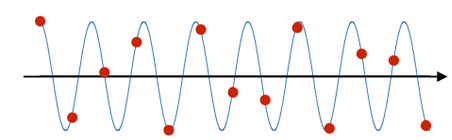
Fermions

Disorder

Random



Quasiperiodic



Many recent works:

P. Anderson, Phys rev, **1958** (Nobel in 1977)

D. Basko, I. Aleiner, and B. Altshuler, Annals of Physics 321, 1126 (2006).

V. Oganesyan and D. A. Huse, Phys. Rev. B 75, 155111 (2007).

M. Znidaric, T. c. v. Prosen, and P. Prelovsek, Phys. Rev. B 77, 064426 (2008).

A. Pal and D. A. Huse, Phys. Rev. B 82, 174411 (2010).

J. H. Bardarson, F. Pollmann, and J. E. Moore, Phys. Rev. Lett. 109, 017202 (2012).

J. A. Kjall, J. H. Bardarson, and F. Pollmann, Phys. Rev. Lett. 113, 107204 (2014).

E. Altman and R. Vosk, Annual Review of Condensed Matter Physics 6, 383 (2015),

INTRODUCTION - Quantum thermalization

How does an isolated quantum many-body systems thermalize?

We consider a system of N sites with Hamiltonian $\mathcal{H}$

$$\mathcal{H}|m\rangle = E_m|m\rangle$$

We prepare the system in a non-stationary state $|\psi_i\rangle$

$$|\psi_i\rangle = \sum_m C_m|m\rangle \qquad \overline{E} = \sum_m |C_m|^2 E_m$$

$$\Delta E = \sqrt{\sum_m |C_m|^2 (\overline{E} - E_m)^2}$$

its time evolution is:

$$|\psi(t)\rangle = \sum_m C_m e^{-iE_m t}|m\rangle$$

Time evolution of a generic observable $\hat{O}$

and its long time average

$$O_{mn} = \langle m|\hat{O}|n\rangle$$

$$\hat{O}(t) \equiv \langle\psi(t)|\hat{O}|\psi(t)\rangle$$

$$= \sum_m |C_m|^2 O_{mm} + \sum_{n,m\neq n} C_m^* C_n e^{-i(E_m - E_n)t} O_{mn}$$

$$\overline{O_t} \equiv \lim_{t_0\to\infty} \frac{1}{t_0}\int_0^{t_0} \hat{O}(t) = \sum_m |C_m|^2 O_{mm}$$

Diagonal ensemble

INTRODUCTION - Thermalization in quantum systems

Unproven, tested numerically

Eigenstates Thermalization Hypothesis (ETH)

Ansatz for matrix elements of observables in the basis of the eigenstates of a Hamiltonian

Deutsch, J. M. - Phys. Rev. A 43, (1991). Srednicki, M. - Phys. Rev. E 50, (1994). Rigol, M. - Nature 452, (2008).

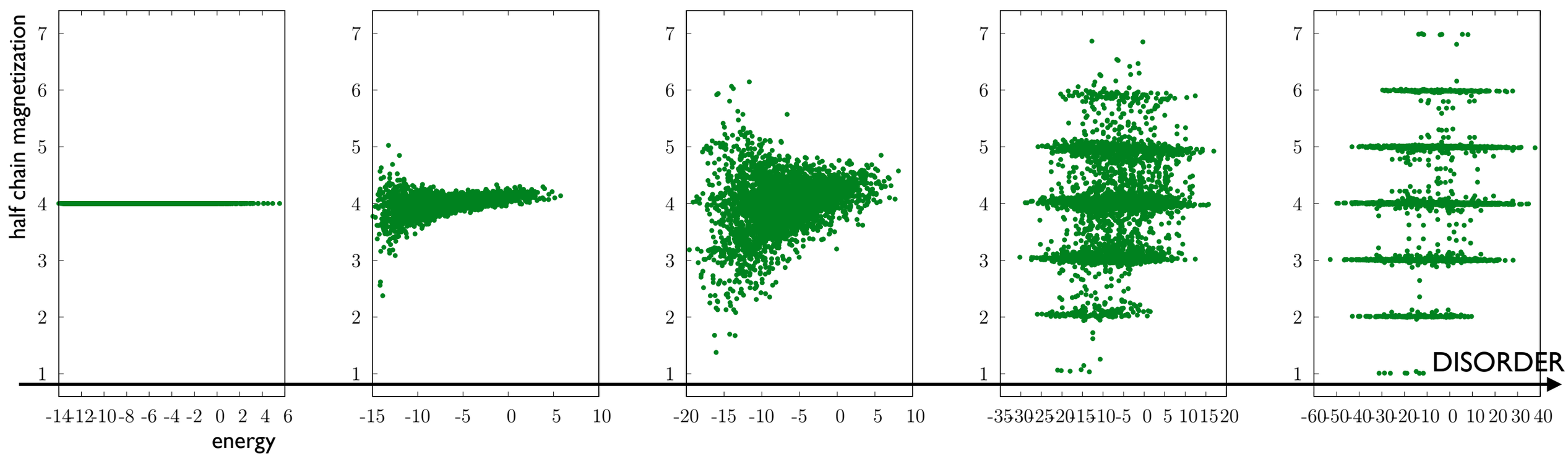
$E_m > E_{m+1} > E_{m+2}$

Diagonal terms almost equal

$$O_{m,m} - O_{m+1,m+1} \sim e^{-L^d}$$

Off-diagonal terms exponentially small

$$O_{m,n} \sim e^{-L^d}$$



INTRODUCTION - Eigenstates Thermalization Hypothesis

* Unproven, widely tested numerically

Eigenstates Thermalization Hypothesis * (ETH)

Ansatz for matrix elements of observables in the basis of the eigenstates of a Hamiltonian

Deutsch, J. M. - Phys. Rev. A 43, (1991). Srednicki, M. - Phys. Rev. E 50, (1994). Rigol, M. - Nature 452, (2008).

$E_m > E_{m+1} > E_{m+2}$

Diagonal terms almost equal

$$O_{m,m} - O_{m+1,m+1} \sim e^{-L^d}$$

Off-diagonal terms exponentially small

$$O_{m,n} \sim e^{-L^d}$$

Long time behavior is fixed by the energy of the initial state

$$\overline{O_t} = \sum_m |C_m|^2 O_{mm} = \sum_{m\in\, \bar{E}\pm\Delta E} |C_m|^2\, O_{mm} \approx O(\bar{E}) \sum_{m\in\, \bar{E}\pm\Delta E} |C_m|^2 = O(\bar{E})$$

The prediction of the microcanonical ensemble

$$\langle \hat{O} \rangle_{MC} = \frac{1}{\mathcal{N}_{\bar{E}}} \sum_{m\in\, \bar{E}\pm\Delta E} O_{mm} \approx \frac{1}{\mathcal{N}_{\bar{E}}} \sum_{m\in\, \bar{E}\pm\Delta E} O(\bar{E}) = O(\bar{E})$$

Ergodicity

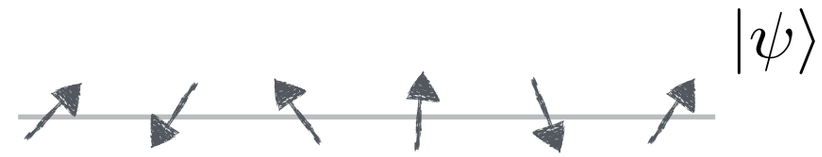
$$\overline{O_t} = \langle \hat{O} \rangle_{MC}(\overline{E})$$

time average = ***ensamble average***

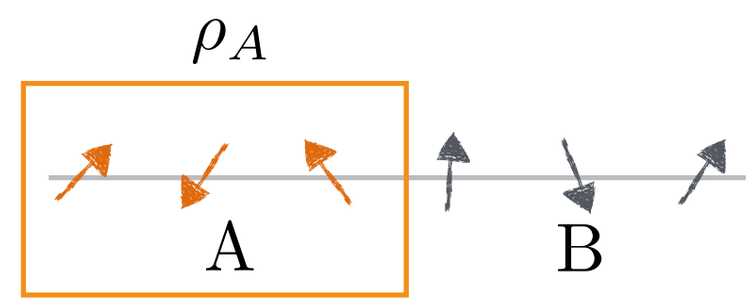
INTRODUCTION - Entanglement Entropy

An ***isolated system*** can reach ***thermal equilibrium*** only if it can acts as its own thermal bath

Pure many-body state $\rho = |\psi\rangle\langle\psi| \quad \mathrm{Tr}\rho^2 = 1$



Can a subsystem act as a bath for the rest of the system?



Reduced density matrix $\rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$

Entanglement entropy $S_A = -\mathrm{Tr}\rho_A \log_2 \rho_A$

$S_A \begin{cases} = 0 & \text{pure state} \\ > 0 & \text{mixed state} \end{cases}$

For systems that fulfill the ETH $\rho_A \approx \exp(-\beta \mathcal{H}_A)$

All the excited eigenstates are THERMAL ⟶ Entanglement Entropy is extensive

VOLUME LAW for ENTANGLEMENT ENTROPY $S_A = s(E)\, L^d$

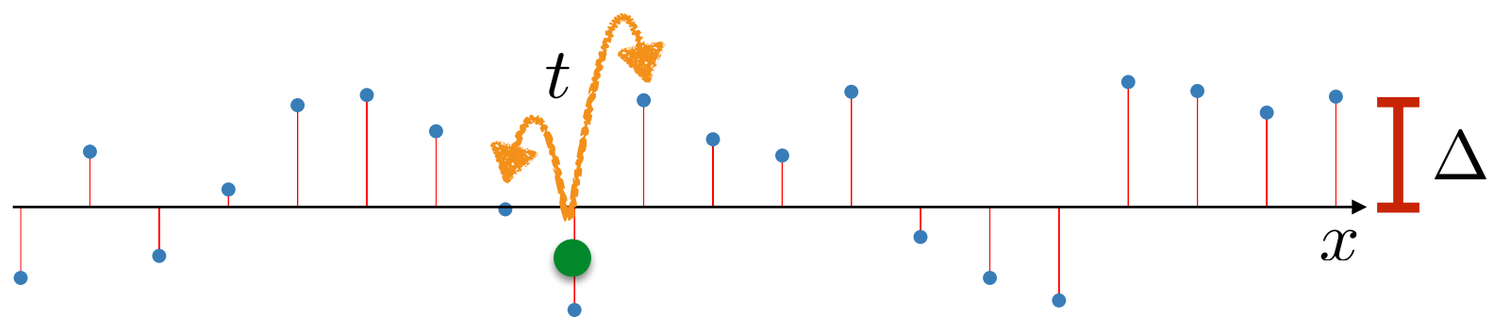
INTRODUCTION - Anderson model

Are all the quantum many-body systems ergodic?

Anderson model

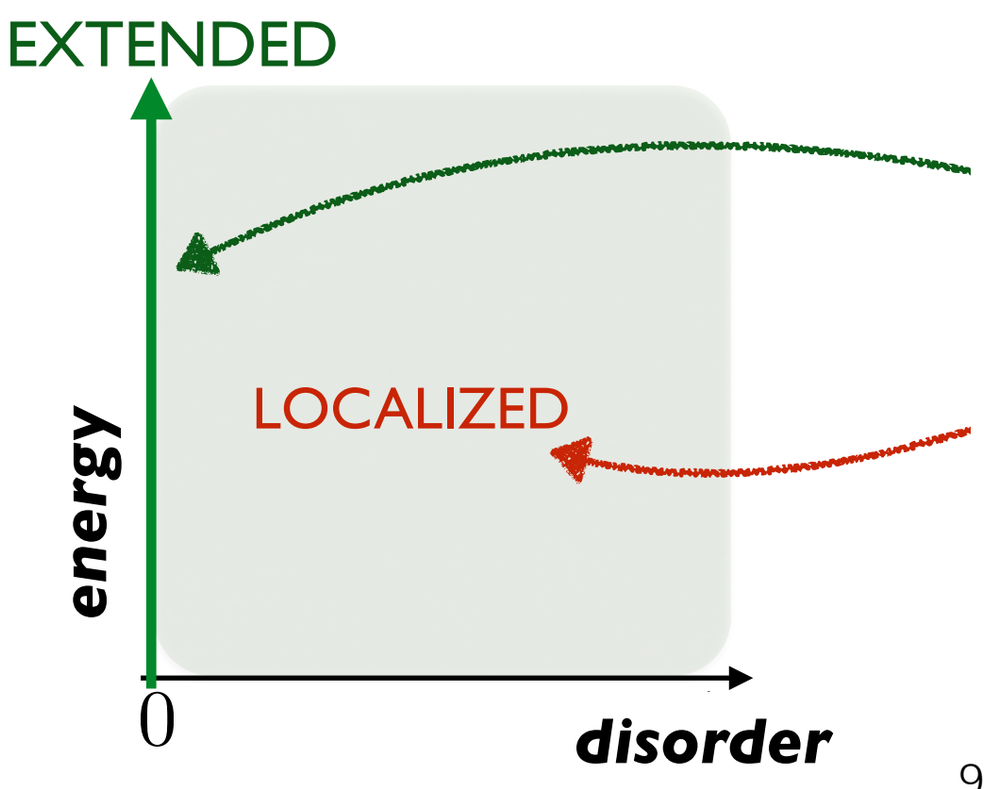
$$\mathcal{H} = \sum_j \boxed{-t\,(c_j^\dagger c_{j+1} + h.c.)} + \boxed{\mu_j\, n_j} \qquad \mu_i \in [-\Delta, \Delta] \;\; \text{random}$$

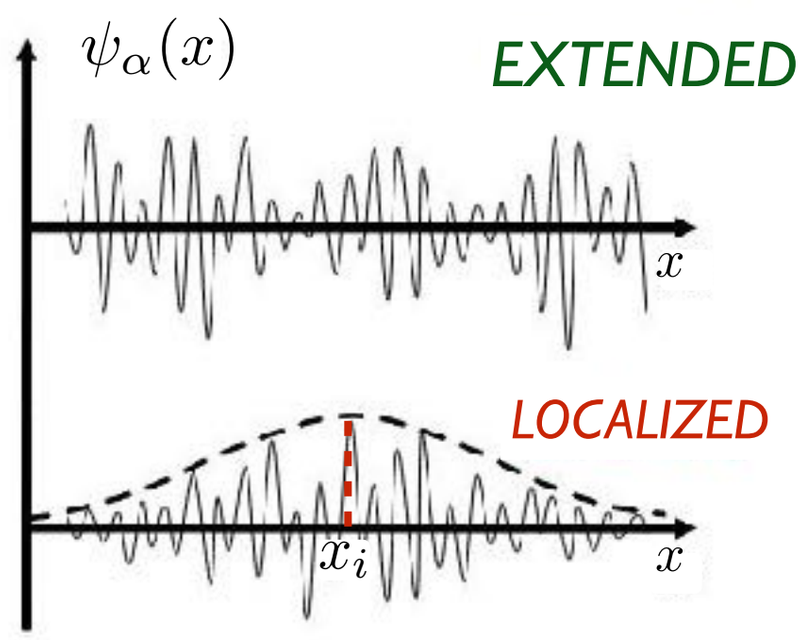
RANDOM POTENTIAL



P. Anderson, Phys rev, 1958
(Nobel in 1977)

$\Delta > 0$ → All eigenstates are LOCALIZED (in 1D e 2D)

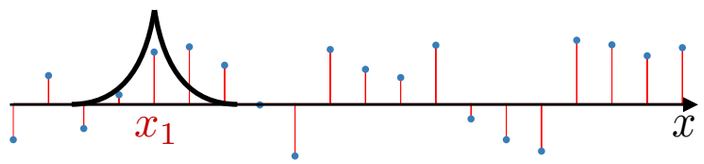




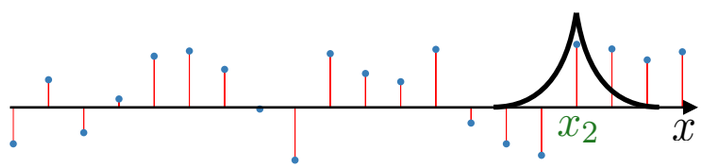
INTRODUCTION - Anderson model

We can prepare the system in a superposition two different localized states:

$|\psi_1\rangle$



$|\psi_2\rangle$



$$|\psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle$$

Particles are localized and will not be free to spread all over the lattice

The evolution of the system will depend strongly on the initial choice of α and β

Evolution of expectation values of states close in energy can be really different!

ETH is not fulfilled by Anderson model in the localized regime

INTRODUCTION - Many-body localization (MBL)

If we add INTERACTIONS to a system of free localized particles…

- Will the interactions destroy the localization?
- Will the localization persists in some regimes (Many Body Localization)?

For strong enough disorder yes!

D. Basko, I. Aleiner, and B. Altshuler, Annals of Physics 321, 1126 (2006).

MANY-BODY LOCALIZATION

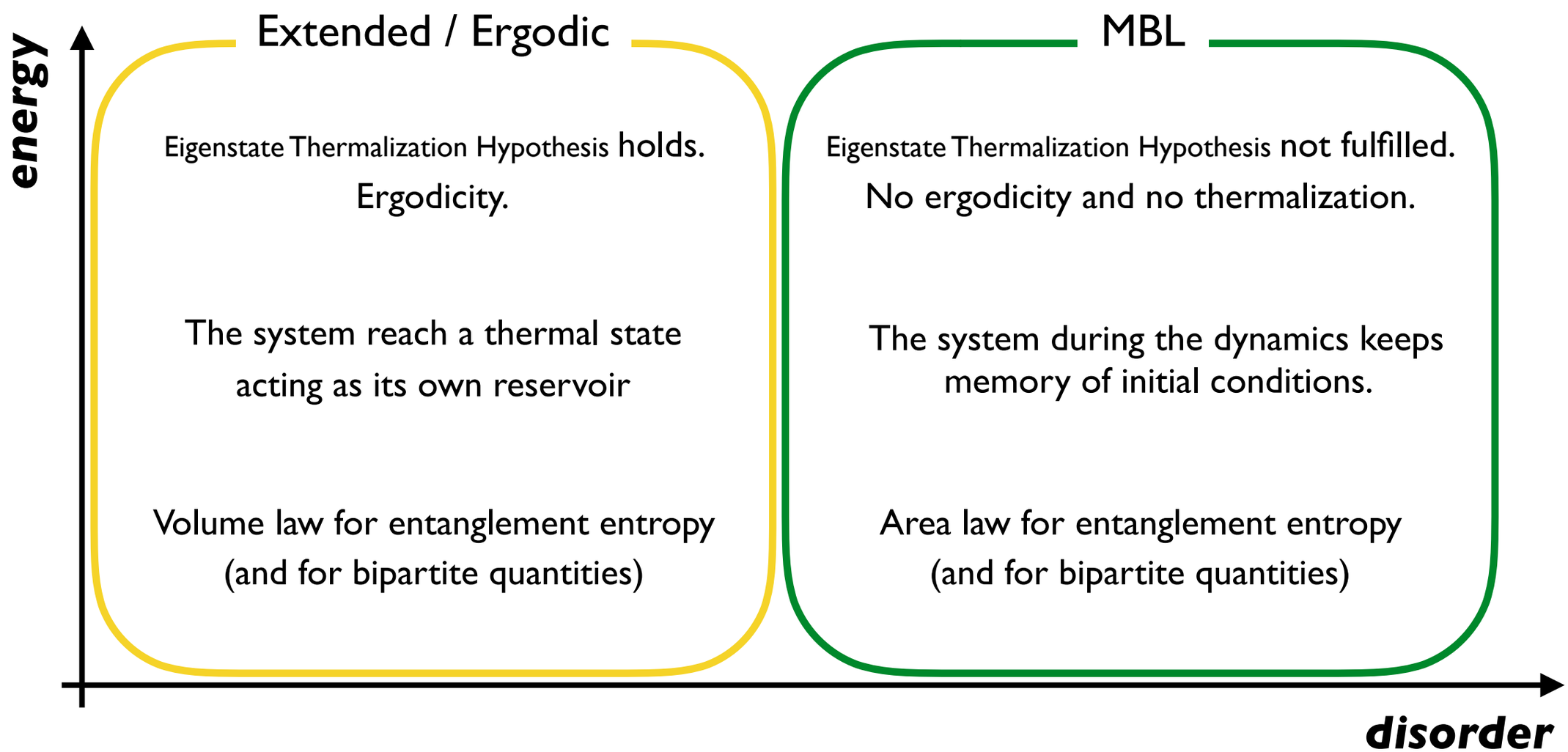
The system during the dynamics keeps memory of initial conditions

MBL states at finite energy density VIOLATE ETH *(not thermal)*

AREA LAW for Entanglement Entropy $S_A \propto L^{d-1}$

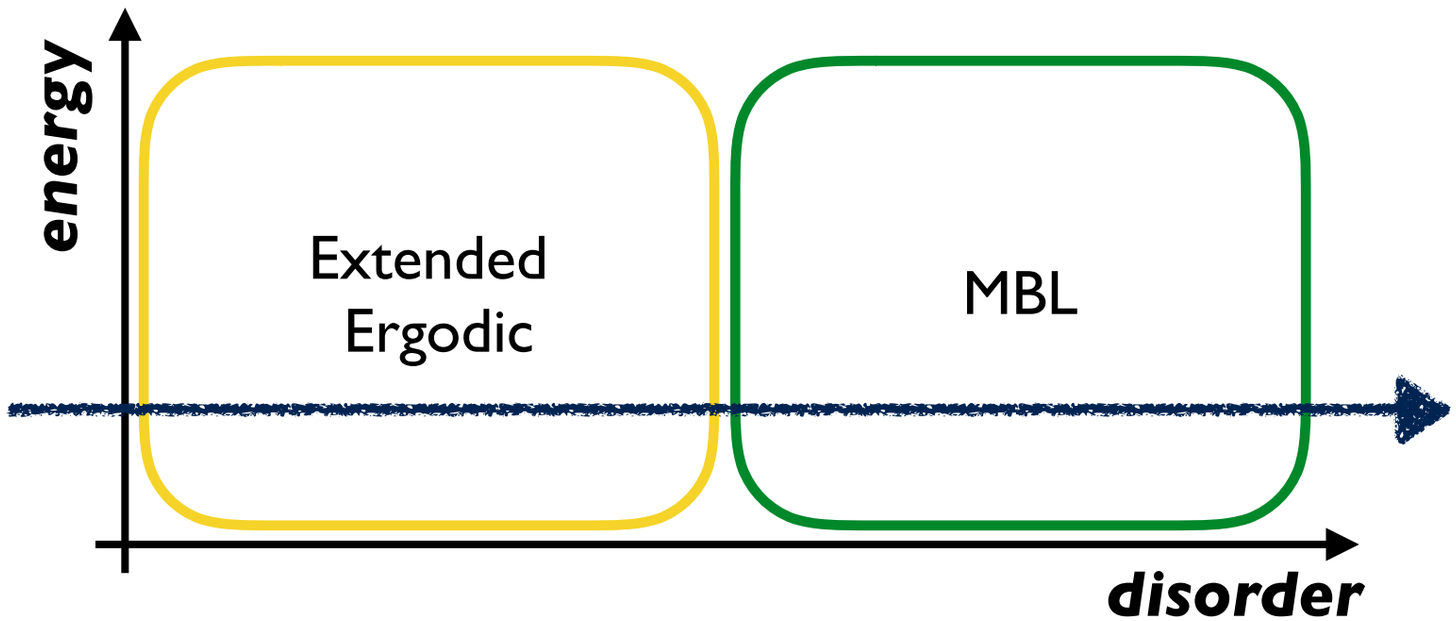
INTRODUCTION - Many-body localization transition

The many-body localization transition is a ***dynamical quantum phase transition*** involving ***highly excited eigenstates*** of a ***disordered quantum many-body Hamiltonian***.

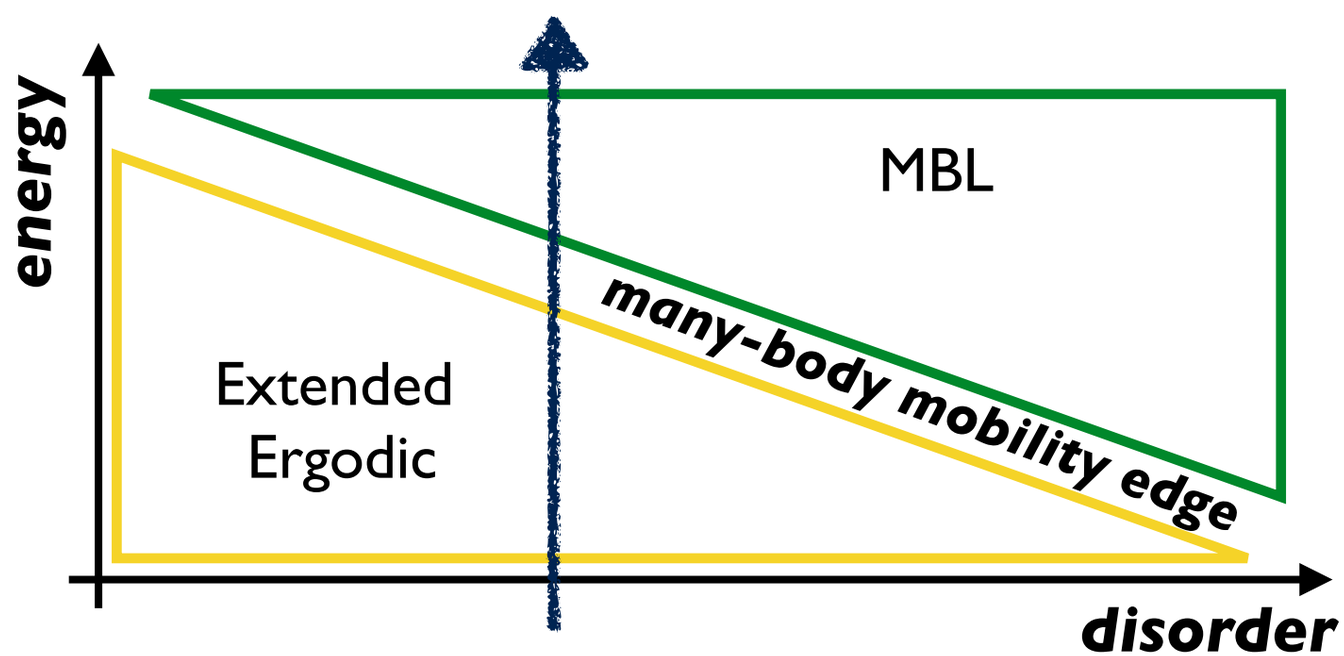


INTRODUCTION - Many-body localization transition

The transition can be driven by the ***strength of disorder*** in a given spectral range ….

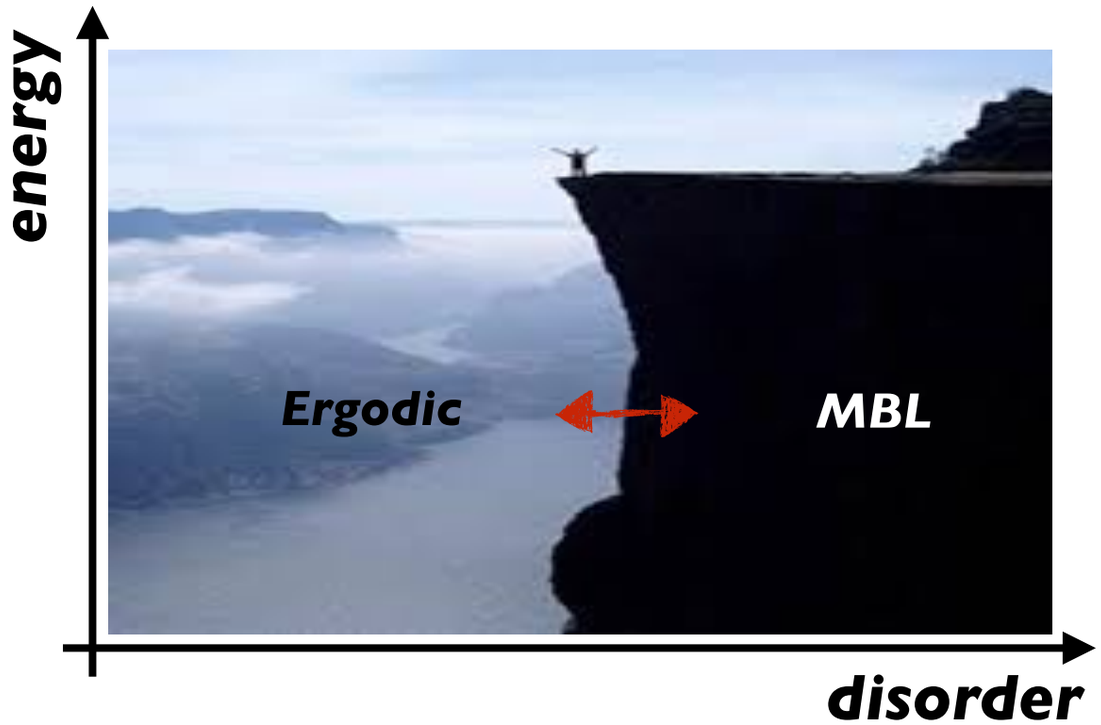


…… or by the *energy (at fixed disorder)* if the system possesses a ***many-body mobility edge***



MBL - Physics in the middle of the spectrum

Highly excited eigenstates of general many-body system are **difficult to study**



Highly excited states are reachable through
EXACT FULL DIAGONALIZATION

$$\dim \mathcal{H}_{tot} = (\dim \mathcal{H}_{loc})^L$$

MAX SIZE: L=22
(disordered systems, fermions, half filling)

Another possibility is to study the dynamics following a ***QUENCH***

$|\psi_0\rangle$ ground state of $\mathcal{H}_i$

$$|\psi(t)\rangle = exp(-i\mathcal{H}_f t)|\psi_0\rangle$$

We can initialize the time evolution in a highly excited state of $\mathcal{H}_f$

MBL - Quench Spectroscopy

P.N., E. Ercolessi, T. Roscilde - SciPost Phys. 1(1), 010 (2016)

QUENCH SPECTROSCOPY

Quenching a parameter in the Hamiltonian $\mathcal{H}(\Delta_i) \to \mathcal{H}(\Delta_f)$

ground state

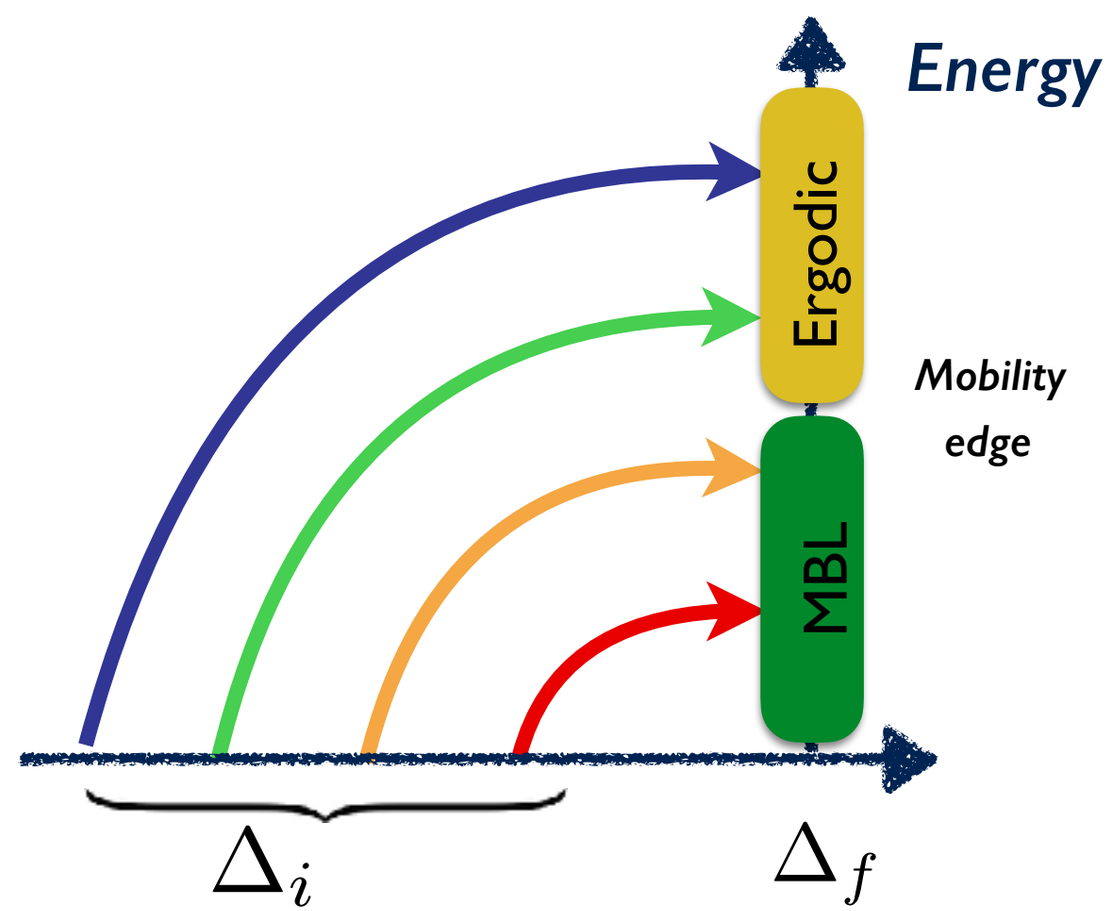
we inject an amount of energy $\epsilon = \langle \psi_i^{(0)} | \mathcal{H}(\Delta_f) | \psi_i^{(0)} \rangle - \langle \psi_f^{(0)} | \mathcal{H}(\Delta_f) | \psi_f^{(0)} \rangle$

that is controlled by the quench amplitude $\epsilon \propto \Delta_f - \Delta_i$

In particular if $\mathcal{H}(\Delta_f)$ has a mobility-edge

varying Δ_i, we can populate highly excited states in different energy windows both in the **EXTENDED** or **MBL** regimes,

and we can study dynamics in the two phases below and above the transition.



MBL - Many-body mobility edge

Interacting fermions in a quasiperiodic potential

$$\mathcal{H} = \sum_j -t\left(c_j^\dagger c_{j+1} + h.c.\right) + \boxed{\Delta}\cos\left(2\pi\,\alpha\,j + \phi\right) n_j \; + \boxed{V}\, n_j n_{j+1}$$

Disorder

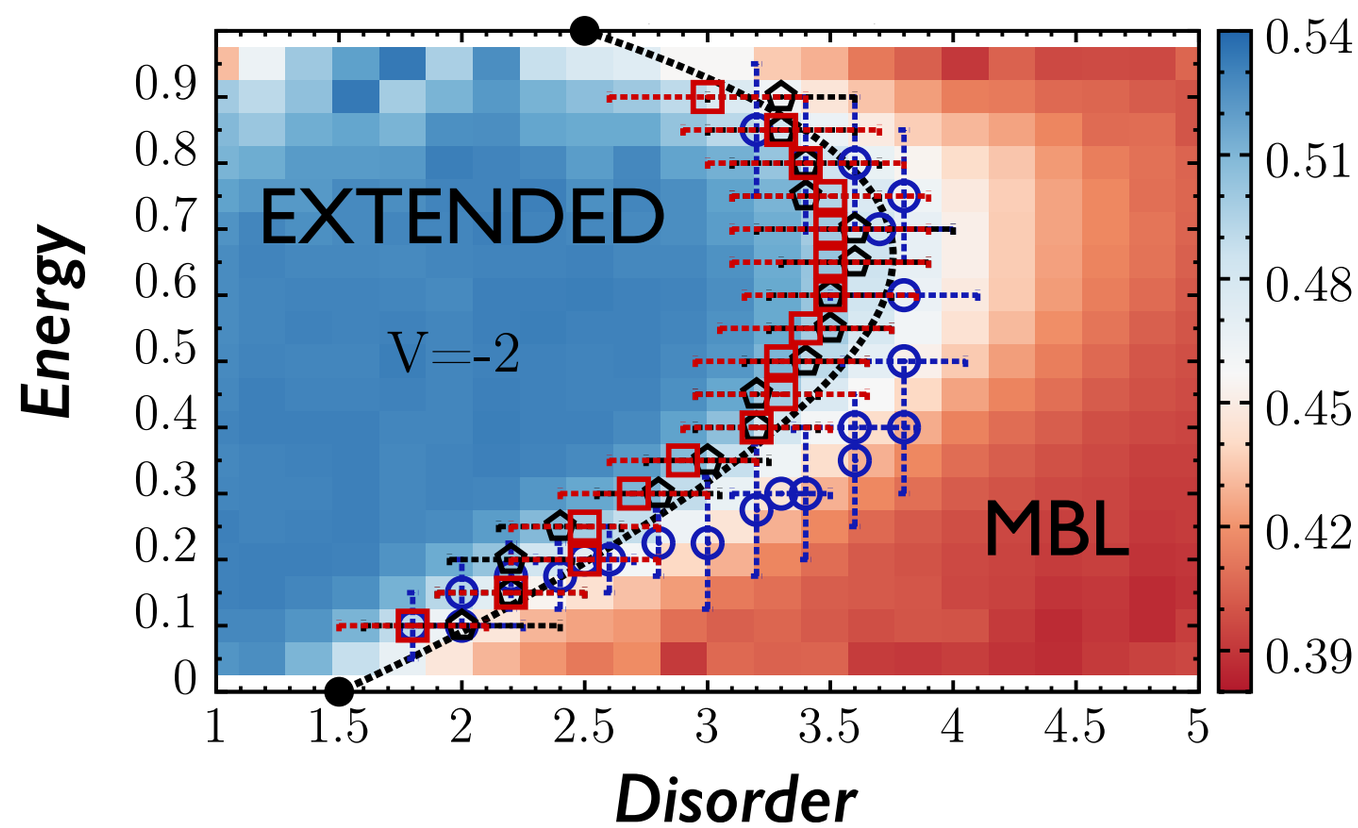
Interactions

Studying several indicators we reconstruct the ***spectral phase diagram***

Energy level statistics

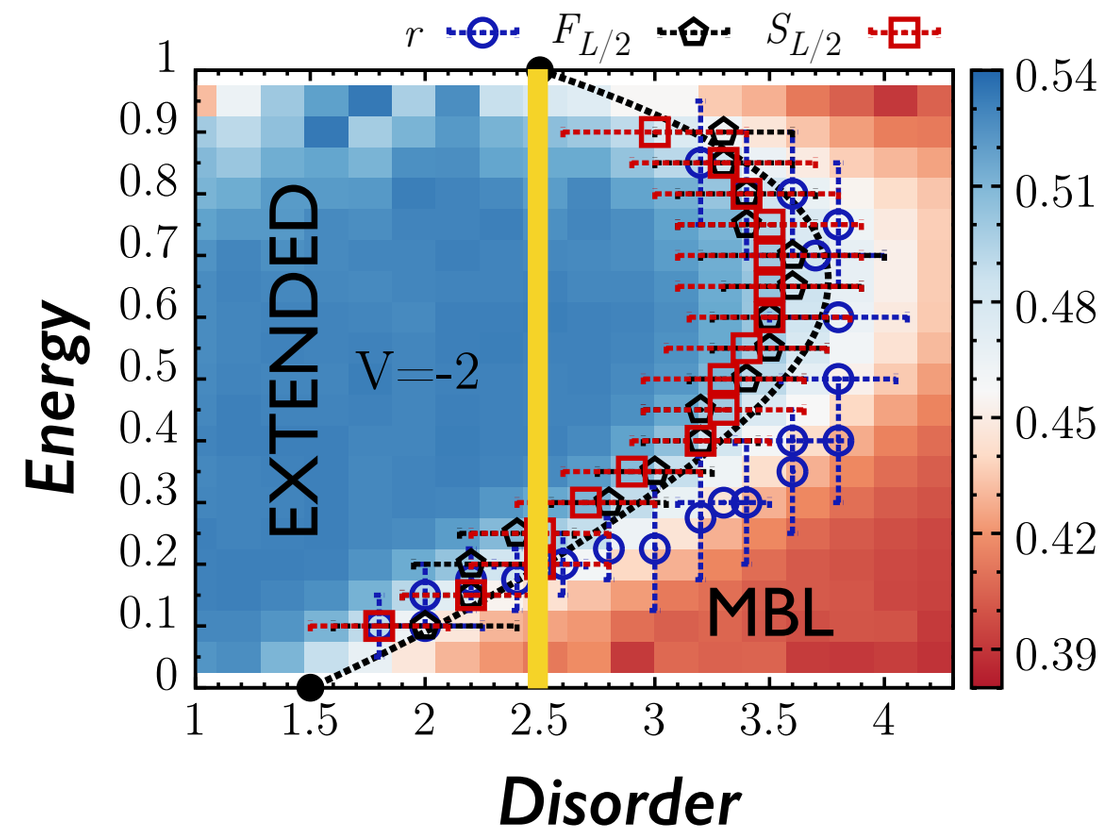
Particle-number fluctuations

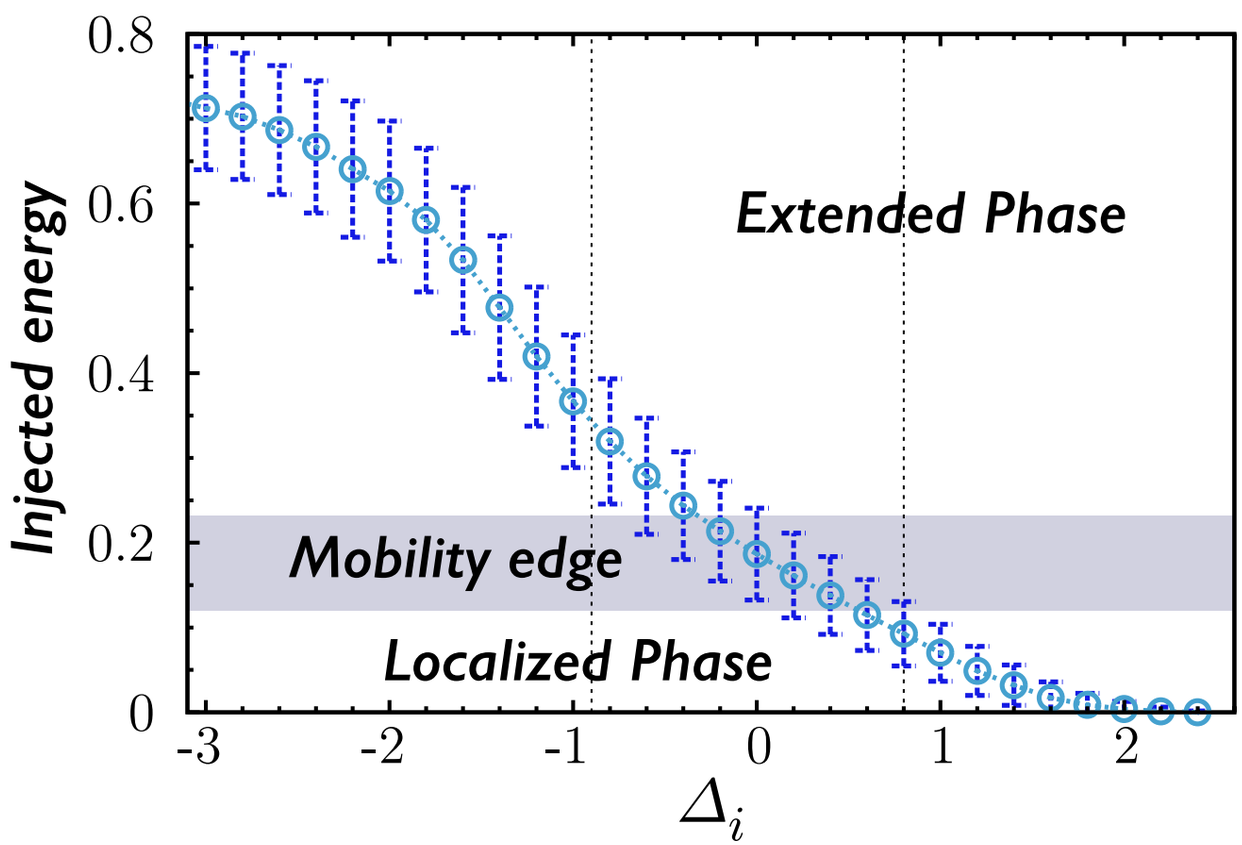
Entanglement entropy



DYNAMICS - Mobility edge via quench spectroscopy

We calculate the energy injected into the system after a quench from an initial potential depth Δ_i to a final value $\Delta_f = 2.5$.

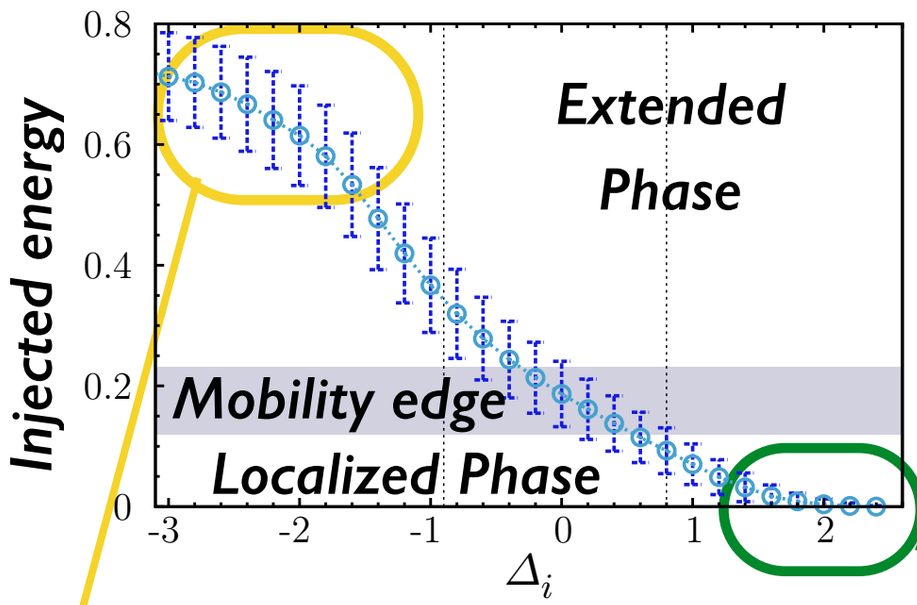




Time-dependent simulations

exact diagonalization	$L \leq 14$
T-DMRG	$L = 20,\ 24,\ 28,\ 34$

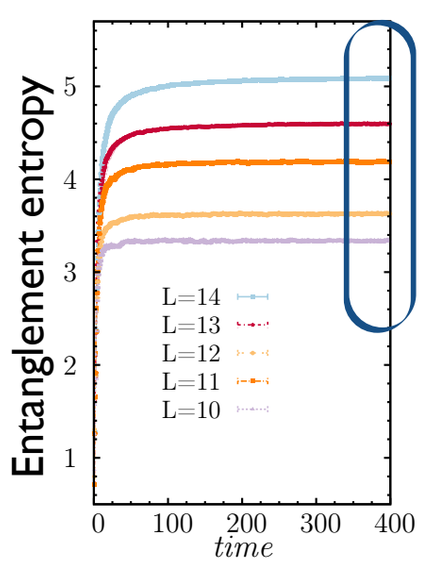
DYNAMICS - Mobility edge via quench spectroscopy

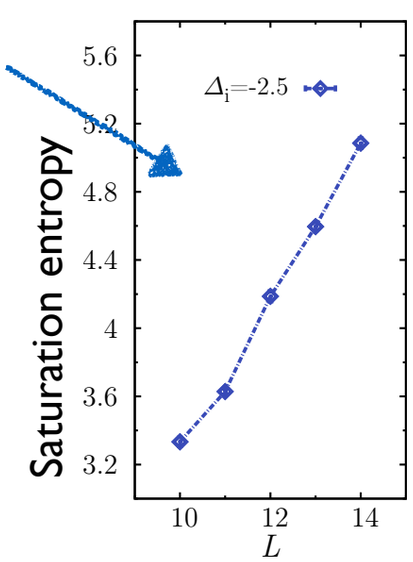


We study the time evolution of entanglement entropy and its saturation value after a quench.

Large quenches

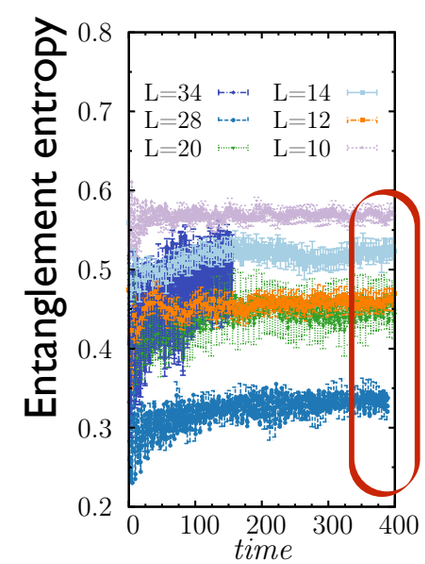
EXTENSIVE ENTROPY → *Extended PHASE*

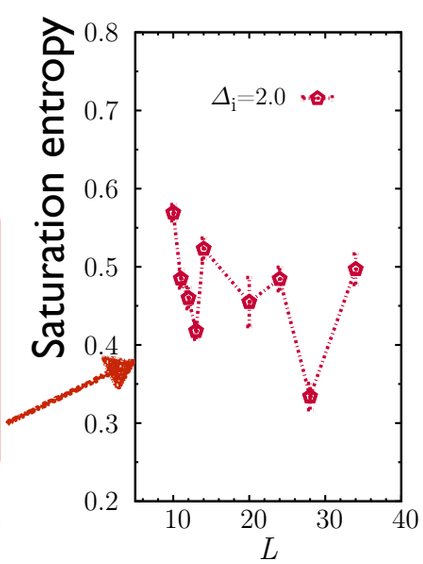




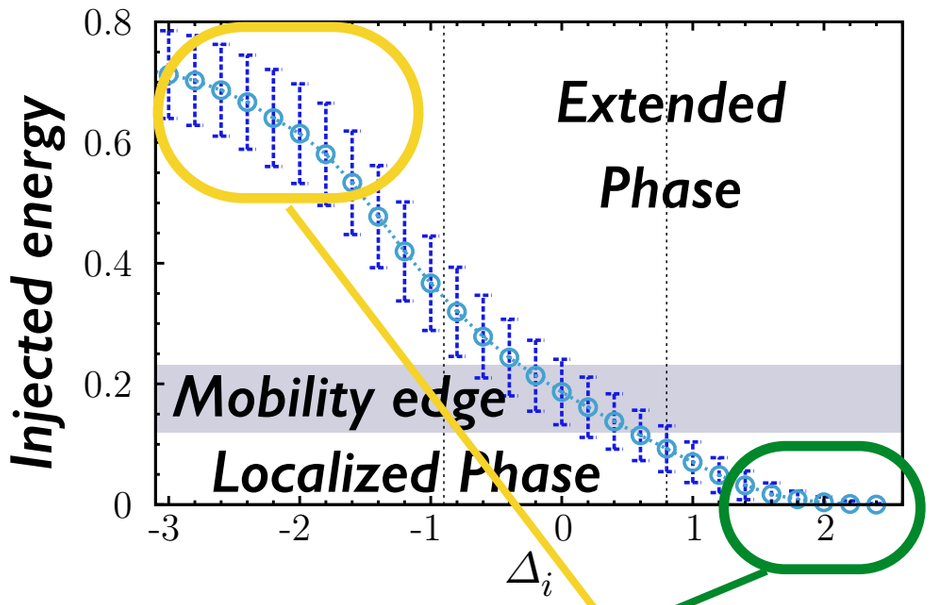
Small quenches

INTENSIVE ENTROPY → *MBL PHASE*





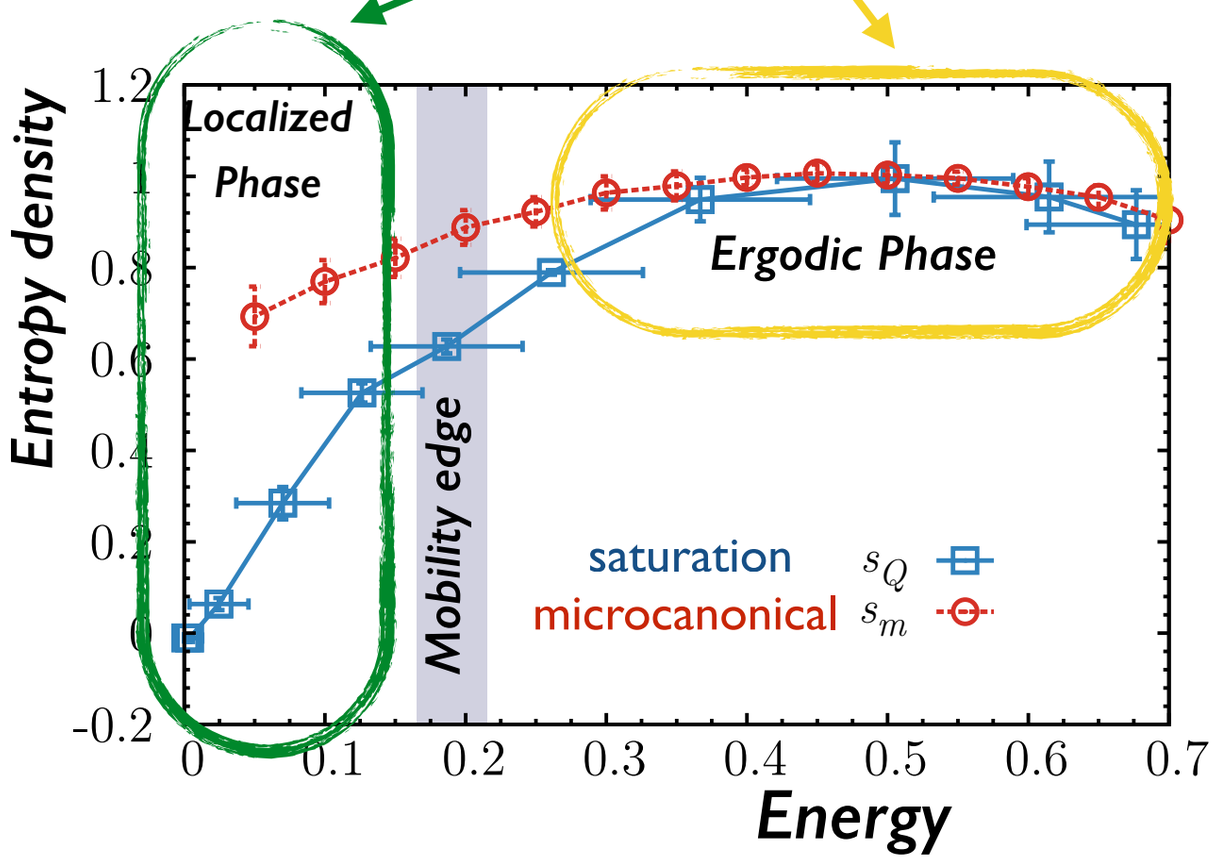
DYNAMICS - Mobility edge via quench spectroscopy



We compare the microcanonical entropy density and entropy density obtained from the scaling of the saturation entropy

$$S(\epsilon; L)_{micro} = s_m(\epsilon) L + \text{const}$$

$$S_\infty = s_Q L/2 + \text{const}$$



Only for a sufficiently large energy density the entanglement entropy density matches the thermal value

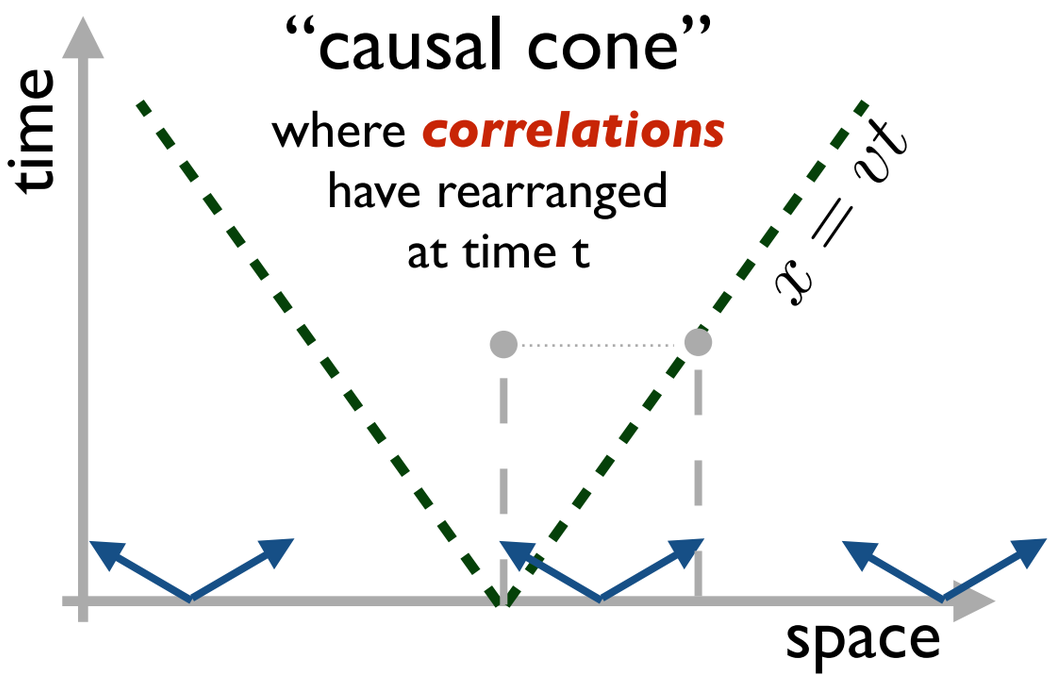
Spreading of correlations

ongoig work!

How does the correlations spread in a Many-body localized system ?

Canonical picture: "light-cone" spreading from <u>quasi-particle emission</u>

Lieb & Robinson, 1972; Calabrese & Cardy, 2004; etc.



One characteristic speed in the system

$$v \approx 2 \max_k v_g(k)$$

maximum group velocity

Spreading of correlations

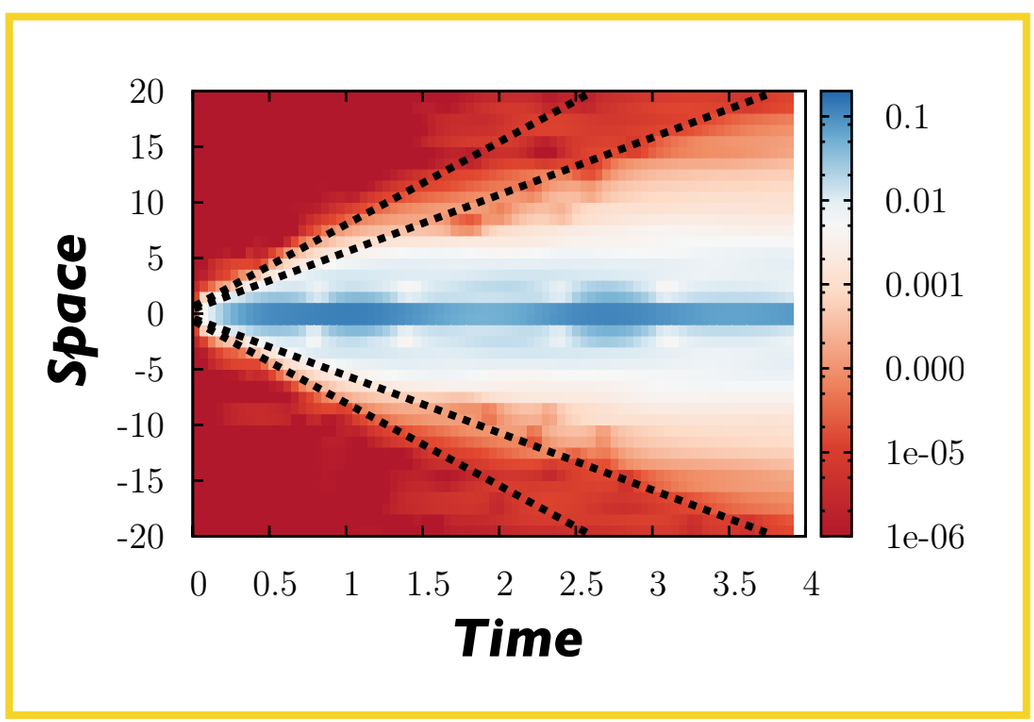
Starting from an uncorrelated product state
we can observe the formation of correlations during the time evolution.

Density-Density correlations
(averaged over translations)

$$C_j(t) = \frac{1}{L}\sum_{i=1}^{L}\left(\langle n_i(t) n_{i+j}(t)\rangle - \langle n_i(t)\rangle\langle n_{i+j}(t)\rangle\right)$$

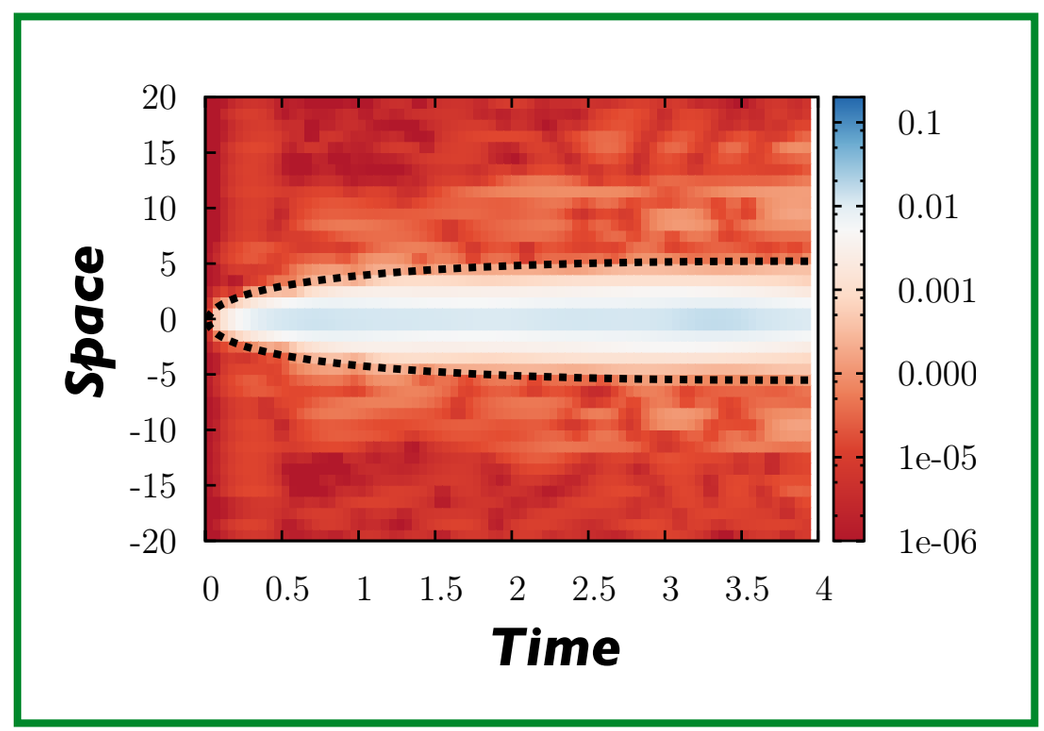
Low disorder - Extended phase

Light-cone spreading of correlations



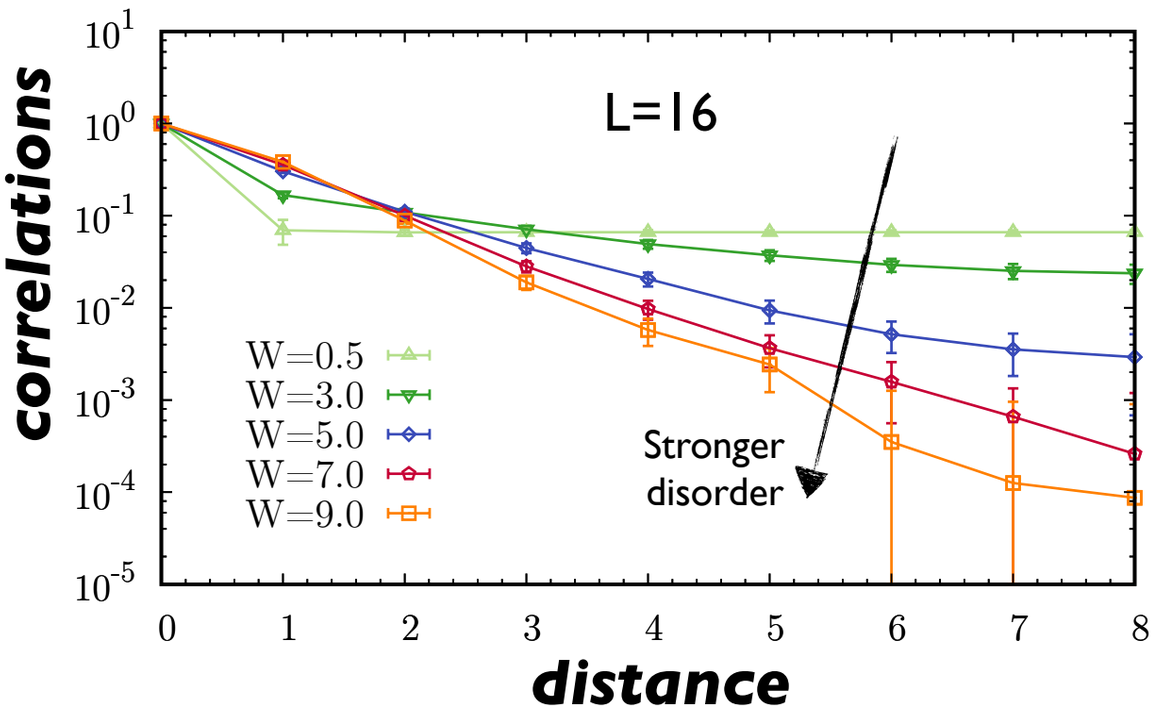
Strong disorder - MBL

No spreading of correlations



Decay of density-density correlations

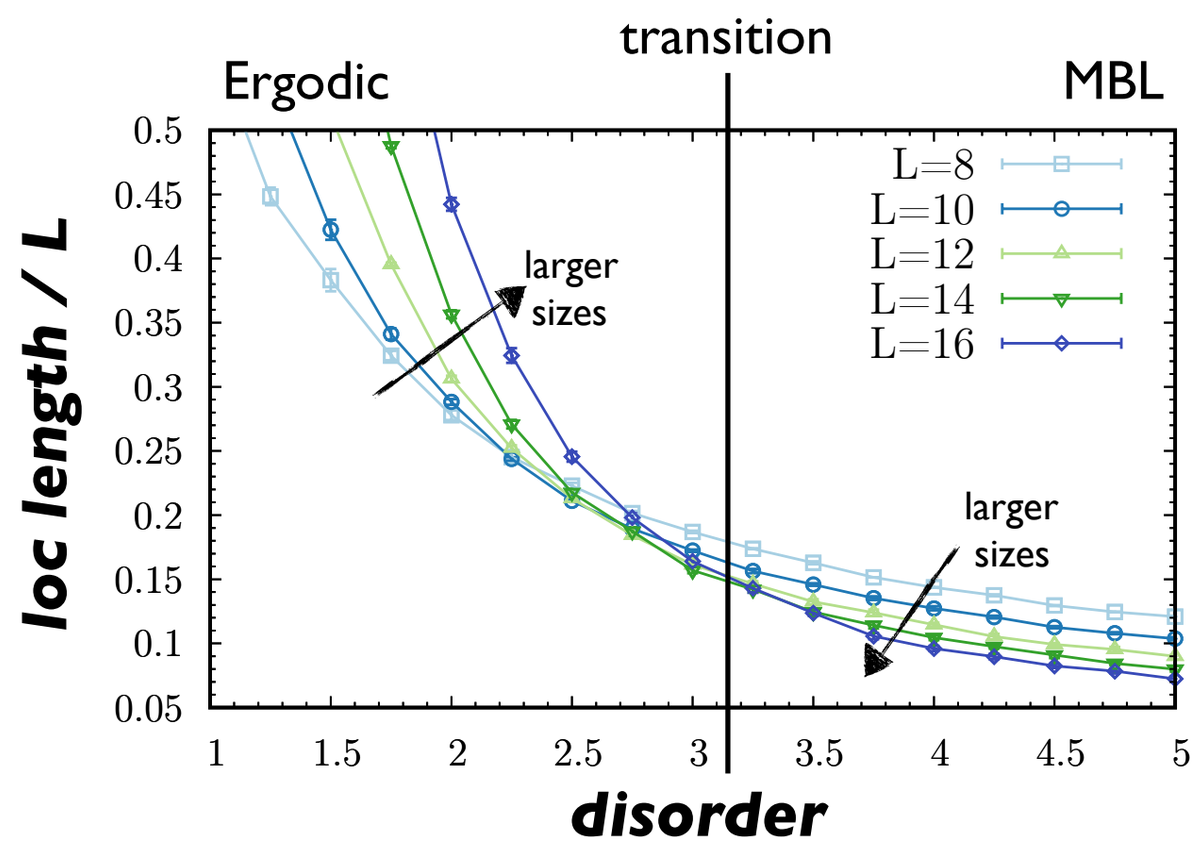
Correlation for the long-time equilibrium state in the ***Diagonal Ensamble***



$$\overline{O_t} \equiv \lim_{t_0 \to \infty} \frac{1}{t_0} \int_0^{t_0} \hat{O}(t) = \sum_m |C_m|^2 O_{mm}$$

Diagonal ensemble

From the correlation functions we can estrapolate a characteristic ***localization length***



Conclusions & Perspectives

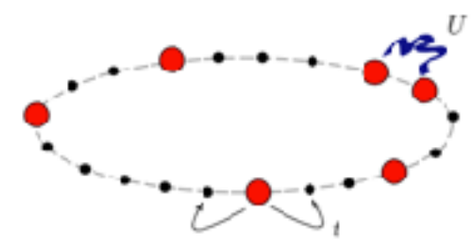
- We propose a method for detecting the many-body mobility edge.

- Quench-spectroscopy for detecting a many-body mobility edge experimentally?

Work in progress

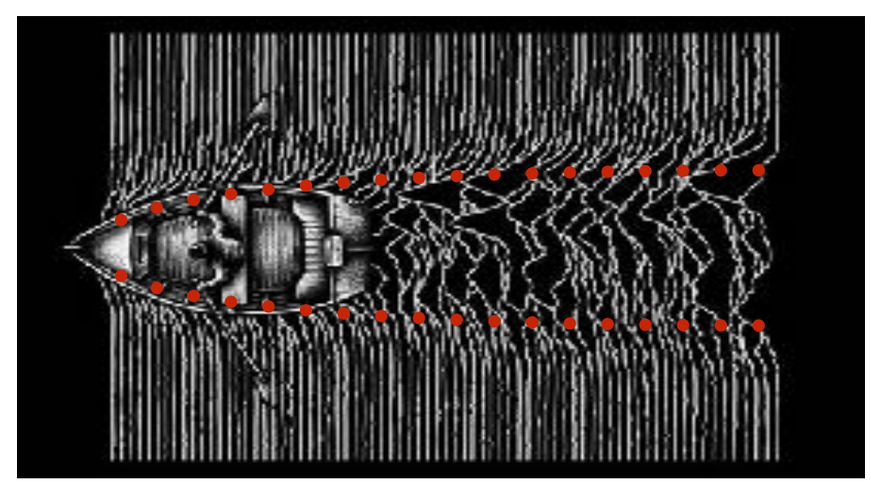
- Transport properties and conduction.

- Spreading of correlations.



Extended

MBL



Thank you for your attention!