Rapidity gap distribution in diffractive deep-inelastic scattering and parton genealogy

Stéphane Munier

Centre de physique théorique
École polytechnique, CNRS, Université Paris-Saclay

Based on work with A.H. Mueller (arXiv:1805.02847,1805.09417)
and with Dung Le (to appear)
Diffraction in optics

Beam of light

Fully absorptive disk

Shadow?
Diffraction in optics

Beam of light

Fully absorptive disk
Diffraction in quantum mechanics

Initial state
proton, electron, photon...

Fully absorptive disk
Diffraction in quantum mechanics

Initial state
proton, electron, photon...

Absorbed

$\sigma_{in}$

50%

Final state
same particle

$\sigma_{el}$

50%

Fully absorptive disk

$\sigma_{in} = \sigma_{el}$
To have diffraction, one needs strong absorption!
Diffraction in particle physics

Initial state
electron/virtual photon

Nucleus
Diffraction in particle physics

Initial state
electron/virtual photon

Final state
meson

Nucleus
Intact in the final state
quasi-elastic
Diffraction in particle physics

Initial state
electron/virtual photon

Final state
several hadrons

Nucleus

Intact in the final state
dissociative
Surprising phenomenon in high-energy DIS!
Its existence seems almost contradictory with the parton model...

*Its observation boosted saturation physics!*

[10% of HERA events were diffractive!]

[Golec-Biernat, Wüsthoff 1998]
Diffraction in deep-inelastic scattering

This talk: Diffractive dissociative electron-nucleus scattering

What is the distribution of the rapidity gap $y_0$?

$$\frac{d \sigma_{\text{diff}}}{dy_0}$$
Diffraction in deep-inelastic scattering

**This talk:** Diffractive dissociative electron-nucleus scattering

\[ \sim q \bar{q} \text{ dipole-nucleus} \]

[ Nikolaev, Zakharov '90, Golec-Biernat, Wüsthoff 1998]

What is the distribution of the rapidity gap \( y_0 \)?

\[
\frac{d\sigma_{\text{diff}}}{dy_0}
\]
Outline

- Equation for the gap distribution
- Picture of onium-nucleus scattering
- Ancestry in branching random walks
- Numerical checks
Equation for the gap distribution

1) Total amplitude: the Balitsky-Kovchegov equation

Forward elastic S-matrix element for dipole-nucleus scattering:

\[ S(r, y = 0) = e^{-\frac{r^2 Q_A^2}{4}} \]

[McLerran, Venugopalan]

\[ \sigma_{tot} = 2 \left( 1 - S \right) \]
Equation for the gap distribution

1) Total amplitude: the Balitsky-Kovchegov equation

Forward elastic S-matrix element for dipole-nucleus scattering:

\[ S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}} \]

\[ \sigma_{tot} = 2 \left(1 - S\right) \]

[McLerran, Venugopalan]

\[ \frac{1}{Q_A} = \text{MV saturation mom.} \]

Color transparency

Small dipoles

Full absorption

Large dipoles

\[ \partial_y S(r, y) = \alpha \int \frac{d^2 r'}{2\pi} \frac{r^2}{r^2(r-r')^2} \left[ S(r', y) S(r-r', y) - S(r, y) \right] \]

[Balitsky, Kovchegov 1996-1999]

\[ \frac{1}{Q_s(y)} = \text{y-dependent saturation mom.} \]
Equation for the gap distribution

2) Diffractive cross section: the Kovchegov-Levin equation

\[ S(r, y = 0) = e^{-\frac{r^2 Q_A^2}{4}} \]

\[ \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r - r')^2} \left[ S(r', y) S(r - r', y) - S(r, y) \right] \]

[Kovchegov, Levin 2000
Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos 2006]
Equation for the gap distribution

2) Diffractive cross section: the Kovchegov-Levin equation

\[ S(r, y = 0) = e^{-\frac{r^2 Q_s^2}{4}} \]

\[ \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \left[ S(r', y) S(r-r', y) - S(r, y) \right] \]

Define \( S_2 \) as

\[ S_2(r, y_0; y_0) = [S(r, y_0)]^2 \]

for \( y > y_0 \):

\[ \partial_y S_2(r, y; y_0) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} \]

\[ \times [S_2(r', y; y_0) S_2(r-r', y; y_0) - S_2(r, y; y_0)] \]

\[ \frac{d \sigma_{\text{diff}}}{d y_0} = -\frac{\partial}{\partial y_0} S_2(r, Y; y_0) \]

The solution to these equations is what we are after, but they are difficult to solve!
Outline

- Equation for the gap distribution
- Picture of onium-nucleus scattering
- Ancestry in branching random walks
- Numerical checks
Picture of onium-nucleus scattering

Total cross section

$$\sigma_{tot}(r, Y) = 2 \left[ 1 - S(r, Y) \right] \quad \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r - r')^2} [S(r', y) S(r - r', y) - S(r, y)]$$

Solution:

$$Q_s^2(Y) \approx Q_A^2 \frac{e^{const \times \bar{\alpha} Y}}{(\bar{\alpha} Y)^{3/2} \gamma_0} \quad 1 - S_{rQ_s(\bar{\gamma})} \ll 1 \quad \ln \frac{1}{r^2 Q_s^2(Y)} \left[ r^2 Q_s^2(Y) \right]^{\gamma_0} \ll 1 \quad (\gamma_0 \approx 0.63)$$
Picture of onium-nucleus scattering

Total cross section

\[ \sigma_{\text{tot}}(r, Y) = 2\left[1 - S(r, Y)\right] \]

\[ \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r - r')^2} [S(r', y) S(r - r', y) - S(r, y)] \]

Solution:

\[ Q_s^2(Y) \approx Q_A^2 e^{\text{const} \times \bar{\alpha} Y} \]

\[ 1 - S(r \bar{Q}_s(Y)) \ll 1 \]

\[ \ln \frac{1}{r^2 Q_s^2(Y)} [r^2 Q_s^2(Y)]^{\gamma_0} \ll 1 \]

\( (\gamma_0 \approx 0.63) \)
Picture of onium-nucleus scattering

Total cross section

\[ \sigma_{tot}(r, Y) = 2 \left[ 1 - S(r, Y) \right] \]

\[ \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r'^2}{r^2 (r - r')^2} [S(r', y) S(r - r', y) - S(r, y)] \]

Solution:

\[ Q_s^2(Y) \approx Q_A^2 \frac{e^{\text{const} \times \bar{\alpha} Y}}{(\bar{\alpha} Y)^{3/2} \gamma_0} \]

\[ 1 - S \ll \ln \frac{1}{r Q_s(\tilde{Y})} \ll 1 \]

\[ \bar{\alpha} \int d^2 r' \frac{r'^2}{r^2 (r - r')^2} \left[ r^2 Q_s^2(Y) \right]^{\gamma_0} \approx 0.63 \]

Onium restframe

1-S is like the transparency of the boosted nucleus
Picture of onium-nucleus scattering

Total cross section

\[ \sigma_{tot}(r, Y) = 2[1 - S(r, Y)] \]

\[ \partial_y S(r, y) = \tilde{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2(r-r')^2}[S(r', y)S(r-r', y) - S(r, y)] \]

Solution:

\[ Q_s^2(Y) \approx Q_A^2 \exp^{\text{const} \times \tilde{\alpha} Y} \]

\[ 1 - S \ll 1 \]

Onium restframe

1-S is like the transparency of the boosted nucleus

Nucleus restframe

1-S is like the probability to have a gluon of transverse momentum larger than the nuclear saturation momentum in the boosted onium Fock state

\[ \gamma_0 \approx 0.63 \]

\[ \ln \left( \frac{r^2 Q_s^2(Y)}{Q_s^2(Y)} \right)^{\gamma_0} \ll 1 \]
Picture of onium-nucleus scattering

Total and diffractive cross section in the $y_0$ frame in which the nucleus has rapidity $y_0$
The scattering between the state of the onium and the evolved nucleus needs to be elastic. This requires an unusually low-$k_T$ gluon at rapidity $y_0$. 
The scattering between the state of the onium and the evolved nucleus needs to be elastic. This requires an unusually low-$k_T$ gluon at rapidity $y_0$.

Probability given by the BK equation:

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 \left( \bar{\alpha} (Y - y_0) \right)} \right]^{3/2}$$
Outline

- Equation for the gap distribution
- Picture of onium-nucleus scattering
- Ancestry in branching random walks
- Numerical checks
Branching random walks
Branching random walks

Distribution of decay time of most recent common ancestor?
Branching random walks

\[
\frac{dp}{dt_0} = \frac{1}{\sqrt{4 \pi}} \times \left[ \frac{t}{t_0(t-t_0)} \right]^{3/2}
\]

Distribution of decay time of most recent common ancestor?

(Derrida, Mottishaw 2016)

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)
Branching random walks

\begin{align*}
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} &= \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2} \\
\bar{\alpha} Y &\sim t \\
\bar{\alpha} y_0 &\sim t_0
\end{align*}

Distribution of rapidity gaps:

\begin{align*}
\frac{dp}{dt_0} &= \frac{1}{\sqrt{4 \pi}} \times \left[ \frac{t}{t_0 (t - t_0)} \right]^{3/2} \\
\text{Distribution of decay time of most recent common ancestor?}
\end{align*}

(Derrida, Mottishaw 2016)

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)
Branching random walks

Distribution of rapidity gaps:

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}
\]

\(\bar{\alpha} Y \Leftrightarrow t\)

\(\bar{\alpha} y_0 \Leftrightarrow t_0\)

Distribution of decay time of most recent common ancestor?

\[
\frac{dp}{dt_0} = \frac{1}{\sqrt{4\pi}} \times \left[ \frac{t}{t_0(t-t_0)} \right]^{3/2}
\]

(Derrida, Mottishaw 2016)

(Physical reason for the correspondence: The common ancestor is an unusually large fluctuation! (here: large value of \(x\))
Outline

- Equation for the gap distribution
- Picture of onium-nucleus scattering
- Ancestry in branching random walks
- Numerical checks
Comparison to numerics

Very large-\(Y\) asymptotics

Theory: \[
\frac{1}{\sigma_{\text{tot}}} \frac{d \sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}
\]

Numerics: Solution to the Kovchegov-Levin eq.
Comparison to numerics

Realistic rapidities (EIC) \( \bar{\alpha} Y = 3 \)

Theory: \[ \frac{d\sigma_{\text{diff}}}{dy_0} \sim \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2} \]

Numerics: Solution to the Kovchegov-Levin eq.
Summary

We have predicted, up to an overall constant, the rapidity gap distribution in diffractive dissociation of a virtual photon off a nucleus:

\[
\frac{1}{\sigma_{\text{tot}}} \frac{d \sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}
\]

Partonic interpretation: the rapidity gap is due to a large fluctuation (unusually low-transverse momentum gluon) in the course of the QCD evolution of the onium Fock state.

There is a deep analogy between this distribution and the distribution of the time at which common ancestors of extreme particles in branching random walks decay.

Outlook

We may have an even closer analogy between diffraction and genealogy: The overall constant in the distributions may be the same!

*Work in progress...*