Chiral inhomogeneous phases in dense quark/ nuclear matter

Hiroaki Abuki
Aichi University of Education

Refs: Y. Takeda, HA, M. Harada, PLD97 (2018)
H. Abuki, JPSC20 (2018); PRD, coming soon

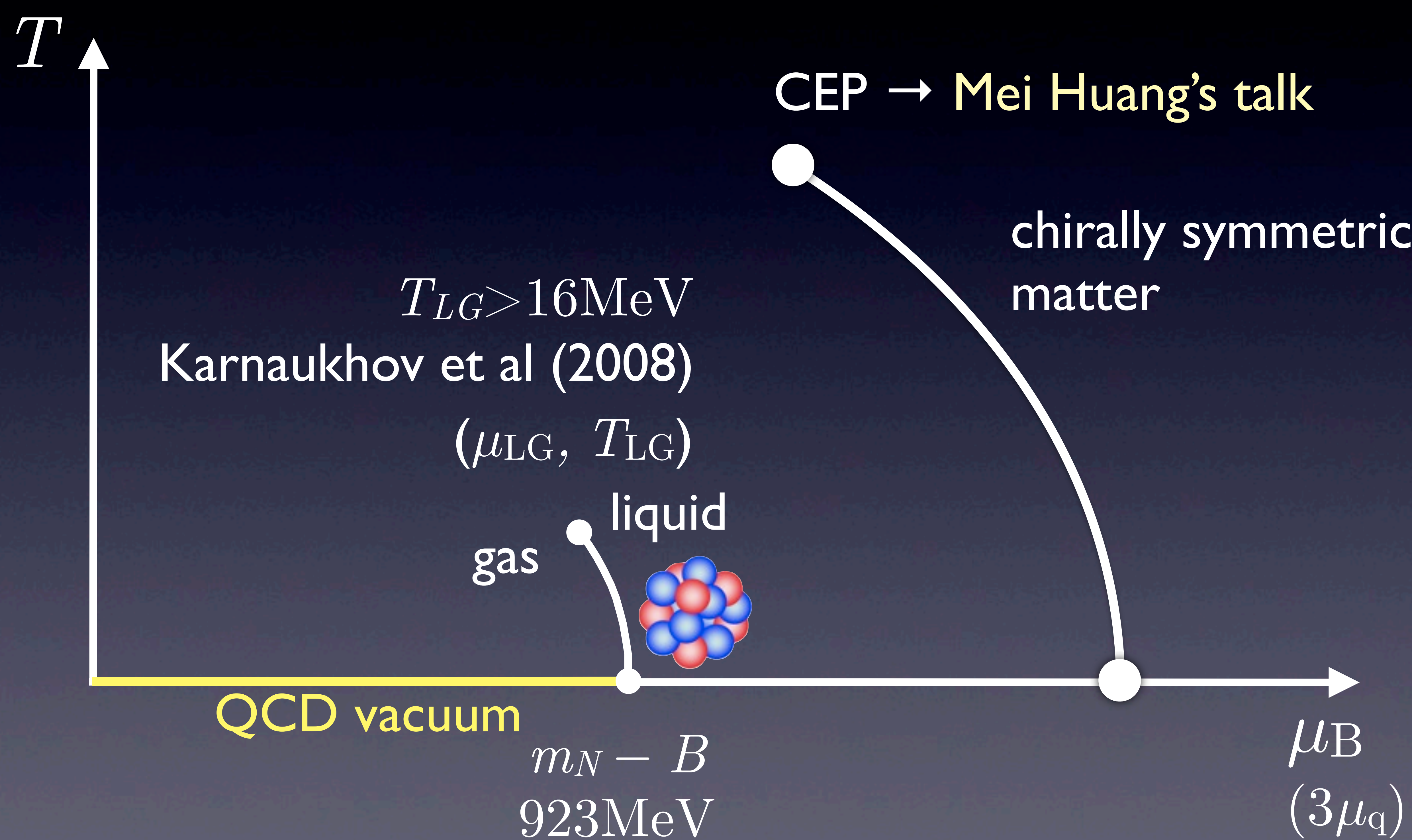
QCD@work 2018: International Workshop on QCD; Theory and Experiment
Matera, Italia, 25-28 June 2018

Thank you for nice workshop every 2 yrs

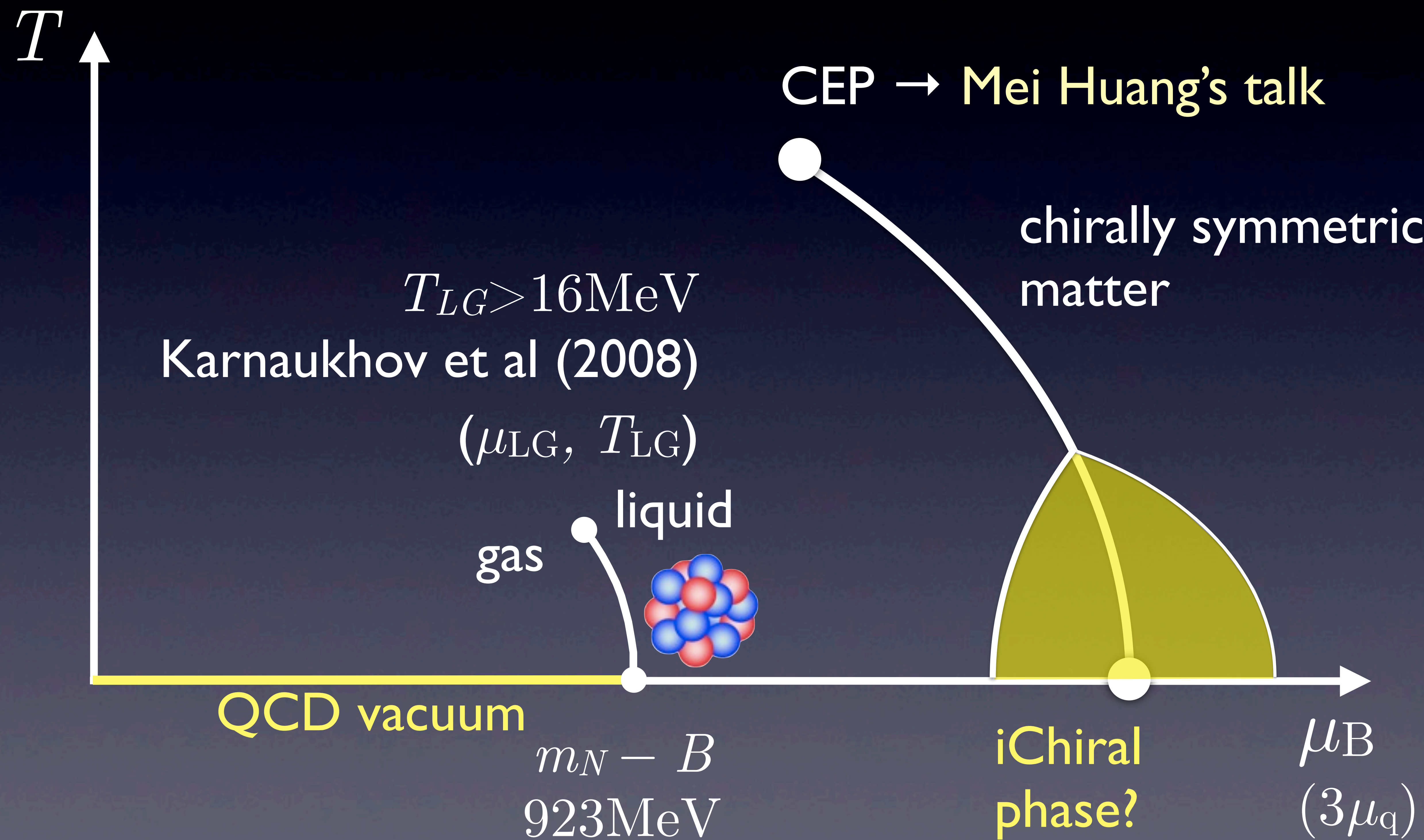
Plan

- What is inhomogeneous chiral phase?
- generalized Ginzburg-Landau (gGL) approach to inhomogeneous phases in quark matter; mass vs magnetic field
- Parity-doublet model applied to DCDW phase in nuclear matter
- Summary and Outlook

QCD phase diagram

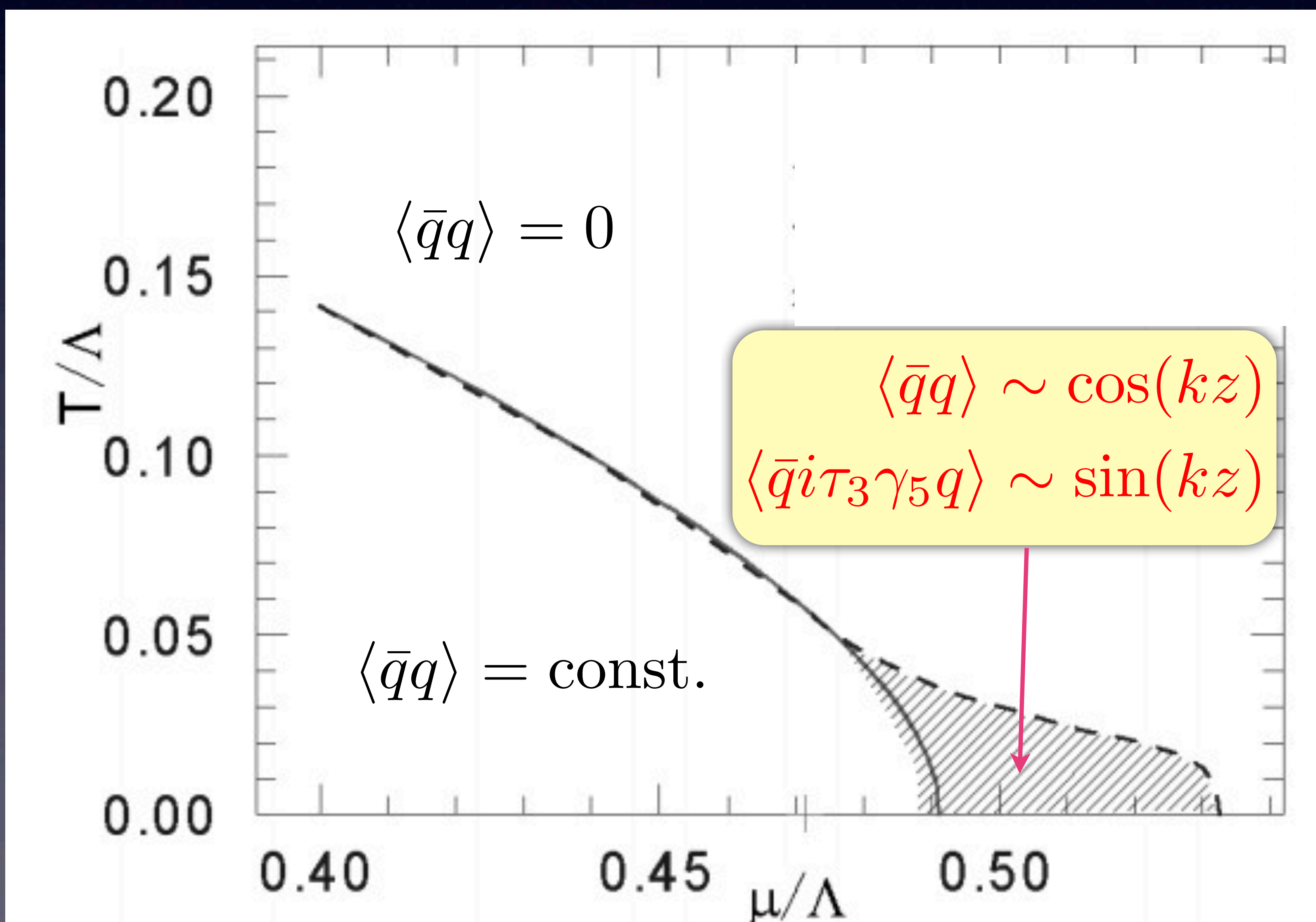


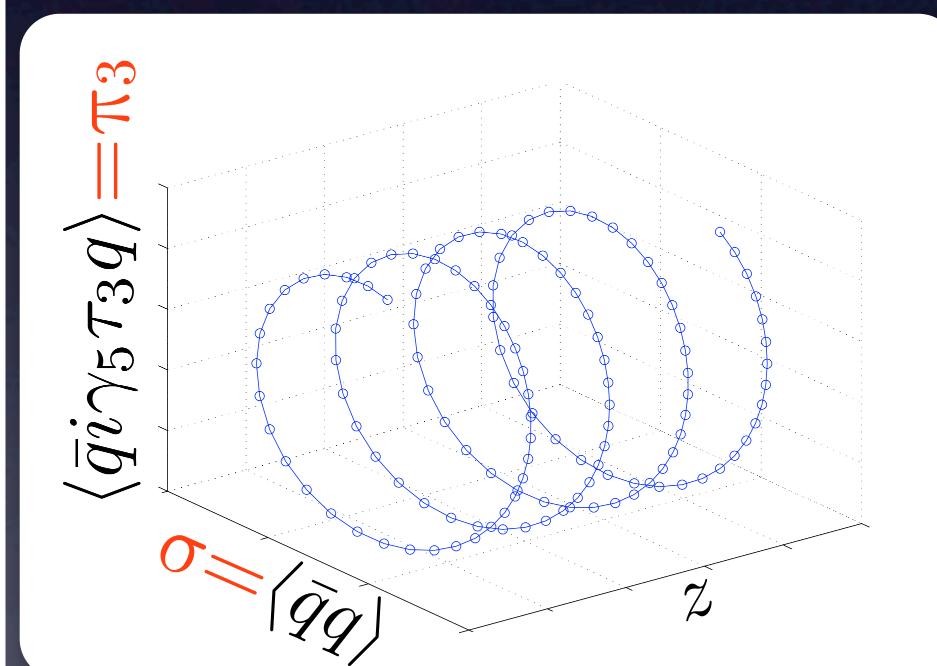
QCD phase diagram



DCDW; Chiral Spiral Phase

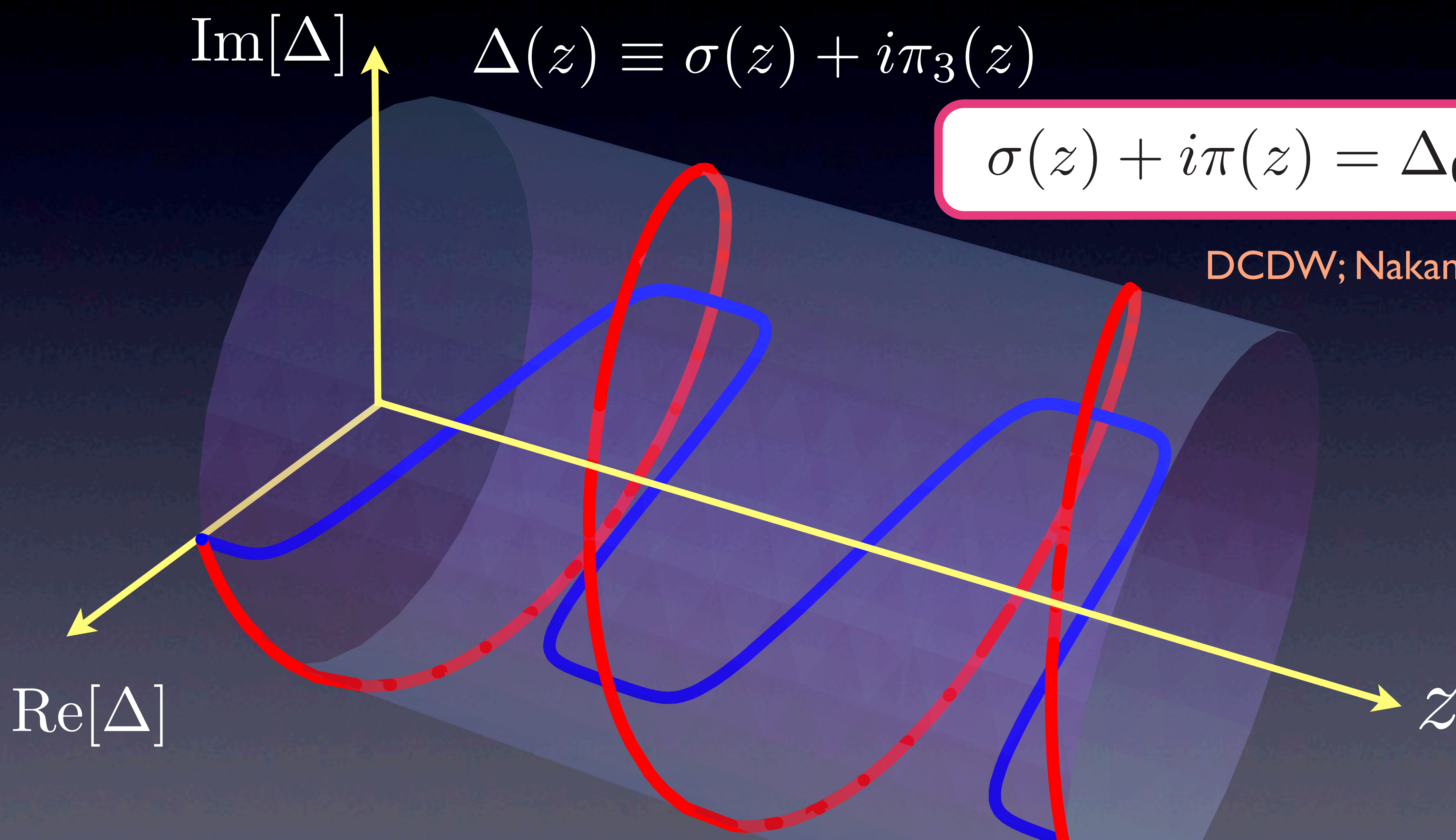
Nakano, Tatsumi, PRD71 (2005) 114006





DCDW or RKC?

$$\Delta(z) \equiv \sigma(z) + i\pi_3(z)$$



$$\sigma(z) + i\pi(z) = \Delta_0 e^{iqz}$$

DCDW; Nakano, Tatsumi

$$\sigma(z) = \sqrt{\nu} q \, \mathrm{sn}(qz, \nu)$$

Real Kink Crystal; Nickel, PRL09, PRD09

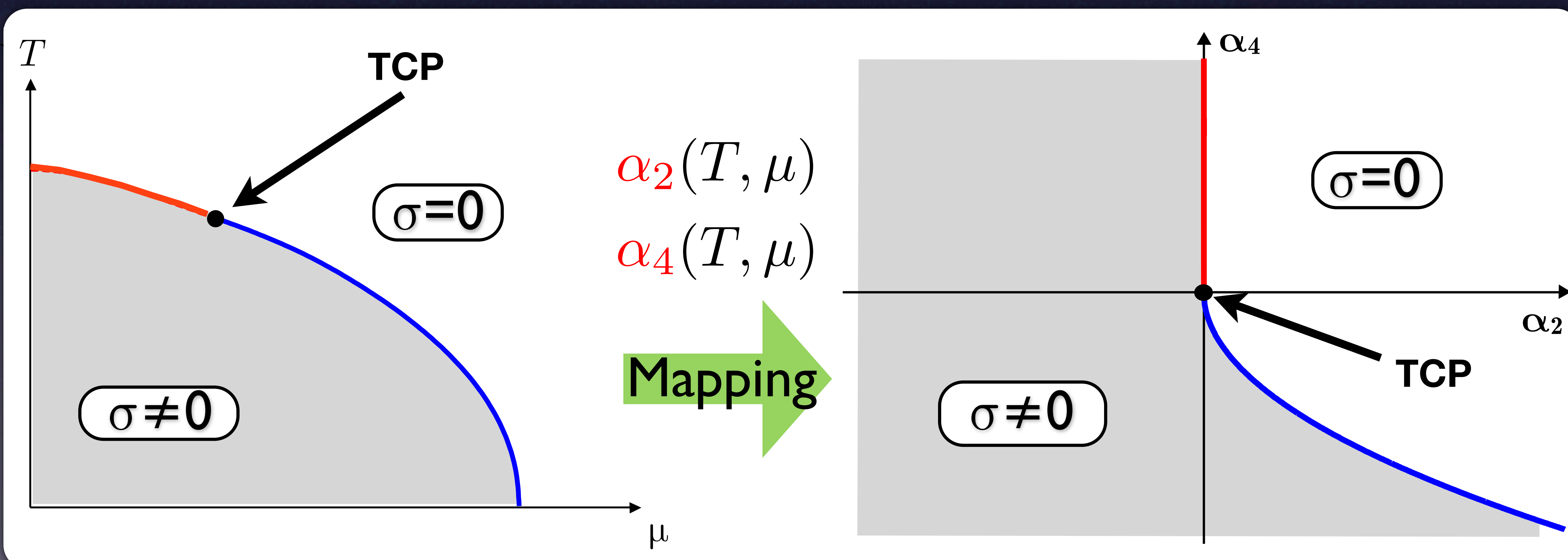
Plan

- What is inhomogeneous chiral phase?
- generalized Ginzburg-Landau (gGL) approach to inhomogeneous phases in quark matter; mass vs magnetic field
- DCDW in nuclear matter? Parity-doublet model approach
- Summary and Outlook

Ginzburg-Landau (GL) approach (homogeneous)

Minimal GL to describe the tricritical point (TCP)

$$\Omega_{\rm GL} \quad = \quad \frac{\alpha_2}{2}\sigma(\mathbf{x})^2 + \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{1}{6}\sigma(\mathbf{x})^6$$



generalized Ginzburg-Landau (gGL) expansion

Nickel, PRL09
Abuki (2014)

mass: $O(4) \mapsto O(3)$ *isospin*

gradient terms

$$\Omega_{\rm GL} = -h\sigma(\mathbf{x}) + \frac{\alpha_2}{2}\sigma(\mathbf{x})^2 + \frac{\alpha_4}{4}\sigma(\mathbf{x})^4 + \frac{\alpha_{4b}}{4}(\nabla\sigma(\mathbf{x}))^2 + \frac{\alpha_6}{6}\sigma(\mathbf{x})^6 + \frac{\alpha_{6b}}{6}\sigma^2(\nabla\sigma)^2 + \frac{\alpha_{6c}}{6}(\Delta\sigma)^2$$

$\alpha_{4b} = 0$: Lifshitz Point (LP)

$h = \alpha_2 = \alpha_4 = 0$: Tricritical Point (TCP)

For *improved* version of gGL→ Mannarelli's talk

Including magnetic field

New couplings: 3D rotational symmetry breaking

$$\delta\Omega = -\frac{1}{8}\frac{\partial \alpha_4}{\partial \mu}(e\mathbf{B}) \cdot (\pi_{\mathbf{3}}\nabla\sigma - \sigma\nabla\pi_{\mathbf{3}})$$

$$= \mathbf{b} \cdot (\pi_3\nabla\sigma - \sigma\nabla\pi_3)$$

Including magnetic field

New couplings: 3D rotational symmetry breaking

$$\delta\Omega = -\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu}(e\mathbf{B})\cdot(\pi_{\mathbf{3}}\nabla\sigma - \sigma\nabla\pi_{\mathbf{3}})$$

$$= \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3)$$

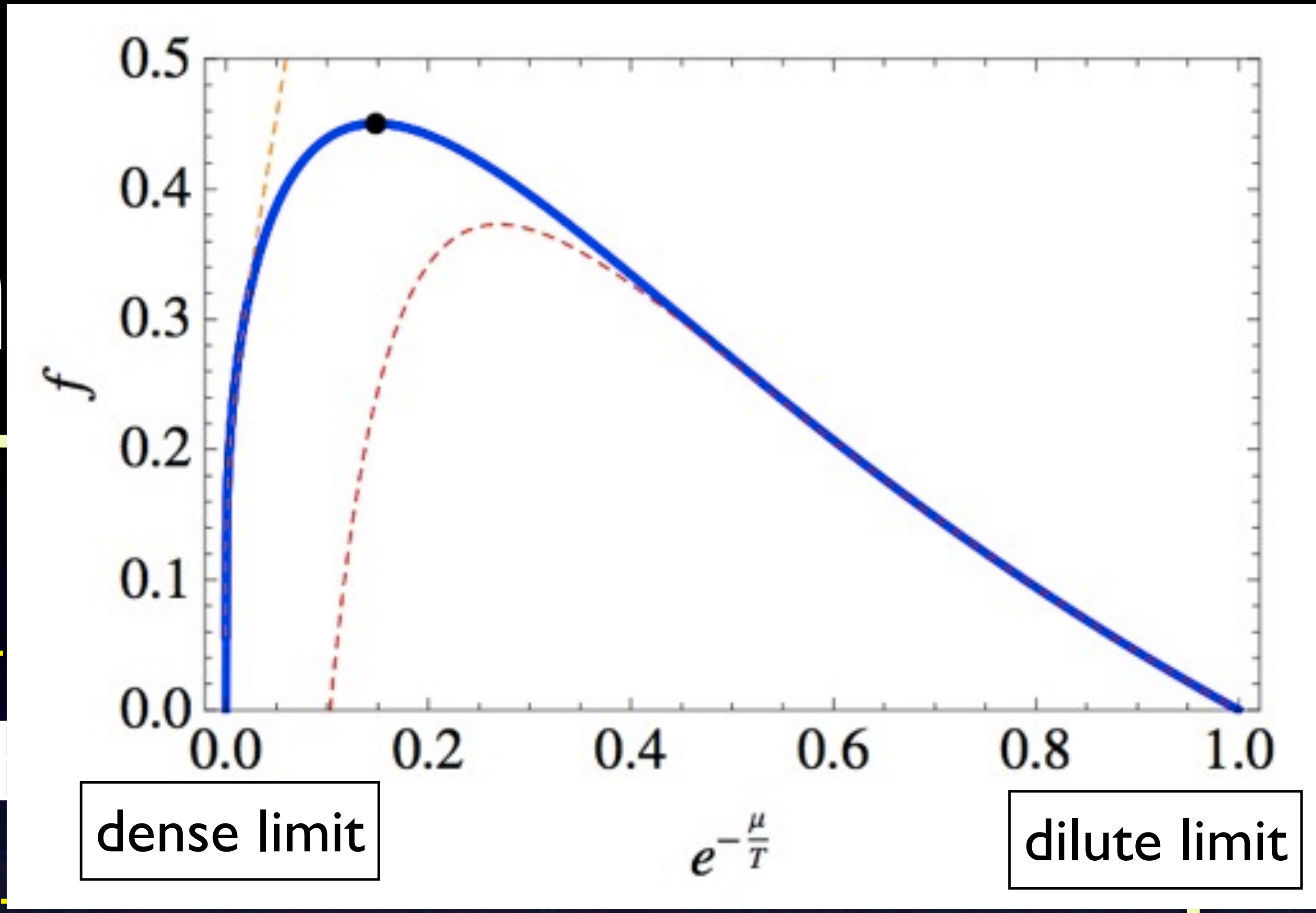
"*Universal*" coupling

$$-\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu} = \frac{N_c}{8\pi^2 T}f(e^{-\mu/T}),$$

$$f(e^{-\mu/T}) = \frac{1}{2\pi}\mathrm{Im}\psi^{(1)}\left(\frac{1}{2} - i\frac{\mu}{2\pi T}\right)$$

Tatsumi, Nishiyama, Karasawa, PLB (2015)

Including m



New couplings: 3D rotational

$$\delta\Omega = -\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu}(e\mathrm{B}$$

$$= \mathbf{b}\cdot(\pi_3\nabla\sigma - \sigma\nabla\pi_3)$$

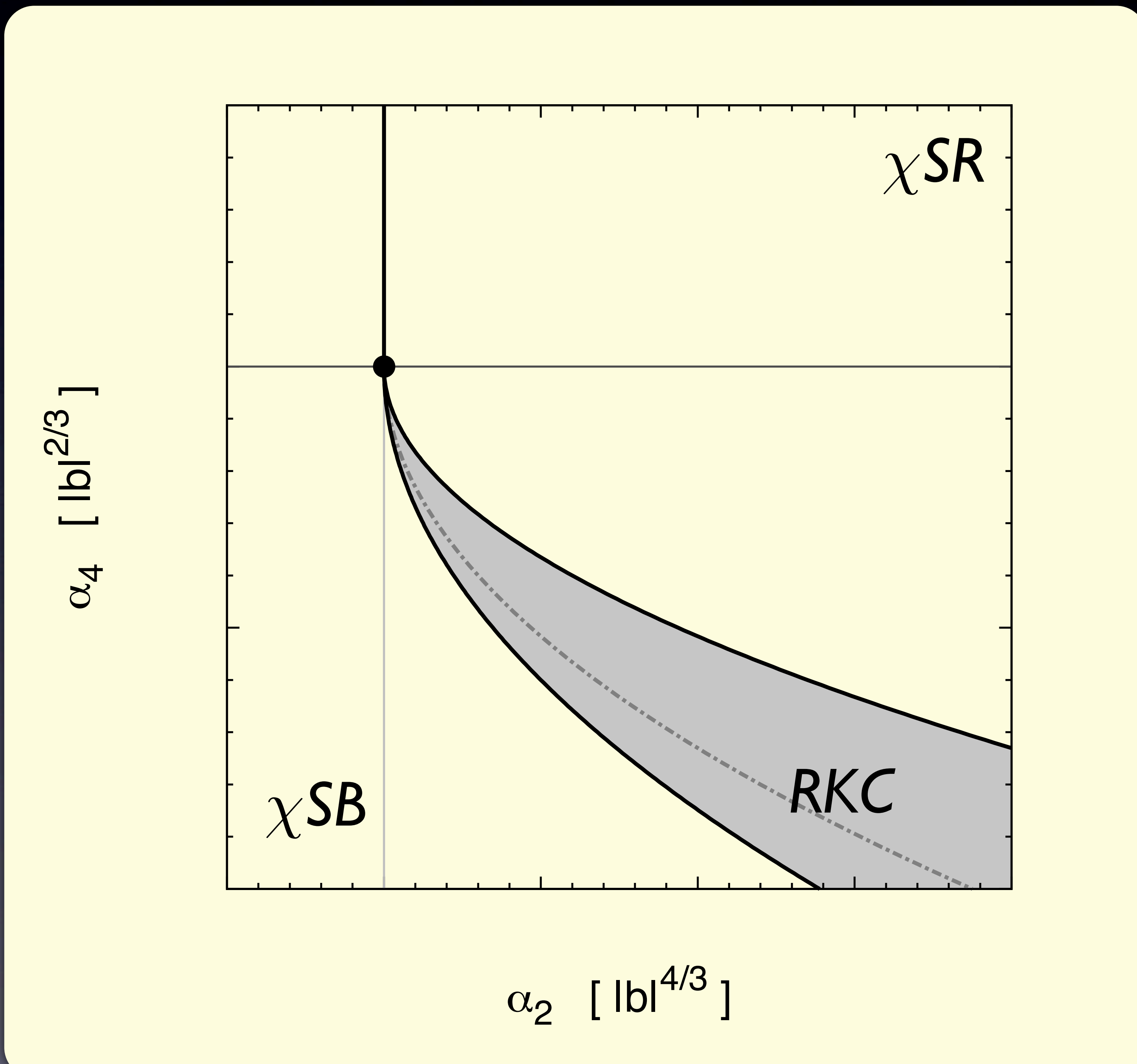
"*Universal*" coupling

$$-\frac{1}{8}\frac{\partial\alpha_4}{\partial\mu} = \frac{N_c}{8\pi^2 T} f(e^{-\mu/T}),$$

$$f(e^{-\mu/T}) = \frac{1}{2\pi}\mathrm{Im}\psi^{(1)}\left(\frac{1}{2} - i\frac{\mu}{2\pi T}\right)$$

Tatsumi, Nishiyama, Karasawa, PLB (2015)

The fate of LTCP

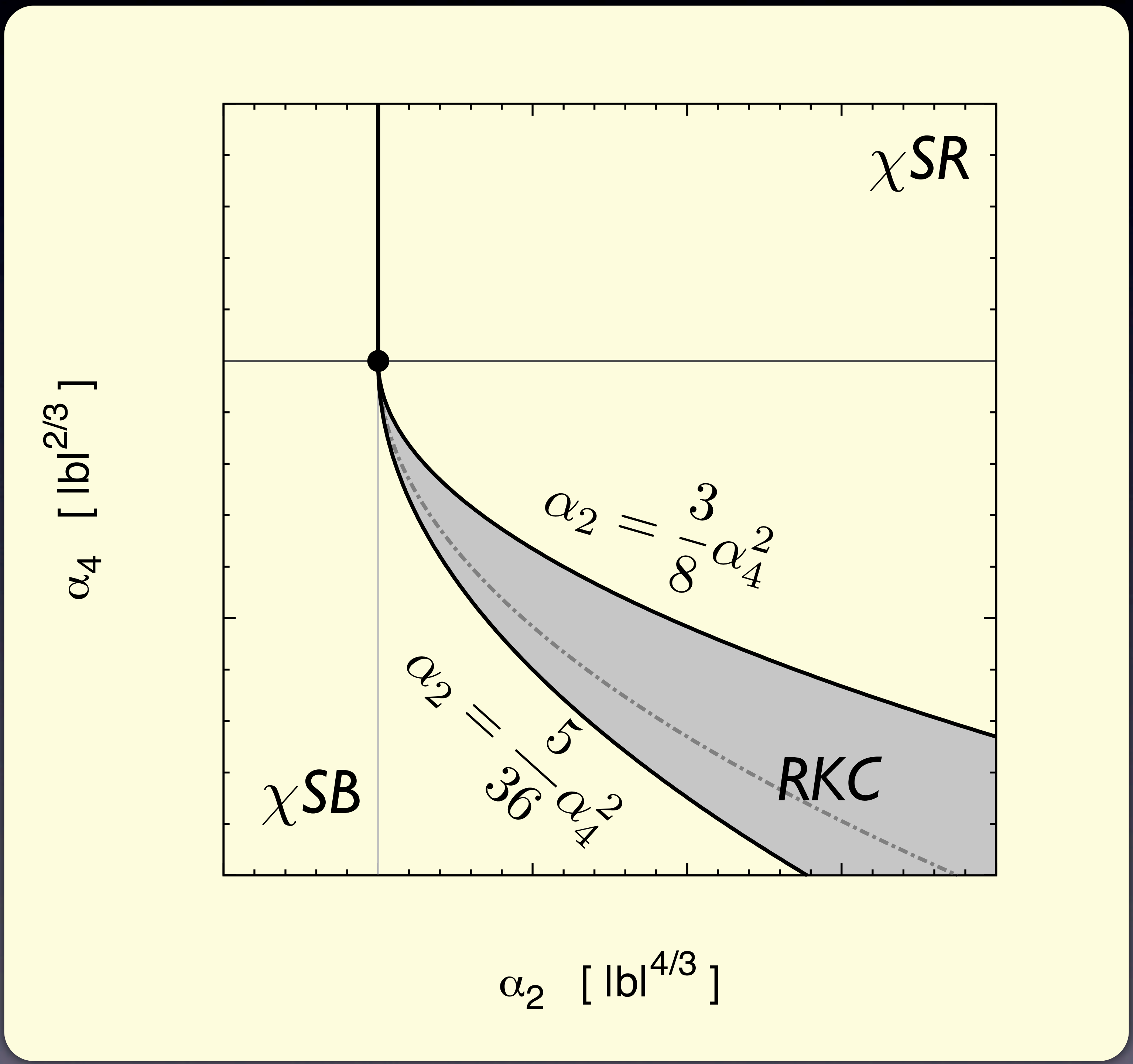


i. *No magnetic field:*

Lifshitz TCP
& RKC phase

$$\sigma = k\nu \mathrm{sn}(kz, \nu)$$

The fate of LTCP

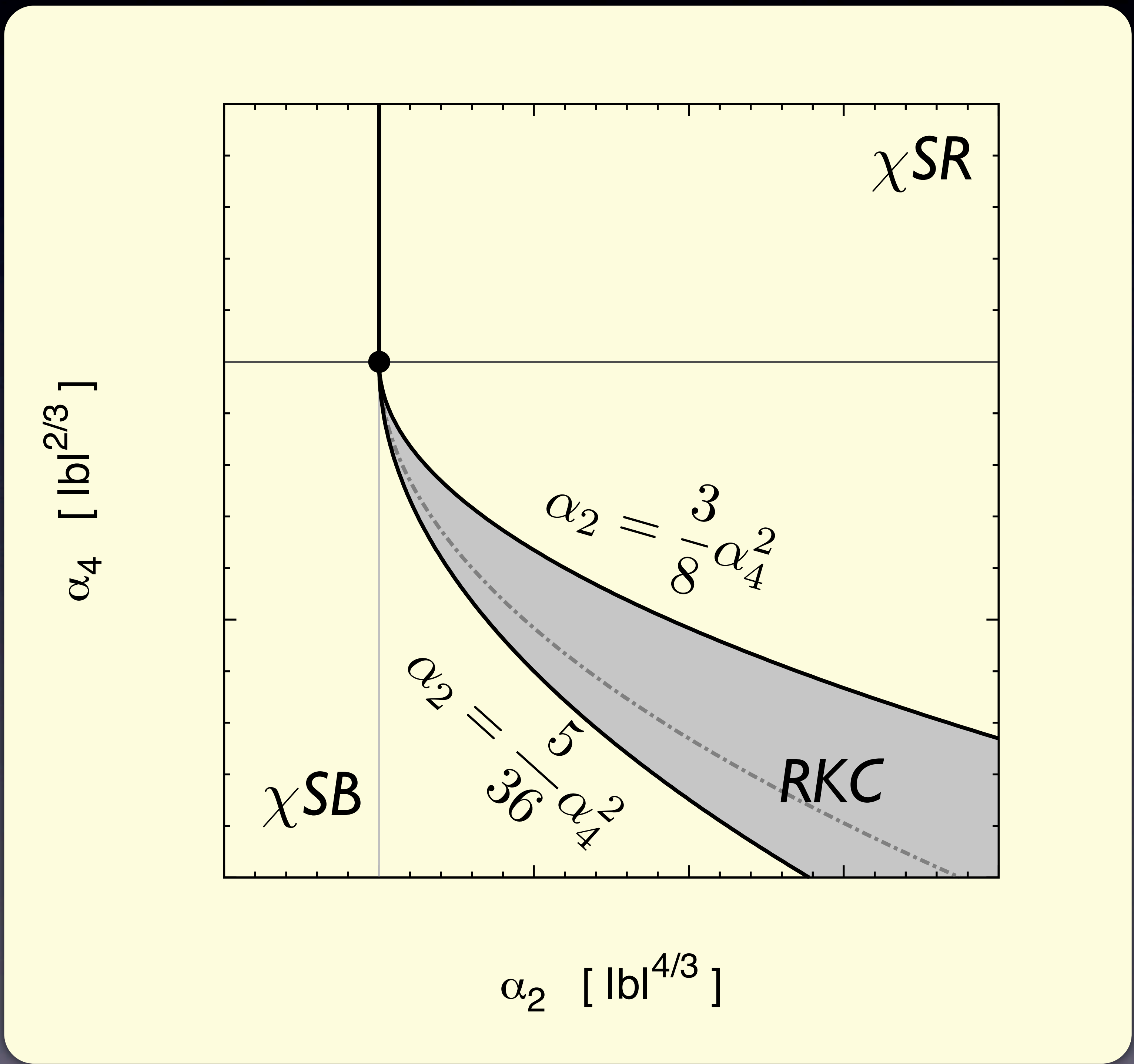


i. *No magnetic field:*

Lifshitz TCP
& RKC phase

$$\sigma = k\nu \operatorname{sn}(kz, \nu)$$

The fate of LTCP



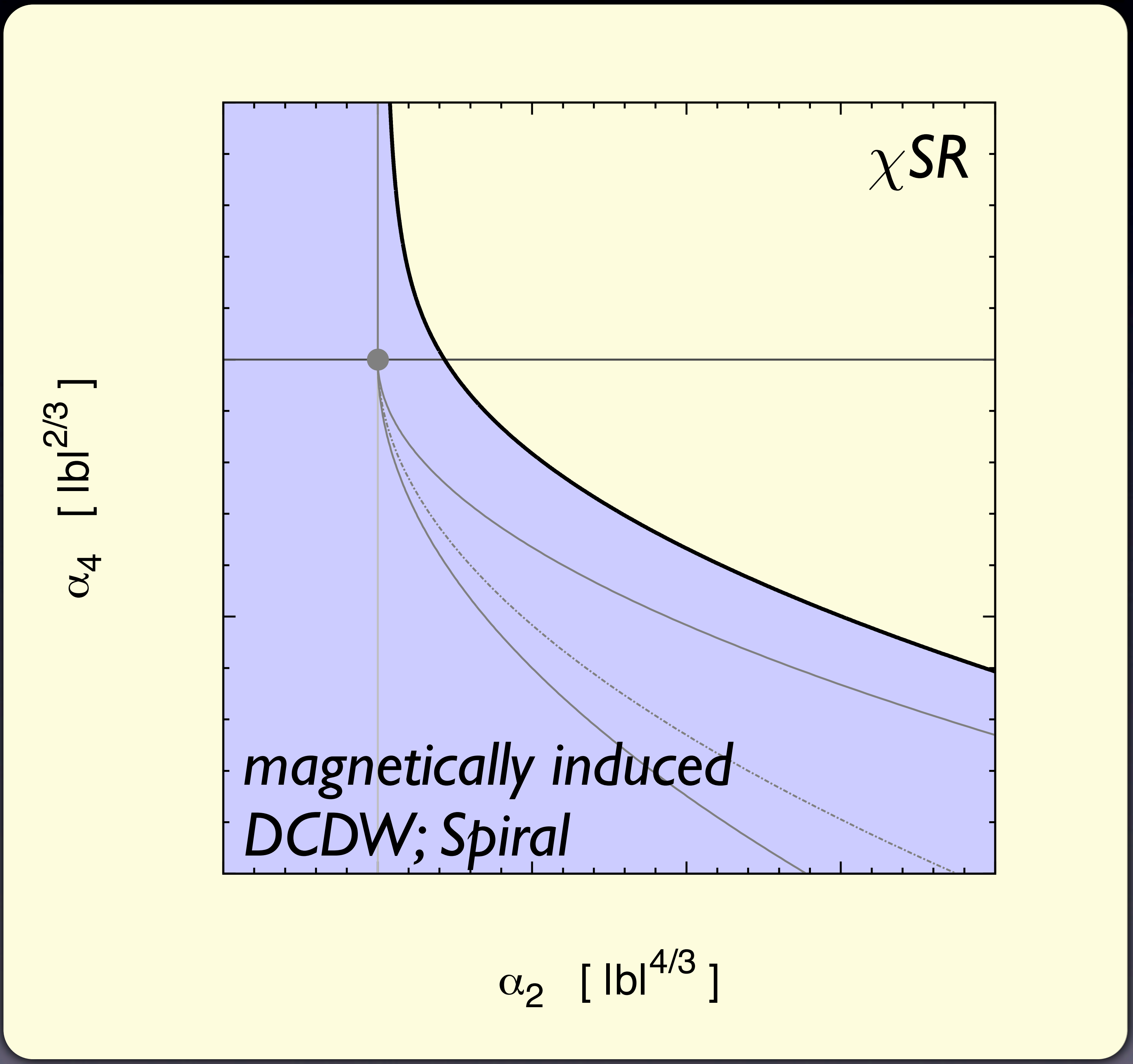
i. *No magnetic field:*

Lifshitz TCP
& RKC phase

$$\sigma = k\nu \mathrm{sn}(kz, \nu)$$

ii. *Magnetic field on:*

The fate of LTCP



i. *No magnetic field:*

Lifshitz TCP
& RKC phase

$$\sigma = k\nu \mathrm{sn}(kz, \nu)$$

ii. *Magnetic field on:*

Chiral spiral covers the whole region

$$\sigma + i\pi_3 = \Delta_0 e^{iqz}$$

Only second order phase transition!

Mass vs Magnetic field

$$\delta\Omega \; = \; \mathbf{b} \cdot (\pi_3 \nabla \sigma - \sigma \nabla \pi_3) - h\sigma$$

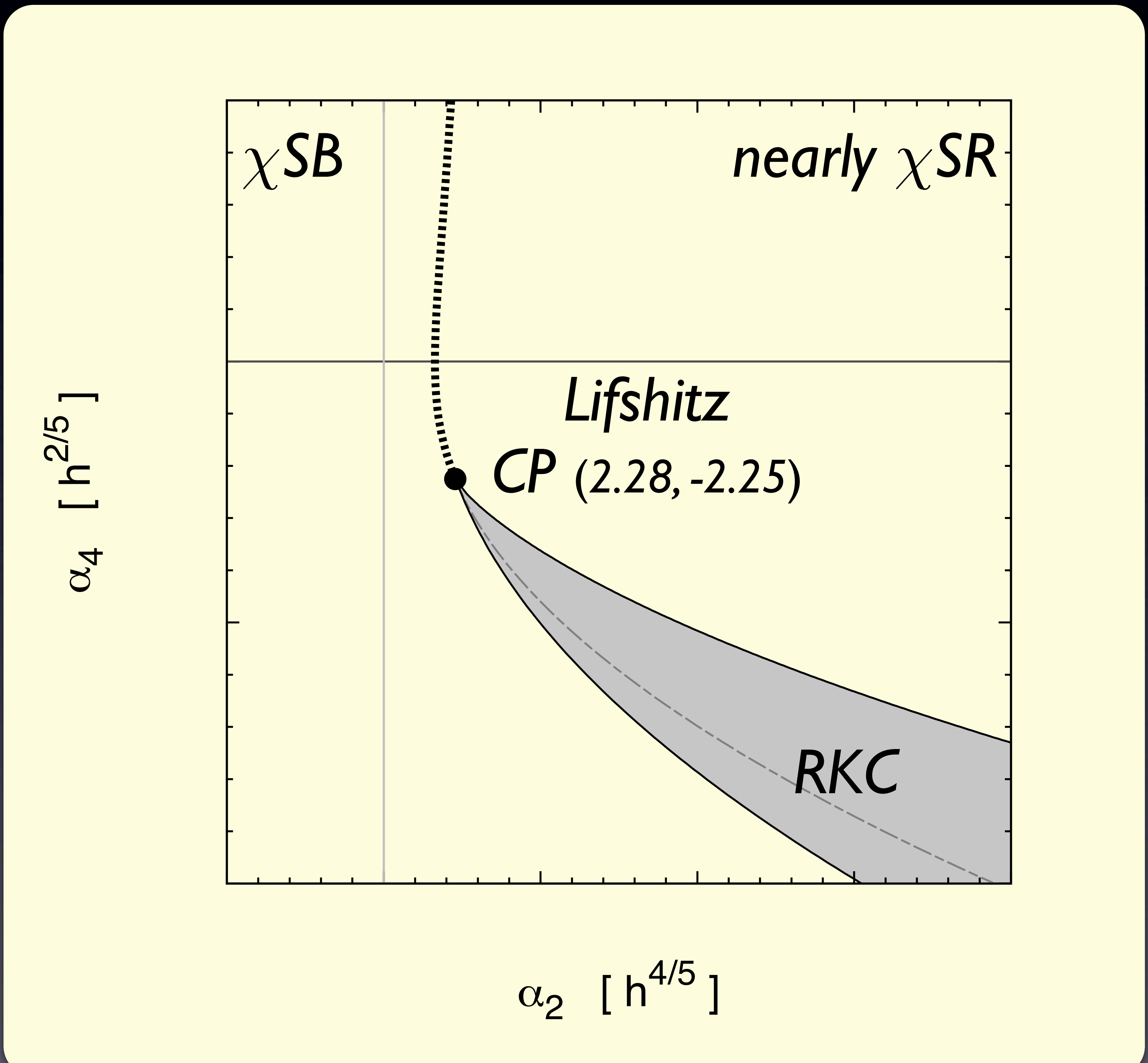
h : mass term

chiral SU(2)xSU(2) ⇒ isospin SU(2)

b : magnetic field

Rotational O(3) symmetry
Time reversal symmetry
Isospin I_3 symmetry

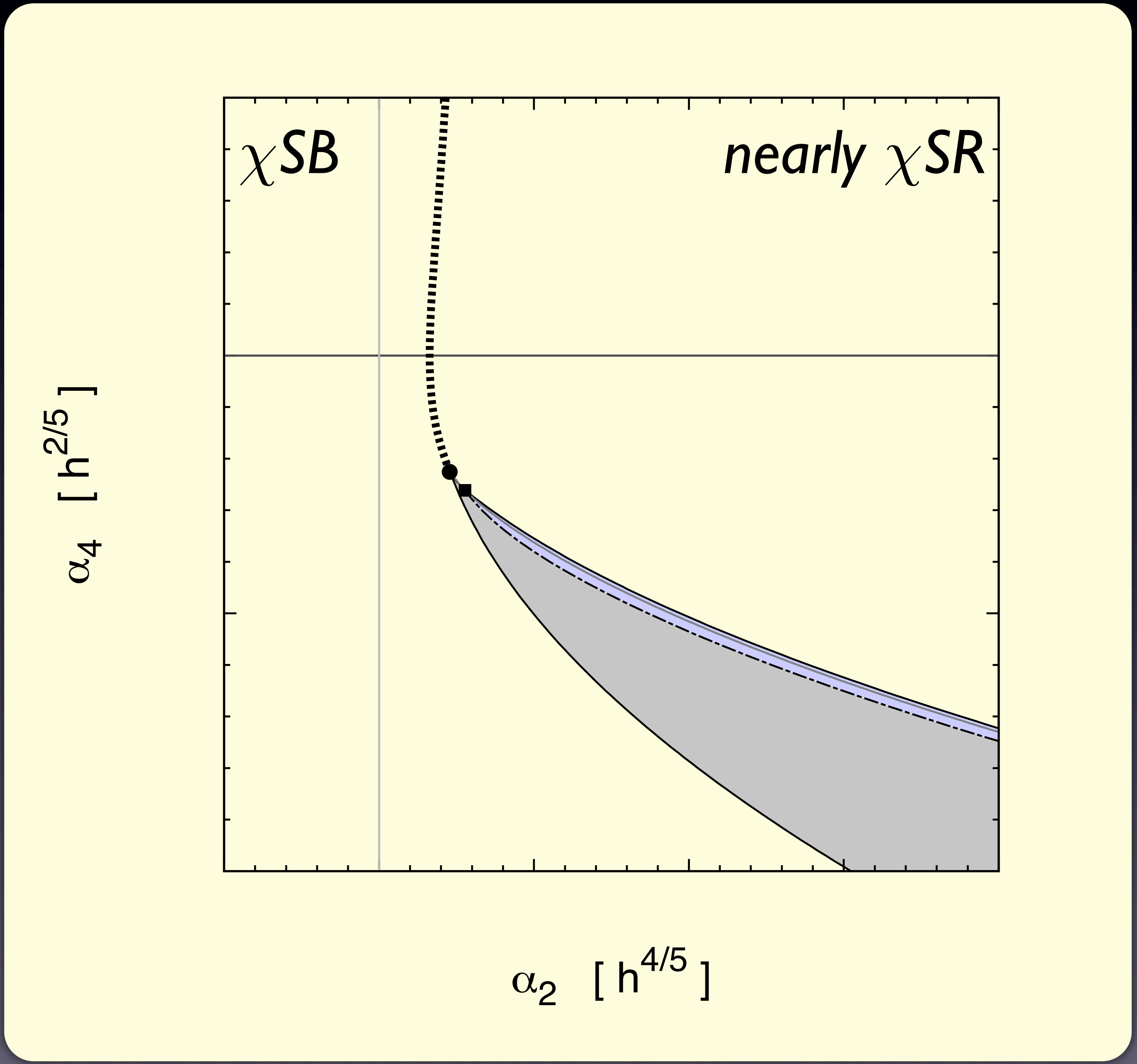
Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

Mass vs Magnetic field



i. *Only mass term on:*

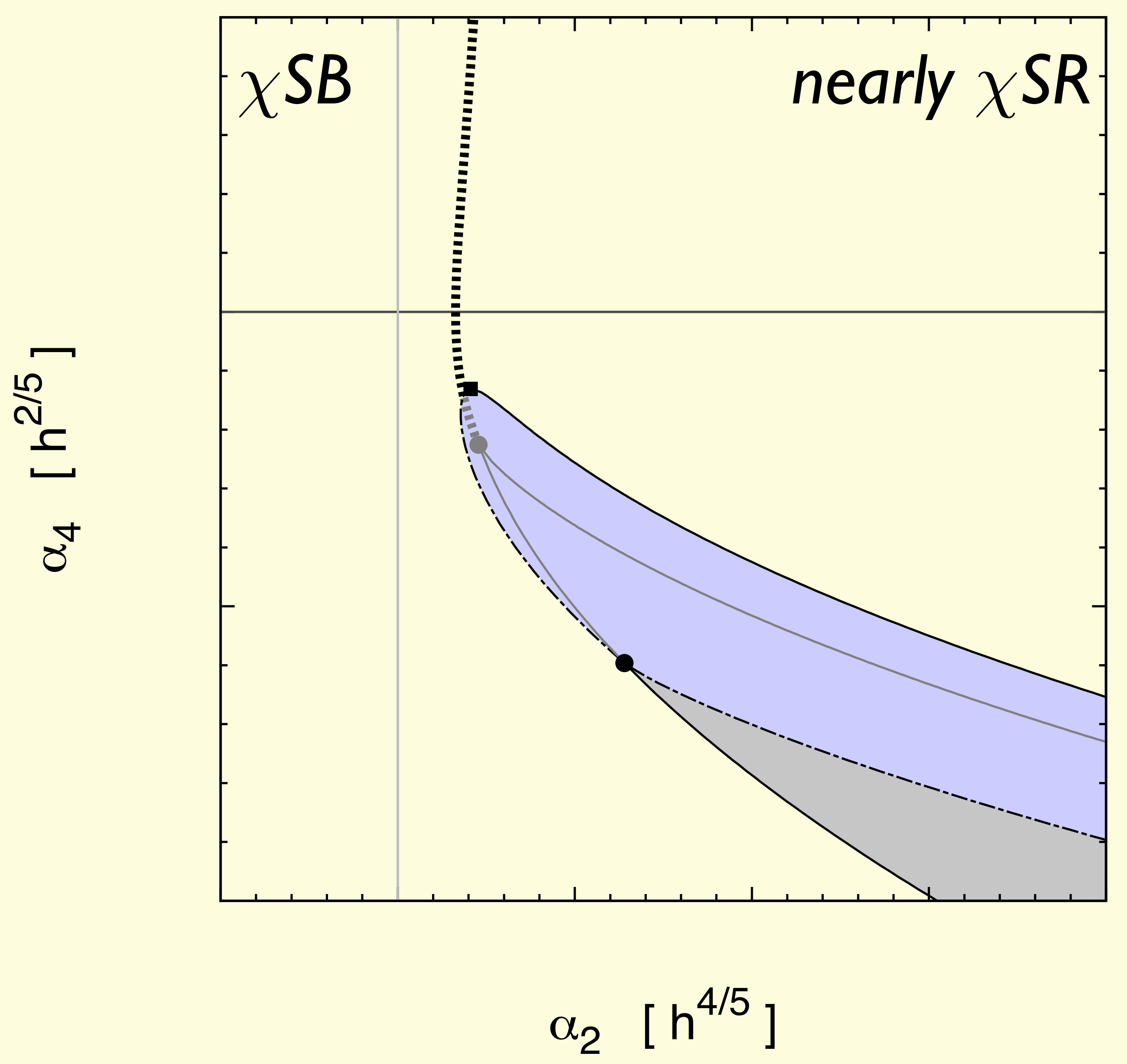
Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

$$8b = \quad 0.5 \times h^{3/5}$$
$$\left(\sqrt{eB} \sim \quad 1.2\,\mathrm{MeV}\right)$$

$$10^{9}\mathrm{T} \leftrightarrow 0.24\mathrm{MeV}$$
$$10^{11}\mathrm{T} \leftrightarrow 2.4\mathrm{MeV}$$
$$10^{13}\mathrm{T} \leftrightarrow 24\mathrm{MeV}$$

Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

$$8b = 5.0 \times h^{3/5}$$

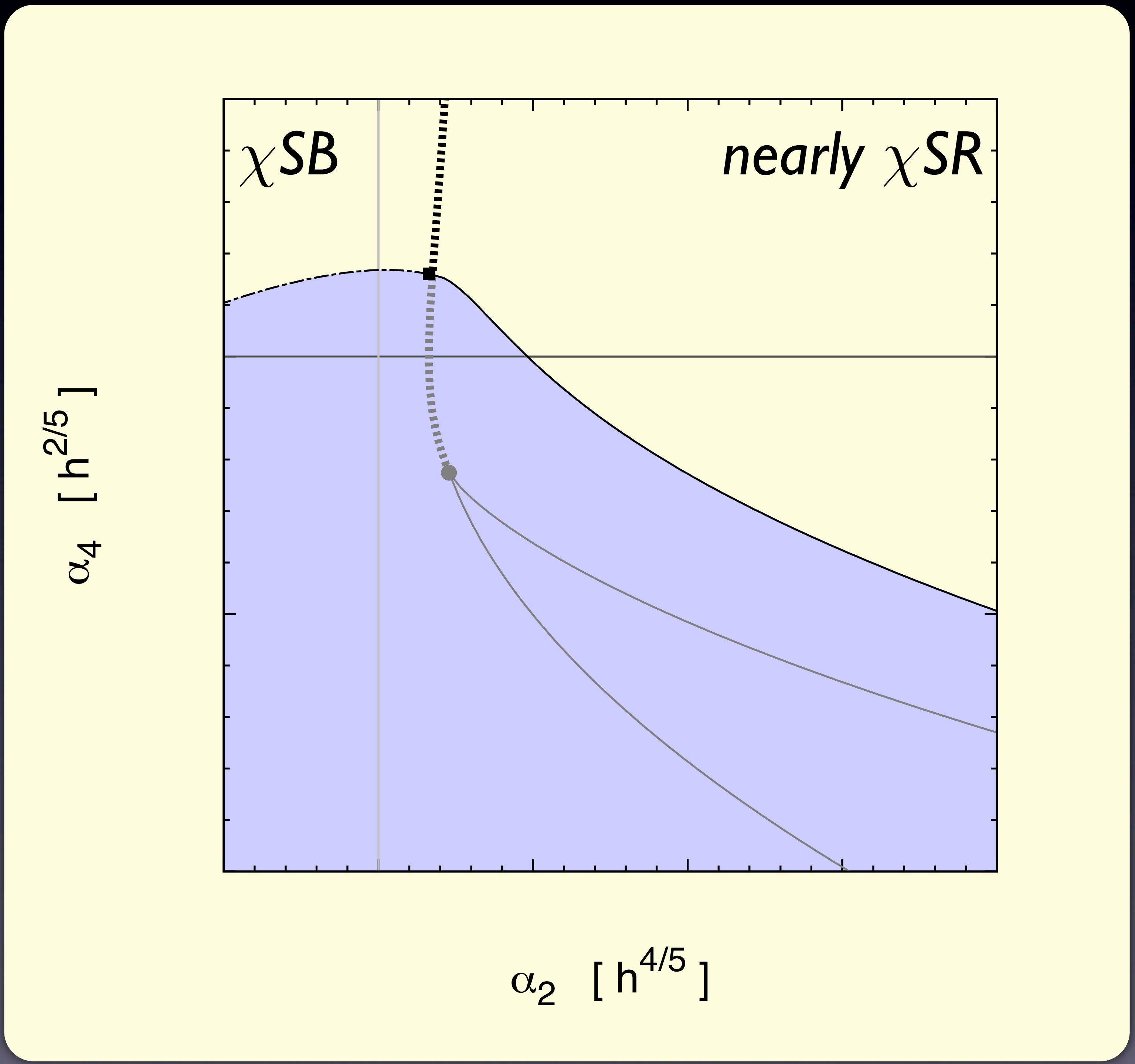
$$\left(\sqrt{eB} \sim 12\,\mathrm{MeV}\right)$$

$$10^9\mathrm{T} \leftrightarrow 0.24\mathrm{MeV}$$

$$10^{11}\mathrm{T} \leftrightarrow 2.4\mathrm{MeV}$$

$$10^{13}\mathrm{T} \leftrightarrow 24\mathrm{MeV}$$

Mass vs Magnetic field



i. *Only mass term on:*

Crossover,
Lifshitz point &
RKC phase

ii. *Magnetic field on:*

$$8b = 15 \times h^{3/5}$$

$$\left(\sqrt{eB} \sim 35\ \mathrm{MeV}\right)$$

$$10^9\mathrm{T} \leftrightarrow 0.24\mathrm{MeV}$$

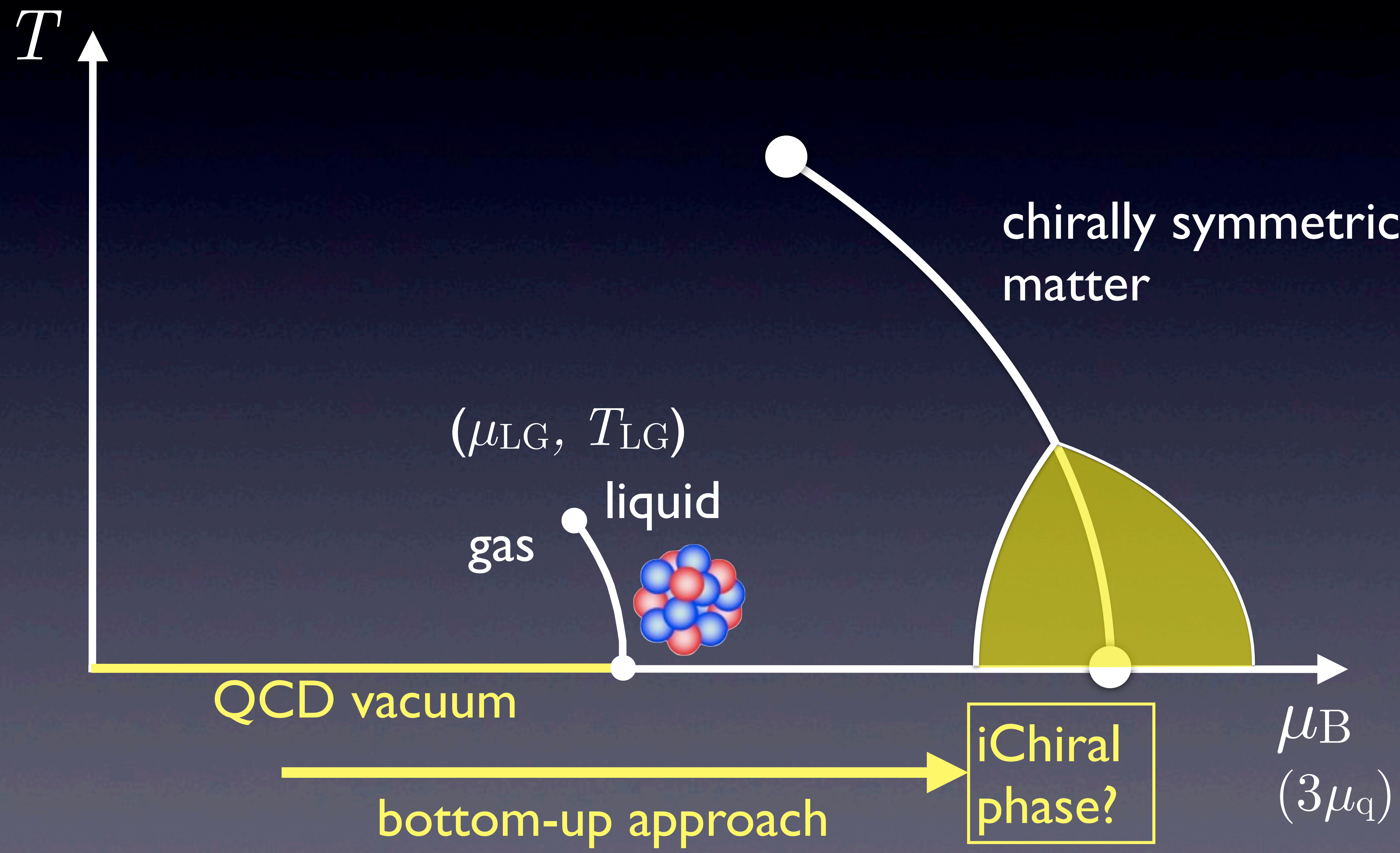
$$10^{11}\mathrm{T} \leftrightarrow 2.4\mathrm{MeV}$$

$$10^{13}\mathrm{T} \leftrightarrow 24\mathrm{MeV}$$

Plan

- What is inhomogeneous chiral phase?
- generalized Ginzburg-Landau (gGL) approach to inhomogeneous phases in quark matter; mass vs magnetic field
- DCDW in nuclear matter? Parity-doublet model approach
- Summary and Outlook

QCD phase diagram



Parity doublet model

DeTar, Kunihiro, PRD39 (1989)
Jido, Oka, Hosaka, PTP106 (2001)

- Regards negative parity N^*(1535) as a chiral partner to positive parity N(939)
- Introduce two nucleon fields (Ψ_1, Ψ_2) and define chiral transformation (mirror assignment)

$$\Psi_{1,r} \to U_R \Psi_{1,r}, \quad \Psi_{1,l} \to U_L \Psi_{1,l} \text{ (parity +)}$$

$$\Psi_{2,r} \to U_L \Psi_{2,r}, \quad \Psi_{2,l} \to U_R \Psi_{2,l} \text{ (parity −)}$$

- chiral invariant mass term allowed

$$m_0 \left(\bar{\Psi}_2 \gamma_5 \Psi_1 - \bar{\Psi}_1 \gamma_5 \Psi_2\right)$$

Baryon bilinear

DeTar, Kunihiro, PRD39 (1989)
Jido, Oka, Hosaka, PTP106 (2001)

- Baryon part

$$\mathcal{L}_{\rm Bar} = \sum_{i=1,2} \bar{\Psi}_i (i\partial\!\!\!/ + \mu\!\!\!/ - g_\omega \gamma_\mu \omega^\mu) \Psi_i$$

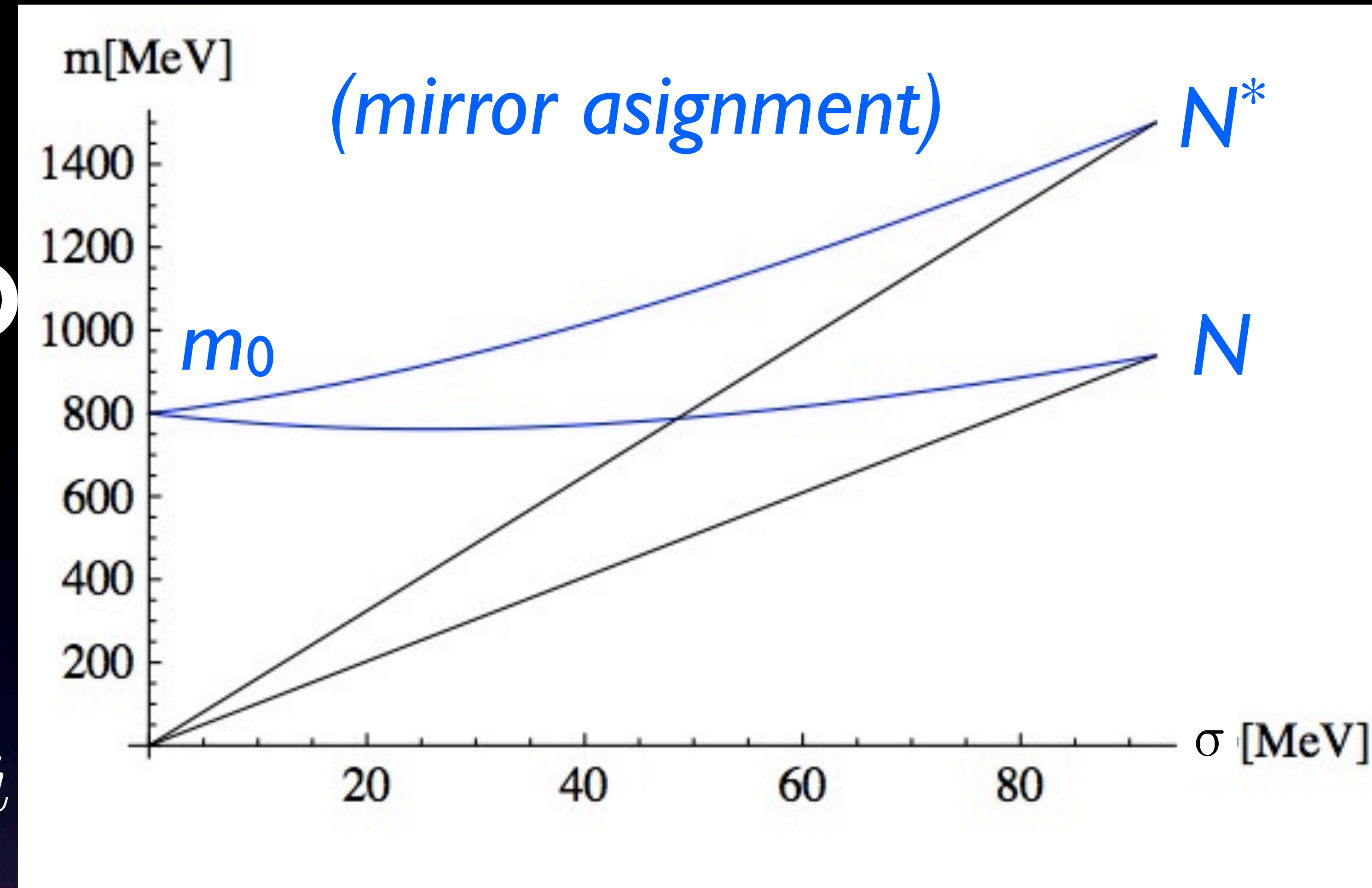
$$- g_1 \bar{\Psi}_1 (\sigma + i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau}) \Psi_1 - g_2 \bar{\Psi}_2 (\sigma - i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau}) \Psi_2$$

$$- m_0 (\bar{\Psi}_2 \gamma_5 \Psi_1 - \bar{\Psi}_1 \gamma_5 \Psi_2)$$

- VEV of sigma, responsible for mass difference

$$m_\pm = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2 \sigma_0 + 4{m_0}^2} \mp |g_2 - g_1| \sigma_0\right]$$

Baryo



S. Gallas, F. Giacosa, arXiv:1308.4817 [hep-ph]

- Baryon part

$$\mathcal{L}_{\mathrm{Bar}} = \sum_{i=1,2} \bar{\Psi}_i (i\partial\!\!\!/ + \mu\!\!\!/$$

$$- g_1 \bar{\Psi}_1 (\sigma + i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\Psi_1 - g_2 \bar{\Psi}_2 (\sigma - i\gamma_5 \boldsymbol{\pi}\cdot\boldsymbol{\tau})\Psi_2$$

$$- m_0 (\bar{\Psi}_2 \gamma_5 \Psi_1 - \bar{\Psi}_1 \gamma_5 \Psi_2)$$

- VEV of sigma, responsible for mass difference

$$m_\pm = \frac{1}{2}\left[\sqrt{(g_1+g_2)^2\sigma_0 + 4{m_0}^2} \mp |g_2 - g_1|\sigma_0\right]$$

Meson Part

Motohiro, Kim, Harada, PRC92 (2015)

$$\mathcal{L}_{\mathrm{Mes}} = \frac{m_\omega^2}{2}\omega_\mu\omega^\mu - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu}$$

HLS in the unitary gauge

$$+ \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi$$

$$+ \frac{\bar{\mu}^2}{2}(\sigma^2+\pi^2) - \frac{\lambda}{4}(\sigma^2+\pi^2)^2 + \frac{\lambda_6}{6}(\sigma^2+\pi^2)^3$$

SSB

scalar six point int.

$$- f_\pi m_\pi^2 \sigma$$

explicit symmetry breaking

Parameter fixing

- Parameters: $\{g_1\,, g_2,\ \bar{\mu},\ g_\omega,\ \lambda,\ \lambda_6\}$ and m_0
  - ✓ Vacuum properties: $\sigma_0 = f_\pi = 93\ \mathrm{MeV}$,

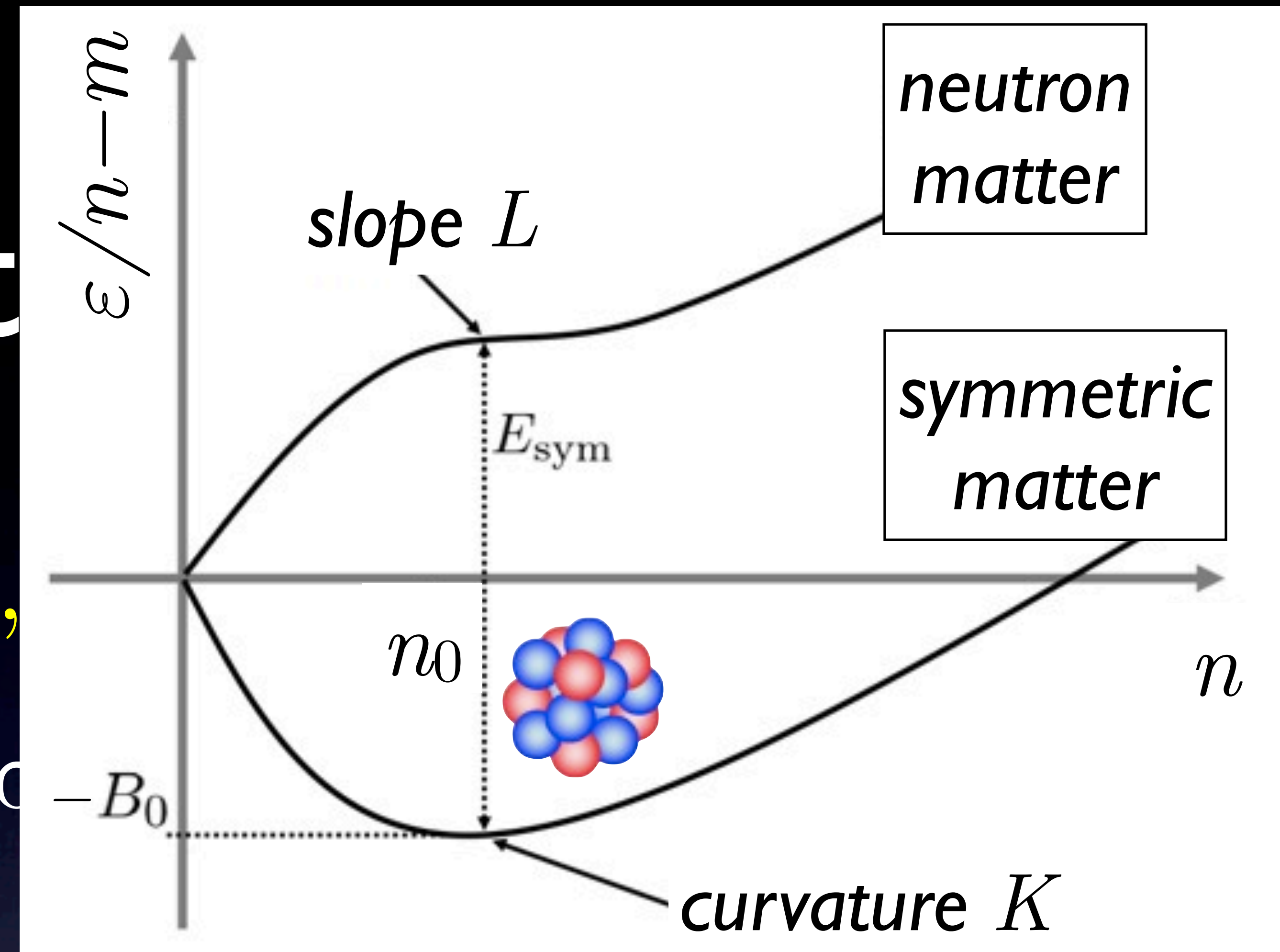
    $m_+ = 939\ \mathrm{MeV},\ m_- = 1535\ \mathrm{MeV}$
  - ✓ Matter properties: $n_0 = 0.16\ \mathrm{fm}^{-3}$,

    $B_0 = m_+ - \mu_0 = 16\ \mathrm{MeV},\ K = 240\ \mathrm{MeV}$

$$\frac{\epsilon}{n} - m_+ = -B + \frac{K}{18{n_0}^2}(n - n_0)^2 + \cdots$$

c.f. ISGMR experiment: K=240 ±10 MeV: Li, Garg, et al., PRC81 (2010)

Paramet



- Parameters: $\{g_1\,, g_2,\ \bar{\mu},$
  - ✓ Vacuum properties: c

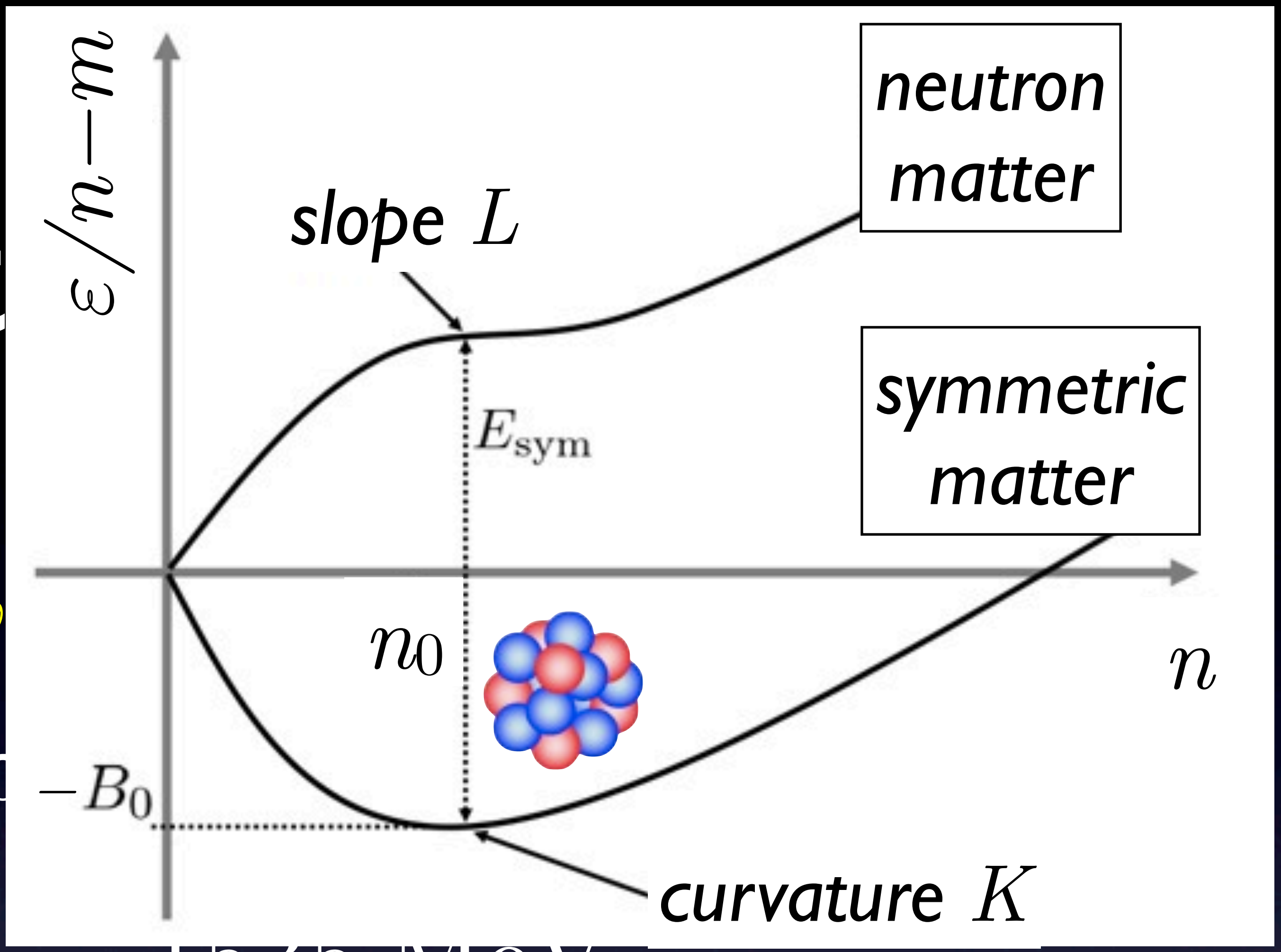
    $m_+ = 939\ \mathrm{MeV},\ m_- = 1535\ \mathrm{MeV}$
  - ✓ Matter properties: $n_0 = 0.16\ \mathrm{fm}^{-3},$

    $B_0 = m_+ - \mu_0 = 16\ \mathrm{MeV},\ K = 240\ \mathrm{MeV}$

$$\frac{\epsilon}{n} - m_+ = -B + \frac{K}{18{n_0}^2}(n - n_0)^2 + \cdots$$

c.f. ISGMR experiment: K=240 ±10 MeV: Li, Garg, et al., PRC81 (2010)

m_0	500	600	700	800	900
g_1	9.03	8.49	7.82	7.00	5.97
g_2	15.5	15.0	14.3	13.5	12.4
$g_{\omega NN}$	11.3	9.13	7.30	5.66	3.52
$\bar{\mu}$ [MeV]	441	437	406	320	114
λ_4	42.2	40.6	35.7	23.2	4.47
$\lambda_6 \cdot f_\pi^2$	17.0	15.8	14.0	8.94	0.644



✓ Vacuum properties:

$m_+ = 939$ MeV, $m_- = 1535$ MeV

✓ Matter properties: $n_0 = 0.16 \text{ fm}^{-3}$,

$B_0 = m_+ - \mu_0 = 16$ MeV, $K = 240$ MeV

$$\frac{\epsilon}{n} - m_+ = -B + \frac{K}{18 {n_0}^2}(n - n_0)^2 + \cdots$$

c.f. ISGMR experiment: K=240 ±10 MeV: Li, Garg, et al., PRC81 (2010)

extended DCDW ansatz

Takeda, Abuki, Harada, PRD97 (2018)

$$\langle\sigma\rangle = \sigma_0 \cos(2fz) + \delta\sigma$$

$$\langle\pi\rangle = \sigma_0 \sin(2fz)$$

shift

shifted DCDW

standard DCDW ansatz

- Solving nucleon eigenstates in the presence of periodic potential (BdG Dirac hamiltonian)

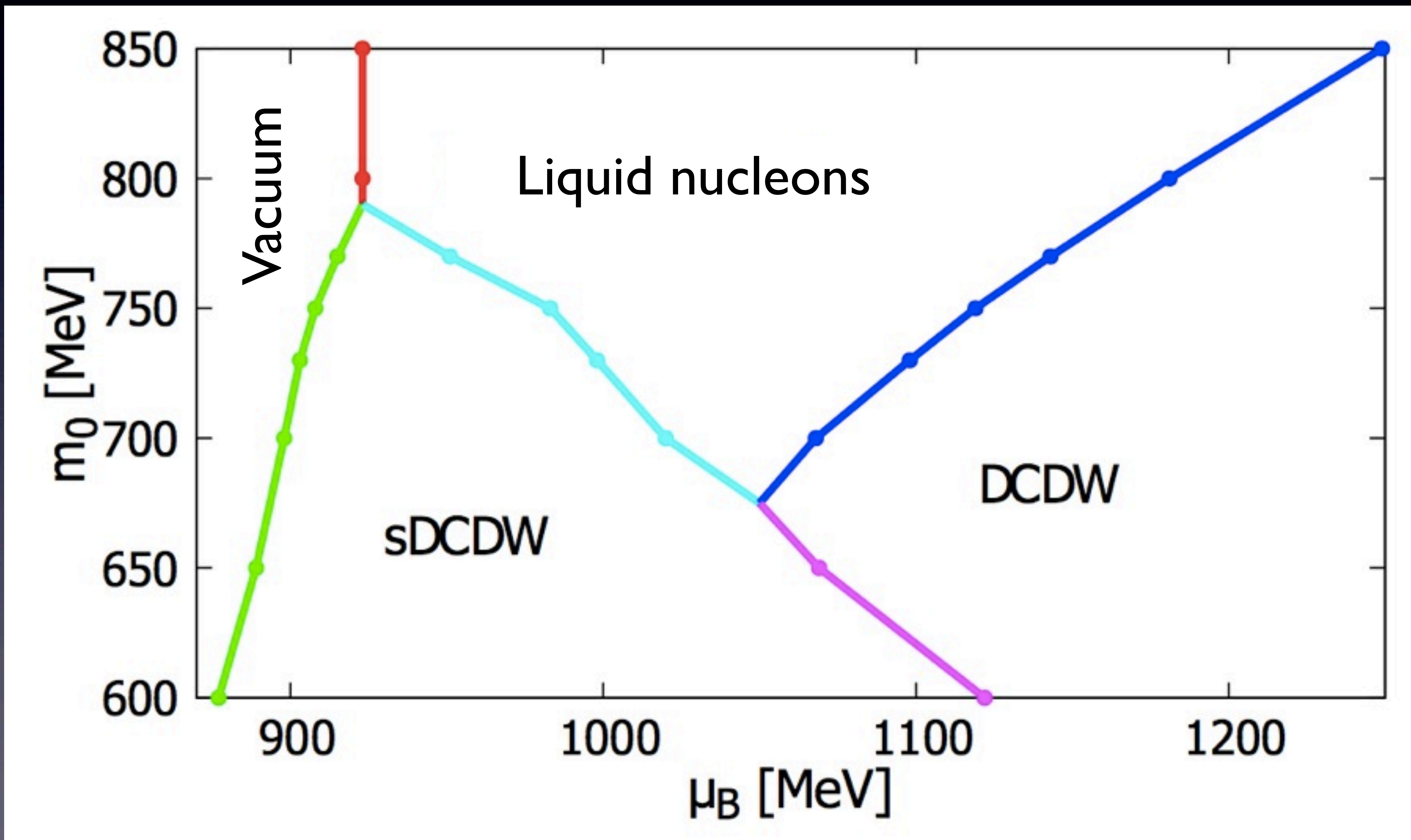
$$\Psi_{\boldsymbol{k},n}(\boldsymbol{x}) = e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} e^{ikz} u_n(z), \quad n = 0, 1, \cdots .$$

- gives .. mean field thermodynamic potential

$$\Omega_{\mathrm{N}} = \sum_{n=0}^{\infty} \int_{-f}^{f} \frac{dk}{\pi} \int \frac{d\boldsymbol{k}_\perp}{(2\pi)^2} (E_{\boldsymbol{k},n} - \mu_B^*) \theta(\mu_B^* - E_{\boldsymbol{k},n})$$

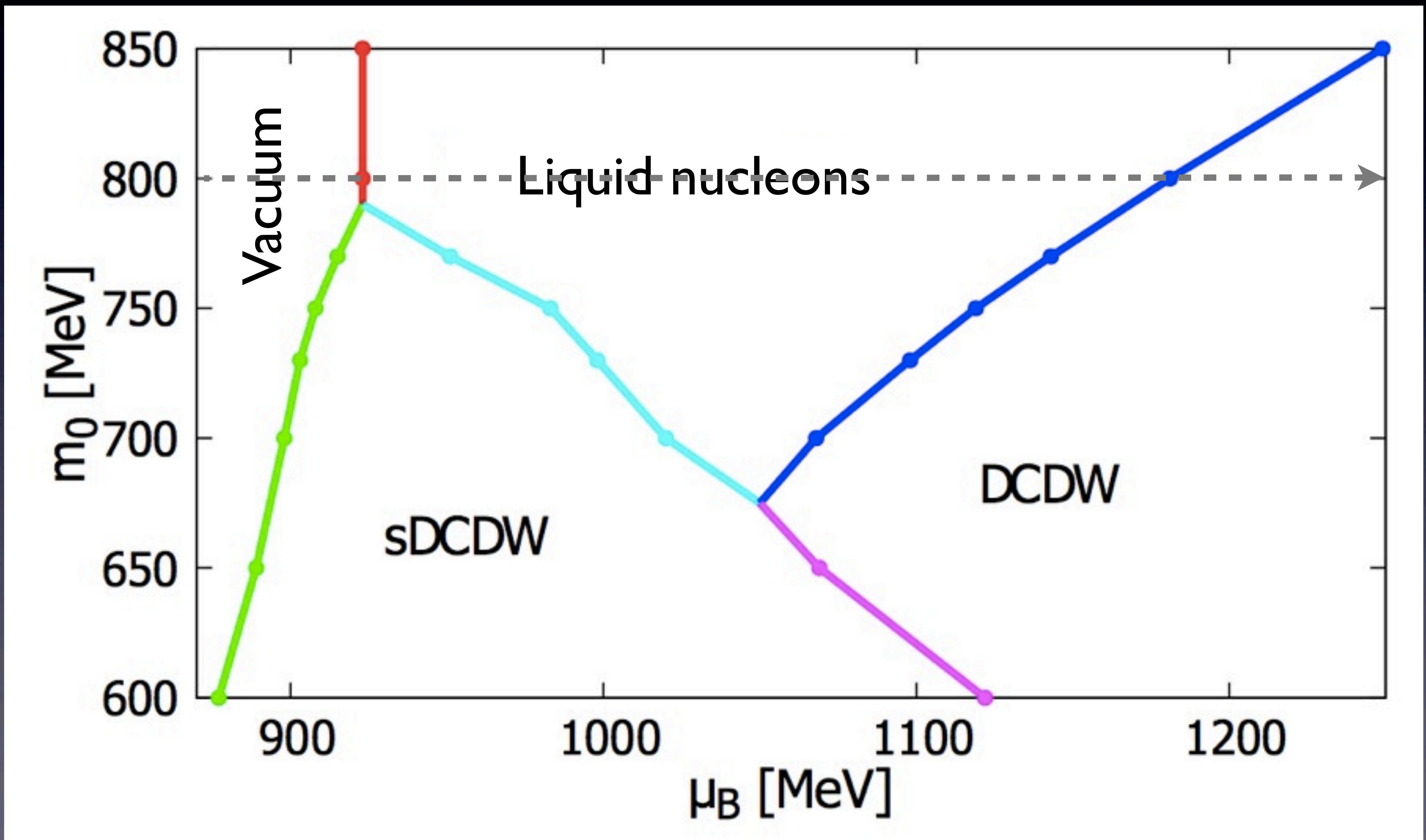
Phases with varying m_0

Takeda, Abuki, Harada, PRD97 (2018)

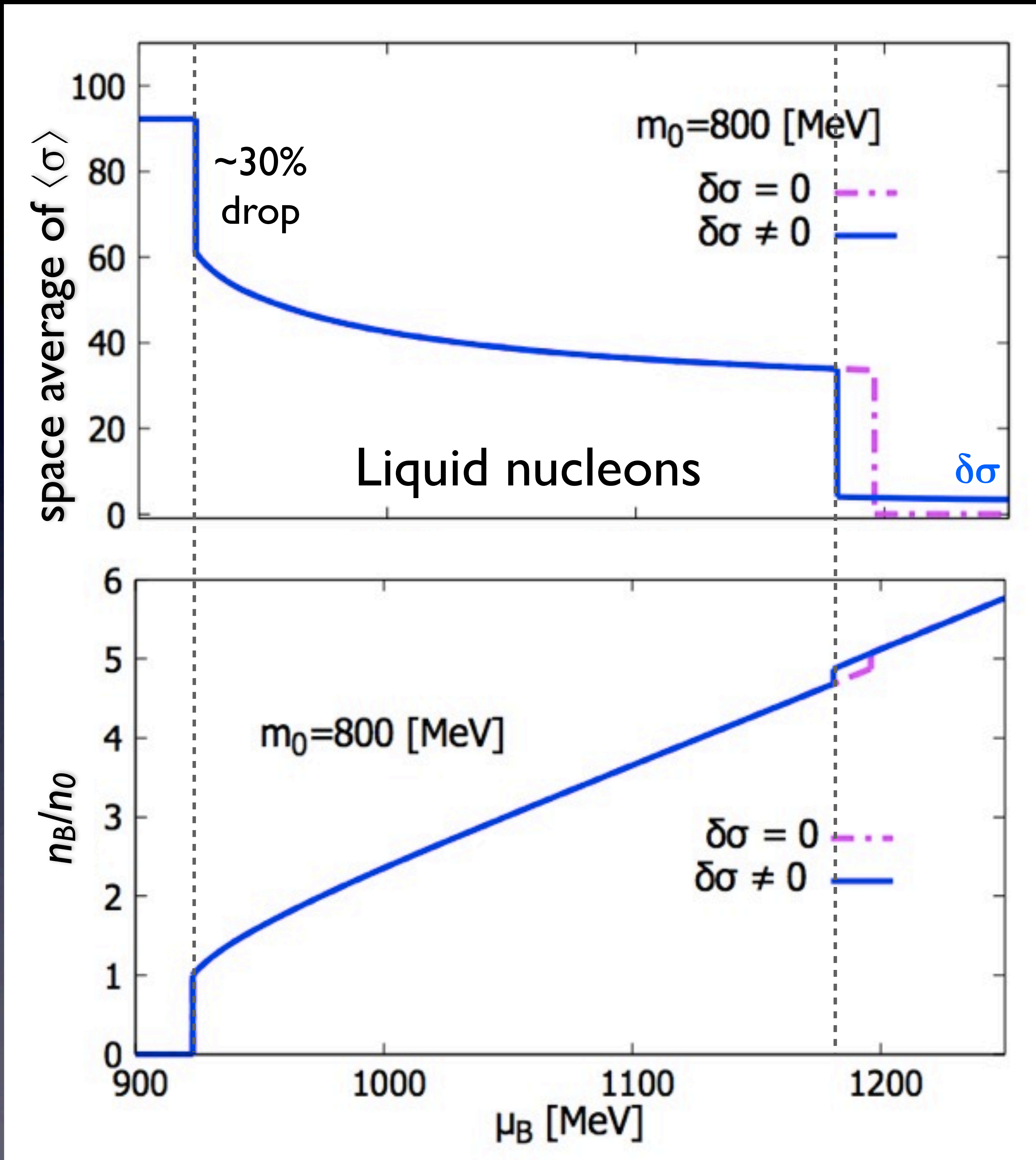


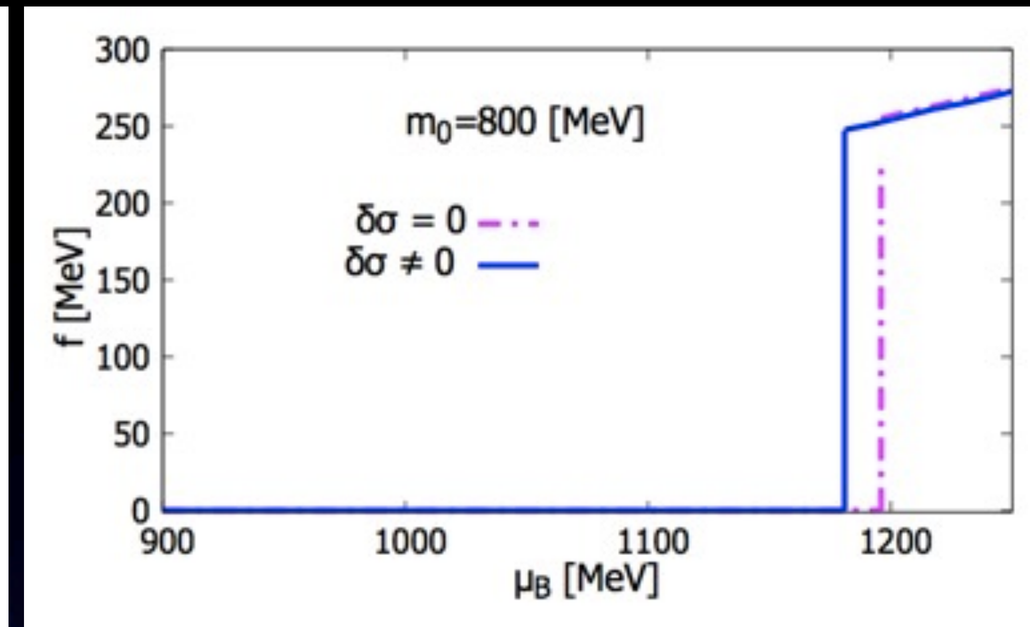
Phases with varying m_0

Takeda, Abuki, Harada, PRD97 (2018)



Takeda, Abuki, Harada, PRD97 (2018)





✓ $q = 2f \sim 500$ MeV

$\lambda \sim 2.5$ fm

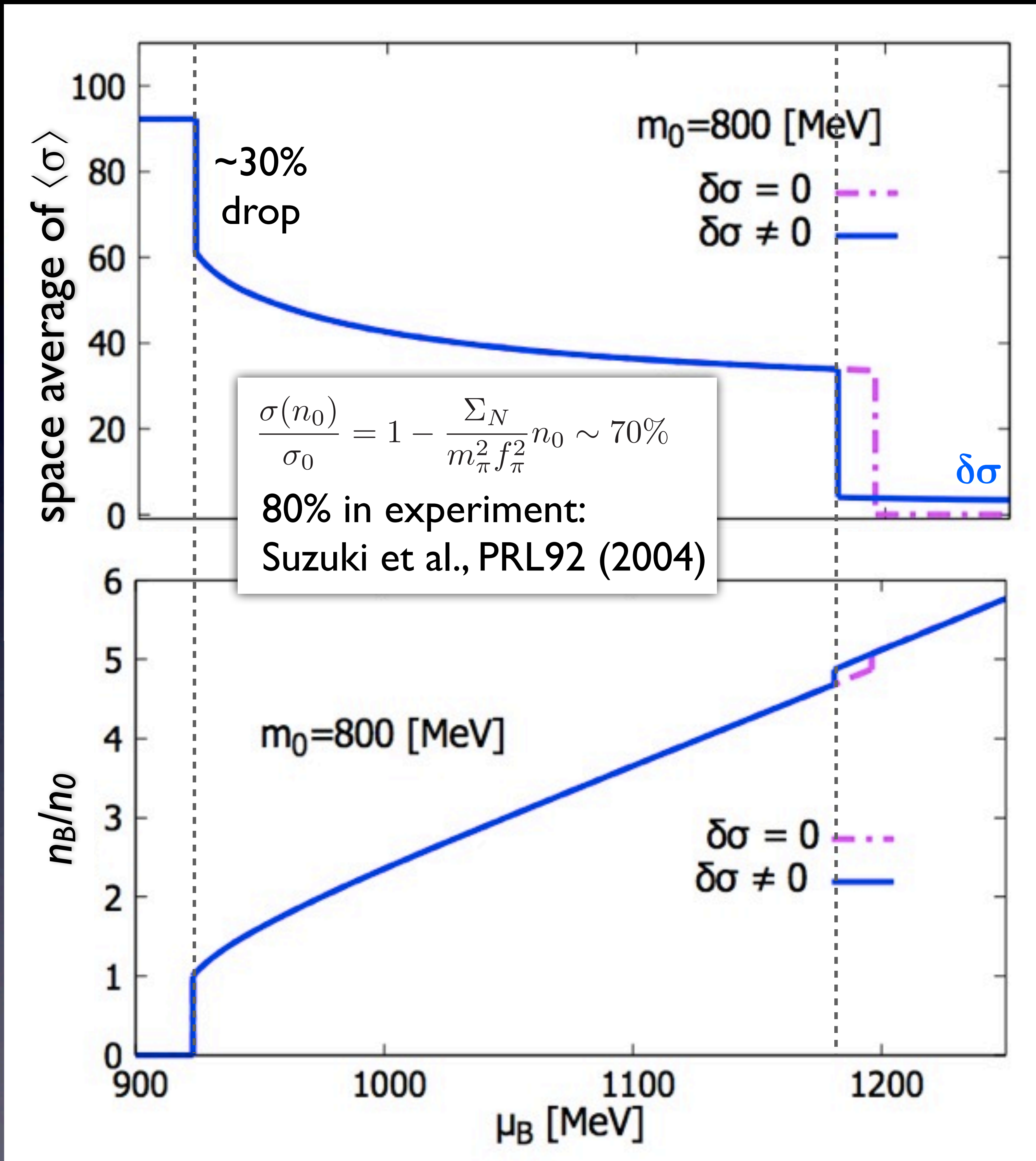
✓ density jump @

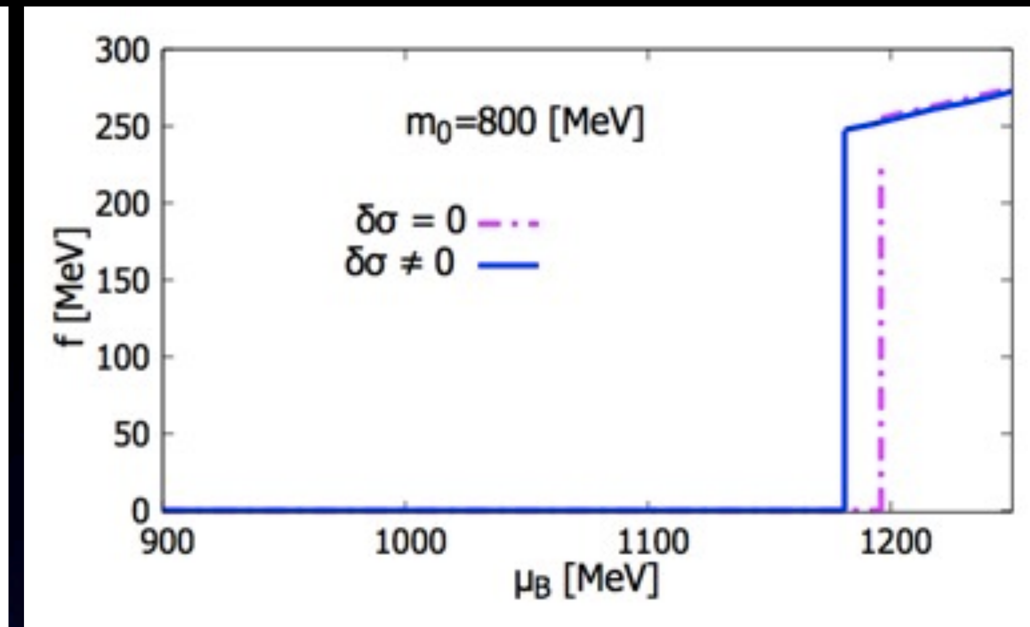
NM → DCDW

4.7 n_0 → 4.8 n_0

(weak first order)

Takeda, Abuki, Harada, PRD97 (2018)





✓ $q = 2f \sim 500$ MeV

$\lambda \sim 2.5$ fm

✓ density jump @

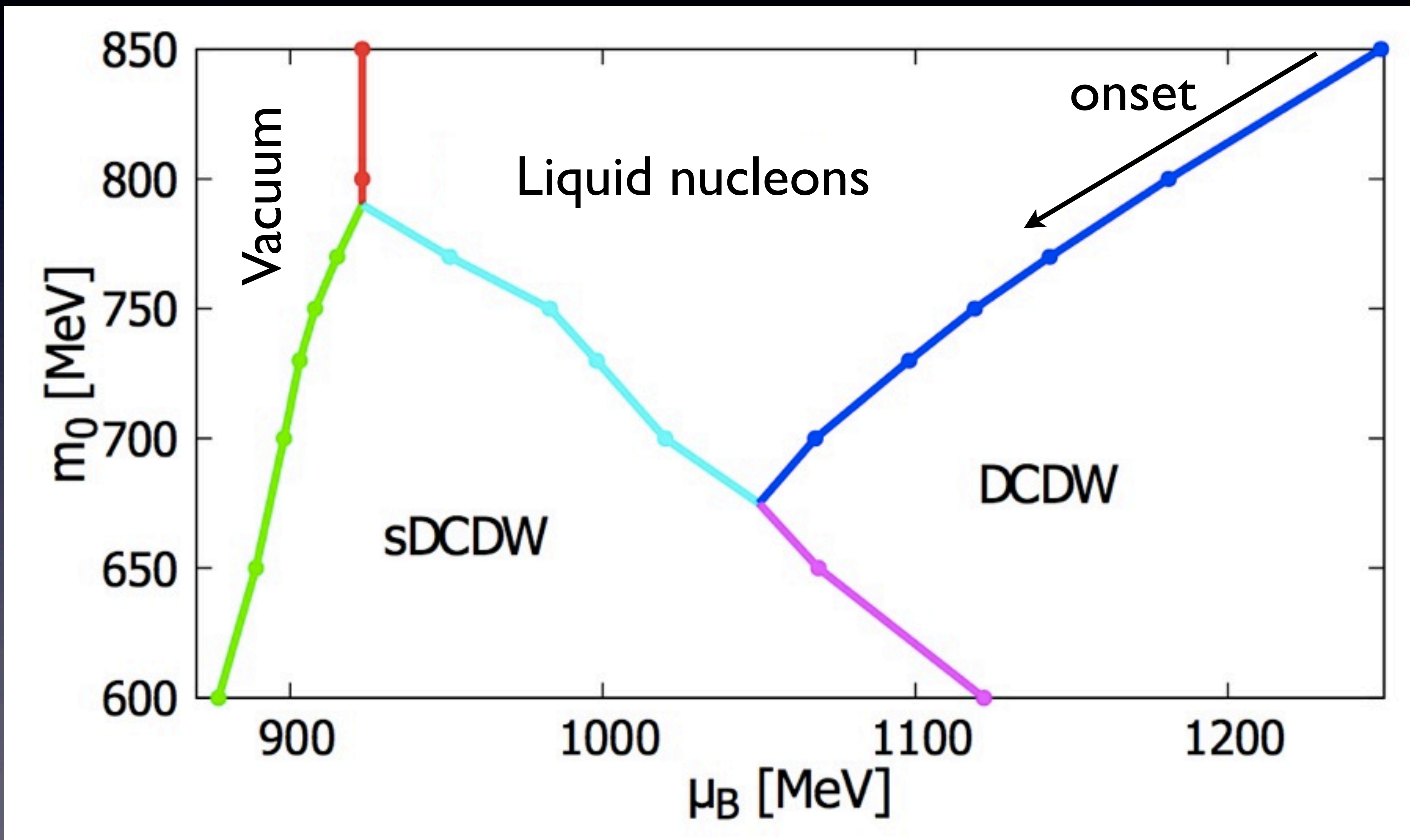
NM → DCDW

4.7 n_0 → 4.8 n_0

(weak first order)

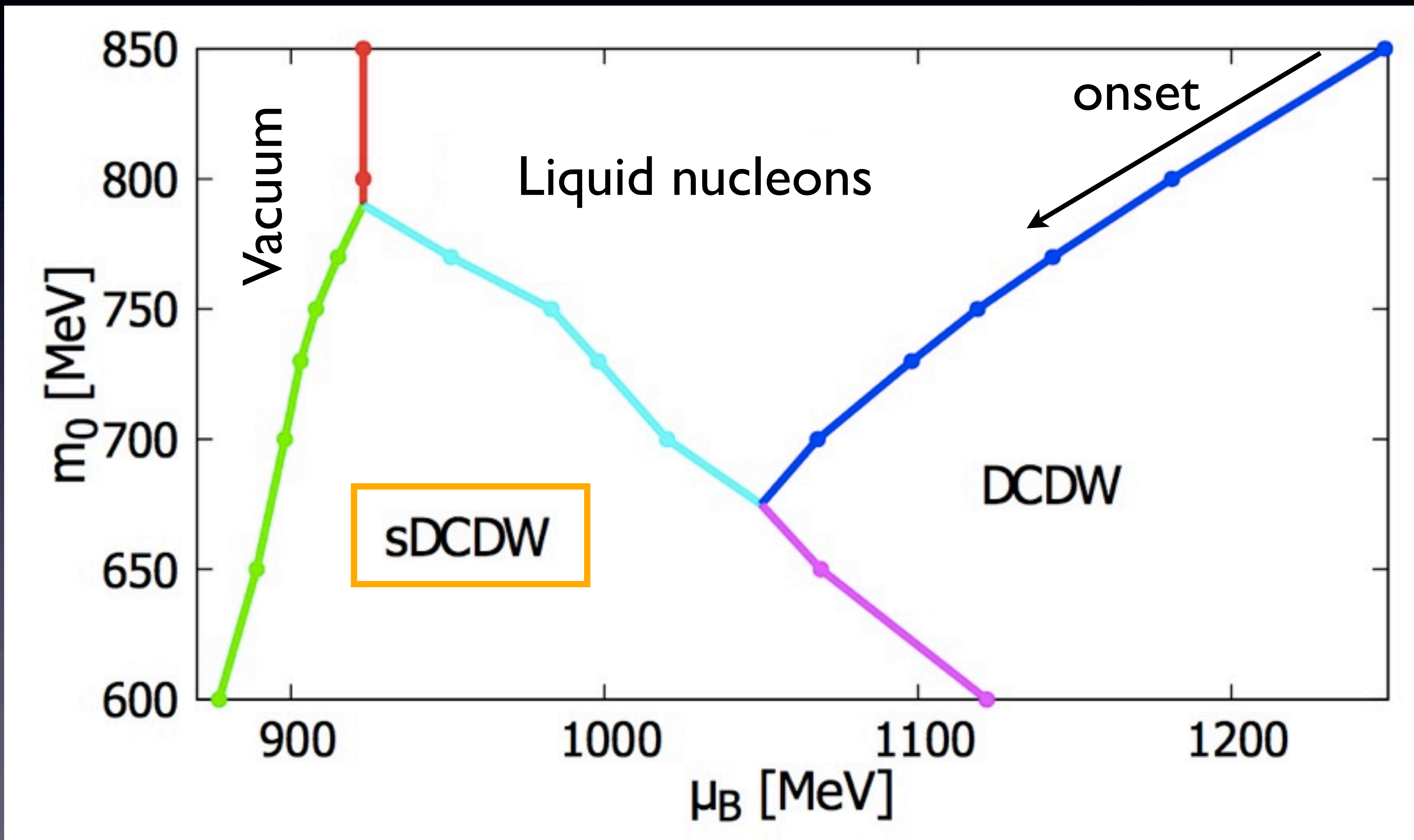
sDCDW phase ?

Takeda, Abuki, Harada, PRD97 (2018)



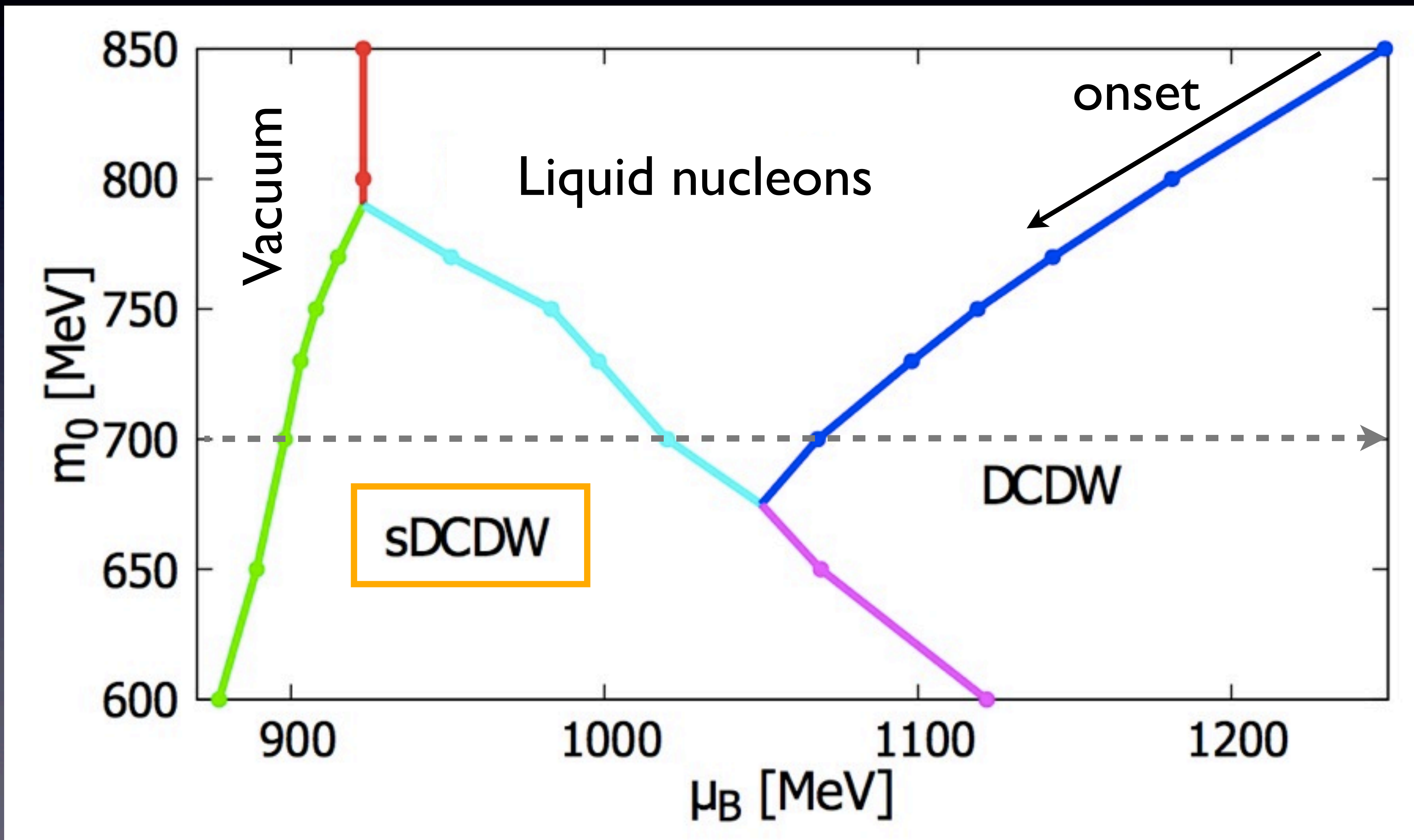
sDCDW phase？

Takeda, Abuki, Harada, PRD97 (2018)

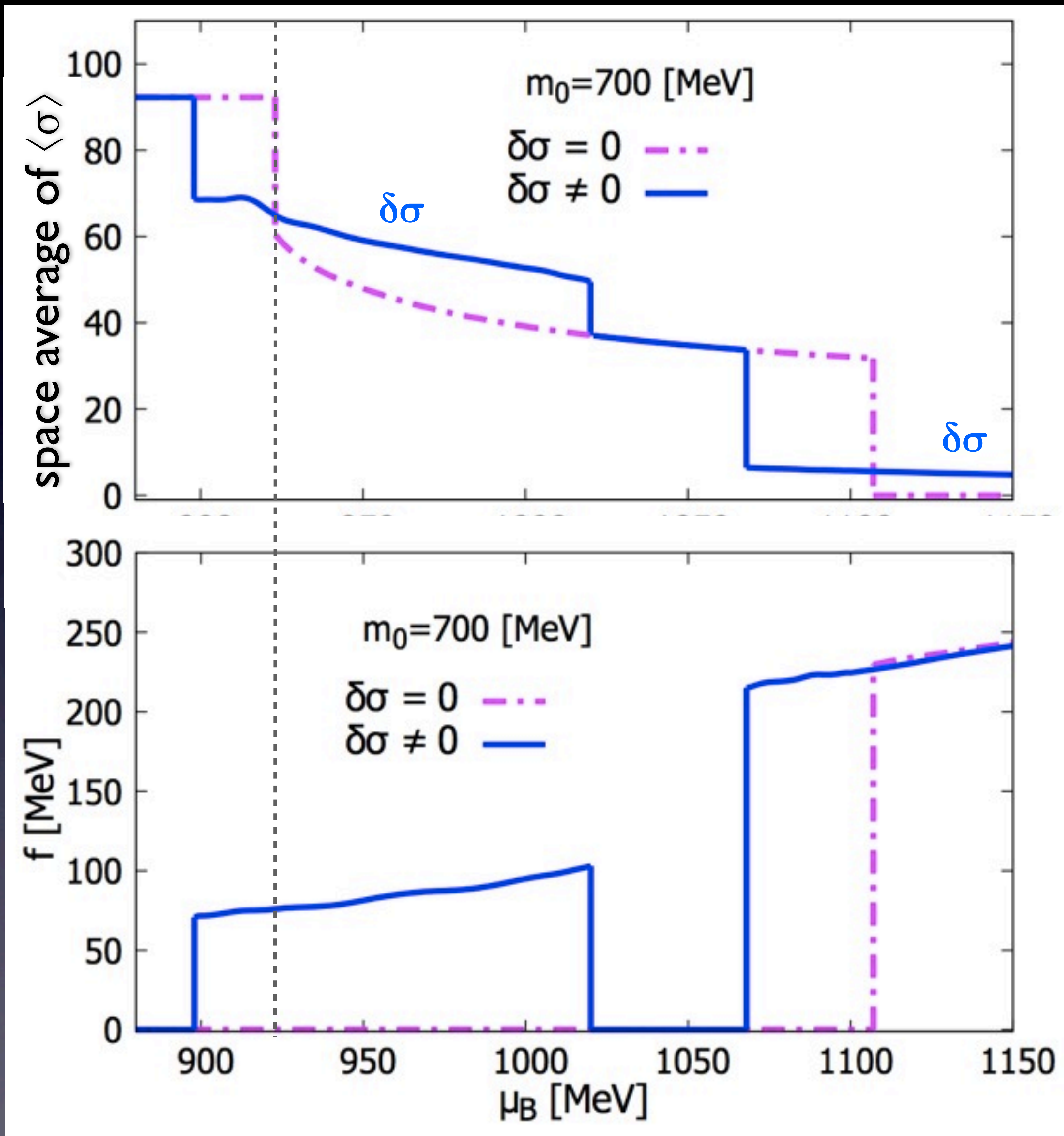


sDCDW phase ?

Takeda, Abuki, Harada, PRD97 (2018)



Takeda, Abuki, Harada, PRD97 (2018)



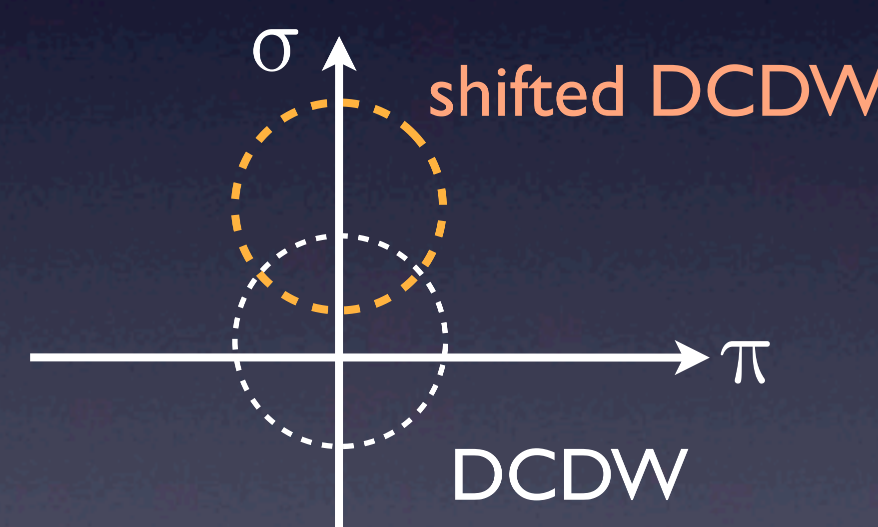
sDCDW phase

✓ *no LG phase transition*

✓ oder parameter

$$\langle\sigma\rangle = \sigma_0 \cos(2fz) + \delta\sigma$$

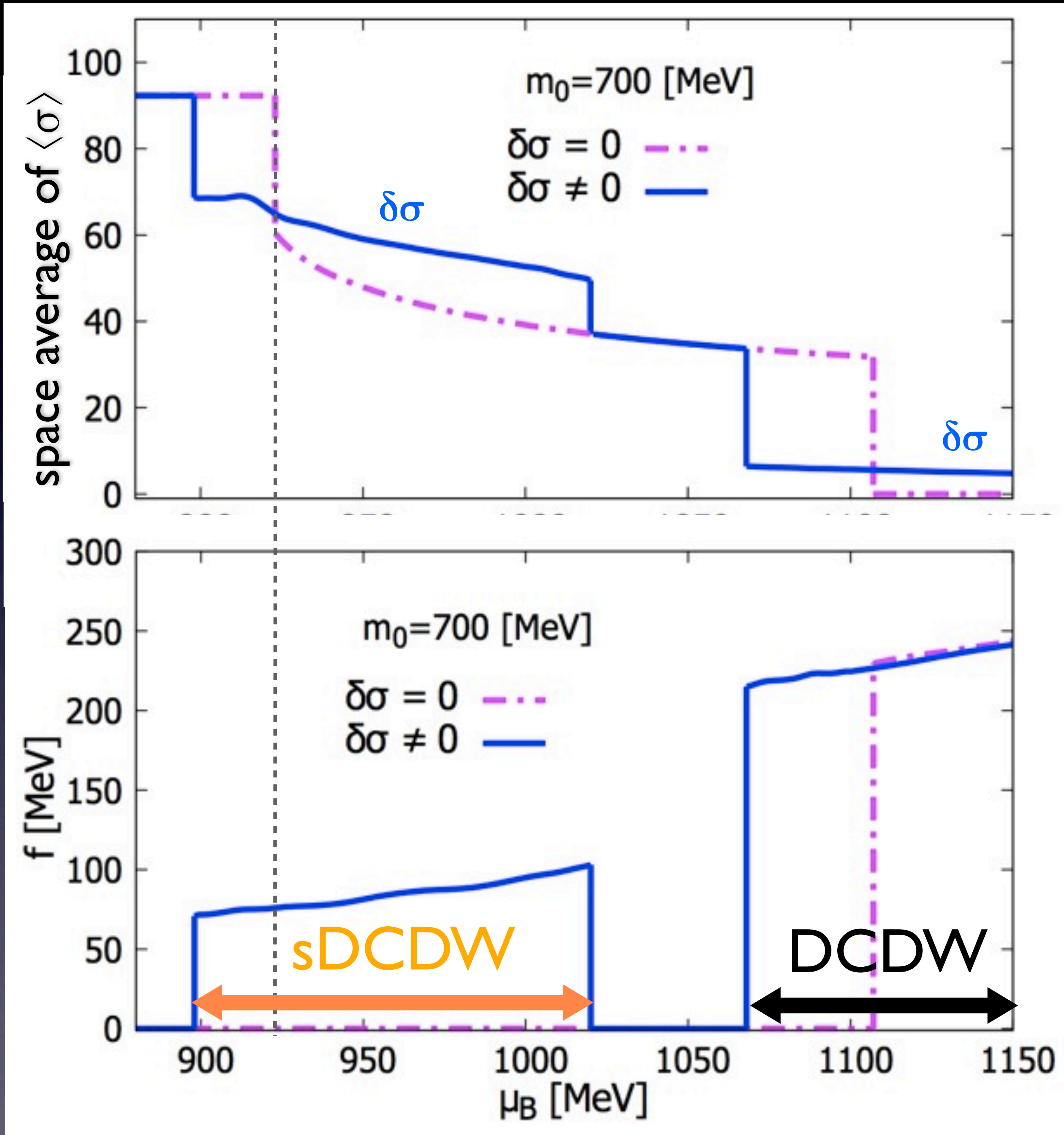
$$\delta\sigma = 50 \sim 70\,\mathrm{MeV} > \sigma_0$$



✓ $q \sim 150 - 200$ MeV

$\lambda \sim 6 - 8$ fm

Takeda, Abuki, Harada, PRD97 (2018)



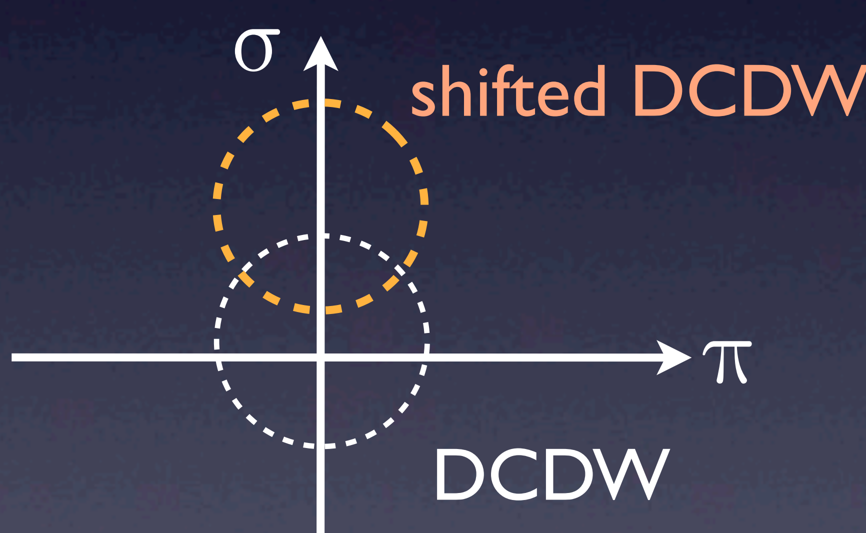
sDCDW phase

✓ *no LG phase transition*

✓ oder parameter

$$\langle\sigma\rangle = \sigma_0 \cos(2fz) + \delta\sigma$$

$$\delta\sigma = 50 \sim 70\,\mathrm{MeV} > \sigma_0$$



✓ $q \sim 150 - 200$ MeV

$\lambda \sim 6 - 8$ fm

Summary

(Magnetic field in quark based model)

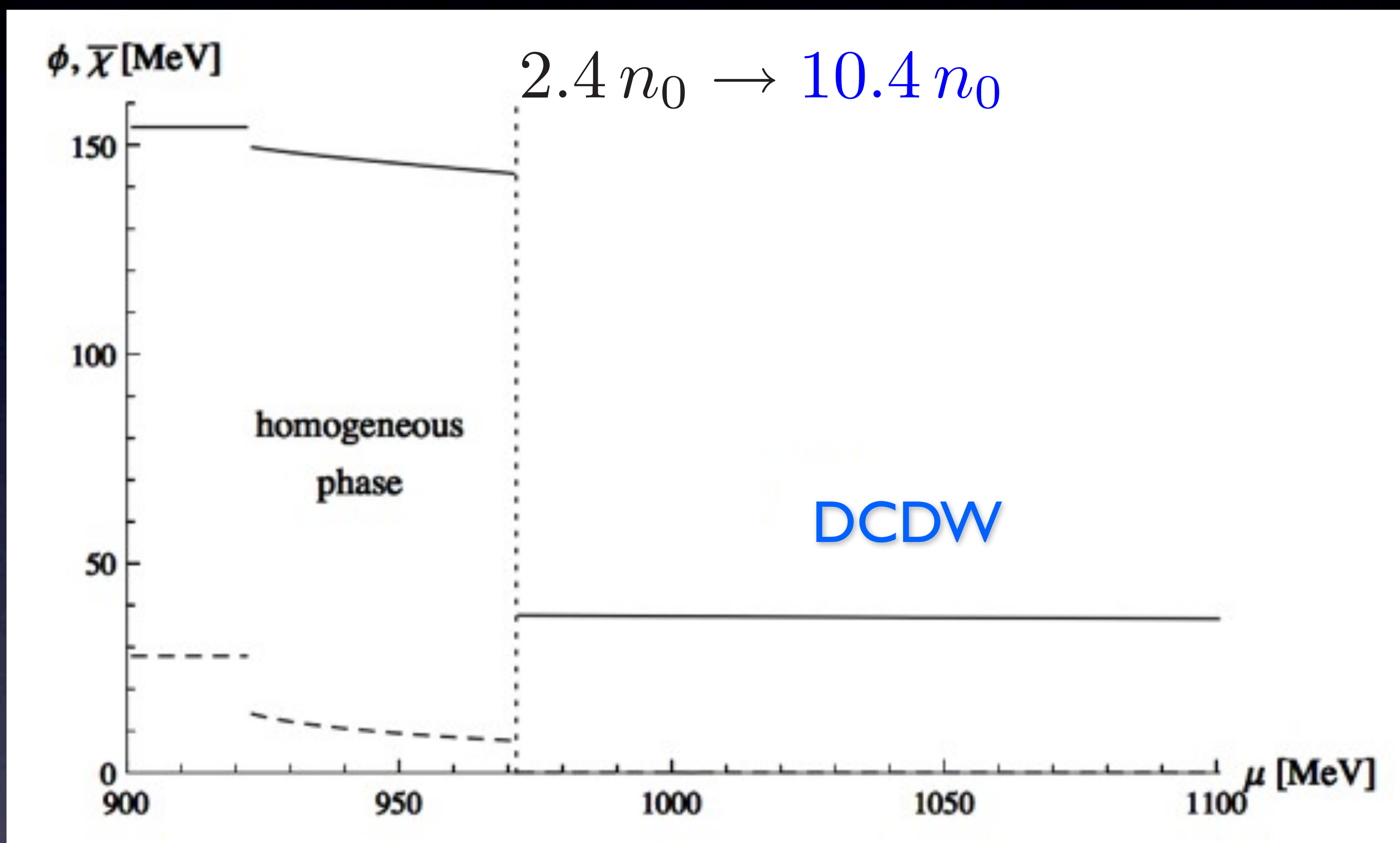
- Studied the response of TCP against m_q and B within a generalized GL approach
- Magnetic field favors DCDW phase (spiral)

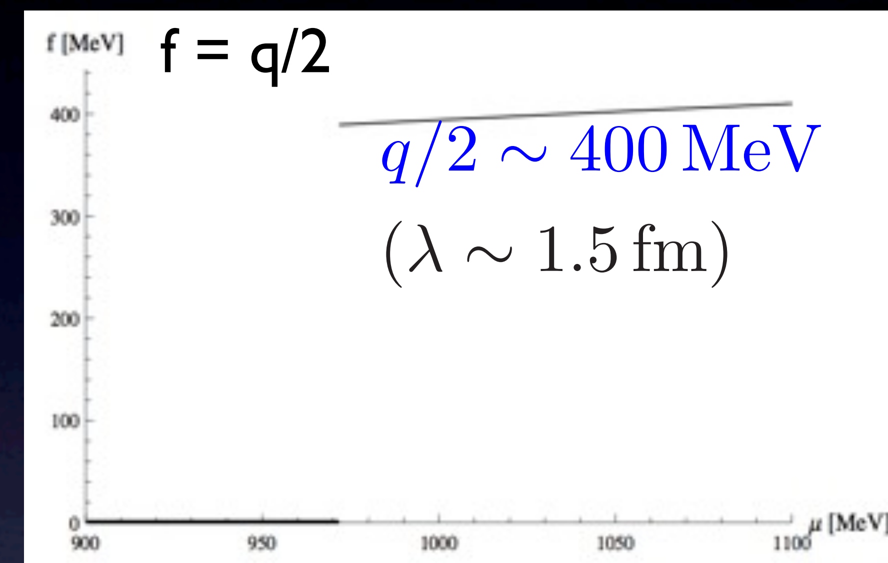
(Nucleon-Meson based model approach)

- Could reproduce matter properties for m_0
- DCDW phase appears @ high denisity
- *shifted* DCDW phase @ $m_0 < 780\mathrm{MeV}$

Backup slides

bottom-up approach: DCDW in nuclear matter?





Heinz, Giacosa, Rischke, NPA (2015)

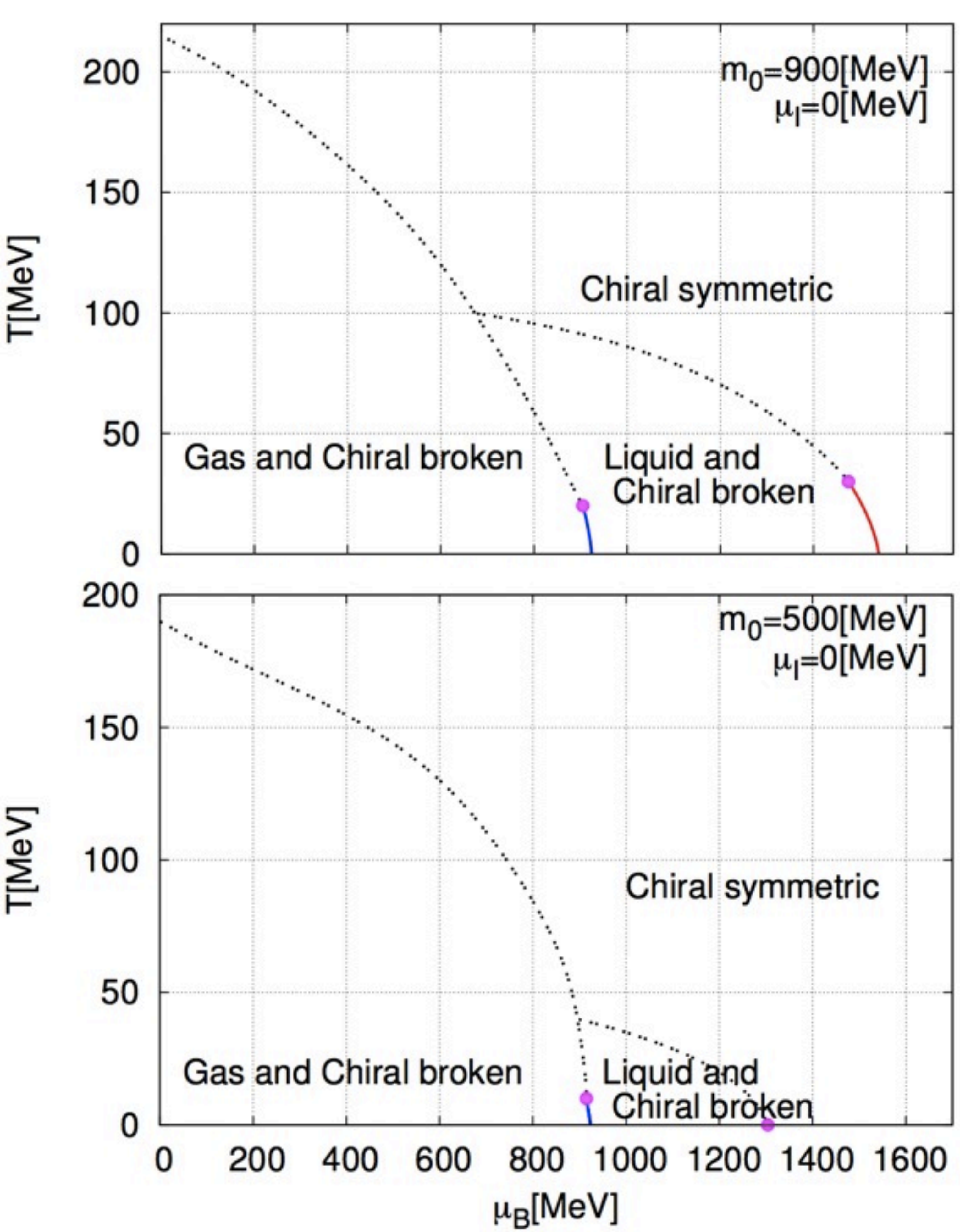
eLSM

$$\begin{aligned}\mathcal{L}_{\text{bar}} &= \overline{\Psi}_1 \imath\gamma_\mu\partial^\mu\Psi_1 + \overline{\Psi}_2 \imath\gamma_\mu\partial^\mu\Psi_2 \\ &\quad - \frac{\widehat{g}_1}{2}\overline{\Psi}_1(\sigma + \imath\gamma_5\tau^3\pi)\Psi_1 - \frac{\widehat{g}_2}{2}\overline{\Psi}_2(\sigma - \imath\gamma_5\tau^3\pi)\Psi_2 \\ &\quad - g_\omega\overline{\Psi}_1\imath\gamma_\mu\omega^\mu\Psi_1 - g_\omega\overline{\Psi}_2\imath\gamma_\mu\omega^\mu\Psi_2 \\ &\quad - a\chi(\overline{\Psi}_2\gamma_5\Psi_1 - \overline{\Psi}_1\gamma_5\Psi_2)\,.\end{aligned}$$

$f_0(500)$ as a second scalar

Diverse m_0 @ vacuum

value of m_0	Inputs (model)	Refs
270 MeV	$\Gamma_{N^* \to \pi N} = 75 \text{ MeV}$	LSM Jido, Oka, Hosaka, PTP106 (2001)
460 MeV	$\Gamma_{N^* \to \pi N} = 67.5 \text{ MeV}$ $\Gamma_{a1 \to \pi\gamma} = 640 \text{ keV}$	LSM w vector meson Gallas, Giacosa, Rischke (2010)
quite a large value, ~900 MeV	Lattice studies	Glozman et al, PRD86 (2012) Aarts et al., PRD92 (2015)
500~800 MeV	Nuclear matter models; best fit of *K*	Zschiesche et al., PRC75 (2007) Sasaki, Mishustin, PRC82 (2010)



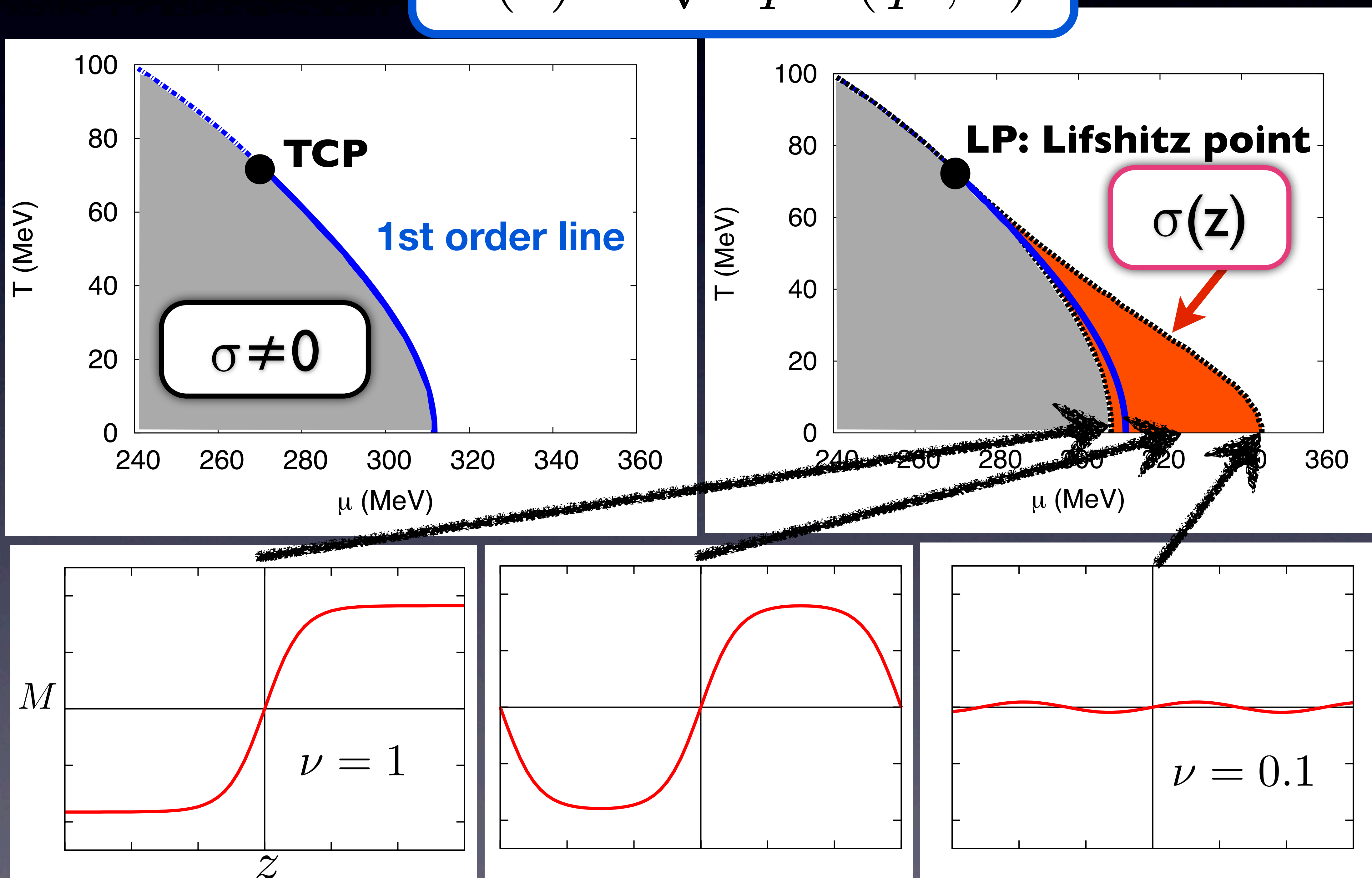
Motohiro, Kim, Harada, PRC92 (2015)

✓ smaller the chiral invariant mass, smaller the critical density for the chiral phase transition

Real Kink Crystal (RKC)

$$\sigma(z) = \sqrt{\nu} q \, \mathrm{sn}(qz, \nu)$$

D. Nickel, PRL09, PRD09



gGL with magnetic field

- How does quark propagator modify?

1. Strong Magnetic Field (LLL approximation)

$$-iS(p_0, p_\parallel, \mathbf{p}_\perp) = 2e^{-\frac{\mathbf{p}_\perp^2}{QB}} \frac{\not{p}+\not{\mu}}{(p_0+\mu)^2 - p_\parallel^2} P_+$$

2. Weak Magnetic Field $10\mathrm{GT} \cong 3 \times 10^{-5} (m_\pi)^2$

$$\begin{aligned} -iS(p) &= \frac{\not{p}+\not{\mu}+m}{(p_0+\mu)^2 - \mathbf{p}^2 - m^2} \\ &\quad + (QB) \frac{\not{p}_\parallel + \not{\mu} + m}{((p_0+\mu)^2 - \mathbf{p}^2 - m^2)^2} (i\gamma^1\gamma^2) \\ &\quad + (QB)^2 \cdots \end{aligned}$$

Gobar, Miransky, Shovkovy, PRD88 (2013)

1+1 dim. NJL/GN model

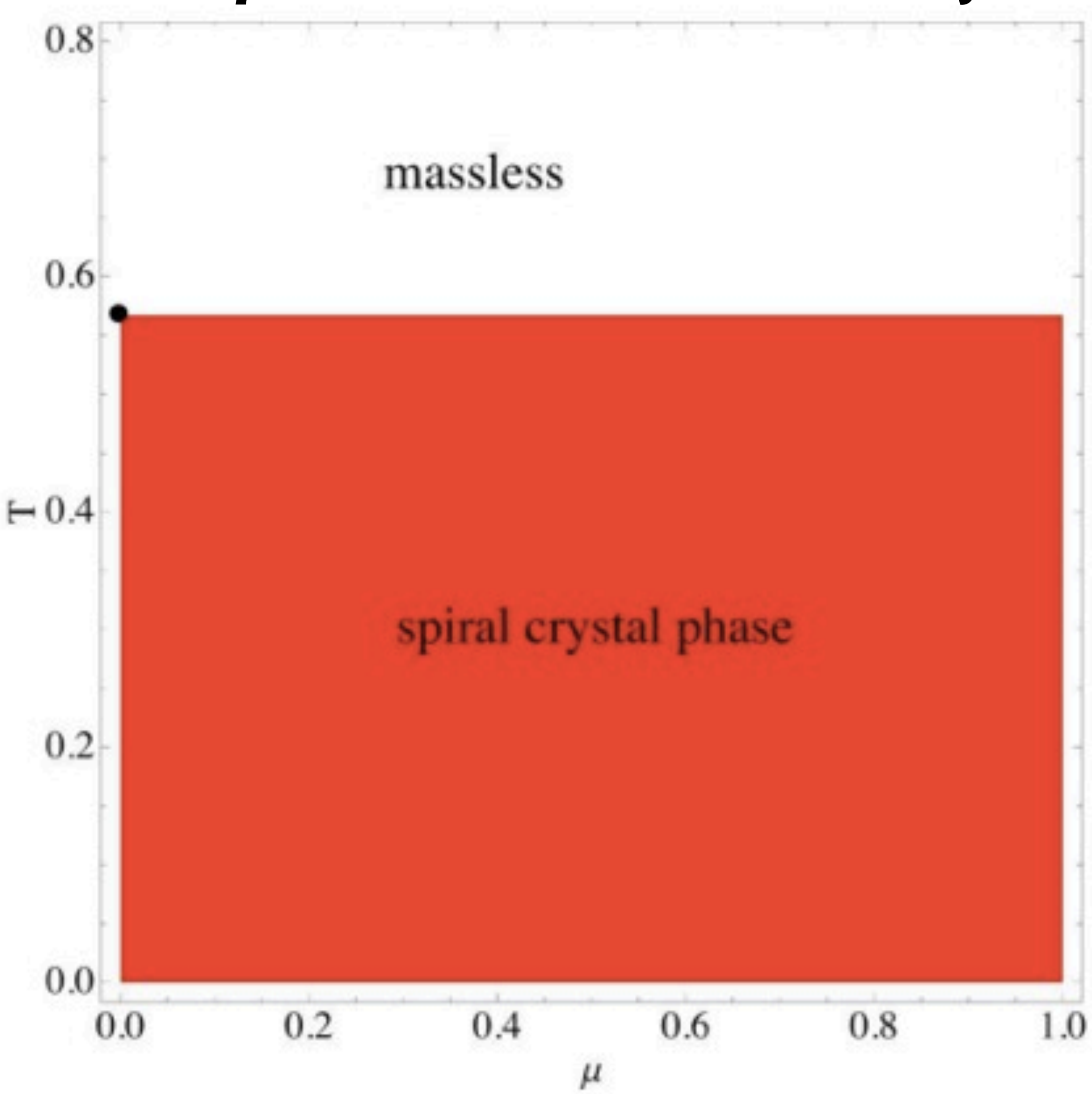
M. Thies, J. Phys. A2006

$$\sigma = \langle \bar{\varphi}\varphi \rangle = \cos(2\mu z)\Delta$$

$$\pi = \langle \bar{\varphi} i\gamma_5 \varphi \rangle = \sin(2\mu z)\Delta$$

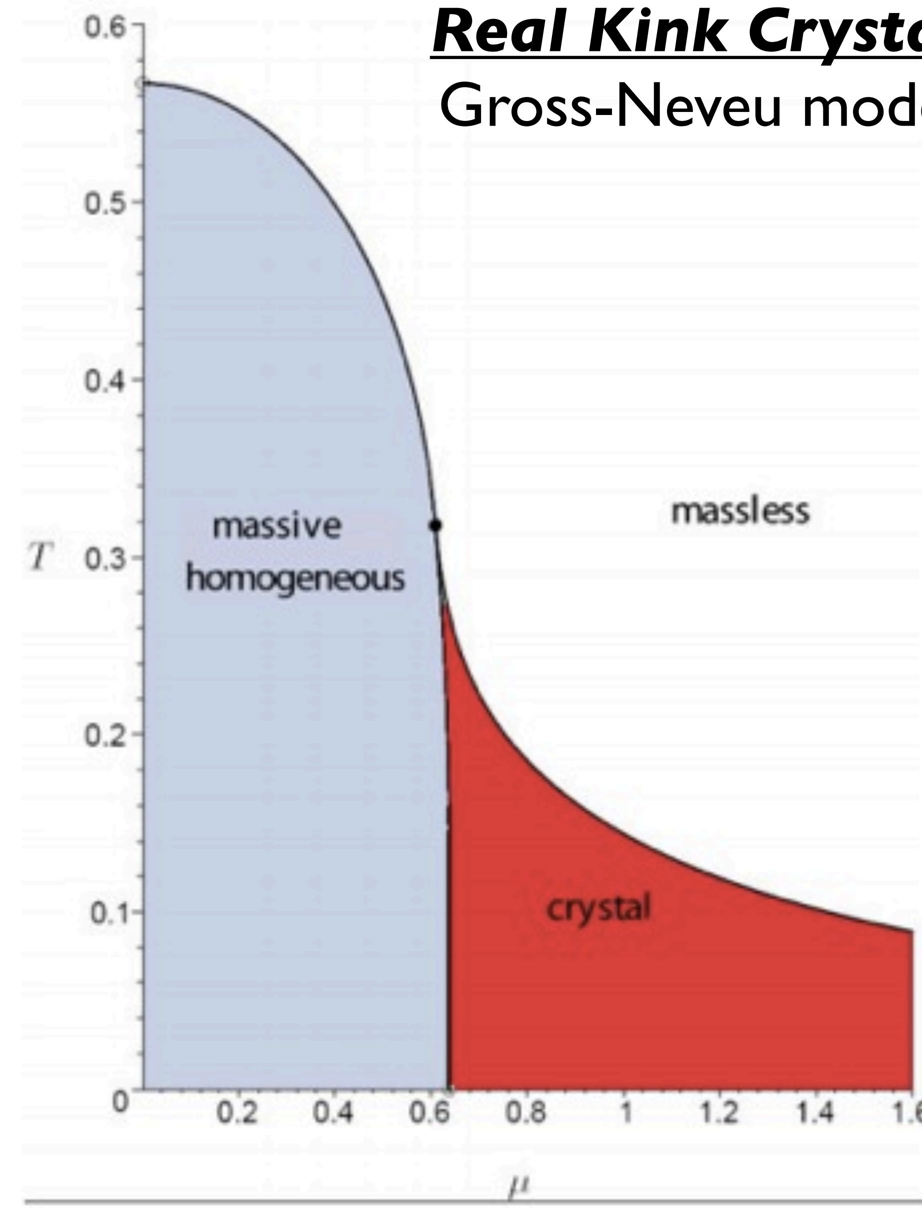
$$\sigma = \langle \bar{\varphi}\varphi \rangle = \sqrt{\nu} q \mathrm{sn}(qz, \nu)$$

Chiral Spiral NJL$_2$



Real Kink Crystal

Gross-Neveu model



Remark

Spiral being always disfavored against RKC in 3D

Boehmer, Thies, Urlichs, PRD75 (2007)

$$\Delta(z) \equiv \sigma(z) + i\pi(z)$$

$$\Omega_{\mathrm{NJL}_2} = \frac{\alpha_2}{2}|\Delta(z)|^2 + \frac{\alpha_3}{3}\mathrm{Im}\left[\Delta^*\Delta'\right] + \frac{\alpha_4}{4}\left(|\Delta(z)|^4 + |\Delta'(z)|^2\right) + \frac{\alpha_5}{5}\mathrm{Im}\left(\left(\Delta'' - 3|\Delta|^2\Delta\right)\Delta'^*\right) + \frac{1}{6}\Delta(\mathbf{x})^6$$

For chiral spiral :

$$\Delta(z) \equiv \Delta_0 e^{iqz}$$

$$-\frac{\alpha_5}{5}\left(q^3\Delta_0^2 + 3\Delta_0^4 q\right) \qquad -\frac{\alpha_3}{3}\Delta_0^2 q$$

see Tatsumi, Nishiyama, Karasawa, PLB743 (2015)

reduced gGL potential

Nickel, PRL09
Abuki (2014)

$$\Omega_{\rm GL} = -h\sigma + \frac{\alpha_2}{2}\sigma^2 + \frac{\alpha_4}{4}\left(\sigma^4 + (\nabla\sigma)^2\right) + \frac{\alpha_6}{6}\left(\sigma^6 + 5\sigma^2(\nabla\sigma)^2 + \frac{1}{2}(\Delta\sigma)^2\right)$$

- Four independent GL parameters.

  $[\alpha_6] = \Lambda^{-2}$

- For thermodynamic stability $\alpha_6 > 0$, so use it to set an energy scale; $\alpha_6 \to 1$

  $\sigma[h^{1/5}], \quad \alpha_2[h^{4/5}], \quad \alpha_4[h^{2/5}], \quad \mathbf{x}[h^{-1/5}],$

  $\Omega \to h^{6/5}\Omega_0$

Ansatz for 1D crystal

M. Thies, J. Phys. A2006
D. Nickel, PRL09, PRD09

H. Abuki, PLB (2014)

Saddle point equation (EL) equarion:

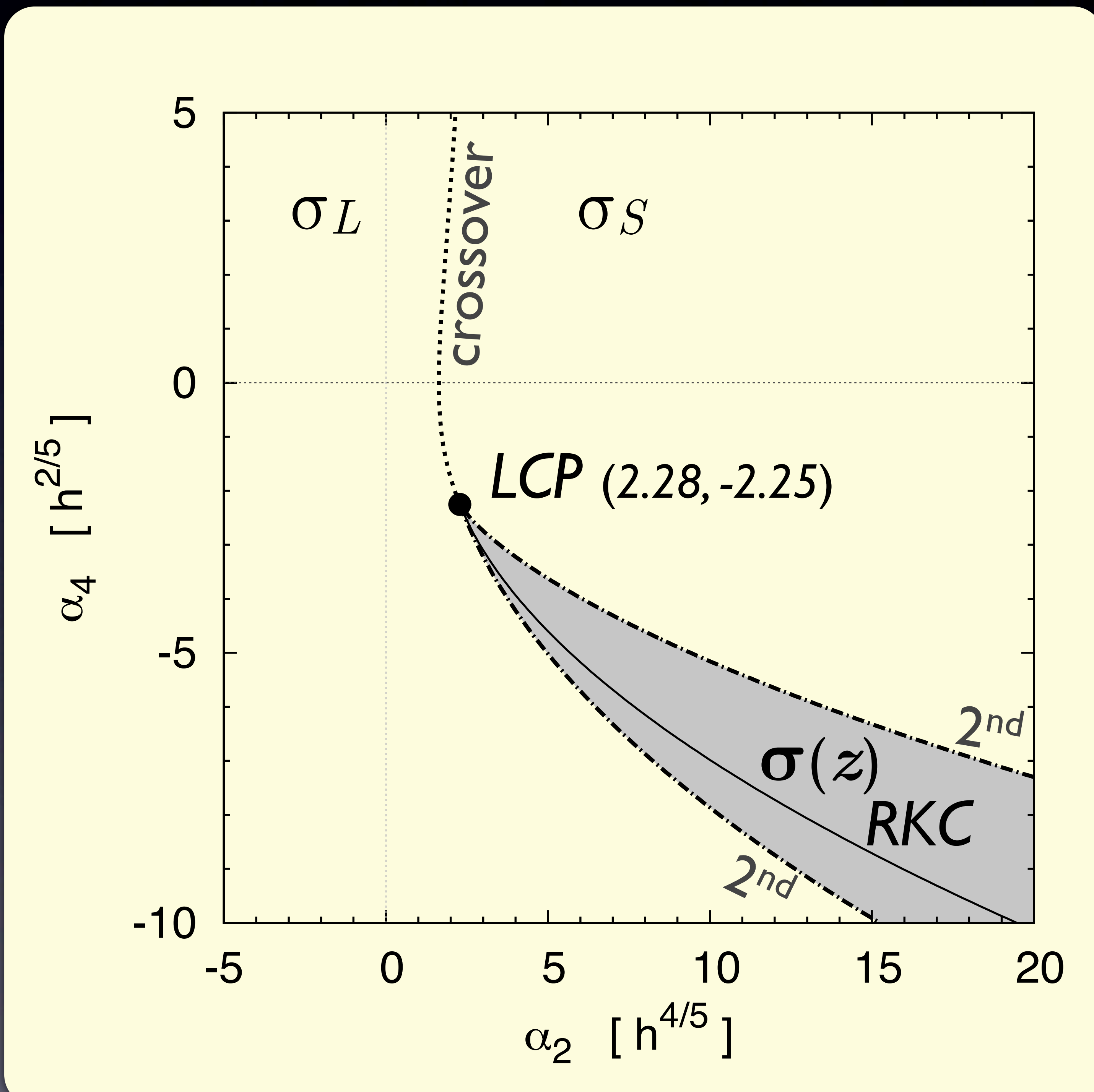
$$h = \sigma^5 + \alpha_4\sigma^3 + \alpha_2\sigma + \frac{1}{6}\sigma^{(4)} - \frac{5}{3}(\sigma^2\sigma'' + \sigma(\sigma')^2) - \frac{1}{2}\alpha_4\sigma''$$

Extended Real Kink Crystal (RKC)

$$\sigma(z) = k\mathrm{sn}(b,\nu)\left(\nu^2\mathrm{sn}(kz,\nu)\mathrm{sn}(kz+b,\nu) + \frac{\mathrm{cn}(b,\nu)\mathrm{dn}(b,\nu)}{\mathrm{sn}^2(b,\nu)}\right)$$

Three parameter solution group: lattice of kink-antikink pairs

$b = 0$:no magnetic case



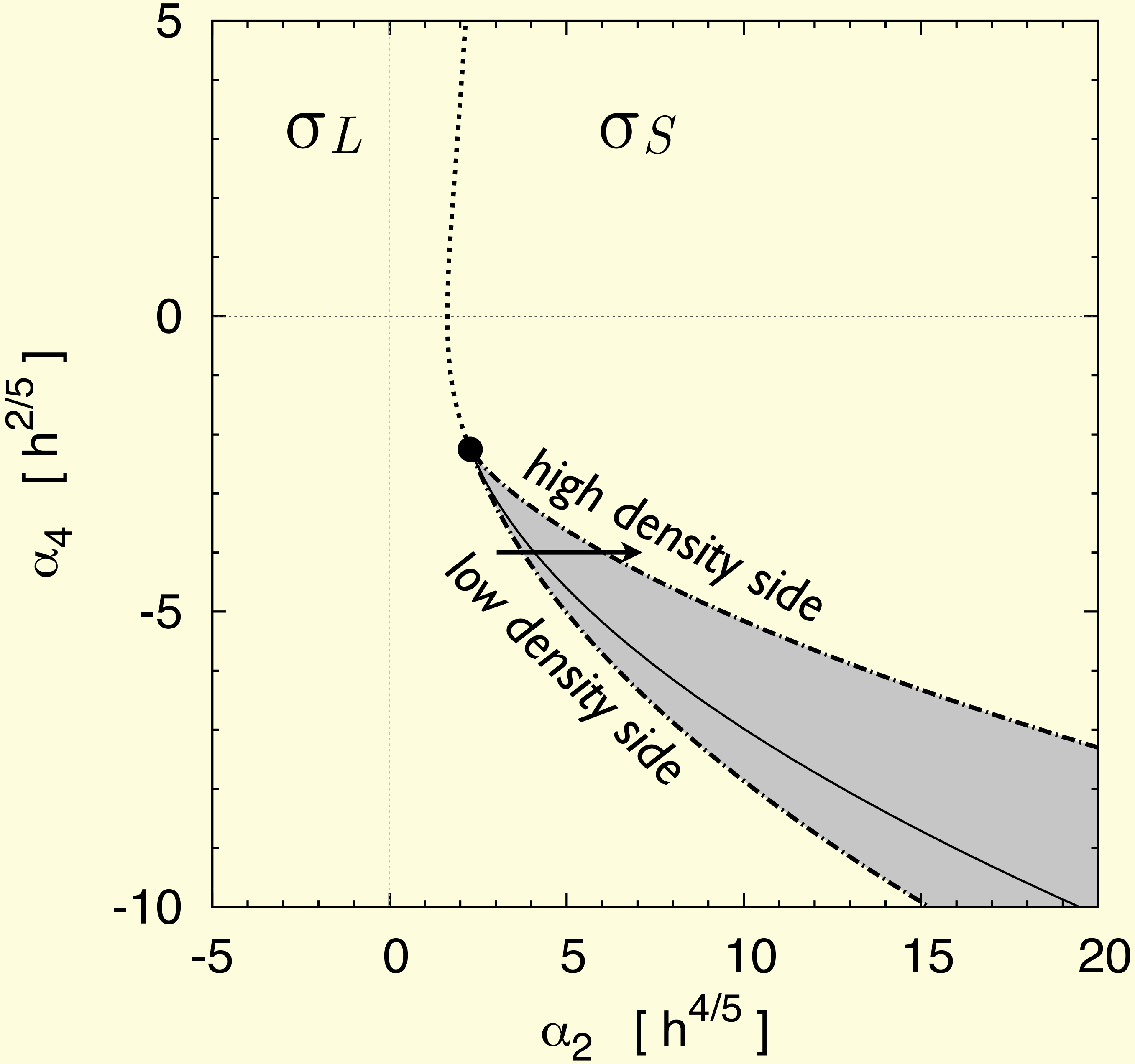
(1) *CP* (2.28, -2.25) *coincides with Lifshitz point*

LCP = Lifshitz CP

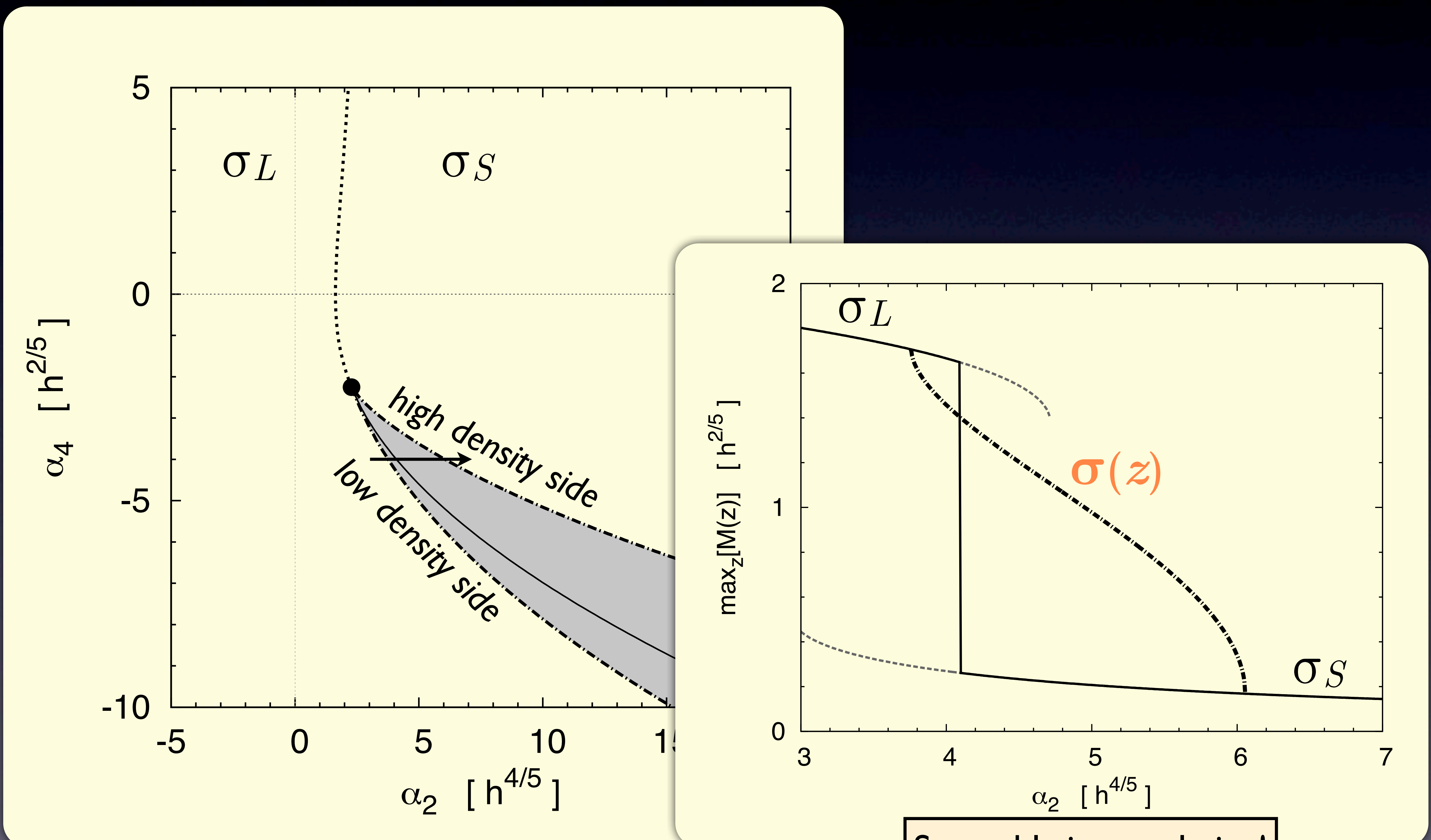
(2) *RKC* stabilized:

Two critical lines enclosing *RKC* are both of 2nd *order*

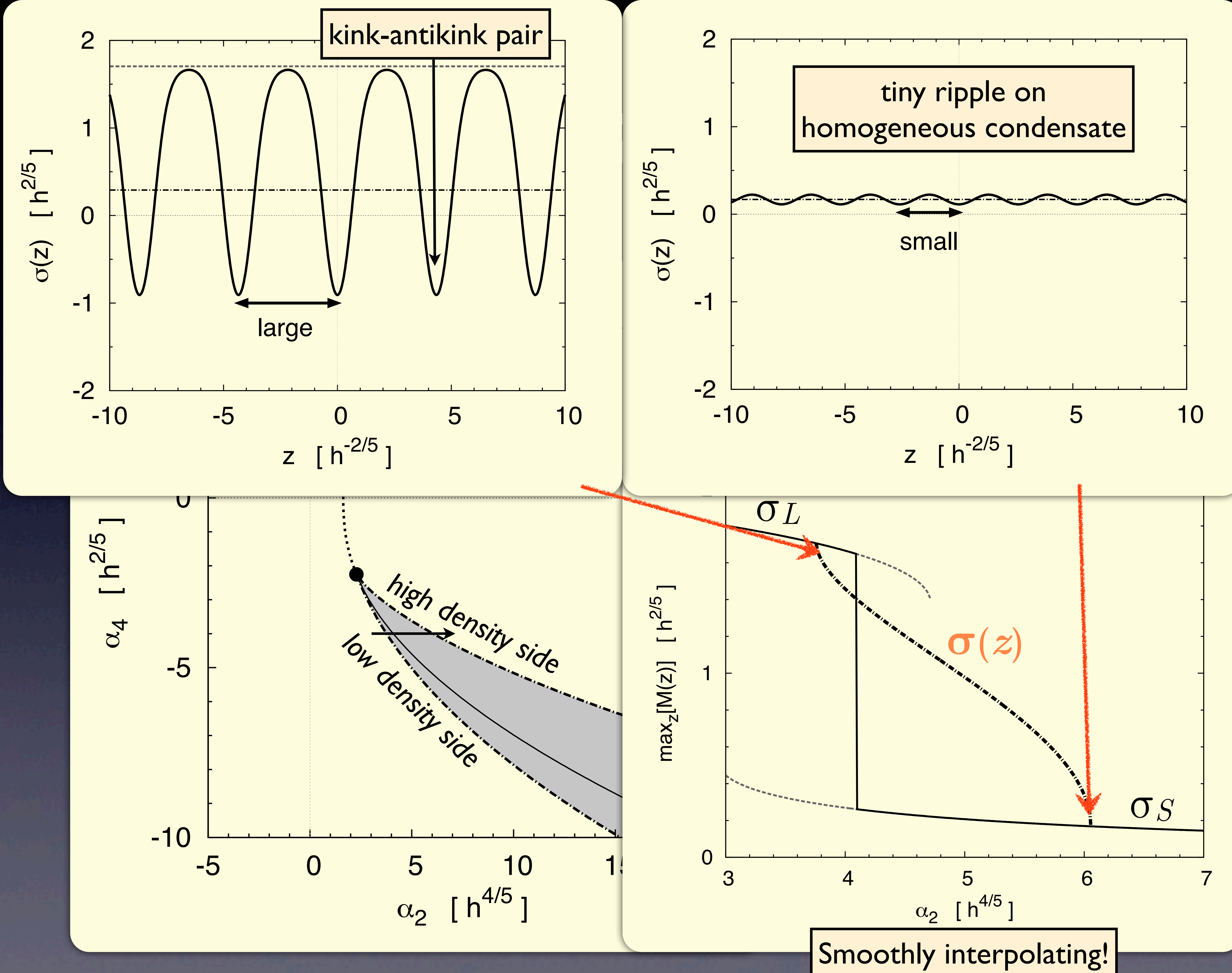
Profile of RKC phase



Profile of RKC phase



Smoothly interpolating!


kink-antikink pair
large
σ(z) [h^2/5]
z [h^-2/5]
tiny ripple on
homogeneous condensate
small
high density side
low density side
α4 [h^2/5]
α2 [h^4/5]
σL
σ(z)
σS
maxz[M(z)] [h^2/5]
Smoothly interpolating!

gGL with magnetic field

- Rewrite 6th gGL @ m_q=0 in the $O(4)$ invariant form. With 4-vector ϕ=(π1, π2, π3, σ)

$$\begin{aligned}\Omega_{\mathrm{GL}} &= \frac{\alpha_2}{2}\phi^2 + \frac{\alpha_4}{4}\left(\phi^4 + (\nabla\phi)^2\right) \\ &+ \frac{\alpha_6}{6}\left(\phi^6 + 5\phi^2(\nabla\phi)^2 + \frac{1}{2}(\Delta\phi)^2\right) \\ &+ \frac{\alpha_{6d}}{6}\left(\phi^2(\nabla\phi)^2 - (\phi\cdot\nabla\phi)^2\right)\end{aligned}$$

- Couplings modifies (time reversal/rotational sym):

$$\alpha_2 \quad \rightarrow \quad \alpha_2 + \frac{5}{27}(eB)^2 + \cdots$$

When may crystals develop?

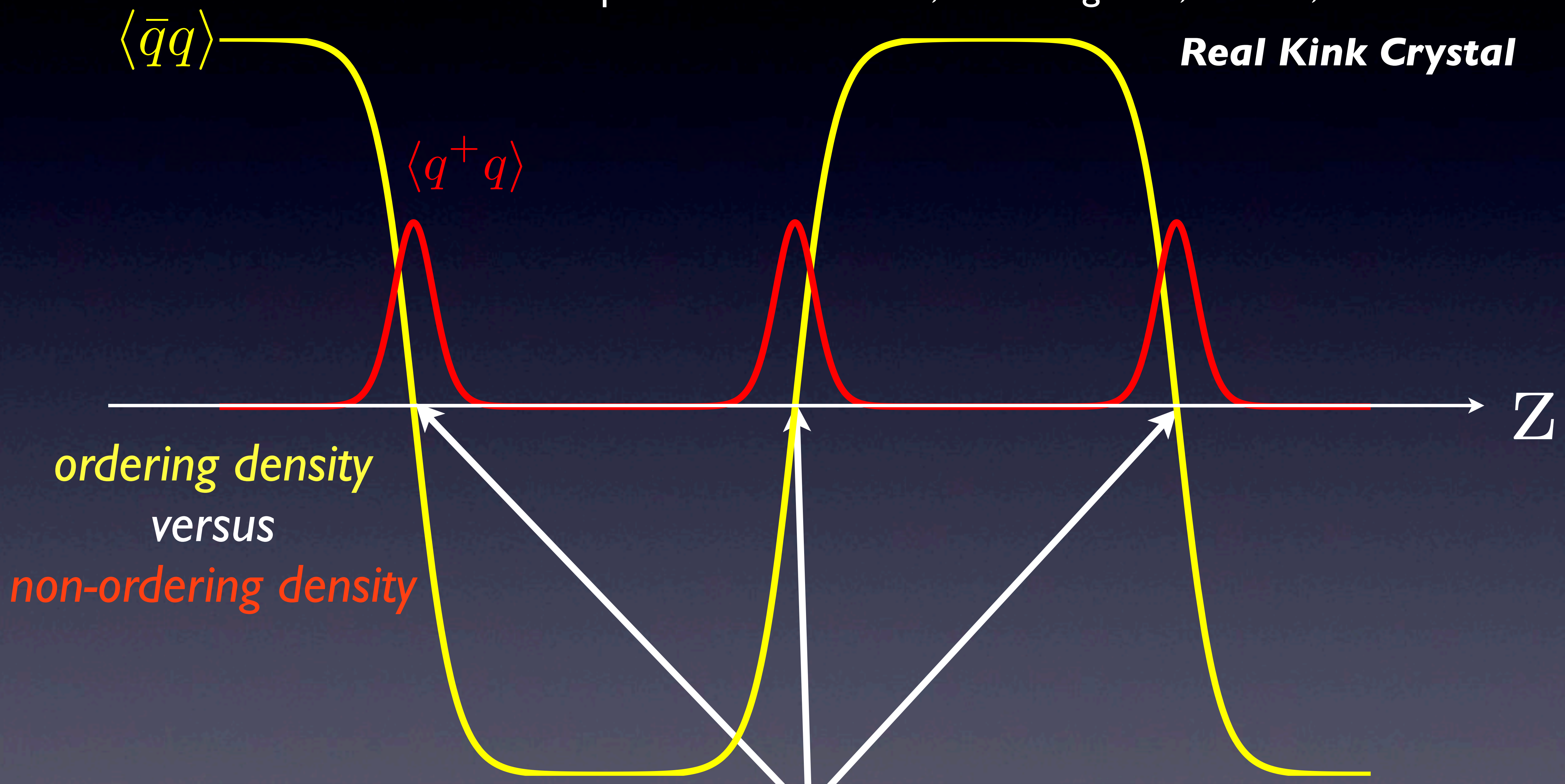
	iChiral Phase	FFLO	Crystalline CSC
order density	$\langle \bar{q} q \rangle$	$\langle \psi_\uparrow \psi_\downarrow \rangle$	$\langle q_A q_B \rangle$
non-ordering density	$\langle q^\dagger q \rangle$ $n_q - n_{\bar{q}}$	$\boldsymbol{n}_\uparrow - \boldsymbol{n}_\downarrow$	$\boldsymbol{n}_A - \boldsymbol{n}_B$
stress force	$\mu_{\mathbf{q}}$	h $\mu_\uparrow - \mu_\downarrow$	$\mu_A - \mu_B$

for CSC and CSC crystals, see Alford, Schmitt, Rajagopal, Schafer, RMP 80 (2008)
Casalbuoni, Mannarelli, Ruggieri et al., RMP86 (2014)

PS with a microscale spatial order

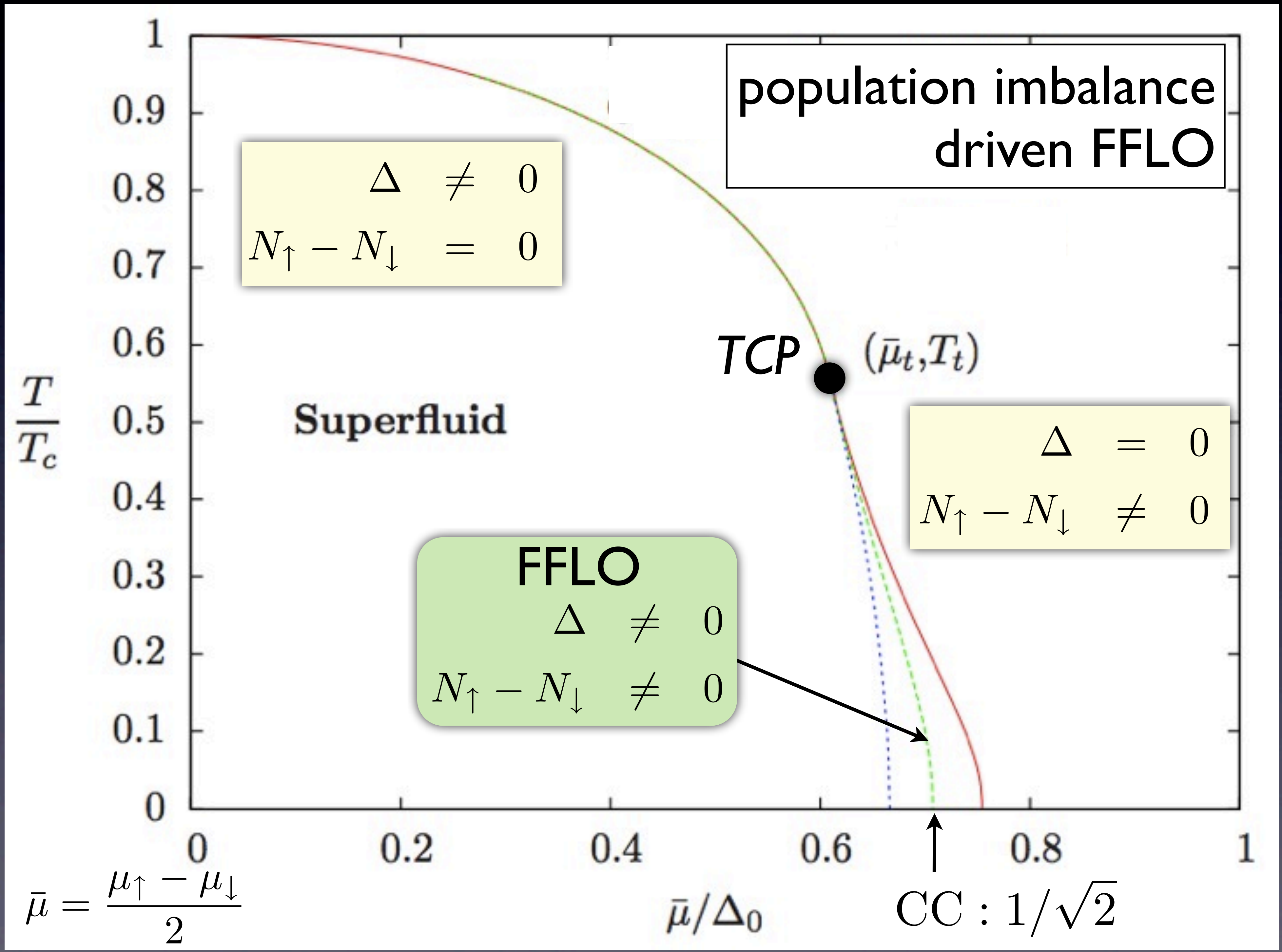
c.f. for explicit demonstration, see Carignano, Buballa, Nickel 2010

Real Kink Crystal



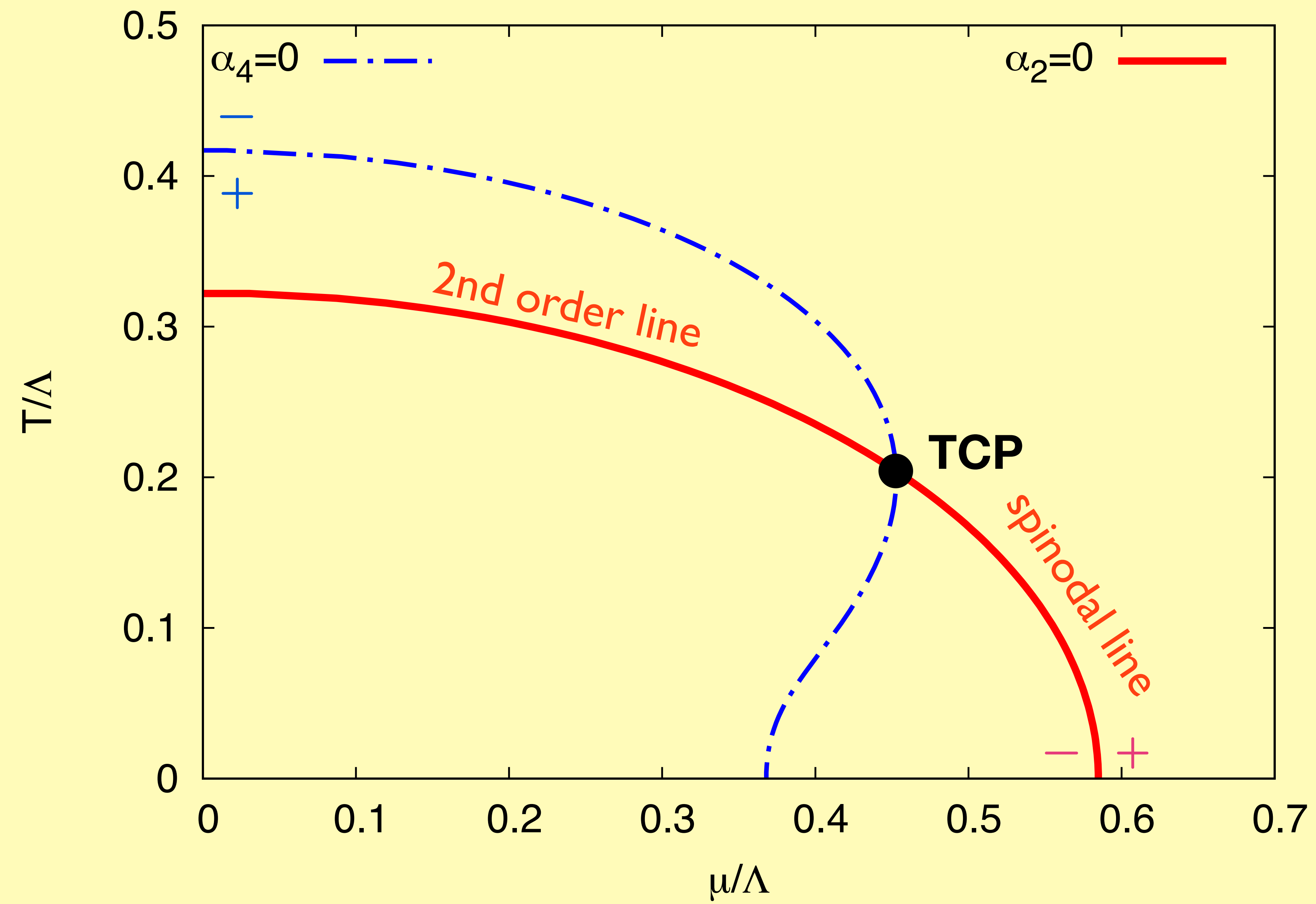
Periodically placed normal phase domains accommodating for a large *q-qbar* imbalance

FFLO @ weak coupling

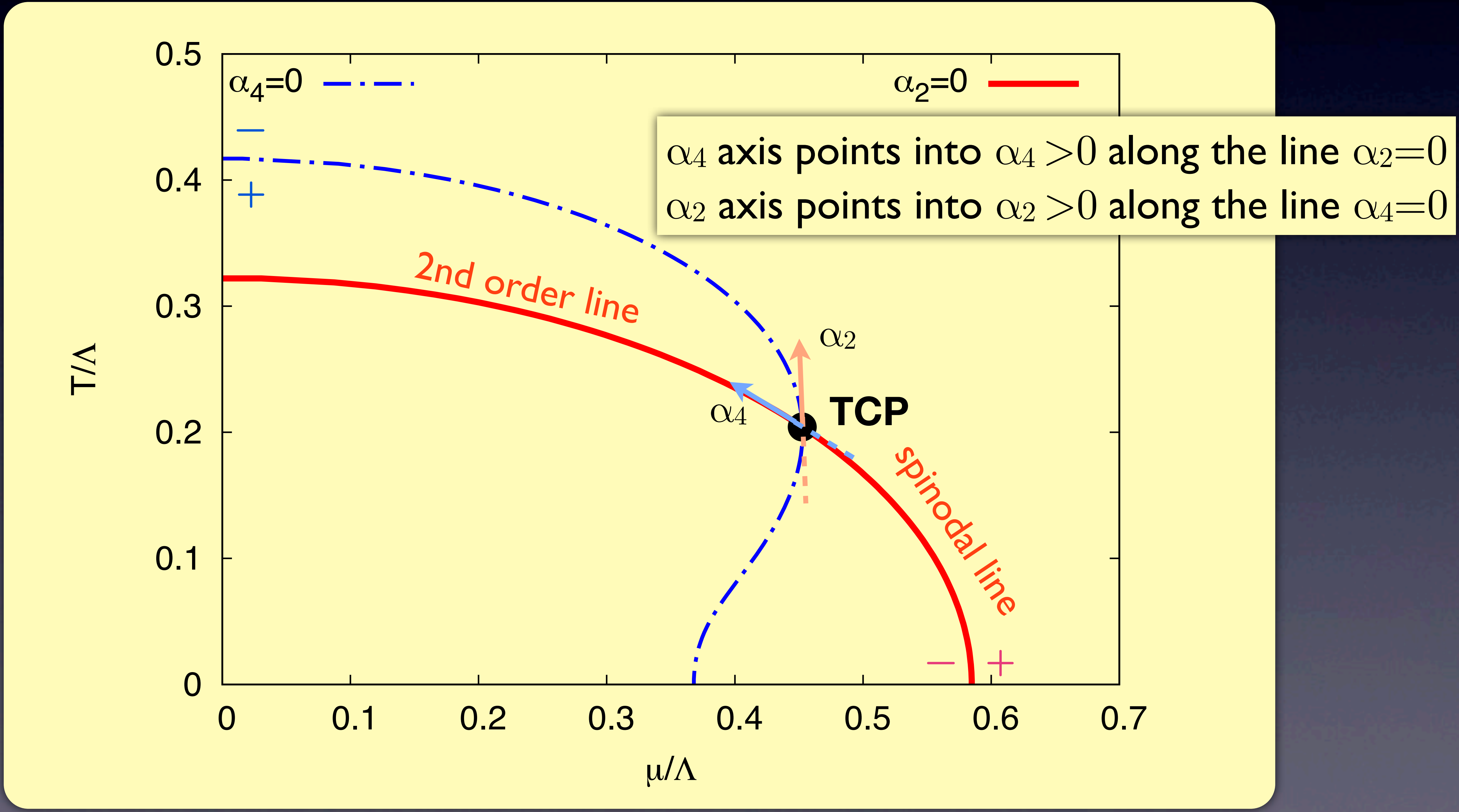


Chevy & Mora, Rep. Prog. Phys. 73 (2010) 112401: Ultra-cold polarized Fermi Gases

$\alpha_2,\ \alpha_4$ in NJL model: How do they map onto (μ, T)?

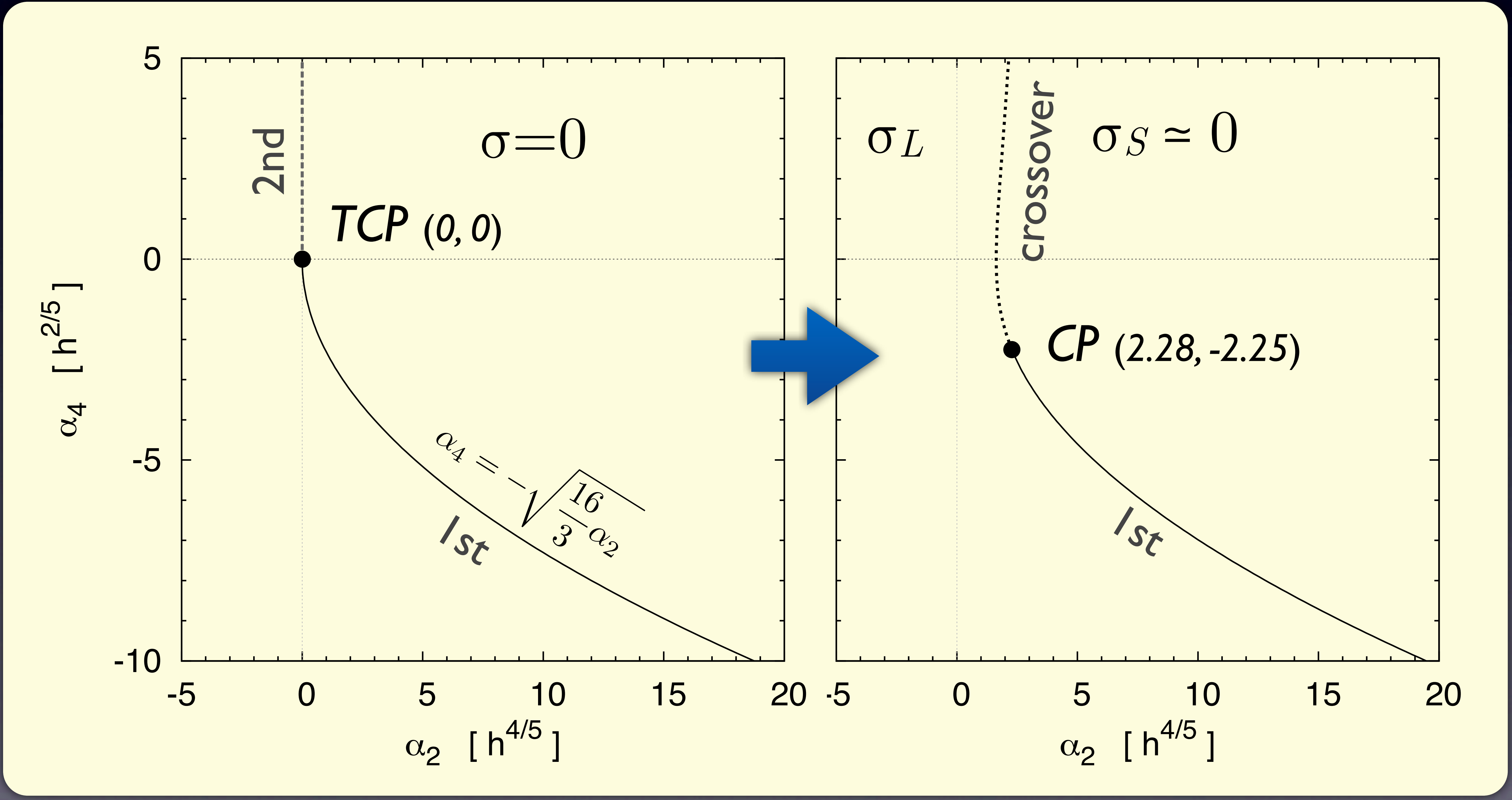


α_2, α_4 in NJL model: How do they map onto (μ, T)?

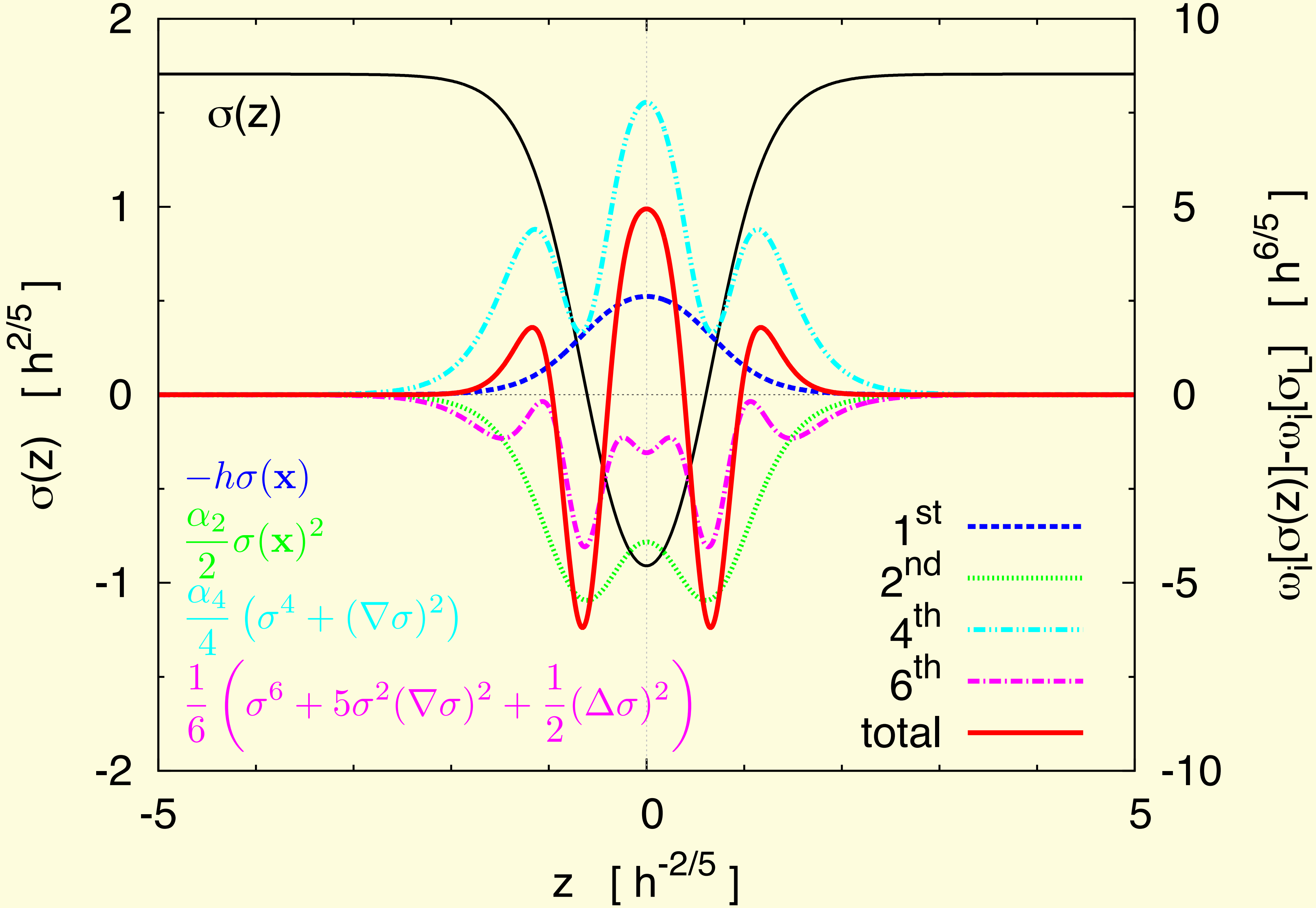


Effect of current mass: m_q

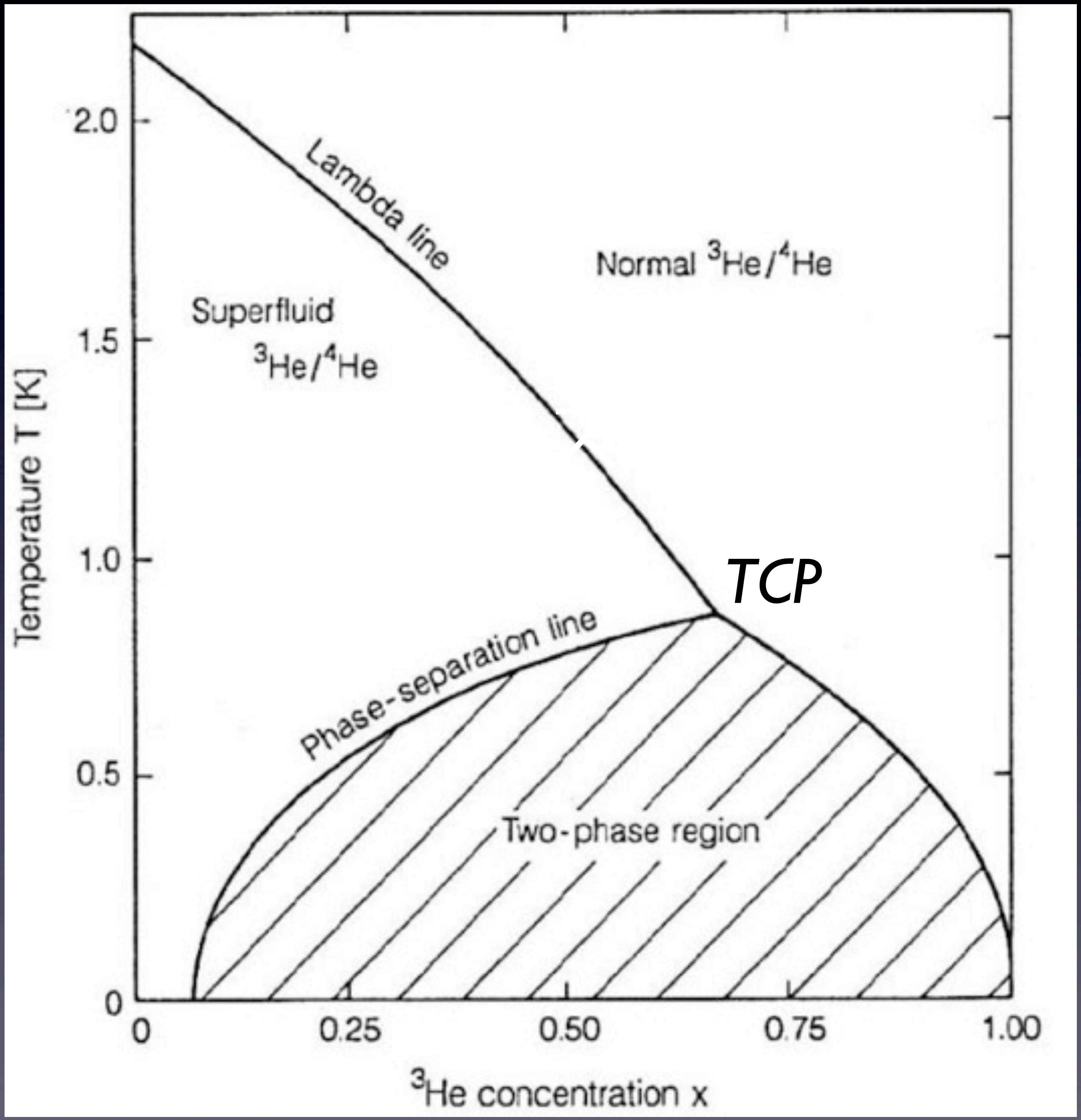
$\delta\Omega = -h\sigma$: *External field* : $O(4) \mapsto O(3) = SU(2)$ *isospin*



@Single Defect Onset

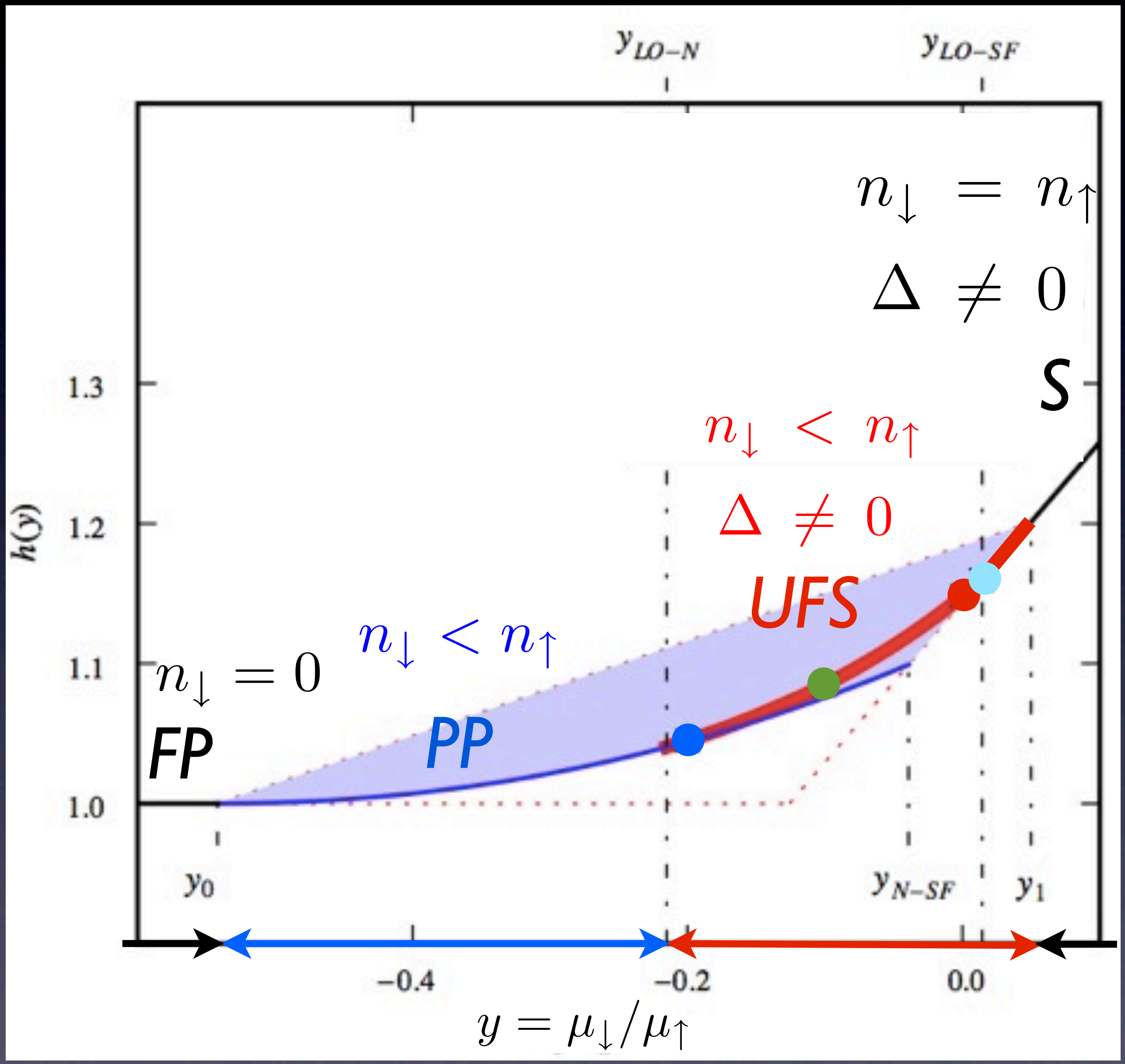


In other Systems?



http://www.cresst.de/refrigerators.php

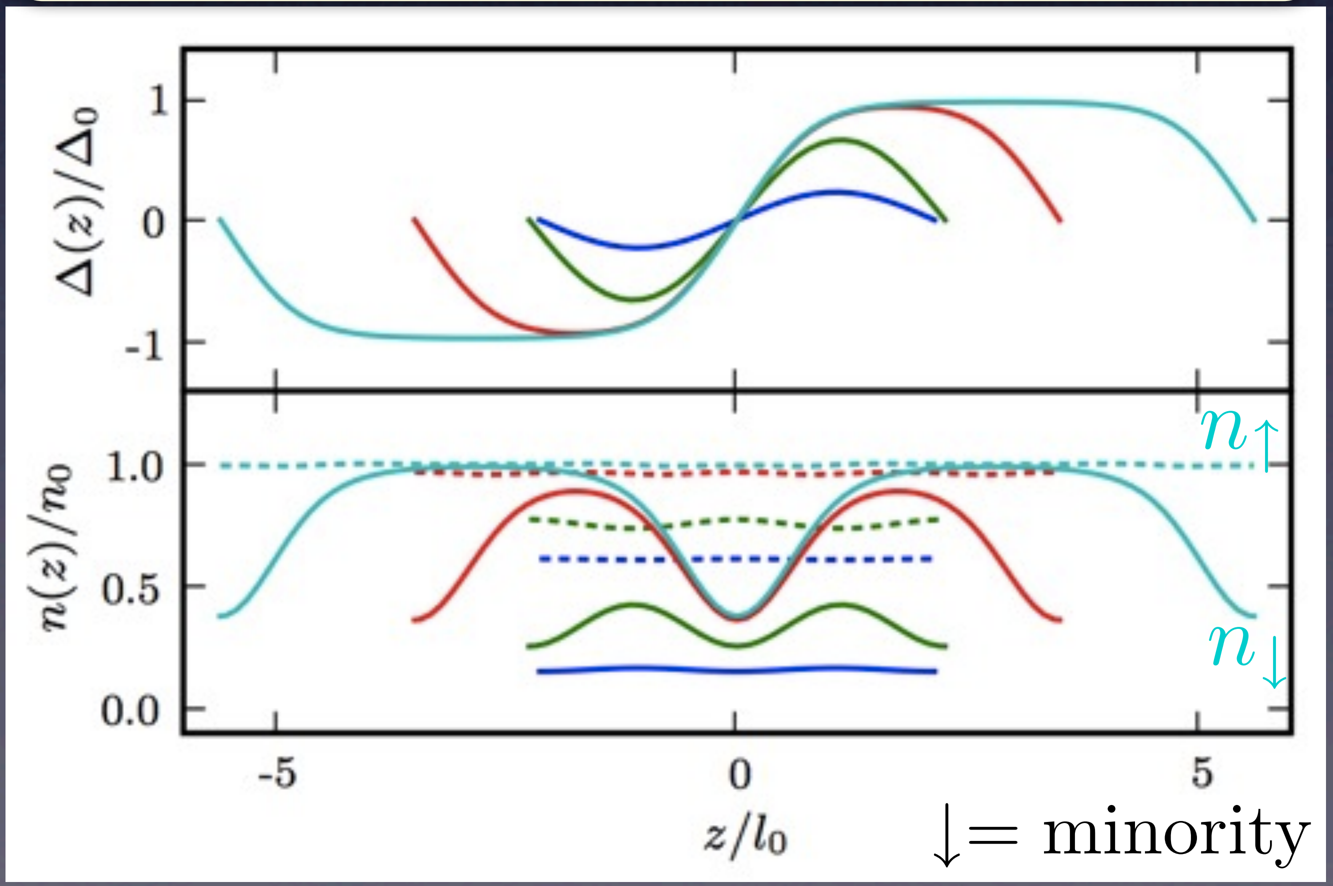
Unitary Fermi Supersolid?



Bulgac & Forbes, RRL (2008)
Hohenberg-Kohn-Sham DFT solved
with DVR (discrete variable representation)

$$h(y) = \frac{P(\mu_\uparrow, \mu_\downarrow)}{P_0(\mu_\uparrow)}$$

$$h(1) = \frac{2^{2/5}}{\xi^{3/5}} \cong 3.1 \ (\xi = 0.42(1))$$



see also: Yoshida & Yip (2007)
MF Bogoliubov-deGennes (B-dG) equation
solved with an ansatz for pair potential

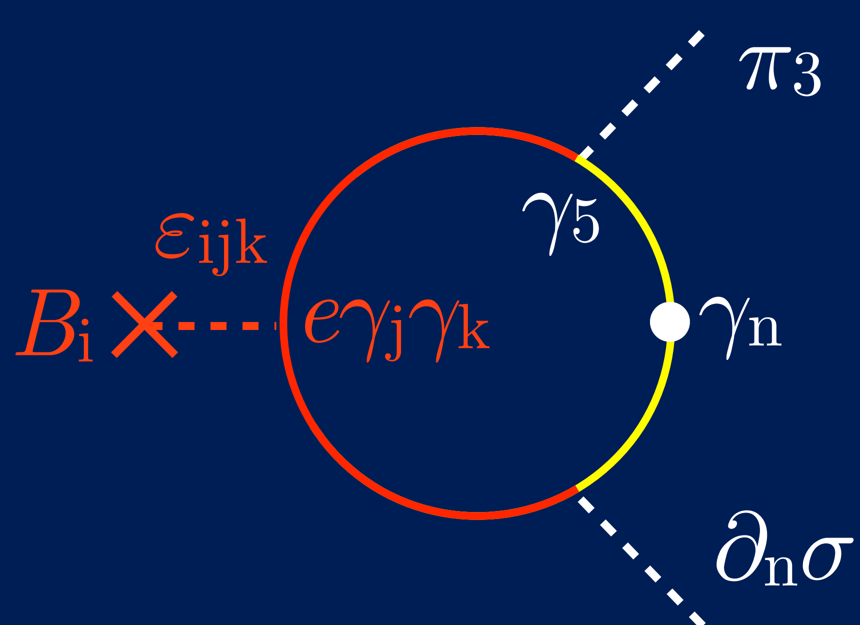
Including magnetic field

New couplings: 3D rotational symmetry breaking

$$\delta\Omega = -\frac{1}{8}\frac{\partial \alpha_4}{\partial \mu}(e\mathbf{B}) \cdot (\pi_3 \nabla\sigma - \sigma\nabla\pi_3)$$

$$= \mathbf{b} \cdot (\pi_3 \nabla\sigma - \sigma\nabla\pi_3)$$

Feynman graph contributing to new deriv. coupling



$$-iS(p) = \frac{\not p + \not \mu}{(p+\mu)^2} + (e_q B_i)\frac{i\epsilon_{ijk}\gamma^j\gamma^k}{2}\frac{\not p_\parallel + \not \mu}{(p+\mu)^4}$$

Tatsumi, Nishiyama, Karasawa, PLB (2015)

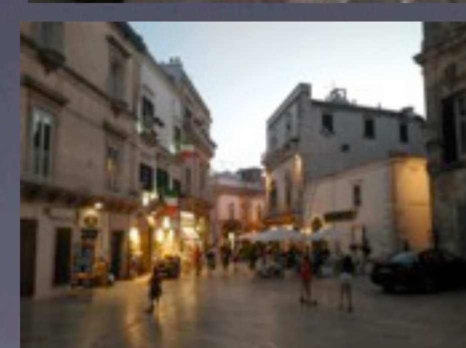
Thank you for nice workshop every 2 yrs

4^{th} meeting 2007 (Martina Franca, trip to Matera)


5^{th} meeting 2010 (Martina Franca)


6^{th} meeting 2012 (Lecce)

7^{th} meeting 2014 (Giovinazzo)




8^{th} meeting 2016 (MF)