

Theory of rare kaon decays

Giancarlo D'Ambrosio
INFN Sezione di Napoli

Closing in on the
radiative weak chiral
couplings Luigi
Capriello, Oscar Cata,
GD
arXiv:
1712.10270,,EPJC

Collaboration with Teppei Kitahara arXiv:
1707.06999 PRL

Collaboration with Crivellin, A., Kitahara, T. and
Nierste, U. e-Print: arXiv:1703.05786 PRD

Flavour issues in warped custodial models: B
anomalies and rare K decays
GD, Abhishek M. Iyer. Dec 21, 2017. 22 pp.
arXiv:1712.08122



Collaboration with Teppei Kitahara, Isabel Fernández Suárez, Miriam Lucio Martínez,
Diego Martínez Santos, Veronika Georgieva Chobanova arXiv:1711.11030

Outline

- QCD, chiral perturbation theory and kaon decays
- $K \rightarrow \pi \nu \nu$
- K-anomalies
- $K_{S,L} \rightarrow \mu \mu$

QCD and EFT

Chiral Perturbation theory

χ PT effective field theory approach based on **two** assumptions

- π 's Goldstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ $SU(3)_L \times SU(3)_R$ symm. $\mathcal{L}_{QCD}$ $m_q = 0$
- **(chiral) power counting** There is a small expansion parameter $p^2/\Lambda^2_{\chi SB}$ $\Lambda_{\chi SB} \approx 4 \pi F_\pi \sim 1.2 \text{ GeV}$

Chiral sym. breaking through dim. parameter F_π , χ related to
 $\langle 0 | J_{5\mu} | \pi \rangle$, $\langle 0 | q_L q_R | 0 \rangle$

$F_\pi \approx 93 \text{ MeV}$

$$\mathcal{L}_S = \mathcal{L}_S^2 + \mathcal{L}_S^4 + \dots = \frac{F_\pi^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \rightarrow l\nu, \pi\pi \rightarrow \pi\pi, K \rightarrow \pi\pi} + \sum_i \overbrace{L_i O_i}^{K \rightarrow \pi\pi} + \dots$$

Fantastic chiral prediction

$$A_{\pi\pi} \sim (s - m_\pi^2)/F_\pi^2$$

L_i Gasser Leutwyler coeff
determined from expts.
 O_i p^4 operator

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \dots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + G_8 F^2 \sum_i \underbrace{N_i W_i}_{K^+ \rightarrow \pi^+ \gamma\gamma, K \rightarrow \pi l^+ l^-} + \dots$$

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations

$$F_V = 2G_V = \sqrt{2}f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2}M_V$$

KSFR: $G_V = \sqrt{2} F_\pi$
determined by dominance
of pion, V,A to recover
QCD short distance
constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \quad L_9^V = \frac{F_V G_V}{2M_V^2}, \quad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

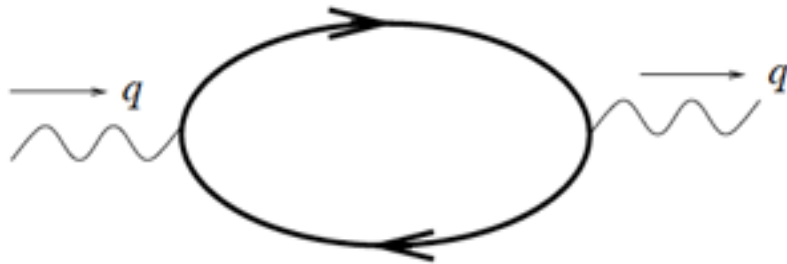
QCD inspired relations

$$L_1^V = L_2^V / 2 = -L_3^V / 6 = L_9^V / 8 = -L_{10}^{V+A} / 6 = f_\pi^2 / (16M_V^2)$$

Holographic QCD at low energies

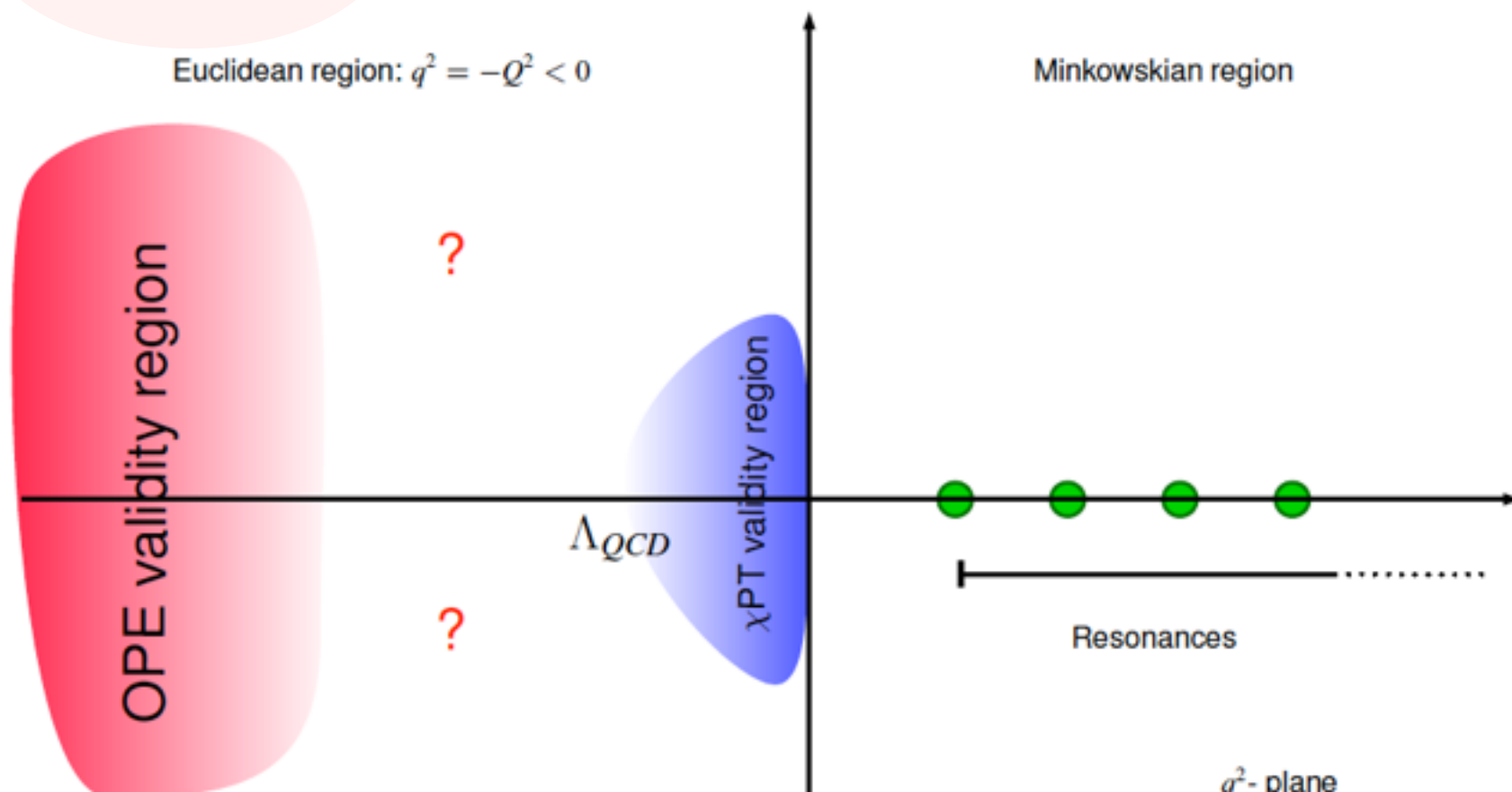
Large N_c QCD properties $\Rightarrow$ conformal symmetry in 5d

$$\int d^4x e^{ip \cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi(p^2)$$

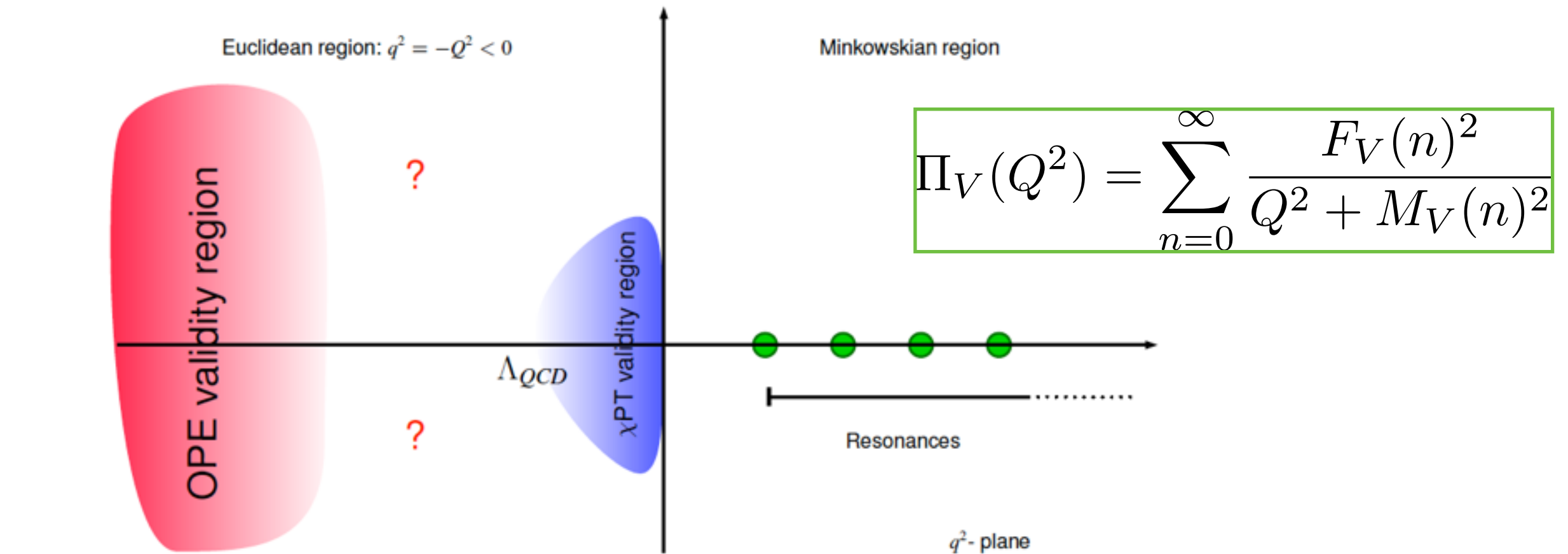


$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$

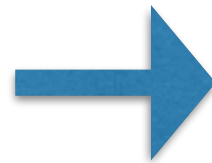
confinement: either IR cut-off or
field modifying metric



Holographic QCD: Hard Wall



$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$



IR very good, but

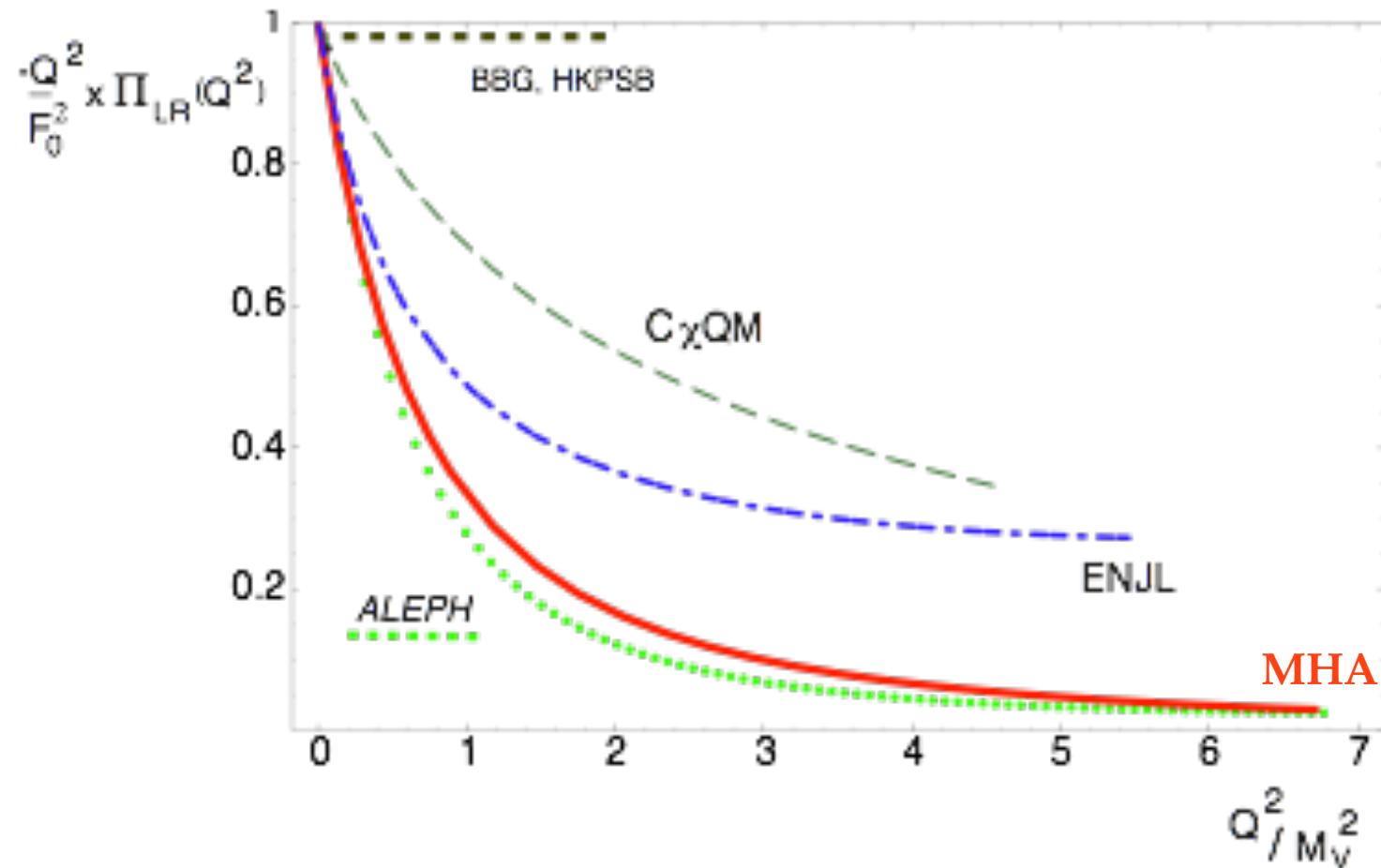
$$M_V^2(n) \sim n^2$$



$$\Pi_V(q^2) = -\frac{4}{3} \frac{N_c}{(4\pi)^2} \left[\log \frac{q^2}{\mu^2} - \pi \frac{Y_0(qz_0)}{J_0(qz_0)} \right]$$

Minimal Hadronic Ansatz (MHA)

- Traditional wisdom:
low energy VERY
WELL approximated
by π 's ,V,A
- Short distance: QCD
- A good interpolation
among the two
regimes is sufficient
for a good
description of the
correlators



De Rafael

Mei Huang

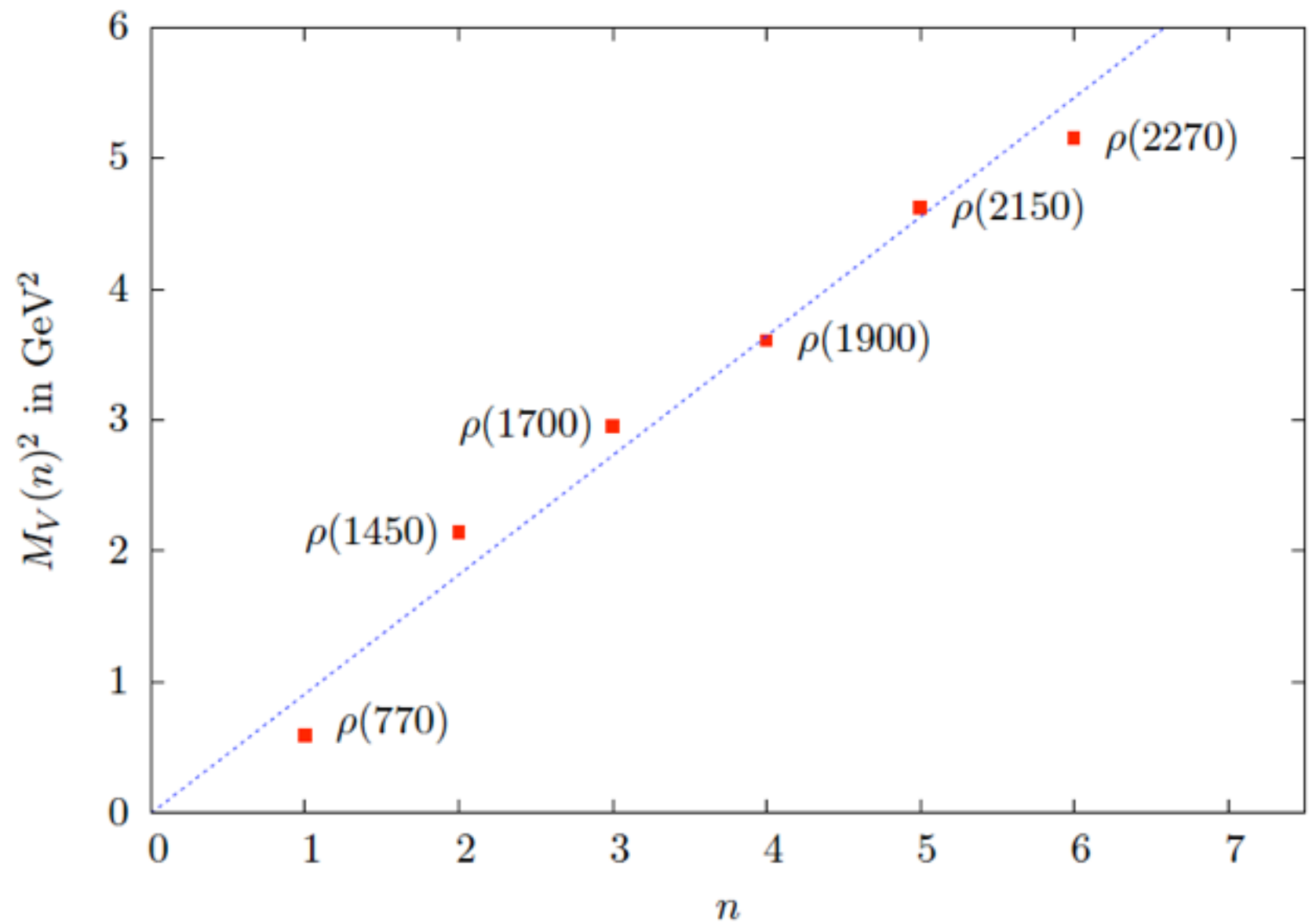
Regge/Veneziano model

$$\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$

$$M_V(n)^2 \underset{n \rightarrow \infty}{\sim} \sigma n$$

σ **0.90 GeV²**
String tension/confining

$F_V(n)$ constant



Strong sector, KSFR: 5D vs 4d

$M_{V_n} \sim n \Leftrightarrow$ first resonances OK
but not Regge behaviour.
UV OK

4d

$$\left(\frac{1}{2}\right) \equiv \frac{g_{V_1}^2 M_{V_1}^2}{f^2}$$

5d

$$\sum_n g_{V_n}^2 M_{V_n}^2 = f^2/3$$

Weak interaction

The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*C_-(\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)$

described in CHPT

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma\gamma, K \rightarrow \pi l^+ l^-} + \cdots$$

VMD not as successful, in particular for $K \rightarrow 3\pi$, where in principle large VMD important

ELECTROWEAK INTERACTIONS OF HADRONS*

BY J.-M. GÉRARD

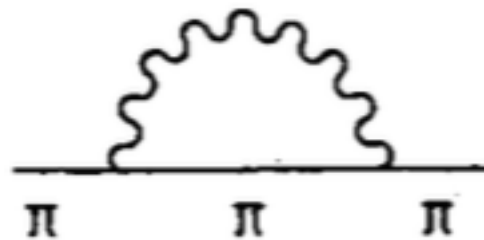
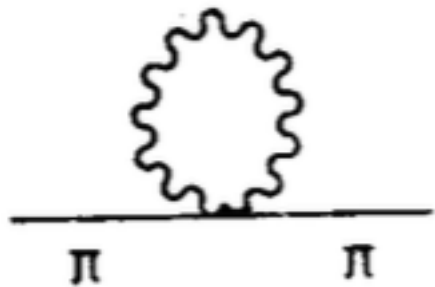
Max-Planck-Institut für Physik und Astrophysik — Werner-Heisenberg-Institut für Physik,
P.O. Box 40 12 12, Munich, West Germany

(Received November 7, 1989)

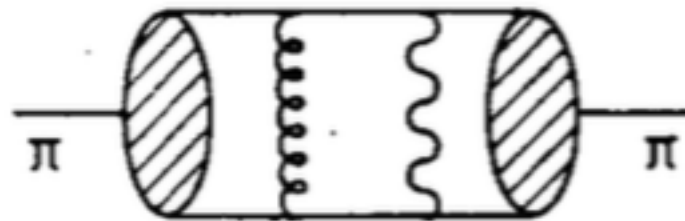
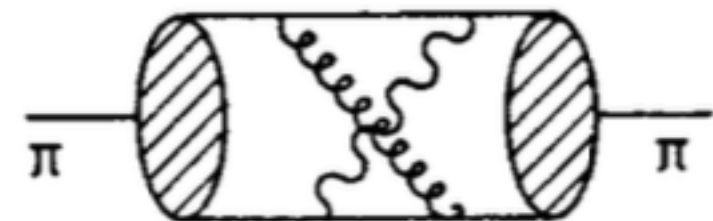
Constraints on physics beyond the standard model require a preliminary understanding of the interplay between the electroweak and strong interactions. Strong interaction effects are in principle non-negligible below the confining scale. A complementary approach, based on the meson-quark duality and applied on the $\pi^+ - \pi^0$ electromagnetic mass difference and the weak K-decays, confirms this expectation. Using this simple non-perturbative framework, we conclude that the $K_L - K_S$ mass difference, the empirical $\Delta I = \frac{1}{2}$ rule and the recently measured ϵ'/ϵ are consistent with the standard model.

$\pi^+ - \pi^0$ e.m. mass difference

$$\Delta m^{\text{exp}} = (4.43 \pm 0.03) \text{ MeV}$$



$$\Delta m_{\pi}^2(0^- \rightarrow +) = \frac{3}{4\pi} \alpha_{\text{QED}} \int_0^{M_2} dq^2,$$

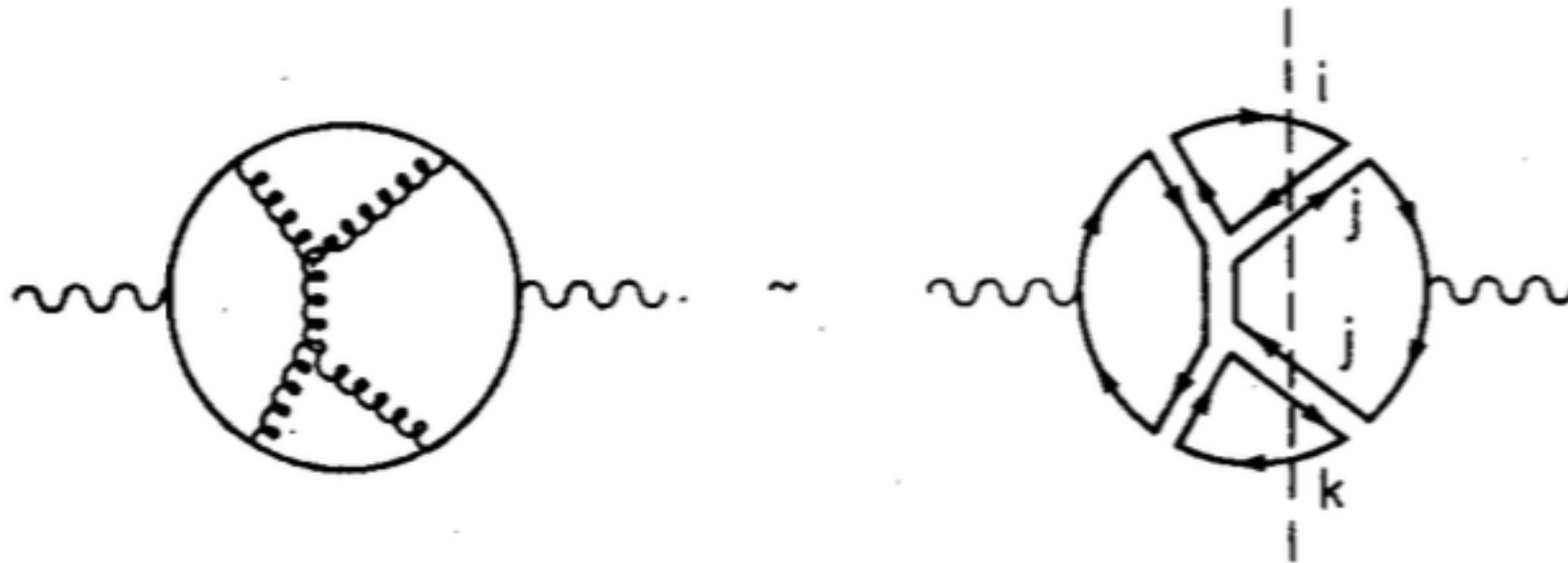


$$\Delta m_{\pi}^2(\text{pert. QCD}) = \frac{3}{8} \alpha_{\text{QED}} f^2(\alpha_{\text{QCD}} r^2) \int_{M_2}^{\infty} \frac{dq^2}{q^4},$$

$$q_{\text{meson}}^2 = q_{\text{quark-gluon}}^2 = q_{\text{photon}}^2$$

$M=0.7 \text{ GeV} \Rightarrow 50\% \text{ LD} \quad 50\% \text{ SD}$

$\pi^+ - \pi^0$ e.m. mass difference

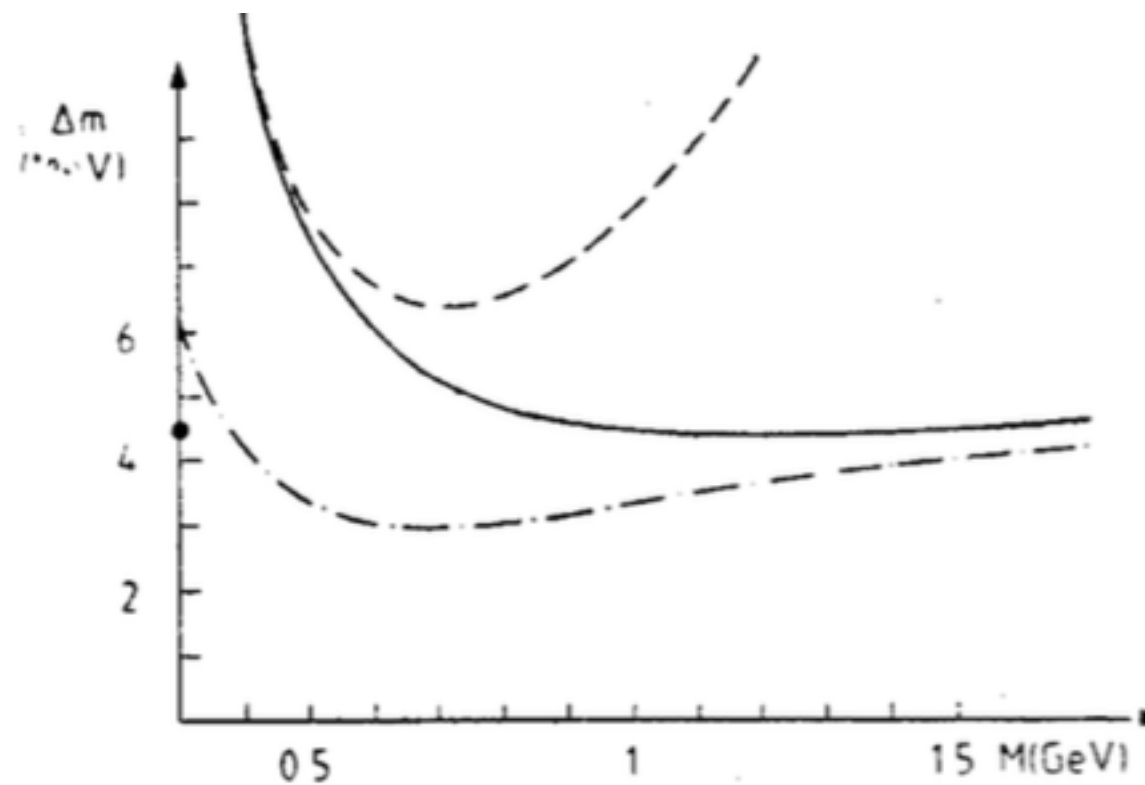
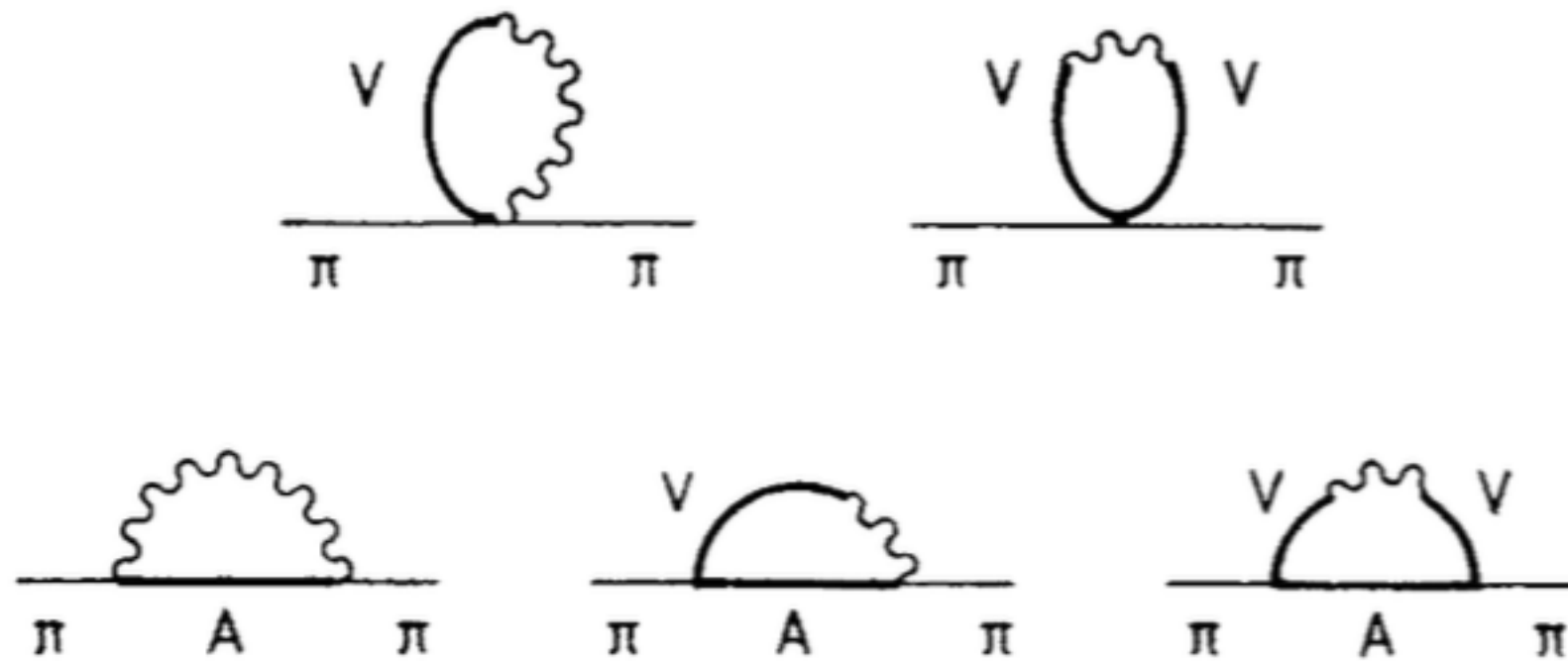


$$\langle J(q)J(-q) \rangle = \sum_n \frac{a_n}{q^2 - m_n^2}$$

$$\langle J(q)J(-q) \rangle \stackrel{q^2 \rightarrow \infty}{\sim} \ln q^2$$

Witten

 $1/N$



Not only a book-keeping but predictive already

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^* (S)$ $\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^* (L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma (S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma (S)$ $\pi^+\pi^-\gamma^* (L)$ $\pi^+\pi^-\gamma^* (S)$ $\pi^+\pi^0\gamma^*$	 $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (L)$ $\pi^+\pi^-\pi^0\gamma (S)$	$N_{14}^r - N_{15}^r$ $K^+ \rightarrow \pi^+ l^+ l^-$ $2N_{14}^r + N_{15}^r$ $K_S \rightarrow \pi^0 l^+ l^-$ $N_{14} - N_{15} - 2N_{18}$ ” $N_{14} - N_{15} - N_{16} - N_{17}$ ” ” $7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17}^r)$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17}^r)$
	$\pi^+\pi^-\gamma (L)$ $\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^-\pi^0\gamma (L)$	$N_{29} + N_{31}$ ” $3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \dots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + G_8 F^2 \underbrace{\sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma \gamma, K \rightarrow \pi l^+ l^-} + \dots$$

Weak chiral couplings

$$K^\pm \rightarrow \pi^\pm \gamma^* :$$

$$K_S \rightarrow \pi^0 \gamma^* :$$

$$K^\pm \rightarrow \pi^\pm \pi^0 \gamma :$$

$$K^+ \rightarrow \pi^+ \gamma \gamma :$$

$$a_+ = -0.578 \pm 0.016 \text{ [3, 4]}$$

$$a_S = (1.06_{-0.21}^{+0.26} \pm 0.07) \text{ [5, 6]}$$

$$X_E = (-24 \pm 4 \pm 4) \text{ GeV}^{-4} \text{ [7]}$$

$$\hat{c} = 1.56 \pm 0.23 \pm 0.11 \text{ [8] .}$$

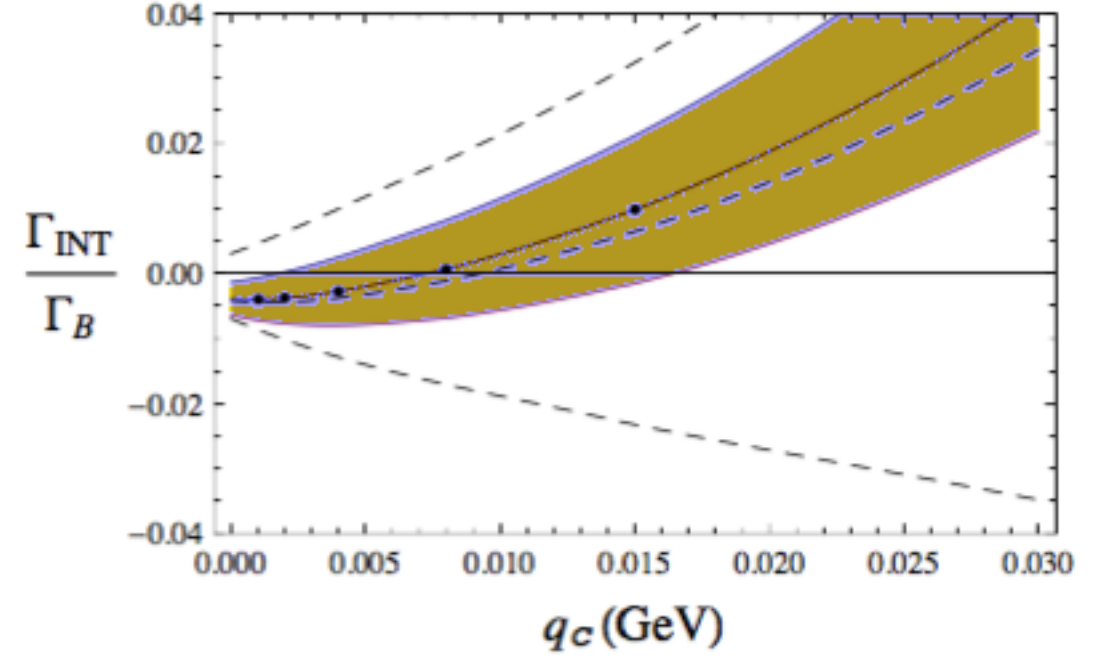
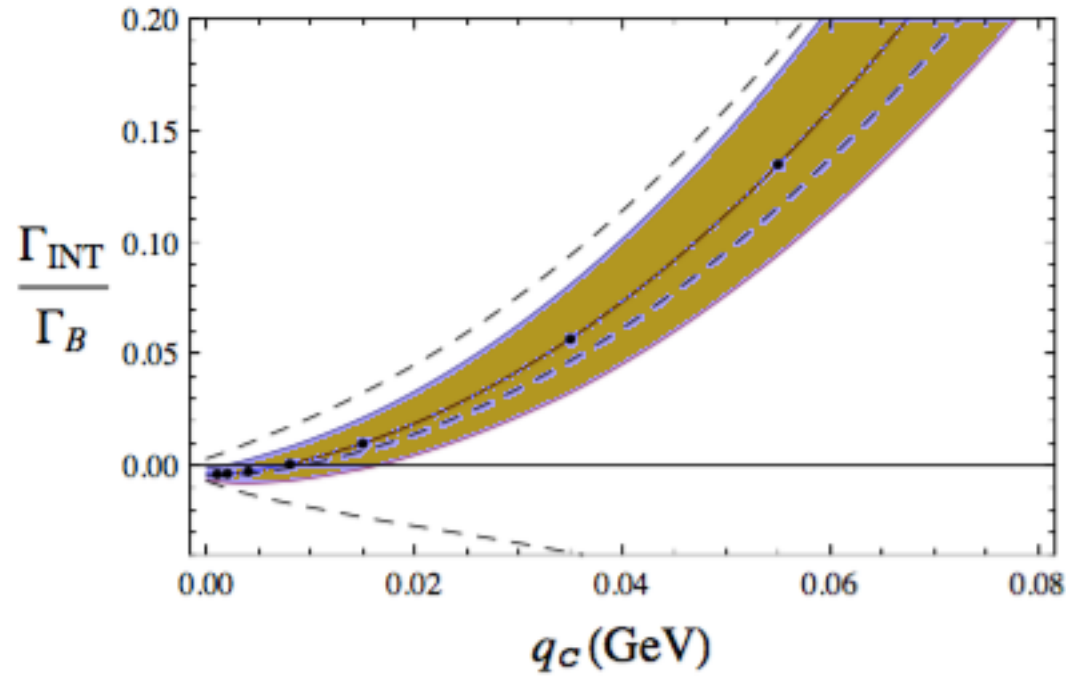
$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2} \left(\frac{1}{3} - \frac{G_F}{G_8} a_+ - \frac{1}{3} \log \frac{\mu^2}{m_K m_\pi} \right) - 3L_9^r ;$$

$$\mathcal{N}_S \equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2} \left(\frac{1}{3} + \frac{G_F}{G_8} a_S - \frac{1}{3} \log \frac{\mu^2}{m_K^2} \right) ;$$

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17}^r = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E ;$$

$$\mathcal{N}_0 \equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2} \hat{c} - 3(L_9^r + L_{10}^r) ,$$

Decay mode	counterterm combination	expt. value
$K^\pm \rightarrow \pi^\pm \gamma^*$	$N_{14} - N_{15}$	$-0.0167(13)$
$K_S \rightarrow \pi^0 \gamma^*$	$2N_{14} + N_{15}$	$+0.016(4)$
$K^\pm \rightarrow \pi^\pm \pi^0 \gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	$+0.0022(7)$
$K^\pm \rightarrow \pi^\pm \gamma \gamma$	$N_{14} - N_{15} - 2N_{18}$	$-0.0017(32)$



q cut in minimum dilepton

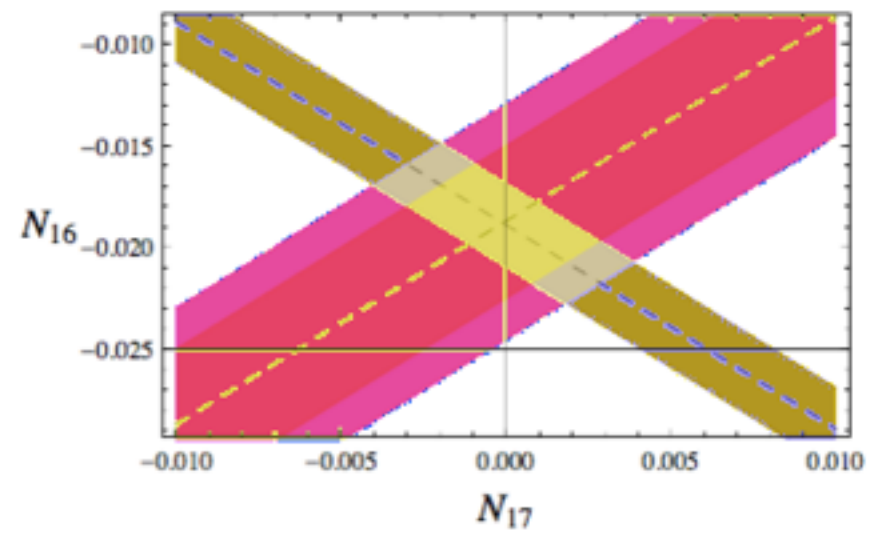
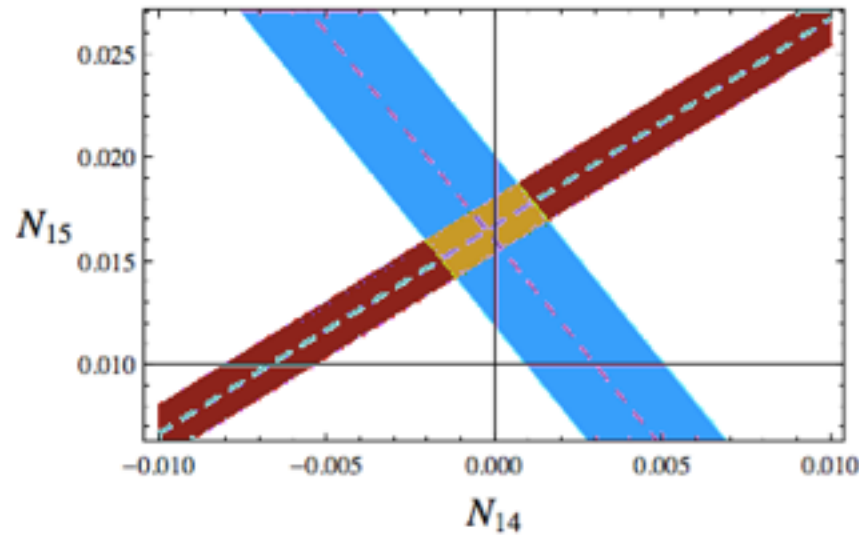


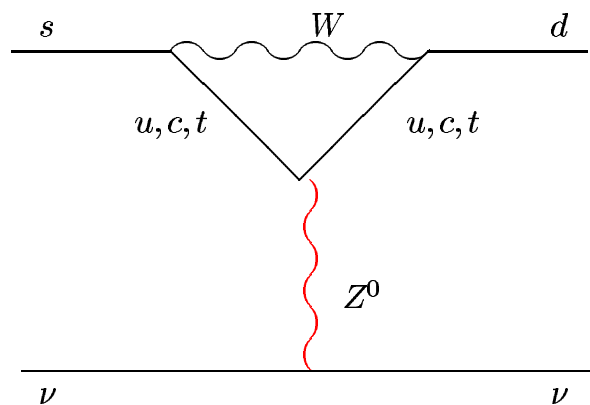
Figure 4: *Left panel: values of N_{14} and N_{15} as given by $K^\pm \rightarrow \pi^\pm \gamma^*$ (blue band) and $K_S \rightarrow \pi^0 \gamma^*$ (violet band). Right panel: values for N_{16} and N_{17} extracted from $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ (blue band) and $K^\pm \rightarrow \pi^\pm \pi^0 e^+ e^-$ (yellow band) measurements. The latter is an educated estimate (see main text).*

Anomalies in Kaons

$$K \rightarrow \pi \nu \bar{\nu}$$

Why we need KOTO and NA62

$$A(s \rightarrow d \nu \bar{\nu})_{\text{SM}} \sim \bar{s}_L \gamma_\mu d_L \quad \bar{\nu}_L \gamma^\mu \nu_L \times \left[\sum_{q=c,t} V_{qs}^* V_{qd} m_q^2 \right]$$



~

$$\left[A^2 \lambda^5 (1 - \rho - i\eta) m_t^2 + \lambda m_c^2 \right]$$

SM

$$\underbrace{V - A \otimes V - A}_{\Downarrow}$$

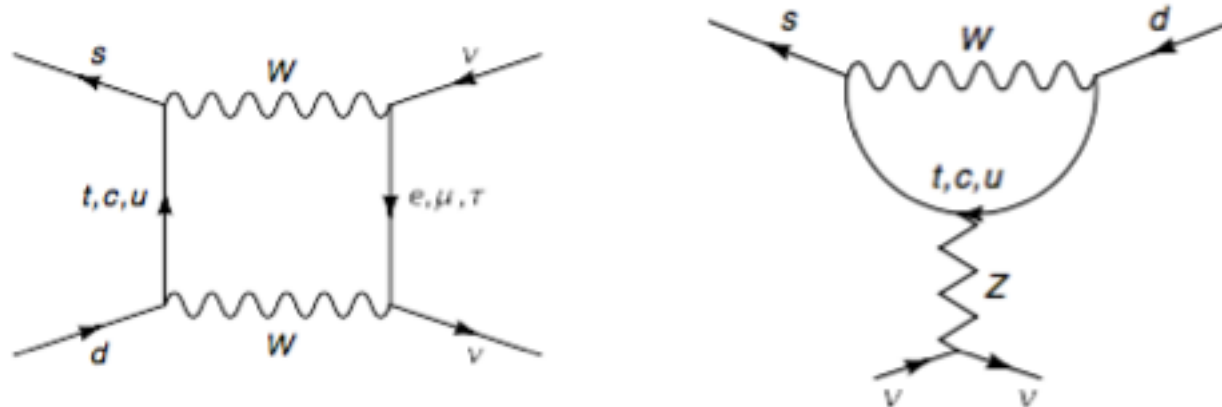
Littenberg

$$\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

$$\left\{ \begin{array}{l} \text{CP violating} \\ \Rightarrow J = A^2 \lambda^6 \eta \\ \text{Only } \textit{top} \end{array} \right.$$

SM

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$



Misiak, Urban; Buras,
Buchalla; Brod, Gorbhan,
Stamou'11, Straub

$$\mathcal{B}(K^+) \sim \kappa_+ \left[\left(\frac{\text{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\text{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right]$$

$\lambda_q = V_{qd}^* V_{qs}$
 $\downarrow$
 K_{l3}

$30\% \pm 2.5\%$
 LD

$$\mathcal{B}(K^\pm) = (8.82 \pm 0.8 \pm 0.3) \times 10^{-11} \quad \text{TH}$$

V_{cb} nonpert QCD

$$(1.73^{+1.15}_{-1.05}) \times 10^{-10} \quad \text{E949}$$

$$< 11 \cdot 10^{-10} \quad 90\% \text{ CL} \quad \text{NA62}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$

$$B(K_L) = (3.14 \pm 0.17 \pm 0.06) \times 10^{-11} \quad \text{TH}$$

$$B(K_L) < 2.6 \times 10^{-8} \text{ at } 90\% \text{ C.L.} \quad \text{E391a}$$

Model-independent bound, based on SU(2) properties
 dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^\pm)_{\text{E949}} \leq 1.4 \times 10^{-9} \text{ at } 90\% \text{ C.L.}$$

UV sensitivity

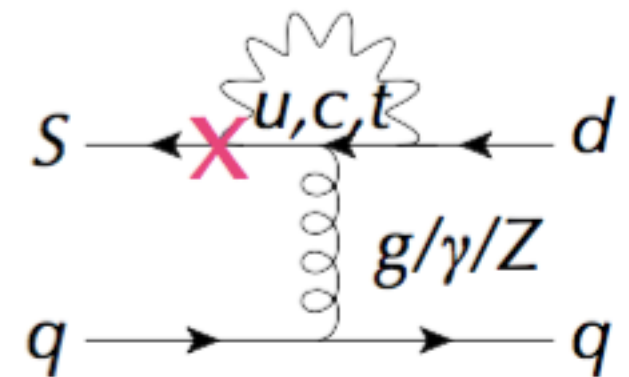
$$\mathcal{L} \sim \frac{1 - 0.3 \, i}{(\textcolor{red}{180 \text{ TeV}})^2} (\bar{s}_L \gamma_\mu d_L \bar{\nu}_L \gamma^\mu \nu_L)$$

News: anomalies in Kaons?

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\text{exp}}} \frac{\omega_{\text{exp}}}{(\text{Re}A_0)_{\text{exp}}} \left(-\text{Im}A_0 + \frac{1}{\omega_{\text{exp}}} \text{Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{\text{Re}A_0}{\text{Re}A_2} = 22.46 \text{ (exp.)}$$

gluon
penguin
 Q_6

EW
penguin
 Q_8



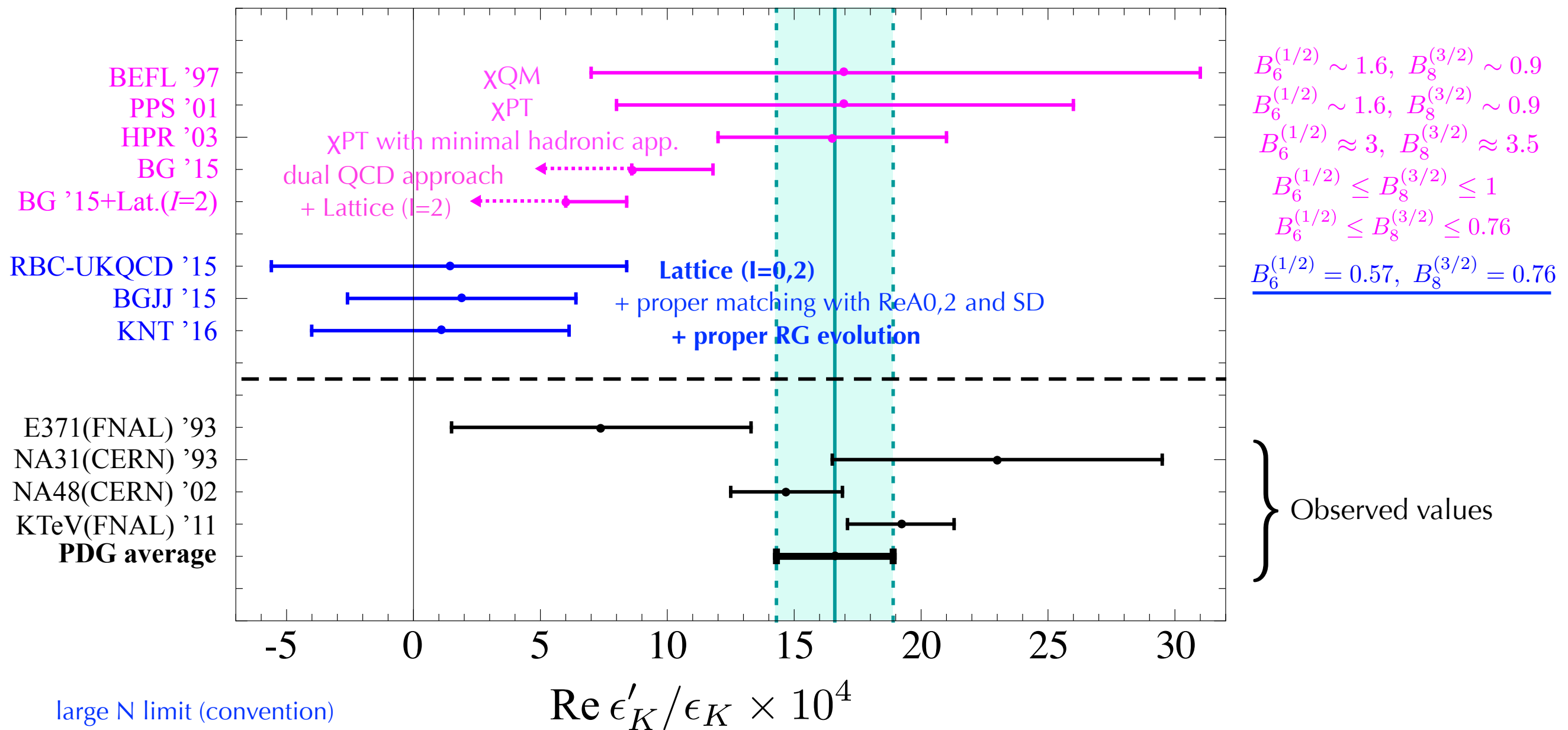
$\langle O_6 \rangle$ and $\langle O_8 \rangle$ have chiral enhancement factor

$$\langle Q_6(\mu) \rangle_0 = -4 \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 (F_K - F_\pi) \underline{B_6^{(1/2)}}$$

$$\langle Q_8(\mu) \rangle_2 = \sqrt{2} \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 F_\pi \underline{B_8^{(3/2)}}$$

**New lattice
result 2015**

Current situation of $\epsilon'_K/\epsilon_K \propto \text{Im}A_0 - \left(\frac{\text{Re}A_0}{\text{Re}A_2}\right) \text{Im}A_2$



large N limit (convention)
 $B_6^{(1/2)} = B_8^{(3/2)} = 1$

dual QCD prediction

$B_6^{(1/2)} \leq B_8^{(3/2)} < 1, B_8^{(3/2)} = 0.8$

	Exp.	χ PT	dual QCD	Lattice (I=0,2)
$\left(\frac{\text{Re}A_0}{\text{Re}A_2}\right)$	22.45 ± 0.05	~ 14	16.0 ± 1.5	31.0 ± 11.1

ϵ'_K from isospin breaking

Kagan Neubert,99, Grossman, Kagan Neubert,99

$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\text{exp}}} \frac{\omega_{\text{exp}}}{(\text{Re}A_0)_{\text{exp}}} \left(-\text{Im}A_0 + \frac{1}{\omega_{\text{exp}}} \text{Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{\text{Re}A_0}{\text{Re}A_2} = 22.46 \text{ (exp.)}$$

Assuming a discrepancy 2.9 sigmas from SM

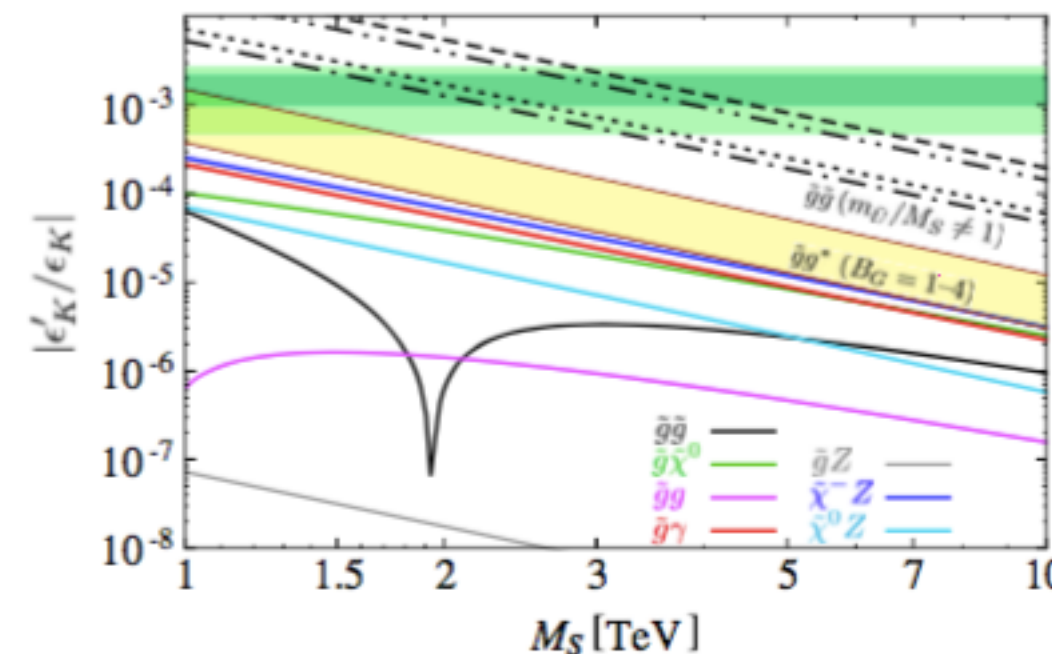
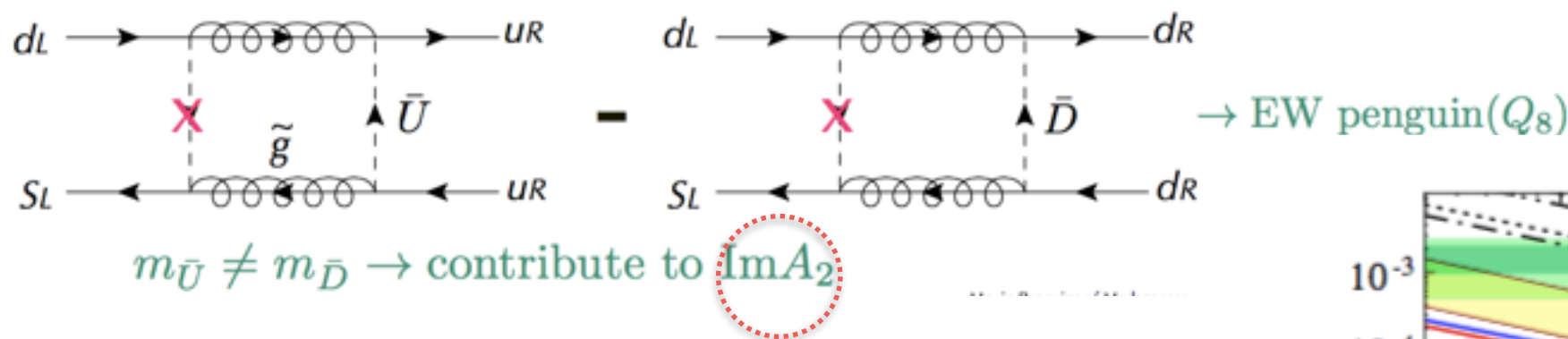


FIG. 3. Individual supersymmetric contributions to $|\epsilon'_K/\epsilon_K|$

Models solving ϵ'/ϵ anomaly

- Several new physics models have been studied to explain ϵ'/ϵ anomaly

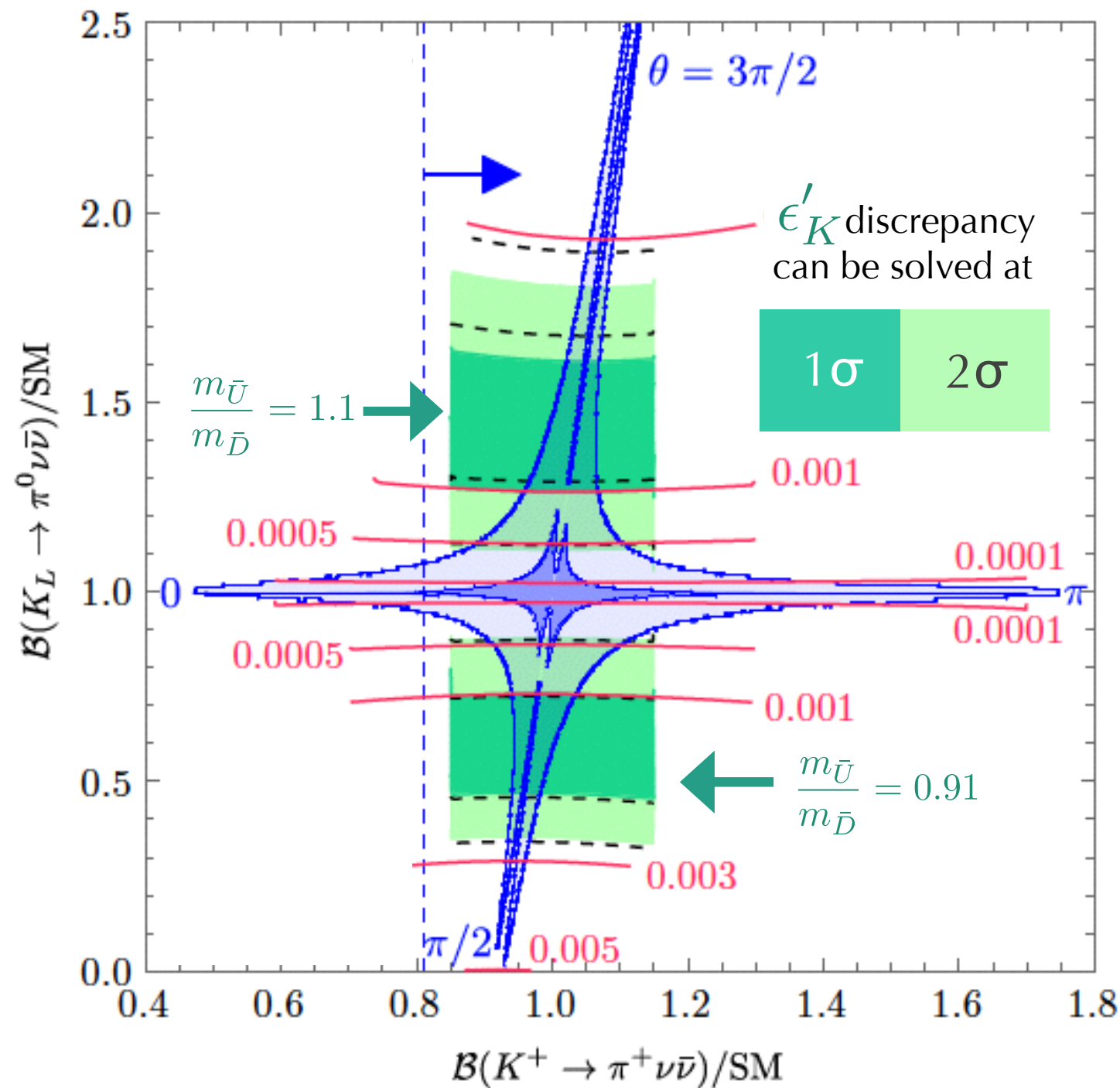
MSSM	-- chargino Z penguin	[M. Endo, S. Mishima, D. Ueda and KY, PLB762(2016)493]
	-- gluino Z penguin	[M. Tanimoto and KY, PTEP(2016)no.12,123B02]
	-- gluino box	[T.Kitahara, U.Nierste and P.Tremper, PRL117(2016)no.9, 091802 A.Crivellin, G.D'Ambrosio, T.Kitahara and U.Nierste, 1703.05786]
Vector-like quarks		[C.Bobeth, A.J.Buras, A.Celis and M.Jung, JHEP1704(2017)079]
Little Higgs Model with T-parity		[M.Blanke, A.J.Buras and S.Recksiegel, EPJ.C76 (2016)no.4,182]
331 model		[A.J.Buras and F.De Fazio, JHEP1603(2016)010 & JHEP1608 (2016) 115]
Right handed current		[V.Cirigliano, W.Dekens, J.de Vries and E.Mereghetti, PLB 767 (2017) 1 S.Alioli, V.Cirigliano, W.Dekens, J.de Vries and E.Mereghetti, JHEP1705 (2017)086]

- Different implications (correlations & predictions) for other observables appear depending on models $\Rightarrow$ Possibility of model discriminations

$B(K \rightarrow \pi \nu \nu)$

[Crivellin, D'Ambrosio, TK, Nierste, '17]

$$m_{\tilde{q}_1} = 1.5 \text{ TeV}, m_L = 300 \text{ GeV}$$



more than 10% mass shift of the gluino mass from $M_3 \simeq 1.45 M_S$ is possible in light of the constraint from ϵ_K

1-10 % mass shift of the gluino mass is possible

$$\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu})/\text{SM} \lesssim 2 \quad (1.2)$$

$$\mathcal{B}(K^+ \rightarrow \pi^+ \nu \bar{\nu})/\text{SM} \lesssim 1.4 \quad (1.1)$$

for a fine-tuning at the 1(10)% level

$m_{\tilde{U}}/m_{\tilde{D}}$ determines a position of the green band

Positive ϵ'_K predicts a strict correlation

$$\text{sgn} [\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) - \mathcal{B}^{\text{SM}}(K_L \rightarrow \pi^0 \nu \bar{\nu})] = \text{sgn} [m_{\tilde{U}} - m_{\tilde{D}}]$$

$$\text{sgn} [m_{\tilde{U}} - m_{\tilde{D}}] \xrightarrow{\epsilon'_K} \arg [m_{Q12}^2] \downarrow \text{sgn} [\mathcal{B}(K_L \rightarrow \pi^0 \nu \bar{\nu}) - \mathcal{B}^{\text{SM}}(K_L \rightarrow \pi^0 \nu \bar{\nu})]$$

Interplay with B-anomalies

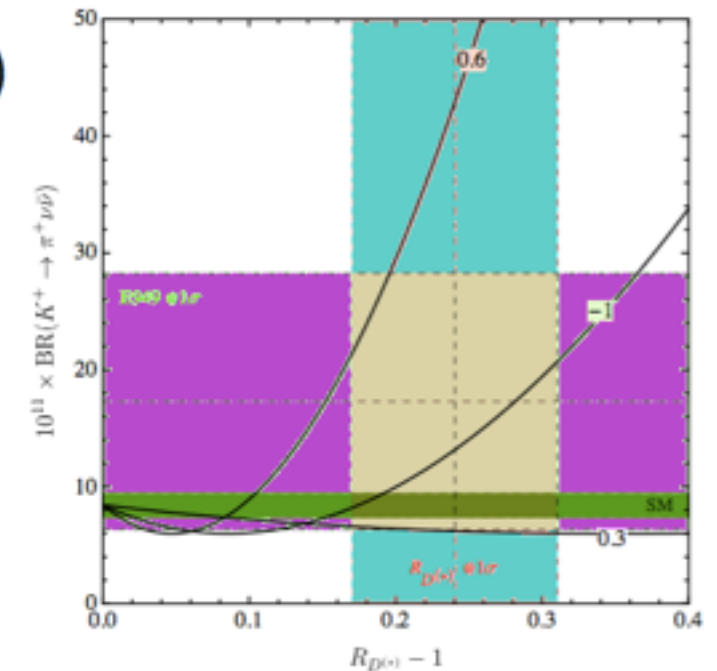
Bordone, Buttazzo, Isidori, Monnard

NP is coupled only to the left-handed third generation flavour-singlets (q_{3L} and l_{3L})

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda^2}(\bar{q}_{3L}\gamma_\mu\sigma^a q_{3L})(\bar{\ell}_{3L}\gamma^\mu\sigma^a \ell_{3L}) - \frac{c_{13}}{\Lambda^2}(\bar{q}_{3L}\gamma_\mu q_{3L})(\bar{\ell}_{3L}\gamma^\mu \ell_{3L})$$

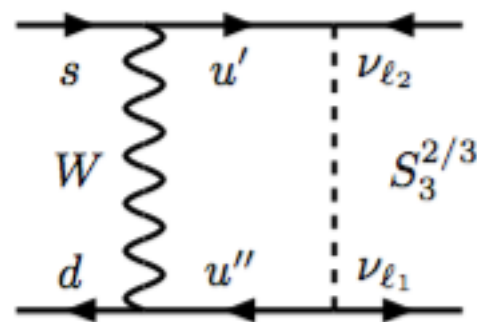
The interference of NP (weak interaction triplets) with the SM amplitude is always destructive.

The suppression could be as large as 30% relative the SM value.



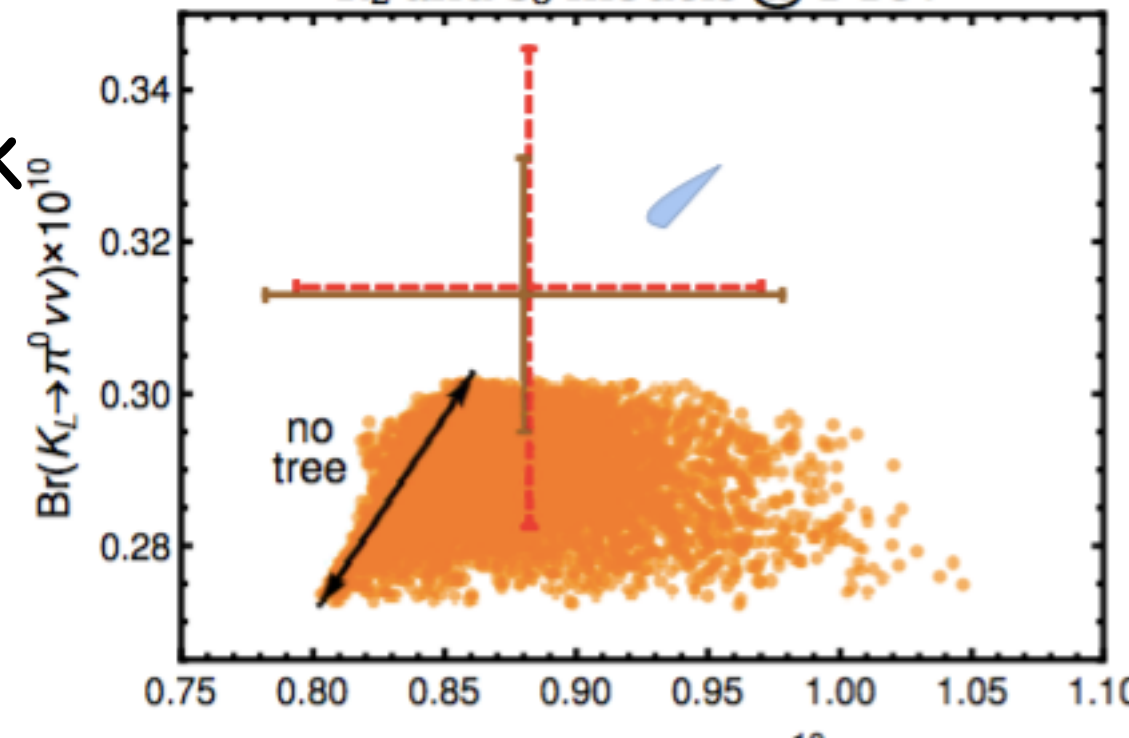
S. Fajfer N. Kořnik, L. Vale Silva

Scalar/triplet leptoquark

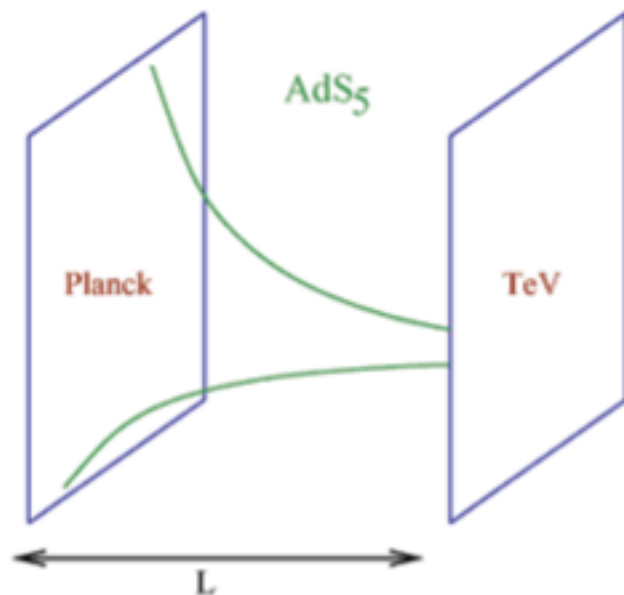


(a) Box diagram (Box).

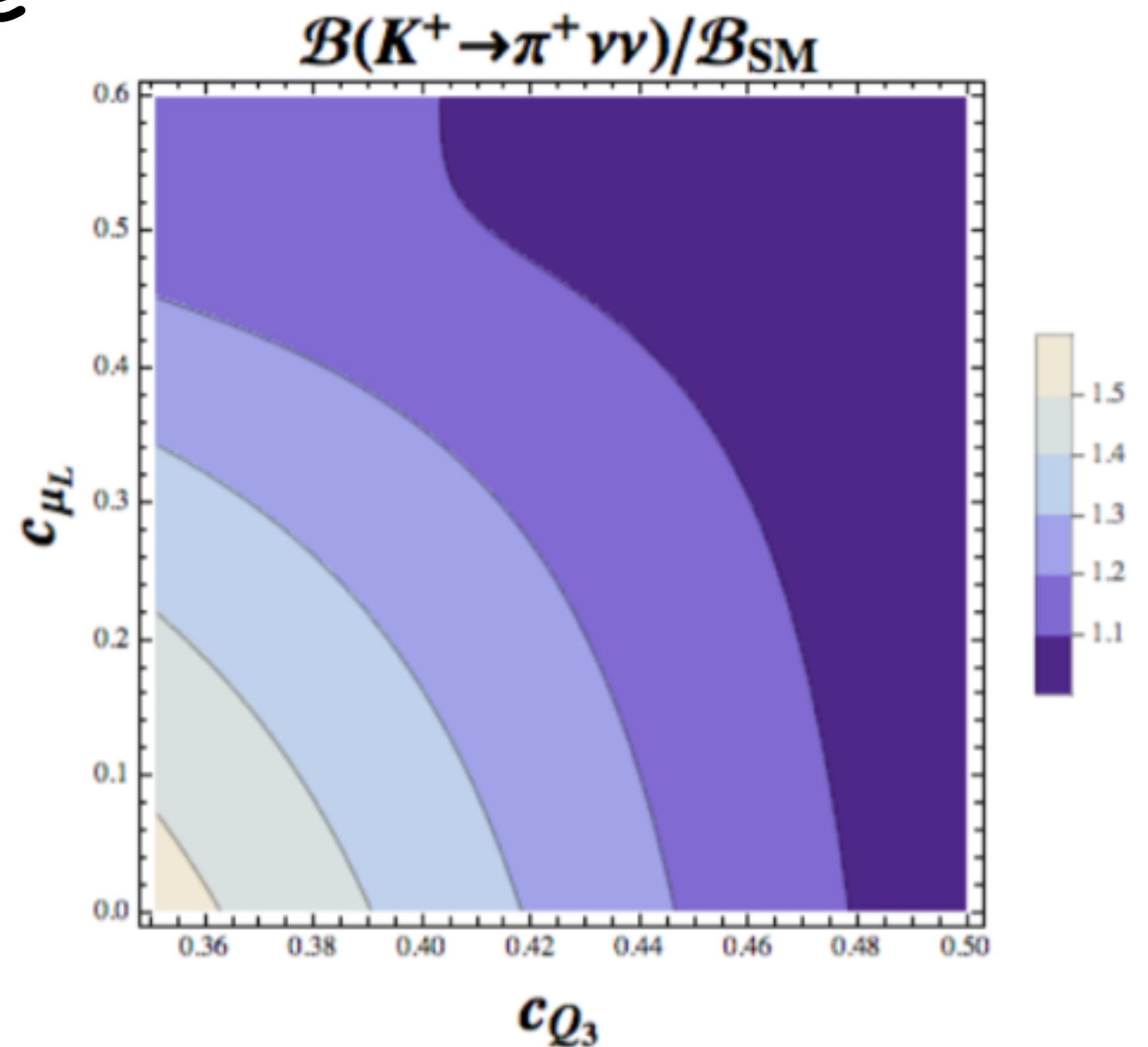
R_2 and S_3 models @ 1 TeV



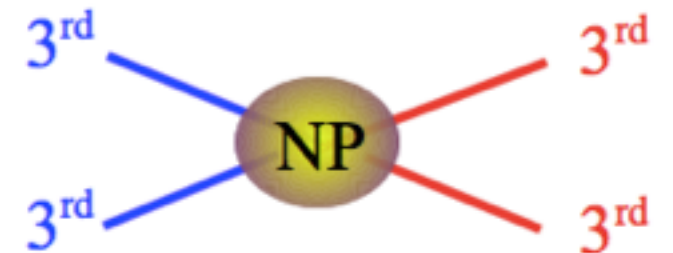
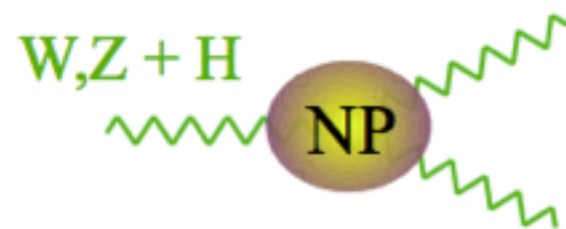
EW vs flavor scale



GD,Iyer



[*a possible shift of parading in model-building*] G. Isidori



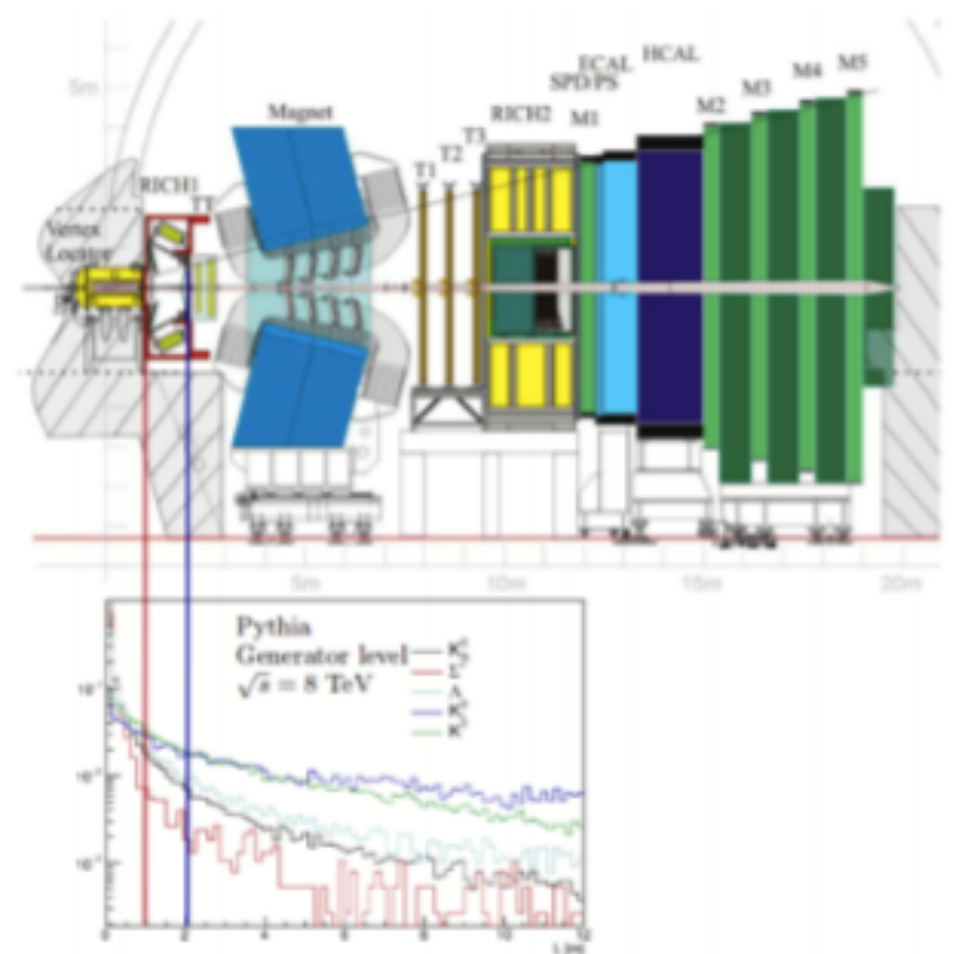
~~large (more interesting...)~~
small (less interesting...)

~~small (less interesting...)~~
large (more interesting...)

Javier Fuentes-Martín

M. Bordone, C. Cornella and G. Isidori

Kaon in LHCb



- LHCb experiment has been designed for efficient reconstructions of b and c
- Huge production of strangeness [$O(10^{13})/\text{fb}^{-1} K_S^0$] is suppressed by its trigger efficiency [$\epsilon \sim 1\text{-}2\%$ @LHC Run-I, $\epsilon \sim 18\%$ @LHC Run-II]
- LHCb upgrade (LS2=Phase I upgrade, LS4=Phase II upgrade) could realize high efficiency for K_S^0 [$\epsilon \sim 90\%$ @LHC Run-III] [M. R. Pernas, HL/HE LHC meeting, Fermilab, 2018]
- In LHC Run-III and HL-LHC, we could probe the *ultra* rare decay $\text{Br} \sim O(10^{-11\sim 12})$

$$K_{L,S} \rightarrow \mu\mu$$

$K_L \rightarrow \mu\mu$

$$\Gamma(K_L^0 \rightarrow \mu^+\mu^-)/\Gamma(K_L^0 \rightarrow \pi^+\pi^-)$$

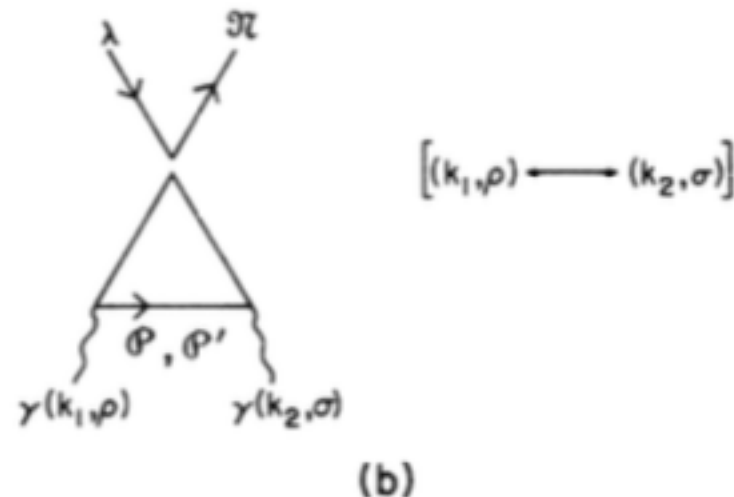
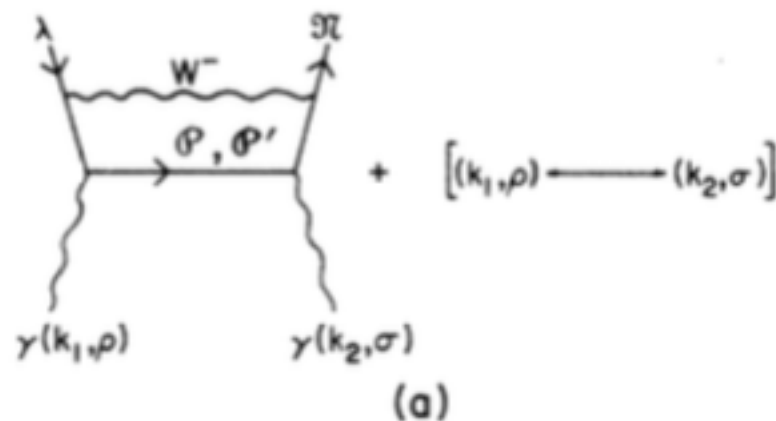


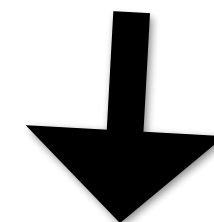
FIG. 7. Leading contributions to $\lambda + \bar{\pi} \rightarrow \gamma + \gamma$. To leading order in M_W^{-2} , the diagrams in (a) reduce to those of (b).

Gaillard Lee

VALUE (10^{-6})	EVTS	DOCUMENT ID	TECN	CO
3.48 ± 0.05	OUR AVERAGE			
3.474 ± 0.057	6210	AMBROSE	2000	B871
3.87 ± 0.30	179	¹ AKAGI	1995	SPEC
3.38 ± 0.17	707	HEINSON	1995	B791
... We do not use the following data for averages, fits, limits, etc. ...				
$3.9 \pm 0.3 \pm 0.1$	178	² AKAGI	1991B	SPEC

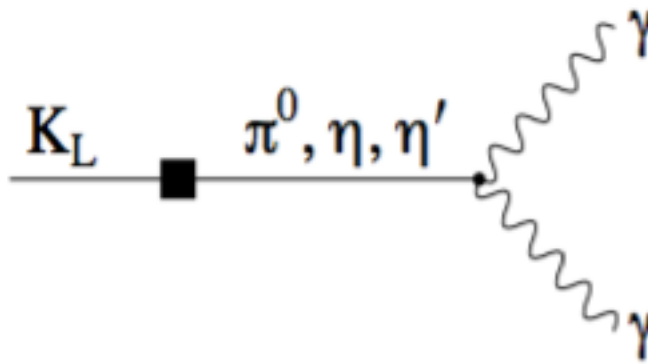
$$\mathcal{B}(K_L \rightarrow \mu^+\mu^-)_{\text{exp}} = (6.84 \pm 0.11) \times 10^{-9}$$

$$K_L \rightarrow \gamma\gamma \big|_{\text{exp}} \text{ known}$$



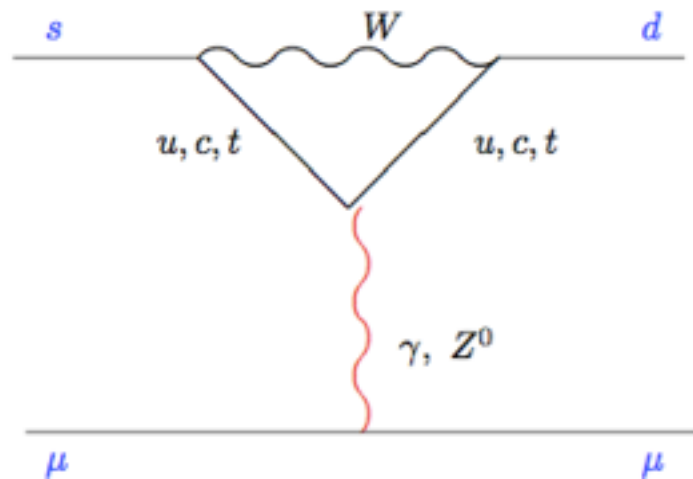
Dispersive calculation: **Re A**, Im A

We do not know the sign of $A(K_L \rightarrow \gamma\gamma)$

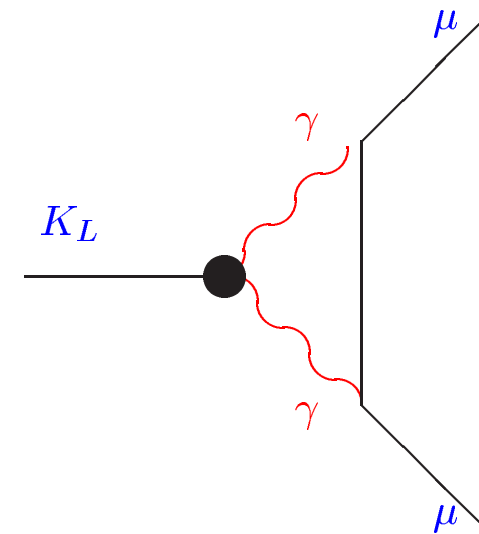


$$\begin{aligned}
 A(K_L \rightarrow 2\gamma_\perp)_{O(p^4)} &= A(K_L \rightarrow \pi^0 \rightarrow 2\gamma_\perp) + A(K_L \rightarrow \eta_8 \rightarrow 2\gamma_\perp) \\
 &= A(K_L \rightarrow \pi^0)A(\pi^0 \rightarrow 2\gamma_\perp) \left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3} \cdot \frac{1}{M_K^2 - M_8^2} \right] \simeq 0
 \end{aligned}$$

$K_L \rightarrow \mu\mu$



$\ll$



$$\frac{\Gamma(K_L \rightarrow \mu\bar{\mu})}{\Gamma(K_L \rightarrow \gamma\gamma)} \sim$$

$$|ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

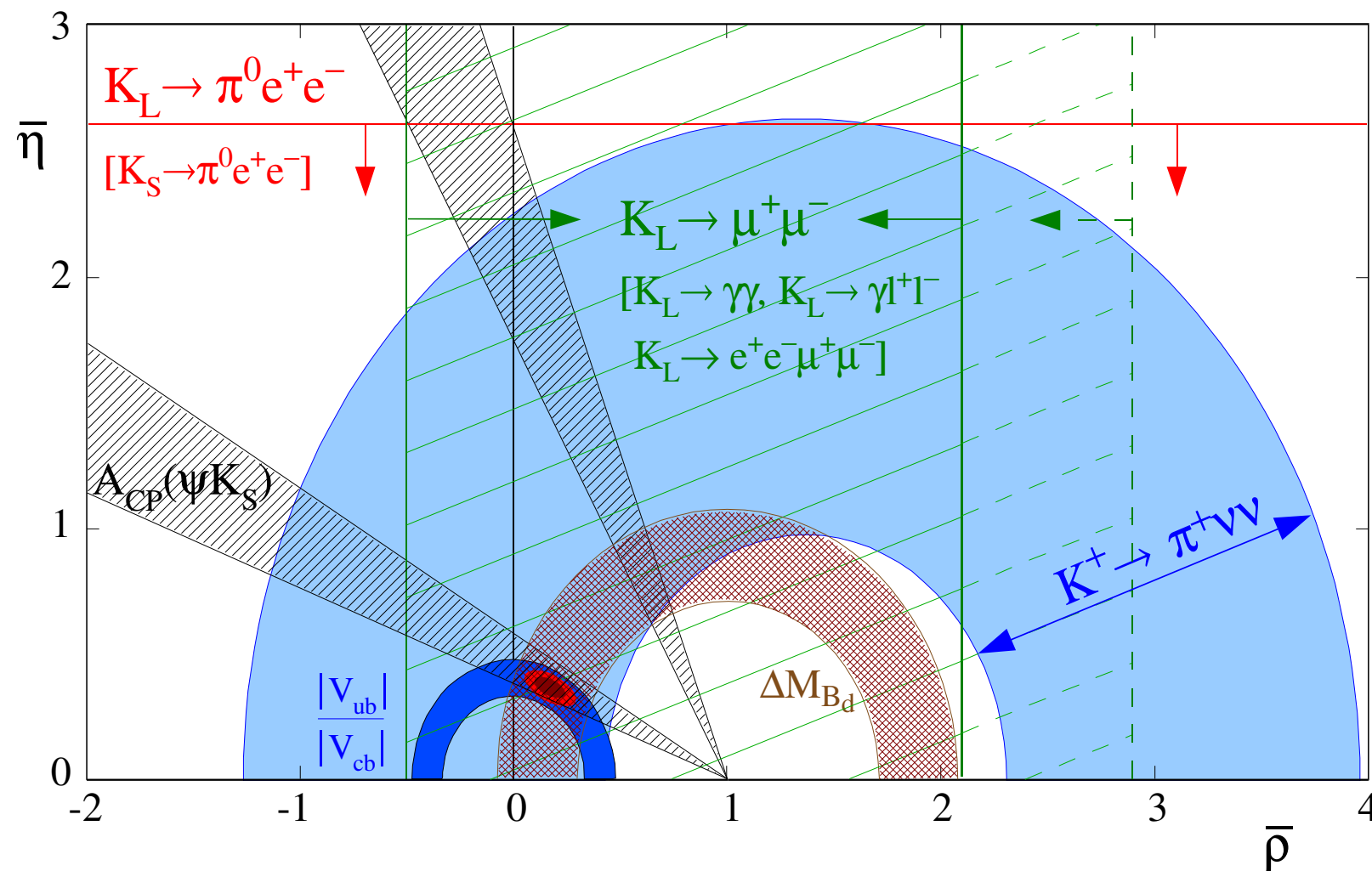
27.14

Subtracting from expt. the Absorptive contribution

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\text{short}} - 5.12)^2$$

$$|\chi_{\text{short}}^{\text{SM}}| = 1.96(1.11 - 0.92\bar{\rho})$$

$K_L \rightarrow \mu\mu$: our sign ignorance



We do not know the sign
We know the sign

$K_S \rightarrow \mu\mu$

PHYSICAL REVIEW D

VOLUME 10, NUMBER 3

1 AUGUST 1974

Rare decay modes of the K mesons in gauge theories

M. K. Gaillard* and Benjamin W. Lee†
National Accelerator Laboratory, Batavia, Illinois 60510‡
(Received 4 March 1974)

Rare decay modes of the kaons such as $K \rightarrow \mu\bar{\mu}$, $K \rightarrow \pi\nu\bar{\nu}$, $K \rightarrow \gamma\gamma$, $K \rightarrow \pi\gamma\gamma$, and $K \rightarrow \pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S|=1$ neutral currents. The experimental suppression of $K_L \rightarrow \mu\bar{\mu}$ and nonsuppression of $K_L \rightarrow \gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda + \bar{\lambda} \rightarrow l + \bar{l}$ and $\lambda + \bar{\lambda} \rightarrow \gamma + \gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K \rightarrow \mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small K_L - K_S mass difference and nonsuppression of $K_L \rightarrow \gamma\gamma$, it is found necessary to assume $m_{\phi'}/m_{\phi} \ll 1$, where m_{ϕ} is the mass of the proton quark and $m_{\phi'}$ the mass of the charmed quark, and $m_{\phi'} < 5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar states lies below 10 GeV. The effective interactions so constructed are then used to estimate the rates of other processes. Some of the results are the following: $K_S \rightarrow \gamma\gamma$ is suppressed; $K_S \rightarrow \pi\gamma\gamma$ proceeds at a normal rate, but $K_L \rightarrow \pi\gamma\gamma$ is suppressed; $K_L \rightarrow \pi\nu\bar{\nu}$ is very much forbidden, and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ occurs with the branching ratio of $\sim 10^{-10}$. $K^+ \rightarrow \pi^+e\bar{e}$ has the

Run1 data (3 fb⁻¹)

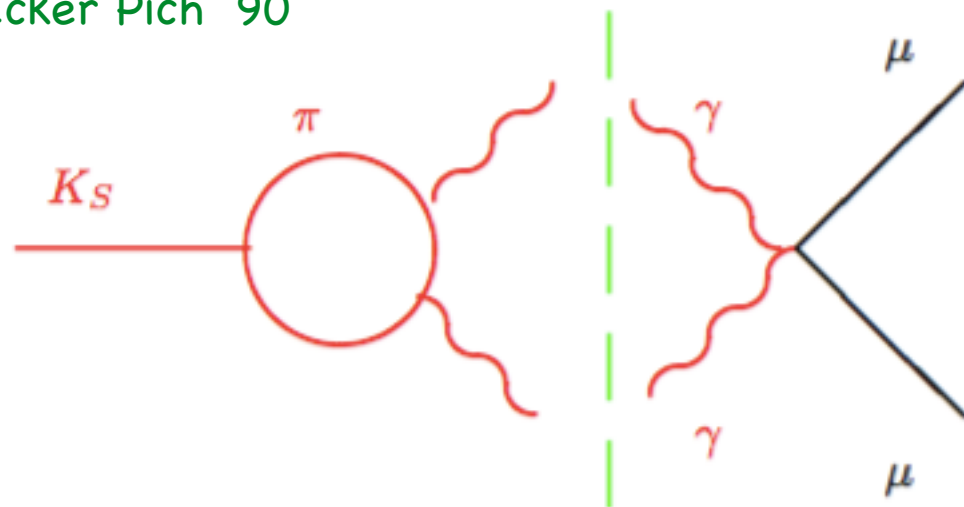
$B(K_S^0 \rightarrow \mu^+\mu^-) < 0.8(1.0) \times 10^{-9}$
90%, 95% CL
factor 11 improvement

VALUE (10 ⁻⁹)	CL%	DOCUMENT ID	TECN
< 9	90	¹ AAIJ	2013G LHCb
... We do not use the following data for averages, fits, limits, etc. ...			
< 0.032 × 10 ⁴	90	GJESDAL	1973 ASPK
< 0.7 × 10 ⁴	90	HYAMS	1969B OSPK

¹ AAIJ 2013G uses 1.0 fb⁻¹ of pp collisions at $\sqrt{s} = 7$ TeV. They obtained $B(K_S^0 \rightarrow \mu^+\mu^-) < 11 \times 10^{-9}$ at 95% C.L.

$K_S \rightarrow \mu\mu$

Ecker Pich '90



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD 5×10^{-12} 30% TH err

Short Distance

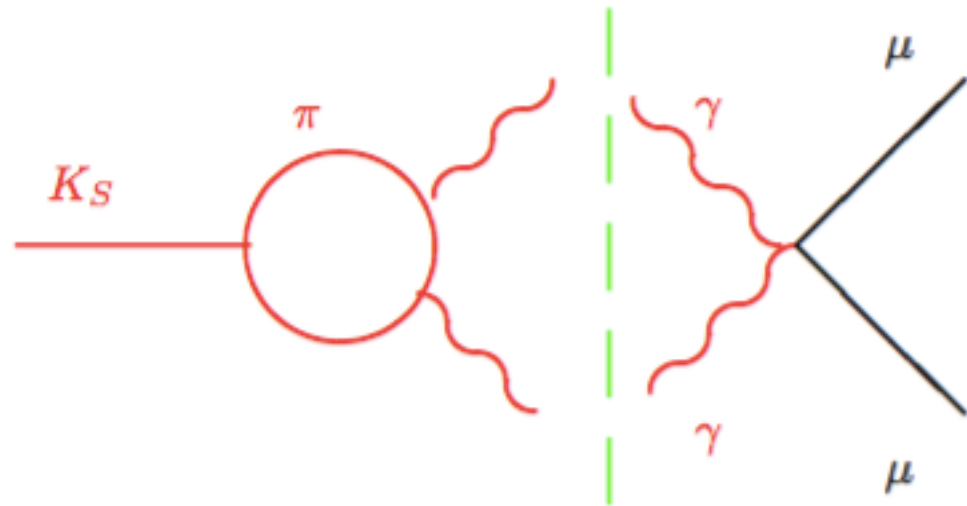
SM $10^{-5} |\Im(V_{ts}^* V_{td})|^2 \sim 10^{-13}$

NP few 10^{-11} allowed

LHCB

$< 8 \times 10^{-10}$ 90%CL

$K_S \rightarrow \mu\mu$: how to improve the THEORY error



Dispersive treatment of $K_S \rightarrow \gamma\gamma$ and $K_S \rightarrow \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

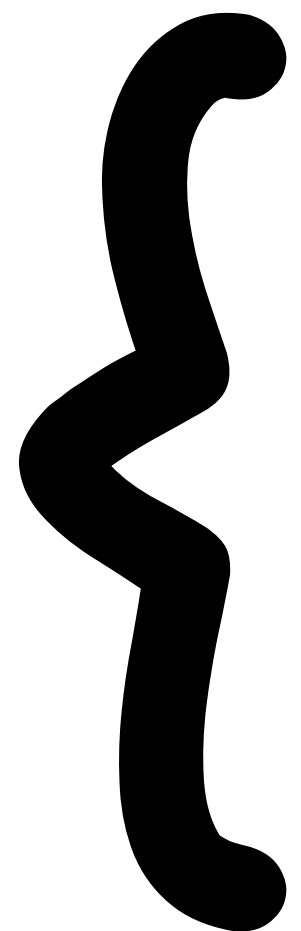
LD 5×10^{-12} 20% TH err

$$K_S \rightarrow \gamma\mu\mu$$

$$K_S \rightarrow \mu\mu\mu\mu$$

$$K_S \rightarrow ee\mu\mu$$

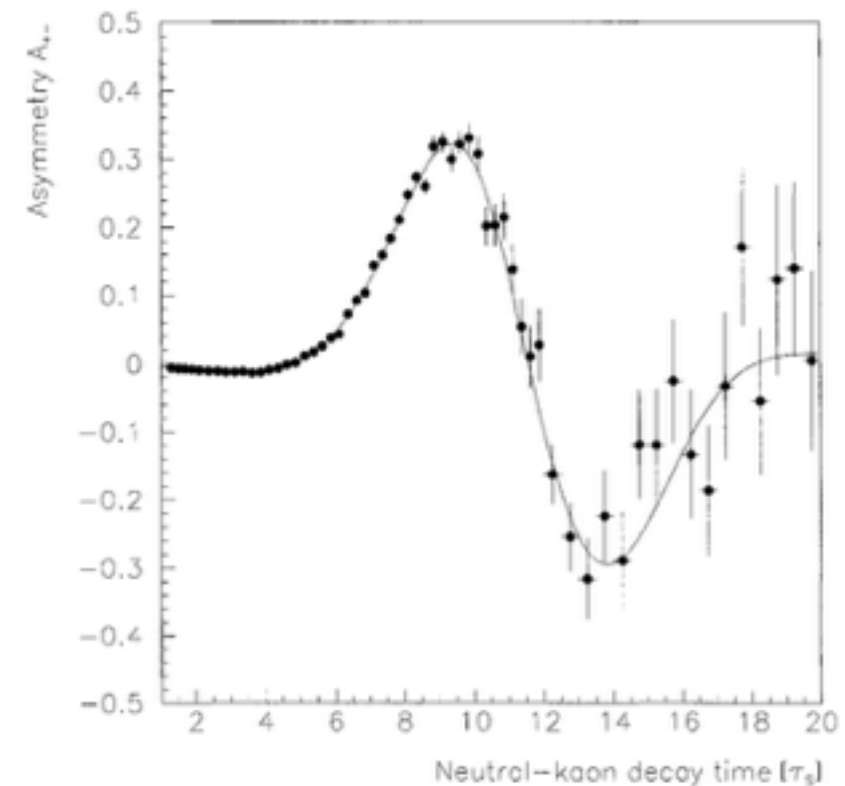
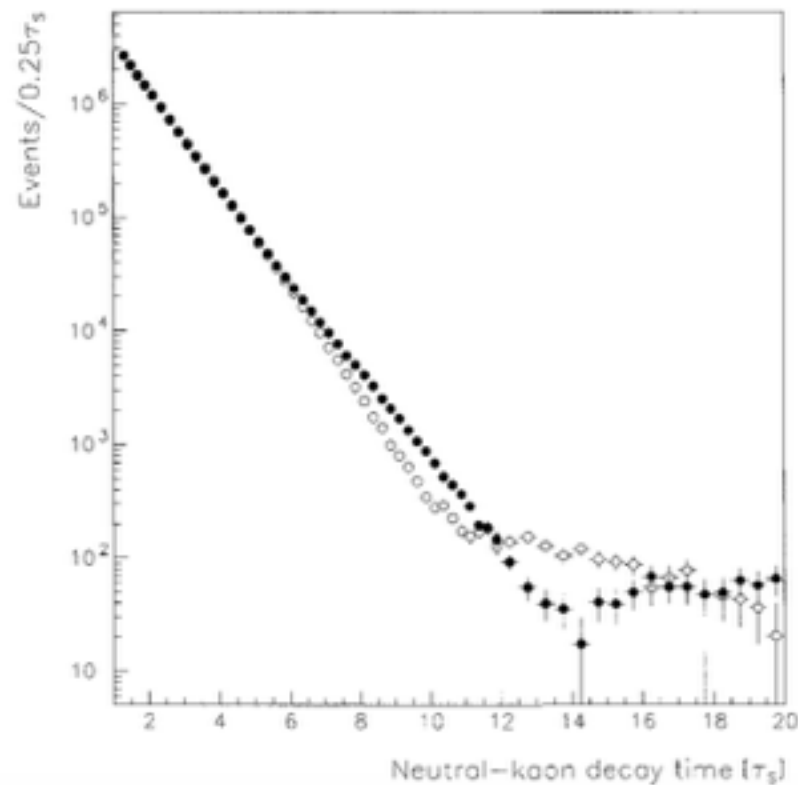
$$K_S \rightarrow \gamma\gamma$$

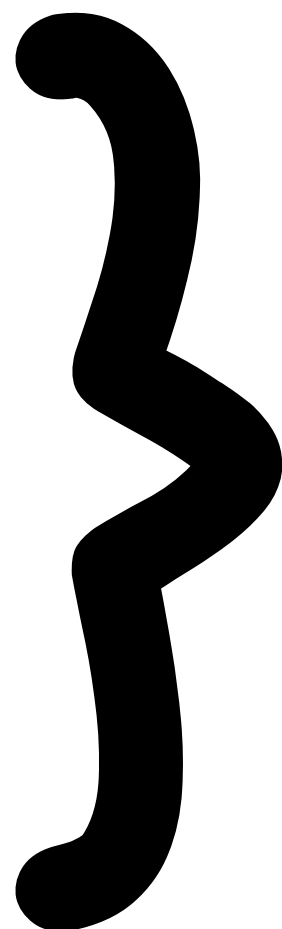


CLEAR Flavor tagging

$$p\bar{p} \rightarrow \begin{array}{c} K^- \pi^+ K^0 \\ K^+ \pi^- \bar{K}^0 \end{array}$$

$$\frac{R(\tau)}{\bar{R}(\tau)} \propto (1 \mp 2\text{Re}(\varepsilon_L))(e^{-\Gamma_S \tau} + |\eta_{+-}|^2 e^{-\Gamma_L \tau} \pm 2|\eta_{+-}| e^{-\frac{1}{2}(\Gamma_S + \Gamma_L)\tau} \cos(\Delta m \tau - \phi_{+-}))$$



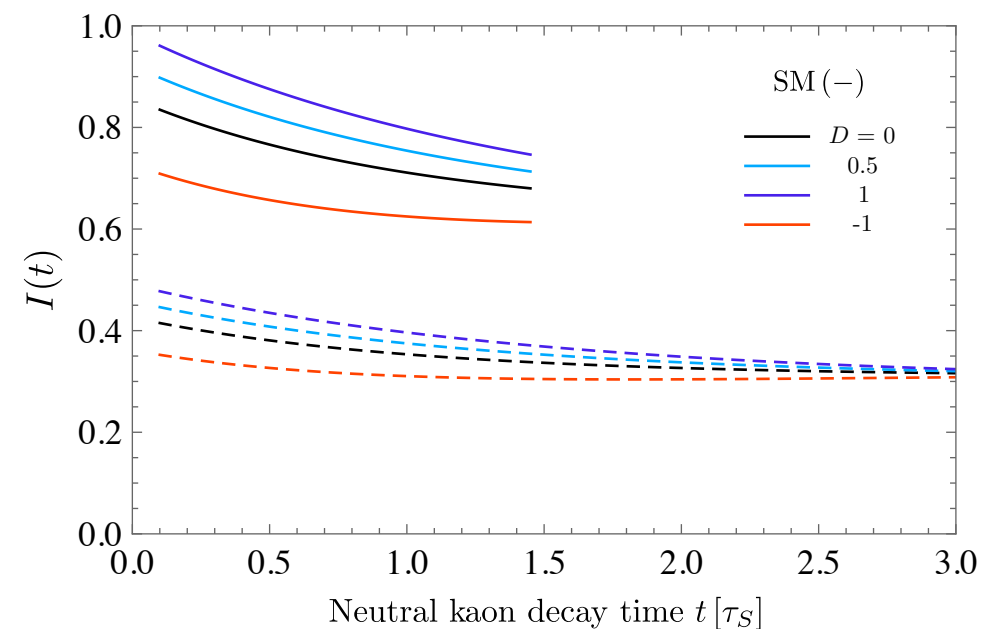
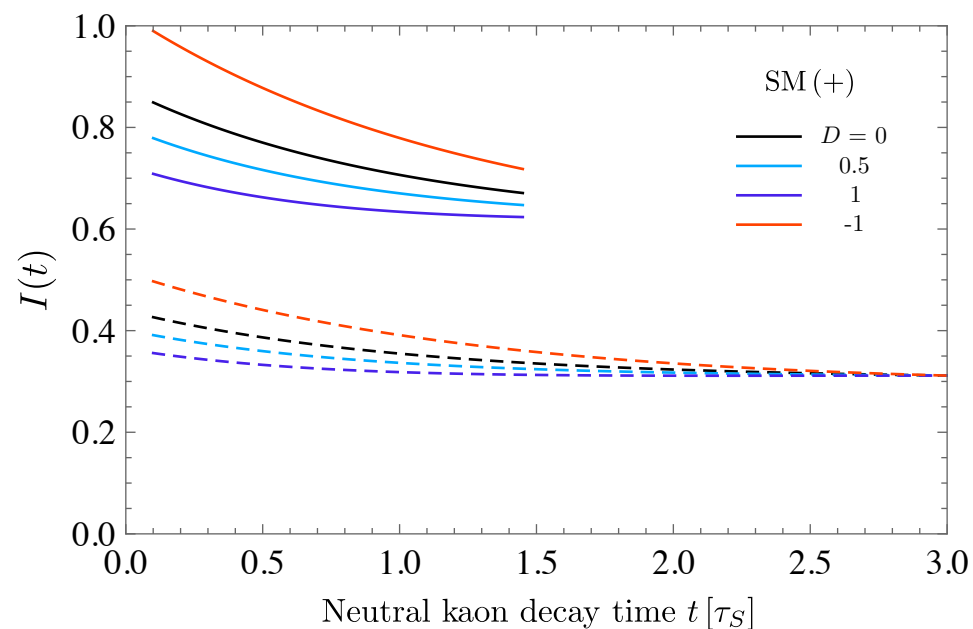


Can we study $K^0(t)$?

GD , Kitahara
1707.06999 PRL

$$pp \rightarrow K^0 \textcolor{red}{K}^- X$$

$$pp \rightarrow K^{*+} X \rightarrow K^0 \textcolor{green}{\pi}^+ X$$



$$|\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2}(1 \pm \bar{\epsilon})} \left[e^{-iH_S t} (|K_1\rangle + \bar{\epsilon}|K_2\rangle) \right. \\ \left. \pm e^{-iH_L t} (|K_2\rangle + \bar{\epsilon}|K_1\rangle) \right]$$

$$D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$$

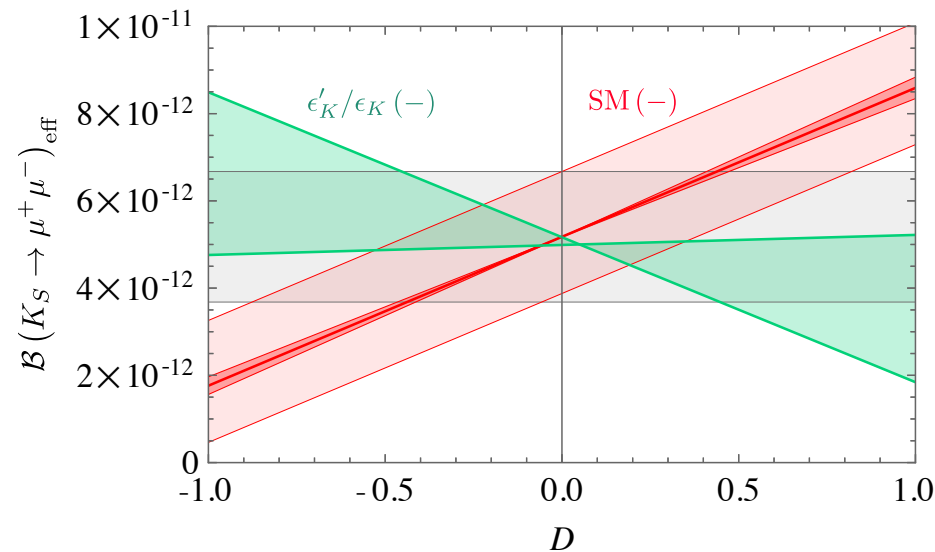
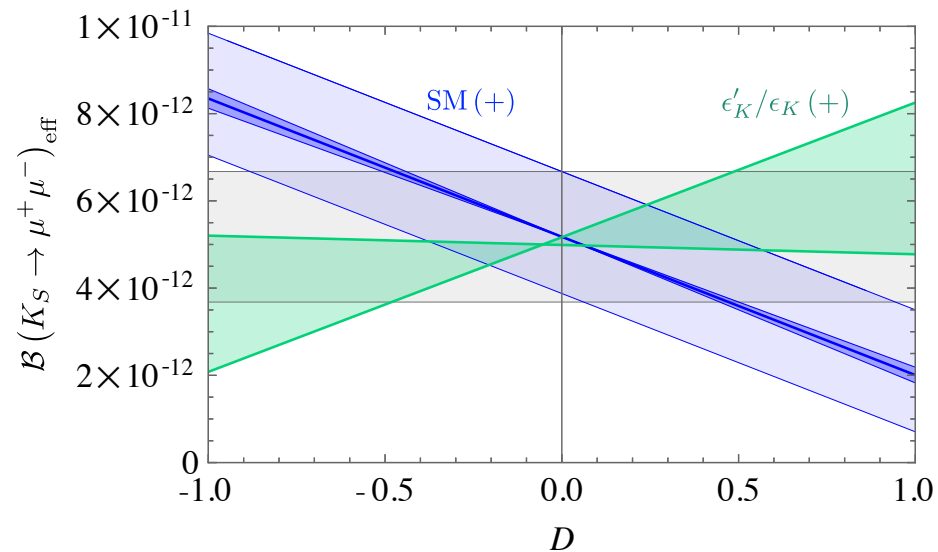
- **Short distance interfering** with Large CP conserving LD contribution !
- We may be able to study the time evolution of K^0 by tracking the associated particles (K^-)

$$\sum_{\text{spin}} \mathcal{A}(K_1 \rightarrow \mu^+ \mu^-)^* \mathcal{A}(K_2 \rightarrow \mu^+ \mu^-)$$

$$\sim \text{Im}[\lambda_t] y'_{7A} \left\{ A_{L\gamma\gamma}^\mu - 2\pi \sin^2 \theta_W (\text{Re}[\lambda_t] y'_{7A} + \text{Re}[\lambda_c] y_c) \right\}$$

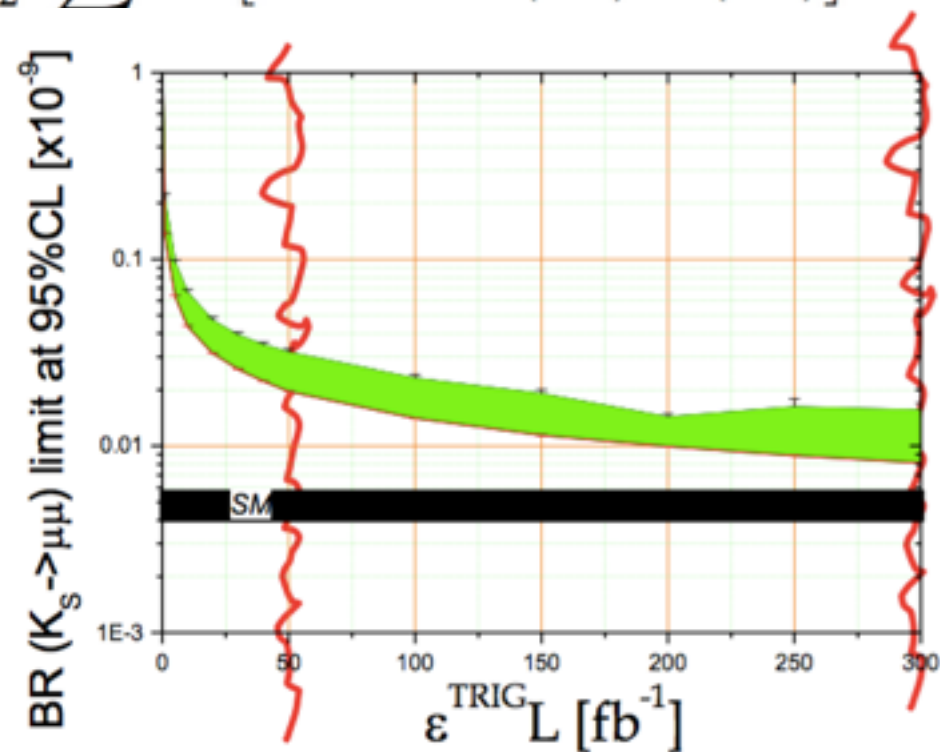
Short distance window

GD , Kitahara
1707.06999 PRL



modified Z-coupling model. ϵ'_K/ϵ_K

$$\mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{eff}} = \tau_S \left[\int_{t_{\min}}^{t_{\max}} dt \left(\Gamma(K_1) e^{-\Gamma_S t} + \frac{D}{8\pi M_K} \sqrt{1 - \frac{4m_\mu^2}{M^2}} \sum \text{Re} [e^{-i\Delta M_K t} \mathcal{A}(K_1)^* \mathcal{A}(K_2)] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right) \varepsilon(t) \right] \times \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t} \varepsilon(t) \right)^{-1},$$

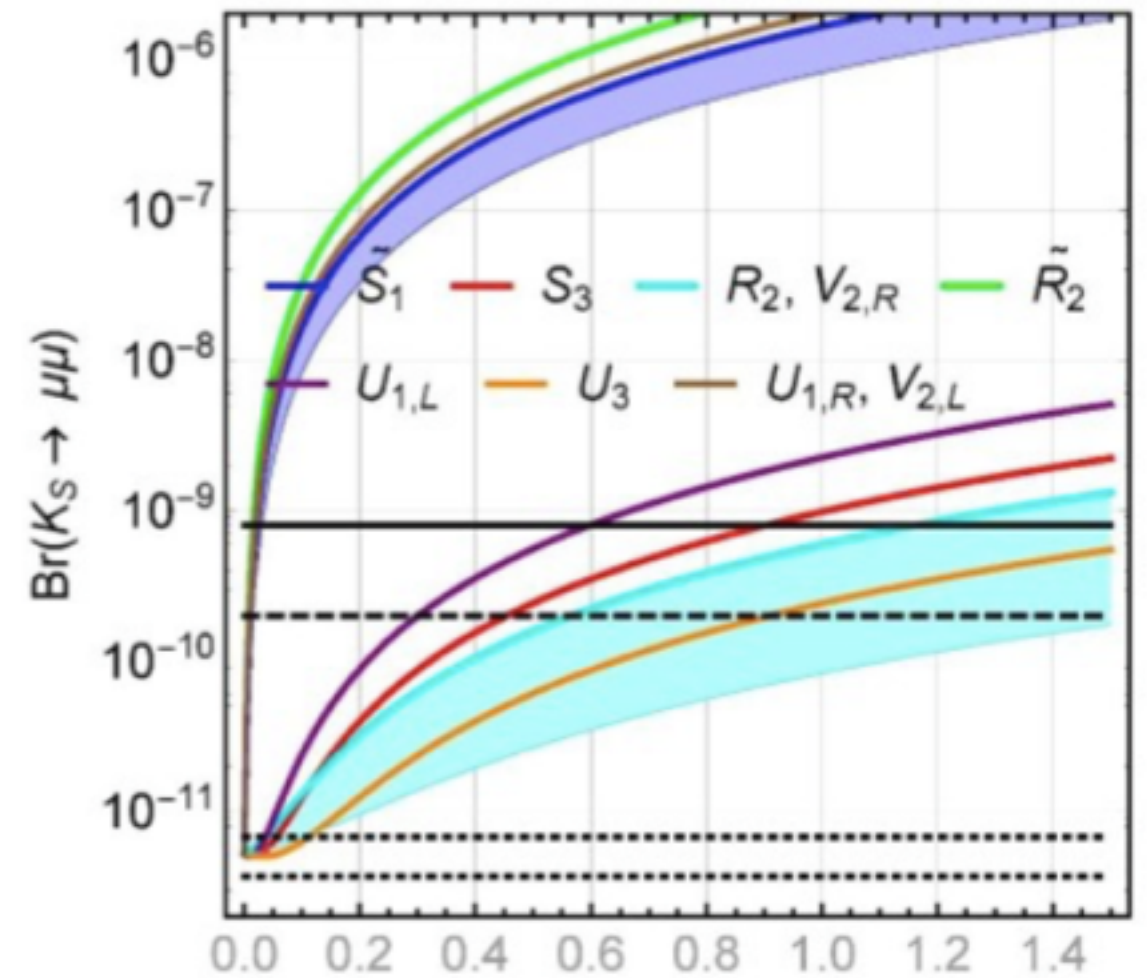
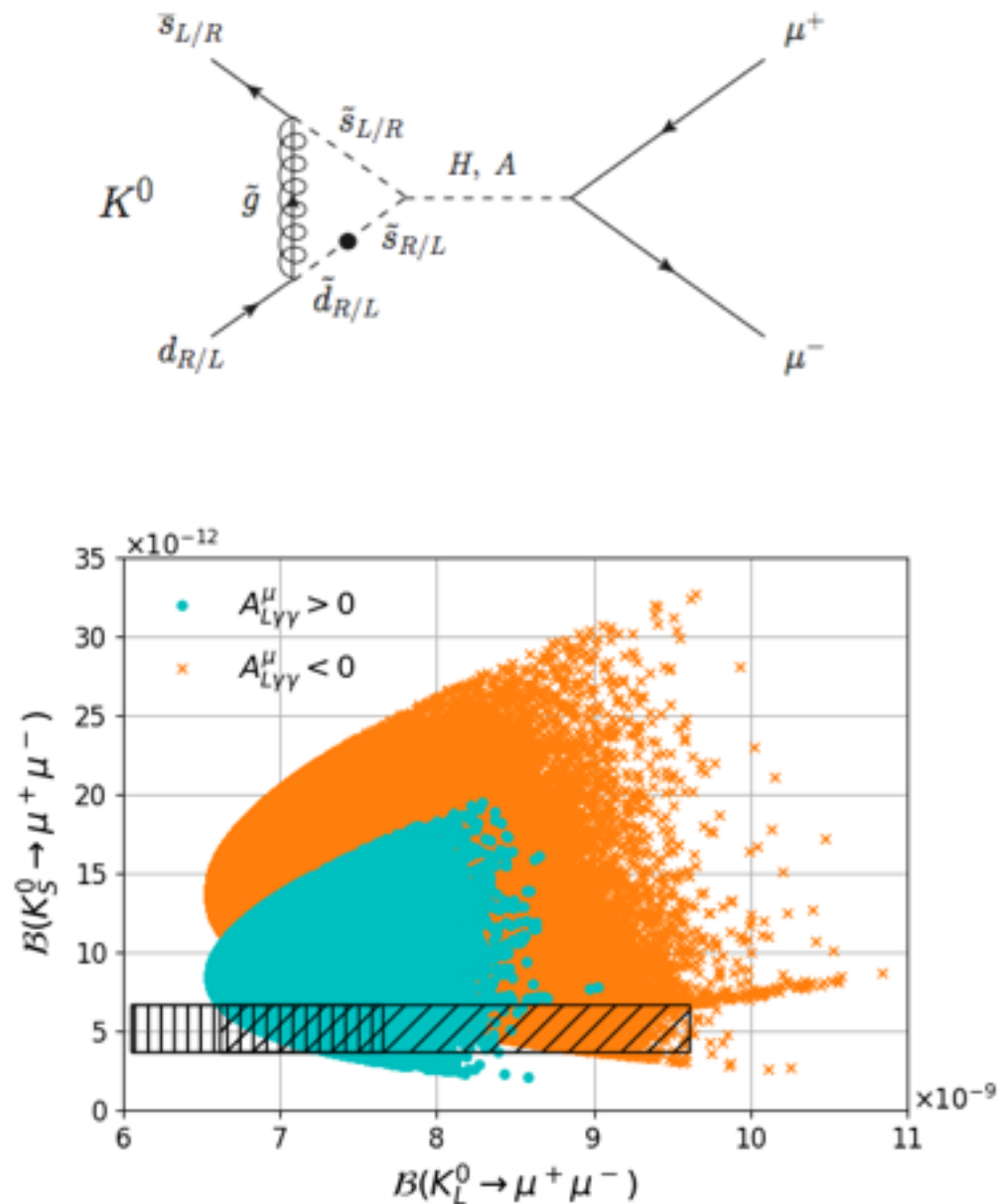


LHCb-upgrade

Phase-II-upgrade?

Collaboration with Teppei Kitahara, Isabel Fernández Suárez, Miriam Lucio Martínez, Diego Martínez Santos, Veronika Georgieva Chobanova arXiv:1711.11030, JHEP

Miriam Lucio

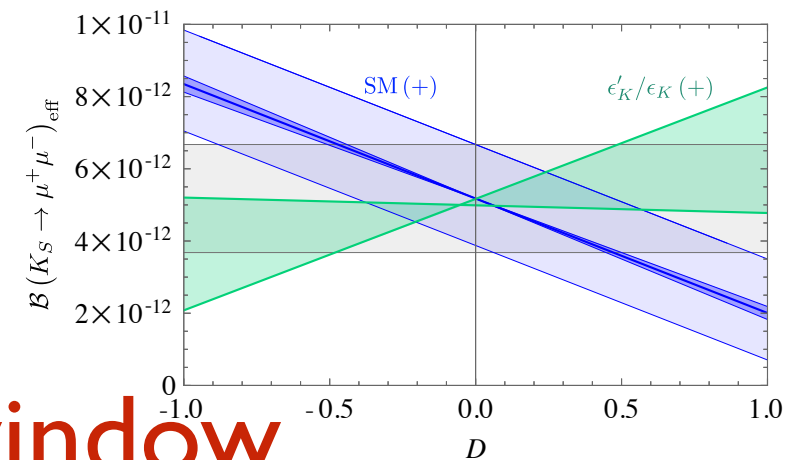


Leptoquark scenarios from Bobeth & Buras, JHEP02(2018)101

Conclusions

- Flavour anomalies: interplay with $K \rightarrow \pi \nu \nu$ but 10% measurement needed!

- LHCb: $K_S \rightarrow \mu \mu$ extraordinary result: interference effect!!! **Short distance window**

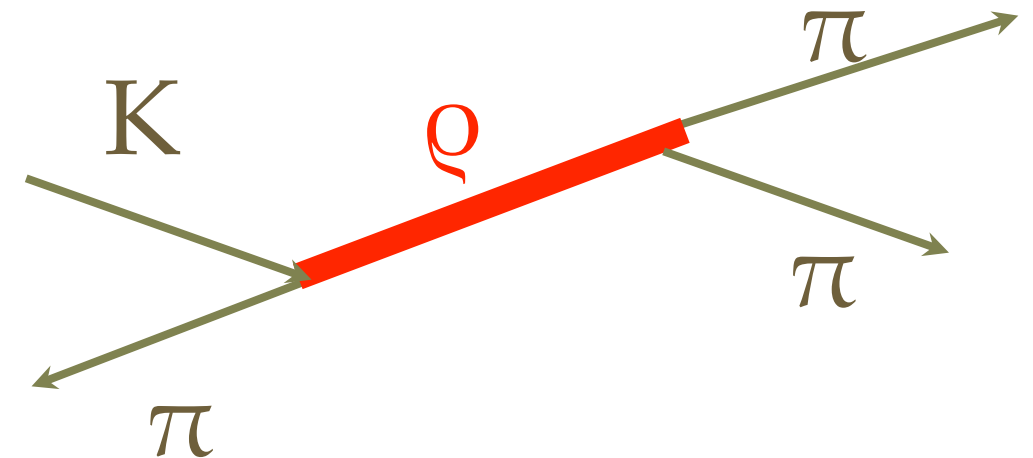


- weak chiral lagrangian
- LFUV in Kaons very useful
- Rich rare kaon program

Hard Wall weak interactions: $K \rightarrow 3\pi$

Luigi Cappiello, Oscar Cata and G.D.

In this channel there is a large VMD in the phenomenological slope



However this is proportional to $L_3 + 3/4 L_9$

$$4D \quad L_3 + 3/4 L_9 = 0$$

5D $L_3 + 3/4 L_9 \neq 0$ and in agreement with phenomenology

K→2 pi/3pi fit

Kambor Missimer Wyler, '90s

$$\mathcal{M}(K_L \rightarrow \pi^+ \pi^- \pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1) u^2 + \frac{1}{3}(\zeta_1 - \xi_1) v^2$$

$$\mathcal{M}(K_L \rightarrow \pi^0 \pi^0 \pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) ,$$

$$\mathcal{M}(K^+ \rightarrow \pi^+ \pi^+ \pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1) u^2 + \frac{1}{3}(2\zeta_1 + \xi_1) v^2 ,$$

$$\mathcal{M}(K^+ \rightarrow \pi^+ \pi^0 \pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1) u^2 - \frac{1}{3}(\zeta_1 - \xi_1) v^2 ,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27f_K f_\pi} m_K^4 \{ (k_1 - k_2) + 24\mathcal{L}_1 \} ,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9f_K f_\pi} m_\pi^2 m_K^2 \{ (k_3 - 2k_1) - 24\mathcal{L}_2 \} ,$$

$$\zeta_1 = -\frac{g_8}{6f_K f_\pi} m_\pi^4 \{ k_2 - 24\mathcal{L}_1 \} ,$$

$$\xi_1 = -\frac{g_8}{6f_K f_\pi} m_\pi^4 \{ k_3 - 24\mathcal{L}_2 \} ,$$

Table 1

The values of the amplitudes in eqs. (4) and (5) obtained from fits to experiment are shown in the first two columns. Our value of $\delta_2 - \delta_0$ is obtained from $K \rightarrow 2\pi$ decays alone, while some additional constraints were used in ref. [8]. The $K \rightarrow 3\pi$ amplitudes $\alpha_1, \dots, \xi'_3$ are in units of 10^{-8} . The results of lowest and next-to-lowest order chiral perturbation theory are displayed in the two columns to the right.

	Devlin and Dickey	Our fit	Lowest order	Order p^4
$a_{1/2}$ [keV]	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4698	0.4698
$a_{3/2}$ [keV]	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0211	0.0211
$\delta_2 - \delta_0$ (deg)	-45.6 ± 5	-61.5 ± 4	0	-29
α_1	91.46 ± 0.24	91.71 ± 0.32	74.0	91.8
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-4.1	-7.6
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-16.5	-25.6
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-1.0	-2.5
γ_3	2.51 ± 0.36	2.26 ± 0.23	1.8	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	-	-0.6
ζ_3	-	-0.21 ± 0.08	-	-0.02
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	-	-1.5
ξ_3	-	-0.12 ± 0.17	-	-0.05
ξ'_3	-	-0.21 ± 0.51	-	-0.08
χ^2/DOF	12.8/3	10.3/2	4121/5	37/13

Vector meson dominance in $K \rightarrow 3\pi$

$$\mathcal{M}(K_L \rightarrow \pi^+ \pi^- \pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1) u^2 + \frac{1}{3}(\zeta_1 - \xi_1) v^2$$

$$\mathcal{M}(K_L \rightarrow \pi^0 \pi^0 \pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) ,$$

$$\mathcal{M}(K^+ \rightarrow \pi^+ \pi^+ \pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1) u^2 + \frac{1}{3}(2\zeta_1 + \xi_1) v^2 ,$$

$$\mathcal{M}(K^+ \rightarrow \pi^+ \pi^0 \pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1) u^2 - \frac{1}{3}(\zeta_1 - \xi_1) v^2 ,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27f_K f_\pi} m_K^4 \{ (k_1 - k_2) + 24\mathcal{L}_1 \} ,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9f_K f_\pi} m_\pi^2 m_K^2 \{ (k_3 - 2k_1) - 24\mathcal{L}_2 \} ,$$

$$\zeta_1 = -\frac{g_8}{6f_K f_\pi} m_\pi^4 \{ k_2 - 24\mathcal{L}_1 \} ,$$

$$\xi_1 = -\frac{g_8}{6f_K f_\pi} m_\pi^4 \{ k_3 - 24\mathcal{L}_2 \} ,$$

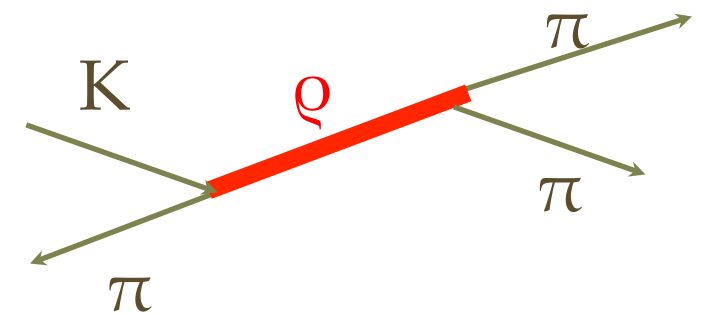
$$u = \frac{s_3 - s_0}{m_\pi^2} , \quad v = \frac{s_1 - s_2}{m_\pi^2} , \quad s_i = (p_K - p_{\pi_i})^2 , \quad s_0 = \frac{1}{3} \sum_{i=1}^3 s_i .$$

Angular momentum decomposition

β_1 should be dominated by
 ρ exchange

$$k_3 = 3(N_1 + N_2 - N_3)$$

It has VMD



Isidori, Pugliese
Ecker Kambor Wyler

We measure the slope, let's check theory predictions

In factorization

$$k_3/24 = 3(N_1 + N_2 - N_3)/24 = L_3 + 3/4 L_9$$

in units 10^{-3}

$$\frac{k_3}{24} = \left(L_3 + \frac{3}{4} L_9 \right) \sim^{expt} 1.7$$

using L_i^{exp}

using L_i^{VMD}

TH VMD 0

5D 1.7

using L_i^{holo}

departures from KSFR

Further NA62 K Physics Program

Decay	Physics	Present limit (90% C.L.) / Result	NA62
$\pi^+ \mu^+ e^-$	LFV	1.3×10^{-11}	0.7×10^{-12}
$\pi^+ \mu^- e^+$	LFV	5.2×10^{-10}	0.7×10^{-12}
$\pi^- \mu^+ e^+$	LNV	5.0×10^{-10}	0.7×10^{-12}
$\pi^- e^+ e^+$	LNV	6.4×10^{-10}	2×10^{-12}
$\pi^- \mu^+ \mu^+$	LNV	1.1×10^{-9}	0.4×10^{-12}
$\mu^- \nu e^+ e^+$	LNV/LFV	2.0×10^{-8}	4×10^{-12}
$e^- \nu \mu^+ \mu^+$	LNV	No data	10^{-12}
$\pi^+ X^0$	New Particle	$5.9 \times 10^{-11} m_{X^0} = 0$	10^{-12}
$\pi^+ \chi \chi$	New Particle	—	10^{-12}
$\pi^+ \pi^+ e^- \nu$	$\Delta S \neq \Delta Q$	1.2×10^{-8}	10^{-11}
$\pi^+ \pi^+ \mu^- \nu$	$\Delta S \neq \Delta Q$	3.0×10^{-6}	10^{-11}
$\pi^+ \gamma$	Angular Mom.	2.3×10^{-9}	10^{-12}
$\mu^+ \nu_h, \nu_h \rightarrow \nu \gamma$	Heavy neutrino	Limits up to $m_{\nu_h} = 350 \text{ MeV}$	
R_K	LU	$(2.488 \pm 0.010) \times 10^{-5}$	$> \times 2$ better
$\pi^+ \gamma \gamma$	χ PT	< 500 events	10^5 events
$\pi^0 \pi^0 e^+ \nu$	χ PT	66000 events	$O(10^6)$
$\pi^0 \pi^0 \mu^+ \nu$	χ PT	—	$O(10^5)$

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^* (S)$ $\pi^+\gamma\gamma$ -----	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^* (L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma (S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma (S)$ $\pi^+\pi^-\gamma^* (L)$ $\pi^+\pi^-\gamma^* (S)$ $\pi^+\pi^0\gamma^*$	 $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (L)$ $\pi^+\pi^-\pi^0\gamma (S)$	$N_{14}^r - N_{15}^r$ $2N_{14}^r + N_{15}^r$ $N_{14} - N_{15} - 2N_{18}$ ----- ” $N_{14} - N_{15} - N_{16} - N_{17}$ ----- ” ” $7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17}^r)$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17}^r)$ -----
	$\pi^+\pi^-\gamma (L)$ $\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^-\pi^0\gamma (L)$	$N_{29} + N_{31}$ ----- ” $3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma \gamma, K \rightarrow \pi l^+ l^-} + \cdots$$

OPE

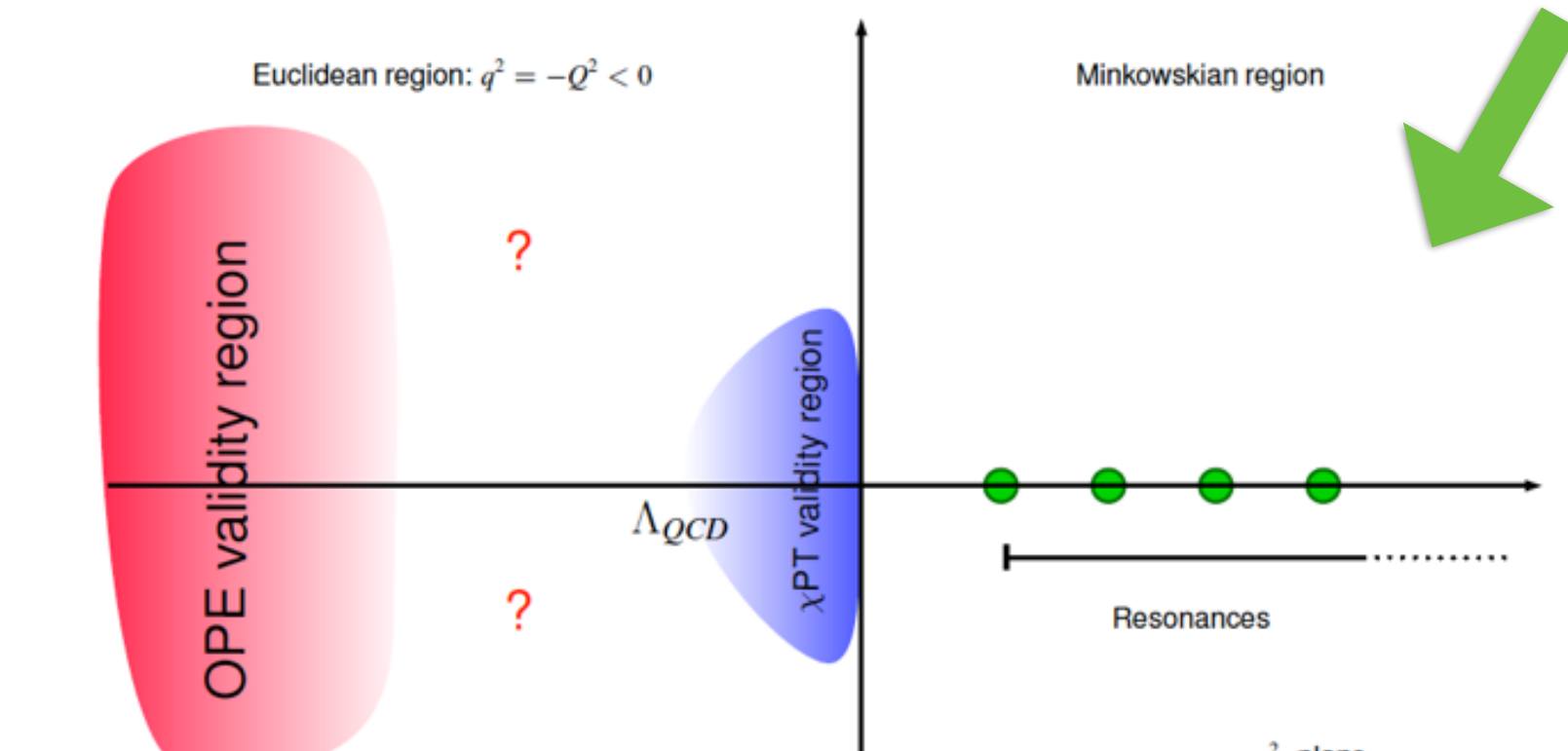
$$\Pi_V(Q^2) \underset{Q^2 \rightarrow \infty}{\sim} \frac{N_c}{24\pi^2} \ln \left(\frac{\Lambda_V^2}{Q^2} \right) + \langle \mathcal{O}_2 \rangle \frac{1}{Q^2} + \langle \mathcal{O}_4 \rangle \frac{1}{Q^4} + \langle \mathcal{O}_6 \rangle_V \frac{1}{Q^6}$$

$$\left\{ \begin{array}{l} \langle \mathcal{O}_2 \rangle = 0 \\ \langle \mathcal{O}_4 \rangle = \frac{1}{12\pi} \alpha_s \langle G^2 \rangle \\ \langle \mathcal{O}_6 \rangle_V = -\frac{28\pi}{9} \alpha_s \langle \bar{\psi}\psi \rangle^2 \end{array} \right.$$

– Second Weinberg's sum rule –

Large N_c QCD

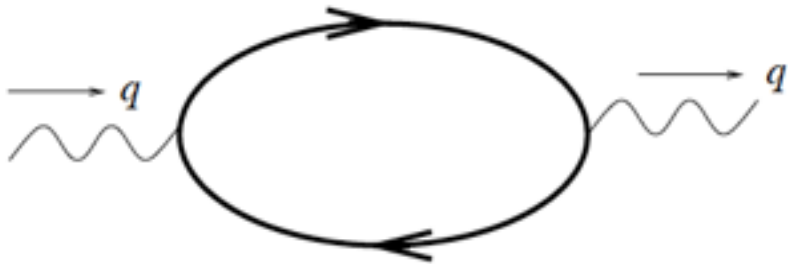
$$\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{F_V(n)^2}{Q^2 + M_V(n)^2}$$



Holographic QCD at low energies

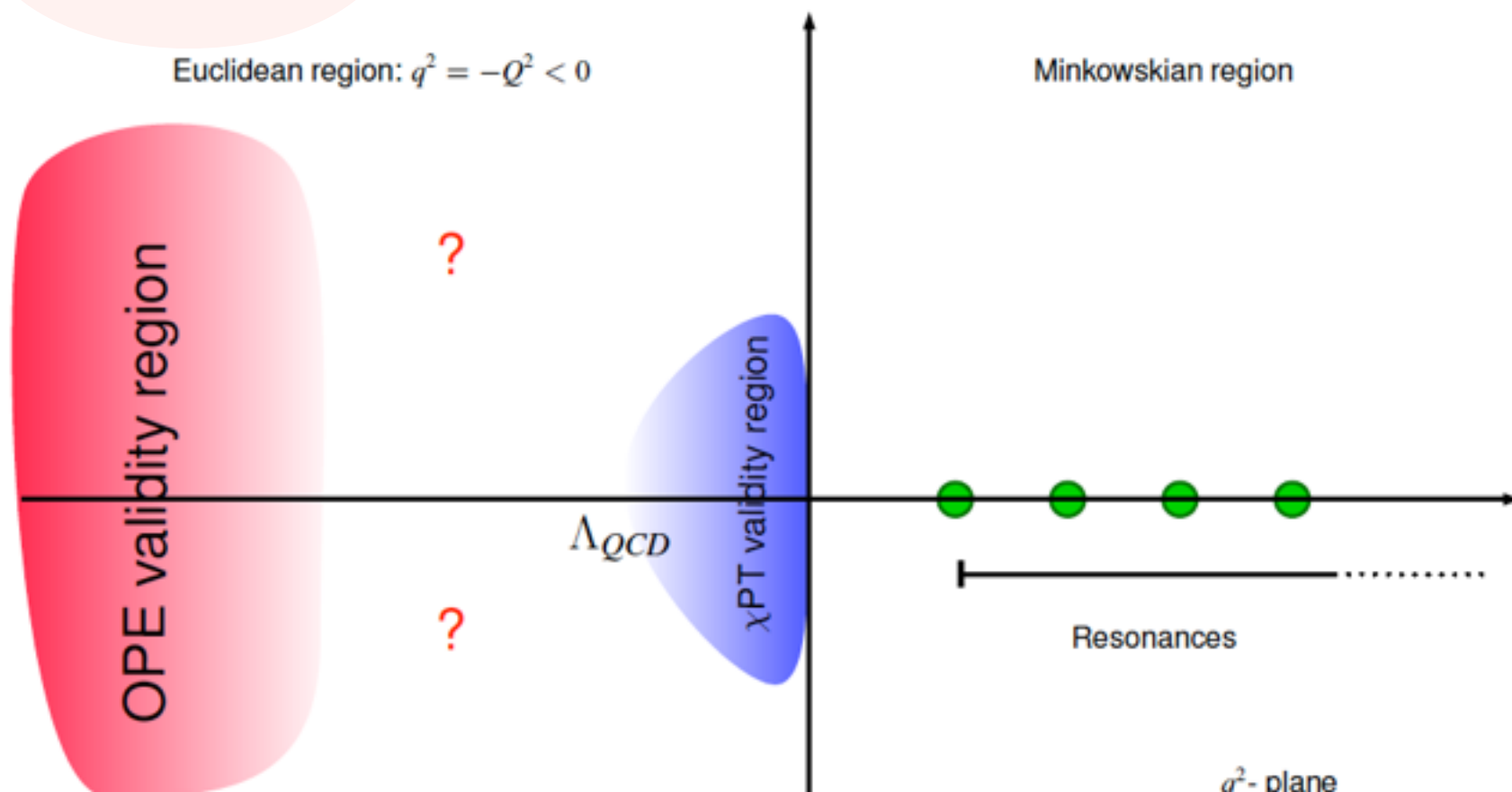
Large N_c QCD properties $\Rightarrow$ conformal symmetry in 5d

$$\int d^4x e^{ip \cdot x} \langle J_\mu(x) J_\nu(0) \rangle = \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Pi(p^2)$$

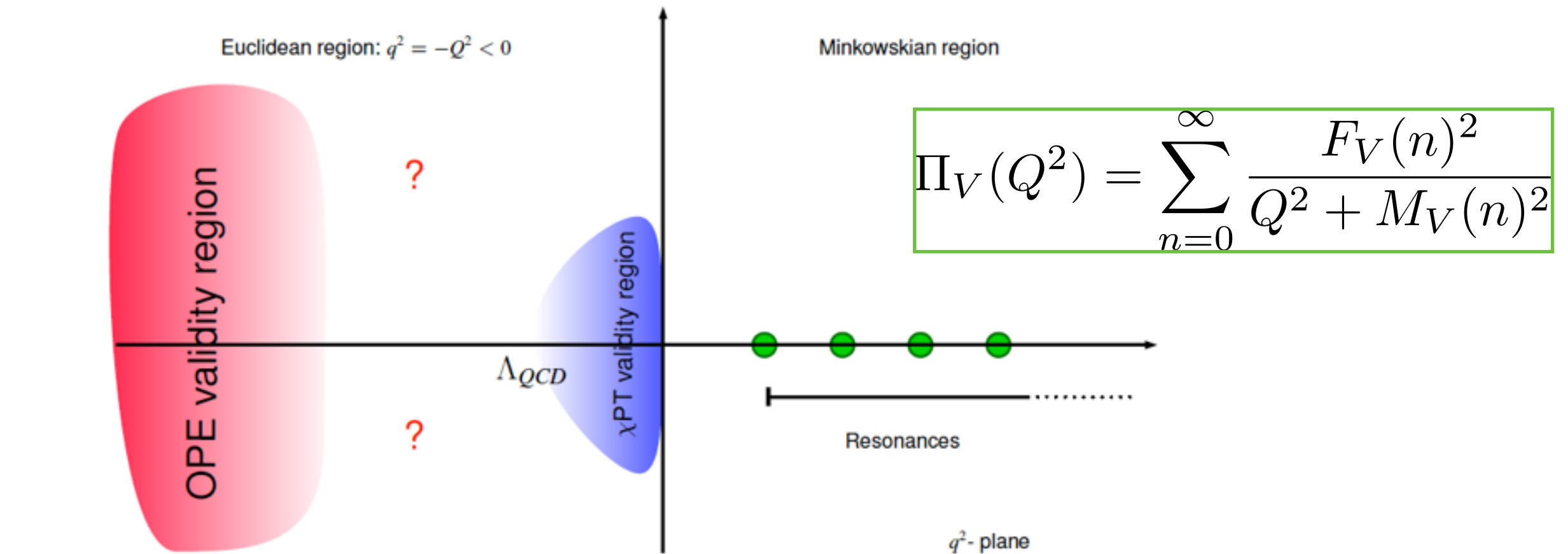


$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$

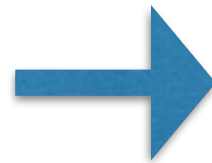
confinement: either IR cut-off or
field modifying metric



Holographic QCD: Hard Wall



$$\Pi_V(Q^2) = -\frac{N_c}{12\pi^2} \ln Q^2 \quad Q^2 \equiv -q^2$$



IR very good, but

$$M_V^2(n) \sim n^2$$



$$\Pi_V(q^2) = -\frac{4}{3} \frac{N_c}{(4\pi)^2} \left[\log \frac{q^2}{\mu^2} - \pi \frac{Y_0(qz_0)}{J_0(qz_0)} \right]$$

Back up

Correlation with different flavor sectors

$\Lambda_{NP}^{b \rightarrow c,s} \sim \mathcal{O}(1, 100) \text{ TeV} \Rightarrow$ direct searches,
low-energy precision observables

GIM suppression and CKM suppression:

$$\mathcal{L}_{\text{eff}} \supset -\frac{1 - 0.3 i}{(\text{180 TeV})^2} (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L) + \text{h.c.}$$

Svjetlana Fajfer and Nejc Kořnik Luiz Vale Silva

Flavour Problem

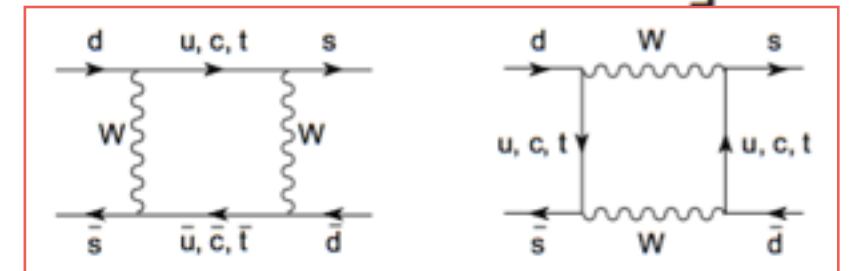
- the SM Yukawa structure

$$\mathcal{L}_{SM}^Y = \bar{Q} Y_D D H + \bar{Q} Y_U U H_c + \bar{L} Y_E E H + \text{h.c.}$$

FCNC

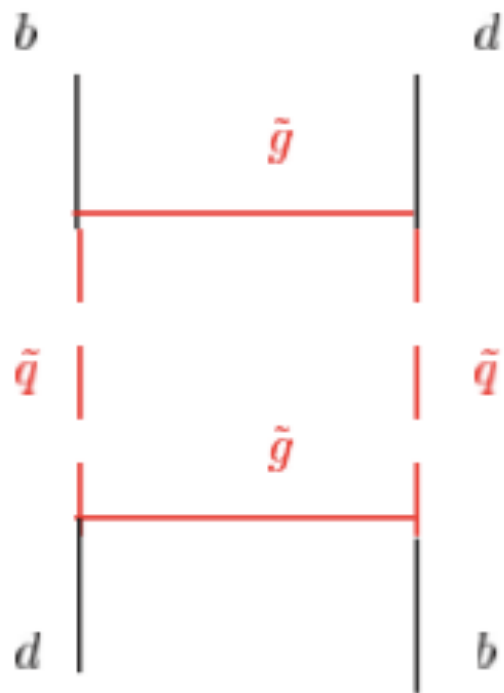
$$\mathcal{H}_{\Delta F=2}^{SM} \sim \frac{G_F^2 M_W^2}{16\pi^2} \left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2 \right] + \text{charm}$$

- Supersymmetry must be broken



$$-\mathcal{L}_{soft} = \tilde{Q}^\dagger m_Q^2 \tilde{Q} + \tilde{L}^\dagger m_L^2 \tilde{L} + \tilde{U} a_u \tilde{Q} H_u + \dots$$

- $m_Q^2, m_L^2, a_u, \dots$ matrices in flavour space additional (to $Y_{u,d,l}$) non-trivial structures



$$\mathcal{H}_{\Delta F=2}^{\tilde{g}} \sim \frac{\alpha_s^2}{9M_{\tilde{Q}}^2} \left[(\delta_{12}^{LL})^2 (\bar{s}_L \gamma_\mu d_L)^2 + \dots \right]$$

δ_{12}^{LL} departure from identity matrix m_Q^2

Gabrielli Masiero
Silvestrini

$$K \Rightarrow \bar{K} \quad \frac{(\delta_{12}^{LL})^2}{M_{\tilde{Q}}^2} \leq \frac{1}{(100\text{TeV})^2} \Rightarrow \text{Naturalness?}$$

obey some Flavour symmetry so that GIM is realized

$$m_Q^2 \sim I$$

$$\mathcal{L}_{\Delta F=2} = \frac{C}{\Lambda_{MFV}^2} \left[\frac{(V_{td}^* m_t^2 V_{tb})^2}{v^4} (\bar{d}_L \gamma^\mu b_L)^2 + \frac{(V_{td}^* m_t^2 V_{ts})^2}{v^4} (\bar{d}_L \gamma^\mu s_L)^2 \right]$$

Traditional solution

Problem already known since '86 technicolour, (Chivukula Georg

susy

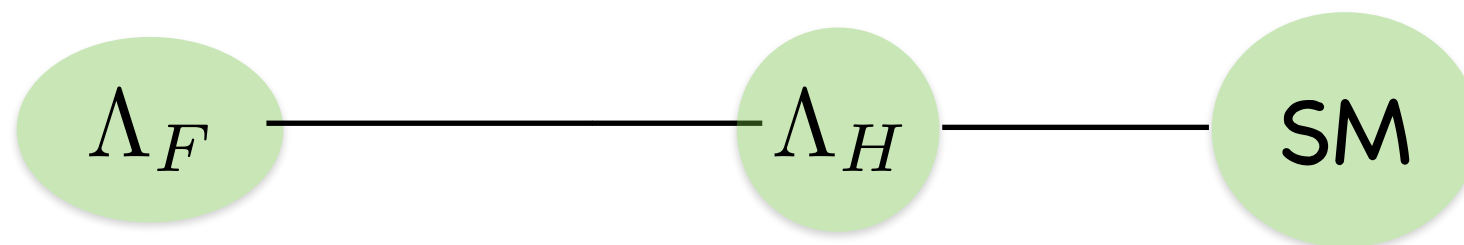
(Hall Randall)

extra dimensions

(Rattazzi Zafferoni)

G.D.,Giudice,Isidori,Strumia; A. Buras, Gambino,Silvestrini

Scale New Physics, stabilizing EW scale, $\Lambda_H \ll \text{scale}$
of the dynamical understanding of Flavor



$$\Lambda_H \ll \Lambda_F$$

CP violation in $K \rightarrow 2\pi$

$$A(K_L \rightarrow \pi^+ \pi^-) \propto \epsilon + \epsilon'$$

$$\epsilon \sim \mathcal{O}(10^{-3})$$

Christenson et al 64

$$\epsilon' \sim \mathcal{O}(10^{-6})$$

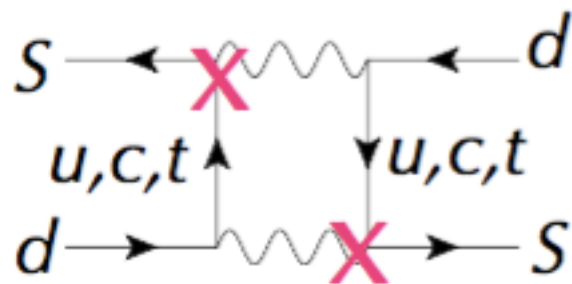
CERN NA31, Fermilab KTeV

$$A(K_L \rightarrow \pi^0 \pi^0) \propto \epsilon - 2\epsilon'$$

$$H_{\Delta S=2}$$

Indirect CP violation

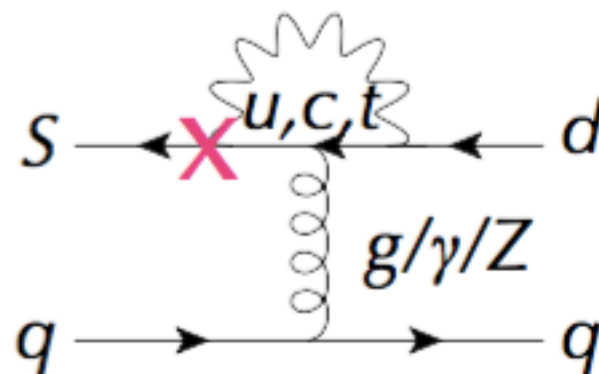
Kaon oscillation



$$H_{\Delta S=1}$$

Direct CP Violation

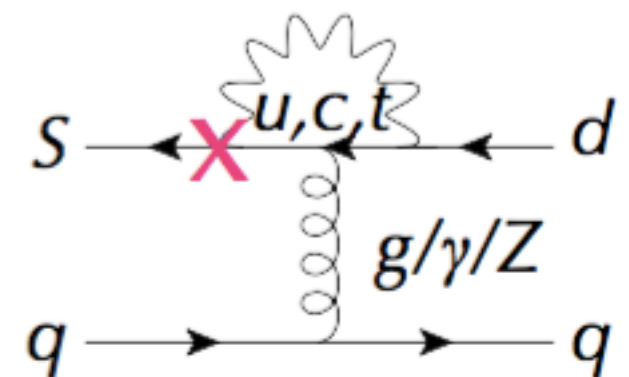
Penguin



$$\frac{\epsilon'_K}{\epsilon_K} = \frac{1}{\sqrt{2}|\epsilon_K|_{\text{exp}}} \frac{\omega_{\text{exp}}}{(\text{Re}A_0)_{\text{exp}}} \left(-\text{Im}A_0 + \frac{1}{\omega_{\text{exp}}} \text{Im}A_2 \right) \quad \text{where} \quad \frac{1}{\omega} \equiv \frac{\text{Re}A_0}{\text{Re}A_2} = 22.46 \text{ (exp.)}$$

gluon
penguin
 Q_6

EW
penguin
 Q_8



$\langle O_6 \rangle$ and $\langle O_8 \rangle$ have chiral enhancement factor

$$\langle Q_6(\mu) \rangle_0 = -4 \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 (F_K - F_\pi) \underline{B_6^{(1/2)}}$$

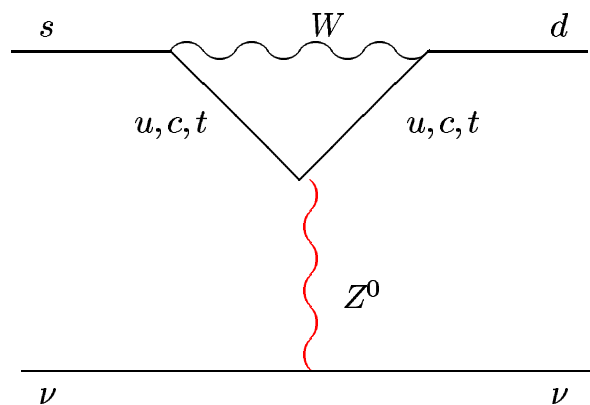
$$\langle Q_8(\mu) \rangle_2 = \sqrt{2} \left[\frac{m_K^2}{m_s(\mu) + m_d(\mu)} \right]^2 F_\pi \underline{B_8^{(3/2)}}$$

**New lattice
result 2015**

$$K \rightarrow \pi \nu \bar{\nu}$$

Why we need to the experiments KOTO and NA62

$$A(s \rightarrow d \nu \bar{\nu})_{\text{SM}} \sim \bar{s}_L \gamma_\mu d_L \quad \bar{\nu}_L \gamma^\mu \nu_L \times \left[\sum_{q=c,t} V_{qs}^* V_{qd} m_q^2 \right]$$



$$\sim [A^2 \lambda^5 (1 - \rho - i\eta) m_t^2 + \lambda m_c^2]$$

SM

$$\underbrace{V - A \otimes V - A}_{\Downarrow}$$

Littenberg

$$\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

$$\left\{ \begin{array}{l} \text{CP violating} \\ \Rightarrow J = A^2 \lambda^6 \eta \\ \text{Only } \underline{top} \end{array} \right.$$

SM

$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

Misiak, Urban; Buras, Buchalla; Brod, Gorbhan, Stamou'11, Straub

$$B(K^+) \sim \kappa_+ \left[\left(\frac{\text{Im}\lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\text{Re}\lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\text{Re}\lambda_t}{\lambda^5} X_t \right)^2 \right]$$

- κ_+ from K_{l3} $\lambda_q = V_{qd}^* V_{qs}$
- P_c : SD charm quark contribution (30%±2.5% to BR)
LD $\delta P_{c,u} \sim 4 \pm 2\%$
- $B(K^\pm) = (8.82 \pm 0.8 \pm 0.3) \times 10^{-11}$ first error parametric (V_{cb}),
second non-pert. QCD
- E949 $B(K^\pm) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

$$K_L$$

$$B(K_L) = (3.14 \pm 0.17 \pm 0.06) \times 10^{-11} \text{ vs}$$

$$\text{E391a } B(K_L) < 2.6 \times 10^{-8} \text{ at 90\% C.L.}$$

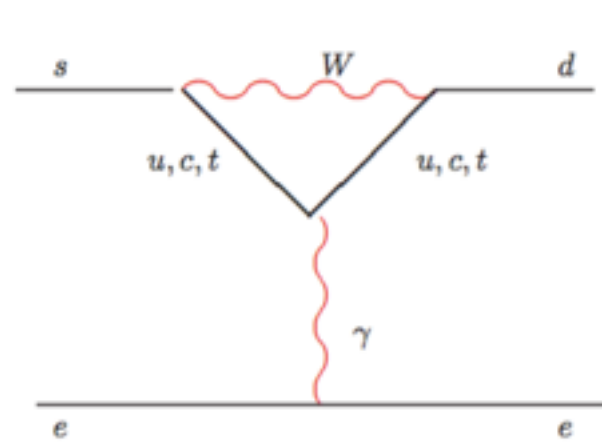
K_L Model-independent bound, based on $SU(2)$ properties dim-6 operators for $\bar{s}d\bar{\nu}\nu$ Grossman-Nir

$$B(K_L) \leq \frac{\tau_L}{\tau_+} \times B(K^\pm)_{\text{E949}} \leq 1.4 \times 10^{-9} \text{ at 90\% C.L.}$$

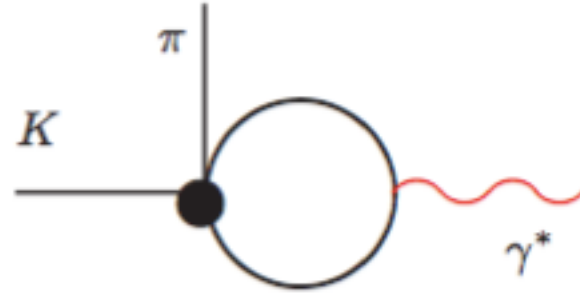
	PDG	Prospects
$K_S \rightarrow \mu\mu$	$< 9 \times 10^{-9}$ at 90% CL	(LD) $(5.0 \pm 1.5) \cdot 10^{-12}$ NP $< 10^{-11}$
$K_L \rightarrow \mu\mu$	$(6.84 \pm 0.11) \times 10^{-9}$	difficult : SD $\ll$ LD
$K_S \rightarrow \mu\mu\mu\mu$	—	SM LD $\sim 2 \times 10^{-14}$
$K_S \rightarrow ee\mu\mu$	—	$\sim 10^{-11}$
$K_S \rightarrow eeee$	—	$\sim 10^{-10}$
$K_S \rightarrow \pi^+\pi^-\mu^+\mu^-$	—	SM LD $\sim 10^{-14}$

$$K^+ \rightarrow \pi^+ e^+ e^-$$

$$K_S \rightarrow \pi^0 e^+ e^-$$



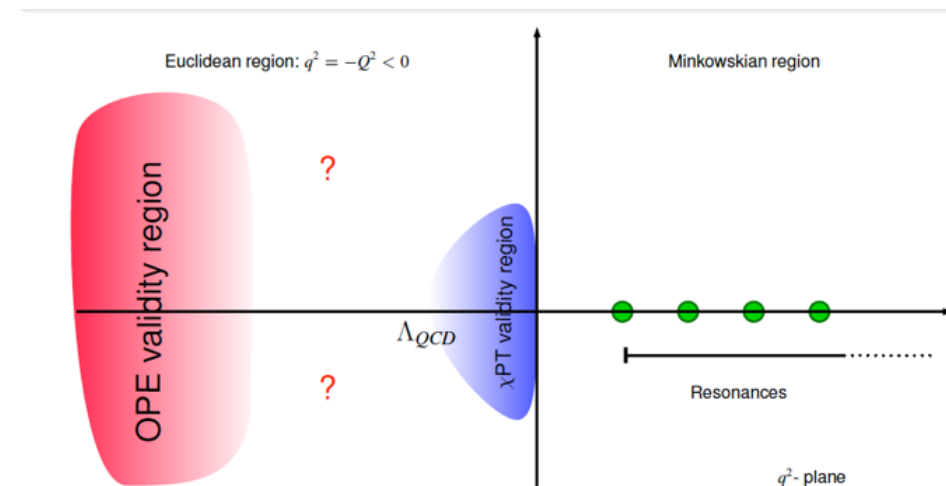
Short distance



$O(p^4)$ ChPT
electrons and μ 's in the final state

constant+ loop

Ecker, Pich, de Rafael



'97 Initial data inconsistency e and μ 's: LFV?

$$K^+ \rightarrow \pi^+ e^+ e^-$$

$$K_S \rightarrow \pi^0 e^+ e^-$$

- gauge+Lorentz inv. => 1 ff

$$i \int d^4x e^{iqx} \langle \pi(p) | T \{ J_{\text{elm}}^\mu(x) \mathcal{L}_{\Delta S=1}(0) \} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} [z(k+p)^\mu - (1 - r_\pi^2)q^\mu]$$

$$W^i = G_F m_K^2 (a_i + b_i z) + W_{\pi\pi}^i(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \quad z = \frac{q^2}{m_K^2}$$

- Observables $\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)$, $\Gamma(K^+ \rightarrow \pi^+ \mu \bar{\mu})$, slopes

- $a_i \quad O(p^4) \quad a_+ \sim N_{14} - N_{15}, \quad a_S \sim 2N_{14} + N_{15}$

Ecker, Pich, de Rafael

- $b_i \quad O(p^6)$

G.D., Ecker, Isidori, Portoles

- a_+, b_+ in general not related to a_S, b_S

Recent lattice determinations Christ et al.

averaging flavour

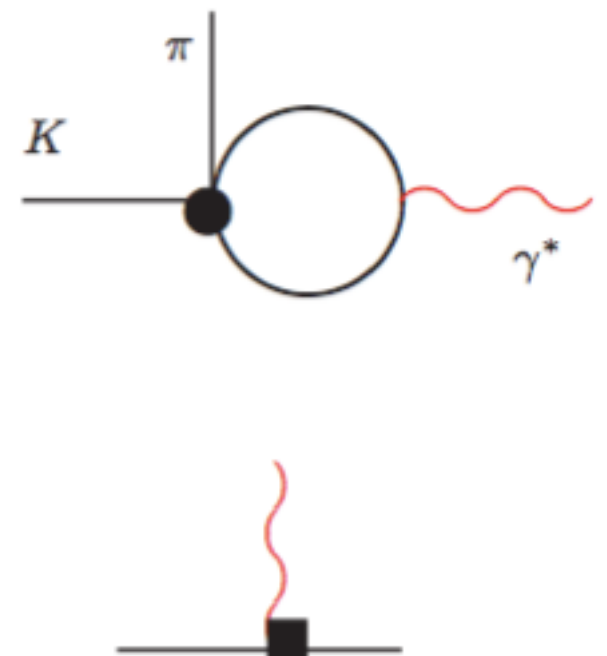
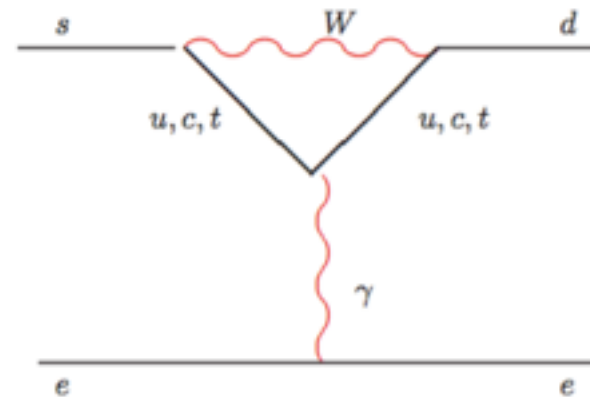
$$a_+^{\text{exp.}} = -0.578 \pm 0.016$$

$$b_+^{\text{exp.}} = -0.779 \pm 0.066$$

LFUV: Kaons

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \mu^+ \mu^-)}{\Gamma(K^+ \rightarrow \pi^+ e^+ e^-)}$$

SM



$$W^i = G_F m_K^2 (a_i + b_i z) + W_{\pi\pi}^i(z)$$

$$i = \pm, S$$

$$a_i, b_i \sim O(1), \quad z = \frac{q^2}{m_K^2}$$

LFUV: Kaons

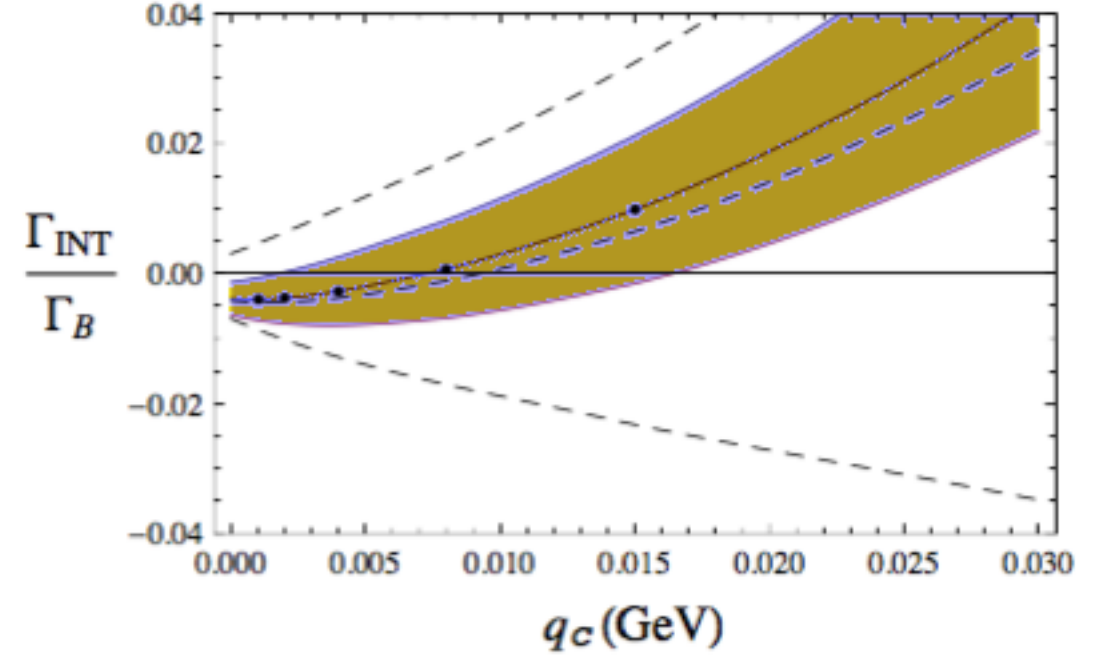
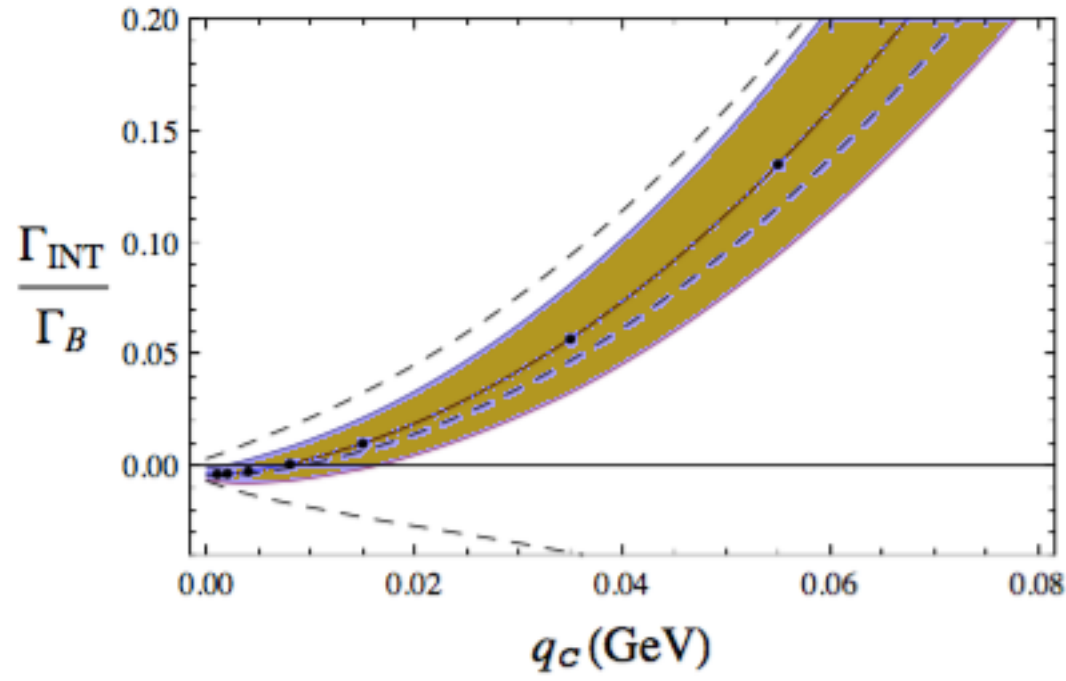
Channel	a_+	b_+	Reference
ee	-0.587 ± 0.010	-0.655 ± 0.044	E865
ee	-0.578 ± 0.016	-0.779 ± 0.066	NA48/2
$\mu\mu$	-0.575 ± 0.039	-0.813 ± 0.145	NA48/2

$$a_+^{\text{NP}} = \frac{2\pi\sqrt{2}}{\alpha} V_{ud} V_{us}^* * C_{7V}^{\text{NP}}$$

$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*} \xrightarrow{MFV} C_{9V}^{B,\mu\mu} - C_{9V}^{B,ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{td} V_{ts}^*} = -19 \pm 79$$

NA62 PLEASE!!

High statistics: nominal # of decays 50 times greater than NA48/2



q cut in minimum dilepton

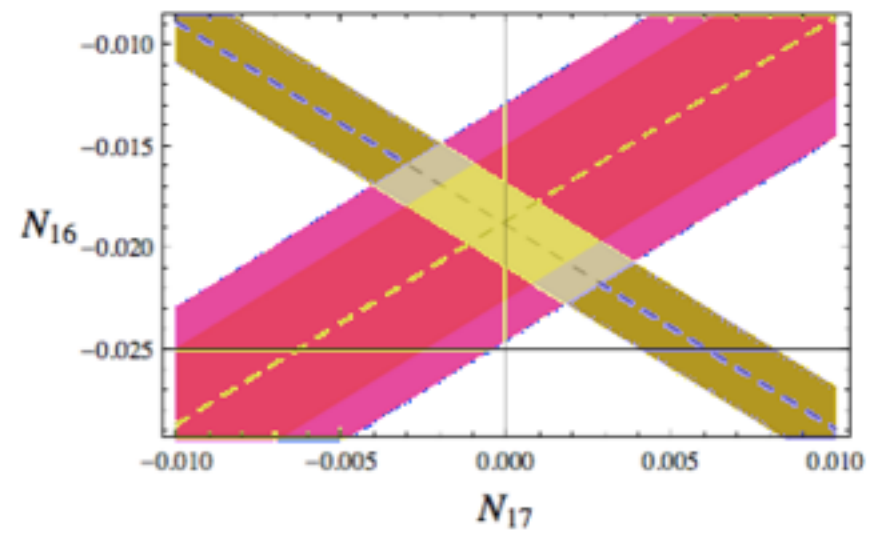
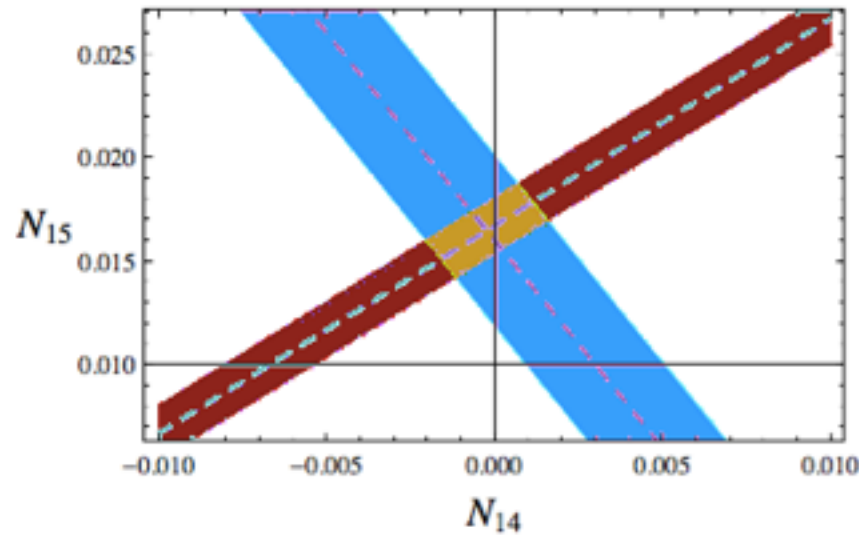


Figure 4: *Left panel: values of N_{14} and N_{15} as given by $K^\pm \rightarrow \pi^\pm \gamma^*$ (blue band) and $K_S \rightarrow \pi^0 \gamma^*$ (violet band). Right panel: values for N_{16} and N_{17} extracted from $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ (blue band) and $K^\pm \rightarrow \pi^\pm \pi^0 e^+ e^-$ (yellow band) measurements. The latter is an educated estimate (see main text).*

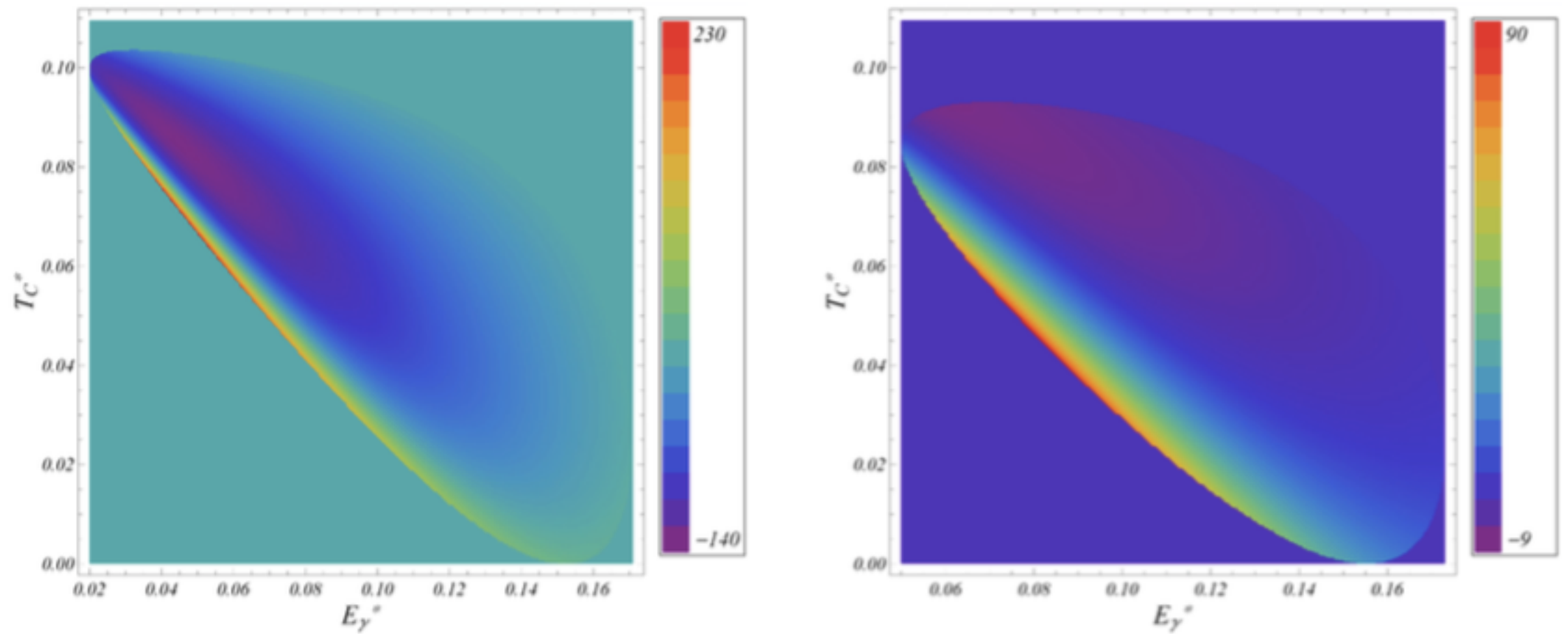


Figure 1: *Dalitz plots for the interference differential decay rate in the (E_γ, T_C) plane for $q = 20$ MeV (left panel) and $q = 50$ MeV (right panel). Numbers are given in units of $10^{-20} \text{ GeV}^{-1}$. The contour plot is 'spikier' the lower the q values are, a pattern mostly dictated by the structure of the Bremsstrahlung term.*

$$BR(K_S \rightarrow \pi^+ \pi^- e^+ e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}}$$

q_c (MeV)	$10^8 \times \Gamma_{\mathcal{B}}$	$\left[\frac{\Gamma_{\mathcal{E}}}{\Gamma_{\mathcal{B}}}\right]^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,1,1)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,0,1)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,1,0)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(0,1,1)}^{-1}$
$2m_l$	418.27	1100	-253	-225	-115	216
2	307.96	821	-265	-226	-98	159
4	194.74	529	-363	-264	-78	101
8	109.60	304	1587	-850	-59	58
15	56.12	161	102	156	-43	31
35	15.50	50	18	21	-26	11
55	5.62	22	7	9	-18	5
85	1.37	8	3	4	-13	3
100	0.67	5	2	3	-11	2
120	0.24	3	1.6	2	-10	1.4
140	0.04	2	1.0	1.1	-8	0.9
180	0.003	1	0.7	0.8	-7	0.7

Table 2: Branching ratios for the Bremsstrahlung and the relative weight of the electric and electric interference terms for different cuts in q , starting at $q_{\min}$ (first row) and ending at 180 MeV. To highlight the role of the different counterterms, the last columns show how the interference term changes when they are switched off one at a time.

QCD at work: Short Distance expansion for weak interaction

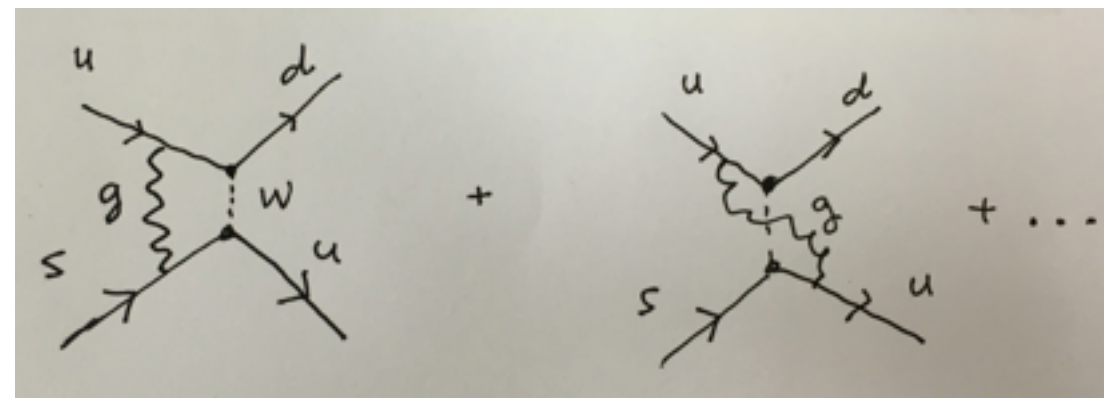
- Fermi lagrangian: description of the $\Delta S=1$ weak lagrangian, in particular the explanation of $\Delta I = 1/2$ rule

$$\frac{A(K^+ \rightarrow \pi^+ \pi^0)}{A(K_S \rightarrow \pi^+ \pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\bar{s}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



QCD at work, theoretical tools

- analytic calculation 't Hooft, large N_c (it explains basic phenomenological facts of QCD, i.e. Zweig's rule) many implications: Skyrme model, VMD, Maldacena
- G. Parisi, '80s lattice: can we predict from QCD the proton mass at 10% level?
- Precise calculation of low energy QCD?

Bardeen Buras Gerard approach to $K \rightarrow \pi\pi$

Also evaluated $\Delta S=2$ transitions, ϵ' (Buras) and $\pi^+ - \pi^0$ mass diff.

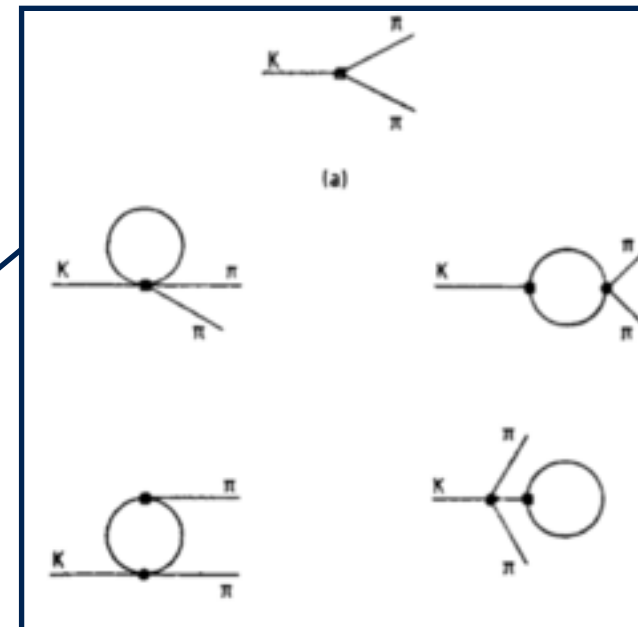
Main idea: phys. amplitudes scale independent

Match SD with LD with a precise prescription for CT

CHPT+Large N_c

$$H_{\text{eff}} = \sum_i C_i(\mu) Q_i(\mu)$$

SD



Can we test somewhere else
the Bardeen Buras Gerard
(BBG) approach?

Coluccio-Leskow, Estefania, GD, Greynat, David and Nath, Atanu

Matching a la BBG for $K^+ \rightarrow \pi^+ e^+ e^-$

Coluccio-Leskow, E. G.D ,Greynat, D and Nath, A

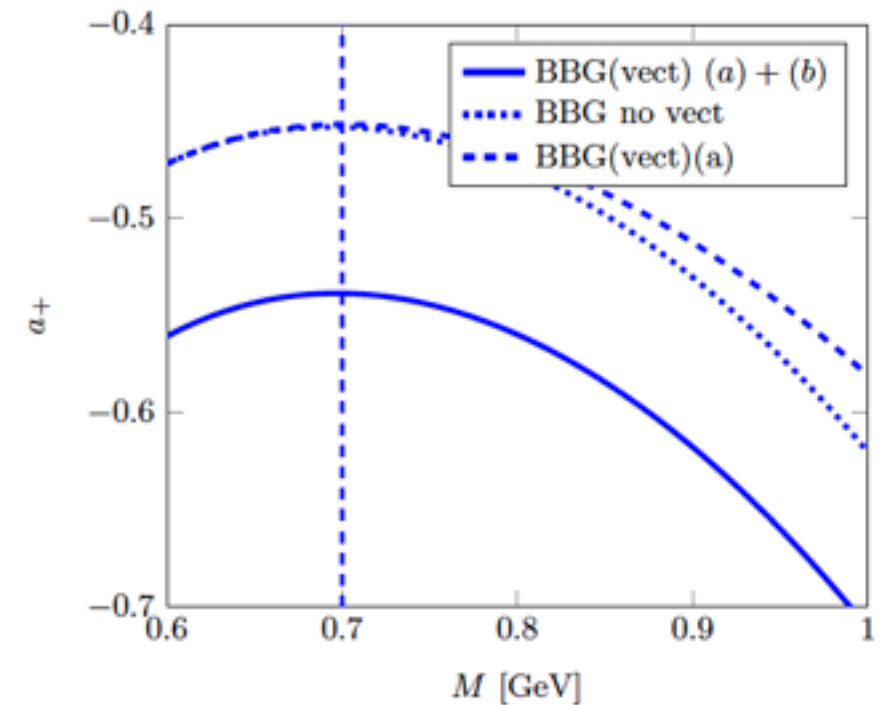
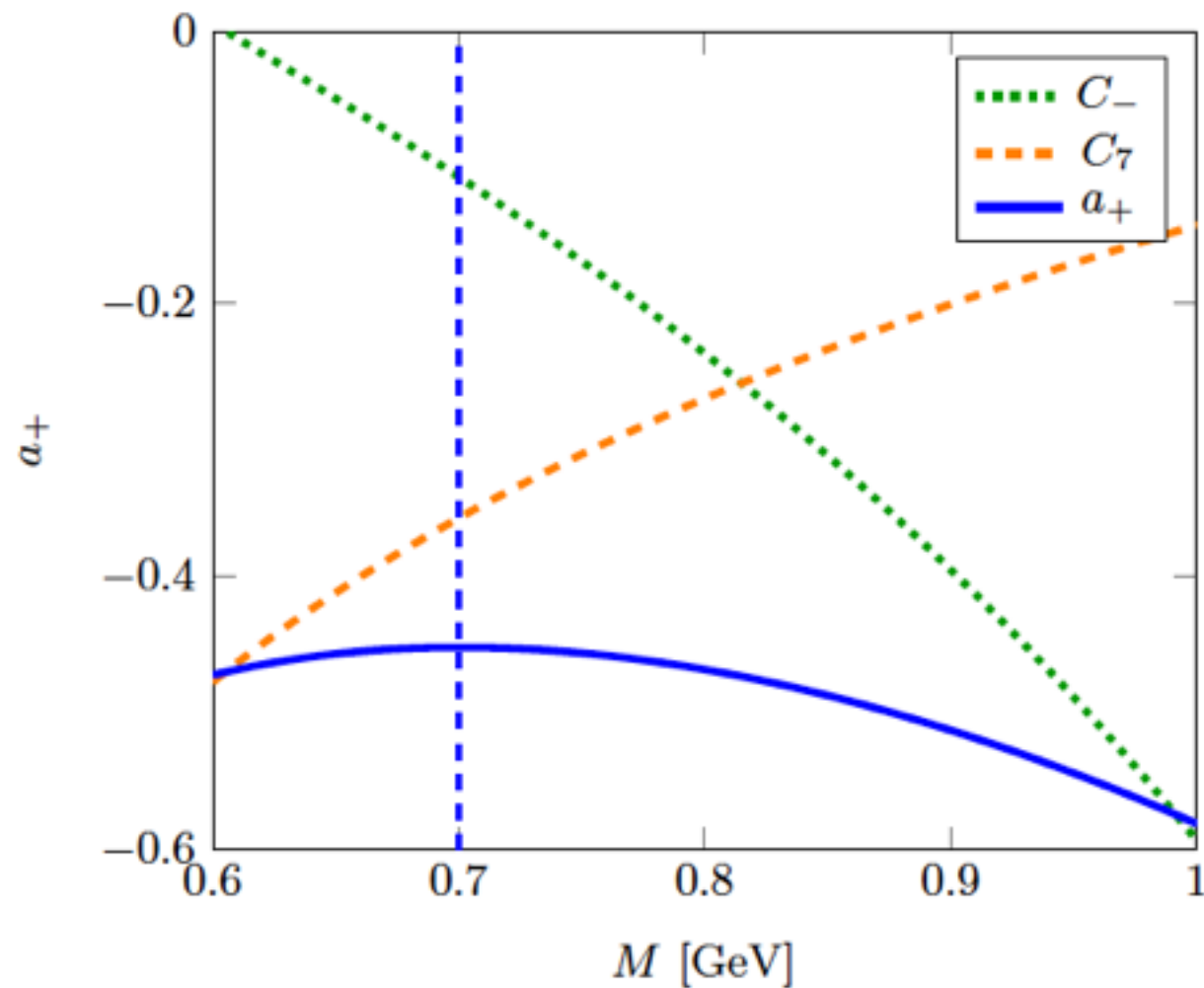
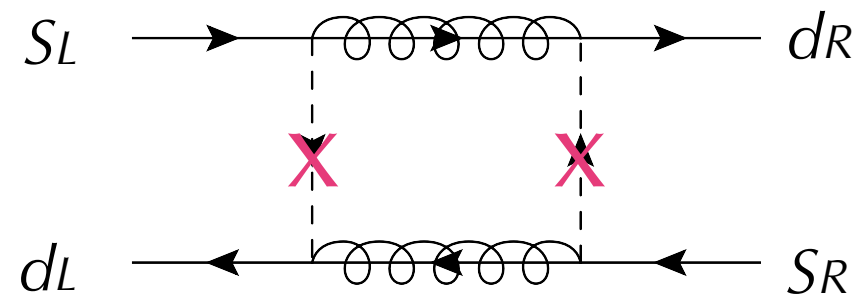


FIG. 5. a_+ as a function of M in the three different frameworks: 'BBG no vect.' where vectors are not included, 'BBG(vect)(a)' represents the contribution coming only from diagrams (a) in Fig. 4 and 'BBG(vect) (a) + (b)' is the case where both (a) and (b) diagrams were included. The vertical line indicates the value $M = 0.7$ GeV.

Main Constraint: ϵ_K ($\Delta S=2$, ID-CPV) cont.

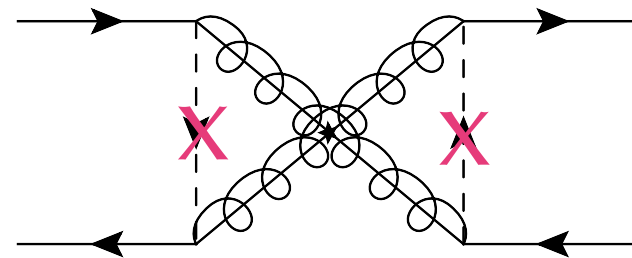
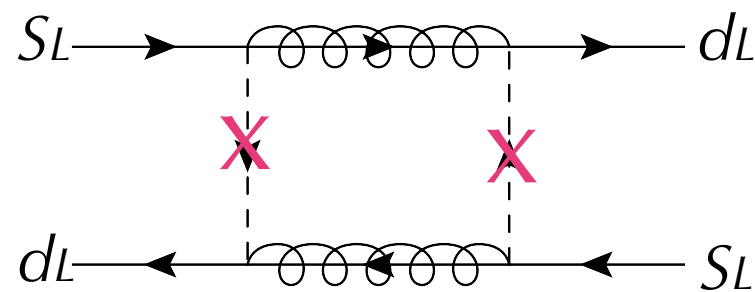
- The leading contribution is given by $\overline{d}_L s_L \overline{d}_R s_R$



$$\propto \left(\frac{m_K}{m_s + m_d} \right)^2$$

this contribution is suppressed when $\Delta_{\bar{D},12} \simeq 0$

- The next contribution is given by $\overline{d}_L s_L \overline{d}_L s_L$



Crossed diagram gives **relatively negative** contributions

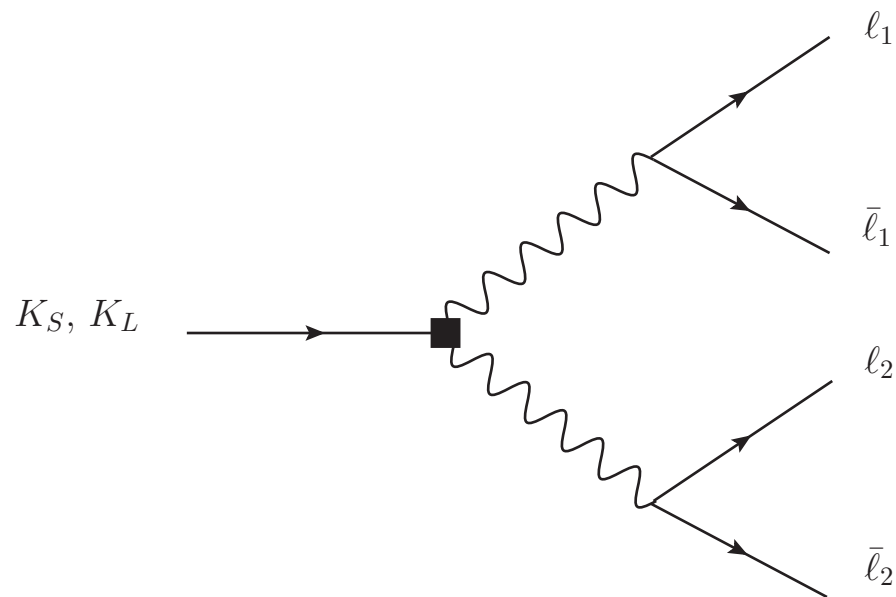
$m_{\tilde{g}} \simeq 1.5 m_{\tilde{q}}$: these contributions almost cancel out

[Crivellin, Davidkov '10]

$m_{\tilde{g}} \gtrsim 1.5 m_{\tilde{q}}$: suppressed by heavy gluino mass

Other interesting channels

$$\begin{array}{lll} K_S \rightarrow \mu\mu\mu\mu & - & \text{SM LD} \sim 2 \times 10^{-14} \\ K_S \rightarrow ee\mu\mu & - & \sim 10^{-11} \\ K_S \rightarrow eeee & - & \sim 10^{-10} \end{array}$$



GD, Greynat, Vulvert