Hybrid and Pentaquark states

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Why Hadron Spectroscopy: Laboratory for studying non-pQCD & confinement.

### Why hadron spectroscopy

- **Asymptotic freedom**
  - High energy
  - Small distance

- **Effective degrees of freedom (models)**
  - Transition
  - 0.1 – 1 fm

- **Confinement**
  - Non-Perturbative
  - Low energy
  - Large distance
  - > 1 fm

#### Perturbative
- pQCD
  - << 0.1 fm

#### Non-Perturbative
- Mesons & Baryons
  - π

- Quantitative understanding of quark and gluon confinement
- Revealing the nature of the mass of the hadrons
- See the QCD degrees of freedom at work
- Validate lattice-QCD predictions
Hadron spectroscopy: lab. for QCD@ work

Bulk of mass of hadrons
Confinement
X,Y, Z, etc. new hadron states
• Finally to claim new physics also in other sectors, a precise knowledge of non perturbative QCD observables is necessary if they are involved!
The gluons and the meson spectrum

Neutralize color

... the simple way

\[ \begin{align*}
\bar{q} & \quad q \\
q & \quad \bar{q} \\
\end{align*} \]

mesons \quad baryons

... or the “exotic” way

\[ S = S_1 + S_2 \]
\[ J = L + S \]
\[ P = - (-1)^L \]
\[ C = (-1)^{L+S} \]

\( J^{PC} = 0^-, 0^+, 1^-, 2^+ \ldots \)

(flavor) exotic

molecules \quad pentaquark \quad glueball meson \quad hybrid meson

exotic of the II kind
Gluonic excitation models

**Flux tube model**
- Gluonic field confined in a tube between q and anti-q
- Linear Regge trajectories
- Hybrid mesons as transverse oscillation of the tube
- Flux-tube breaking give rise to meson decay

**Bag model**
- Quarks confined inside a cavity
- Full relativistic
- Gluonic excitation: gluonic field modes by boundary conditions

**CQM + constituent gluon**
- qq + massive transverse quasi-gluon ($J_g^{PgCg}$)
- Gluon adds in relative S-wave to a qq pair is S-wave or P-wave

qq in S-wave + $J_g^{PgCg}=l^+$ in S-wave

Lightest multiplet $(0,1,2)^+,l^+$

qq in P-wave + $J_g^{PgCg}=l^-$ in S-wave

Lightest multiplet $(0,1,2)^+,l^+$

or in Cb gauge QCD:
P.Guo, A.Szczepaniak, Galatà, Vasallo, E.S., PRD78, 056003(2008)

• Repulsive 3-body force selects $J_g^{PgCg}=l^+$ in relative P-wave added to a qq pair is S-wave or P-wave

qq in S-wave + $J_g^{PgCg}=l^+$ in P-wave

Lightest multiplet $(0,1,2)^+,l^-$

qq in P-wave + $J_g^{PgCg}=l^-$ in P-wave

Lightest multiplet $(0,1,2)^+,l^-$

Lightest multiplet $0^+, (1^+)^3, (2^+)^2, 3^-, 0^+, 0^+ l^+, 2^+$
Starting from the study of the glue-lamp (lamp or "grumo" of gluons or "constituent gluon") as obtained from QCD in physical gauge

it is easy to study the ccbar –gluon system, i.e. the hybrids (next two slides)
Flux tube and strings

\[ P \times C = +1 \]

\[ J^{PC} = 1^- \]

\[ P \times C = -1 \]

\[ J^{PC} = 1^+ \]

R \rightarrow 0  \quad \text{glue-lump}  \quad \text{"constituent gluons"}  \quad \text{R} \rightarrow \infty \quad \text{flux tube}  \quad \text{"gluon chain"}

Gluelamp

Guo, Szczepaniak, Vassallo, E.S., PRD 2008

Gluon chain model

Ostrander, Szczepaniak, Vassallo, E.S., PRD 2014
Charmonia (qq bar) & hybrids (qqg)

<table>
<thead>
<tr>
<th>$J_g^P$</th>
<th>This work [GeV]</th>
<th>$J^{PC}$</th>
<th>Lattice [14] [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1$^+$</td>
<td>4.476</td>
<td>0$^-$, 1-$^+$, 2$^-$, [1-$^-$]</td>
<td>4.291(48), 4.327(36), 4.376(24), (?)</td>
</tr>
<tr>
<td>1$^-$</td>
<td>4.762</td>
<td>1$^{++}$, 2$^{++}$, [0$^{++}$, 1$^{++}$]</td>
<td>4.521(48), 4.508(48), (?)</td>
</tr>
<tr>
<td>2$^+$</td>
<td>5.144</td>
<td>1$^+$, [2$^-$, 2$^-$, 3$^+$]</td>
<td>4.696(103), (??, ??)</td>
</tr>
<tr>
<td>2$^-$</td>
<td>5.065</td>
<td>2$^{--}$, [1$^{++}$, 2$^{++}$, 3$^{++}$]</td>
<td>4.733(42), (??, ??)</td>
</tr>
</tbody>
</table>

First exotic 1$^+$ (in agreement with lattice predictions)

The lightest hybrid supermultiplets

Y(4260)

X(3872)

The lightest hybrid supermultiplet predicted (and explained) for charmonia by QCD in physical gauge, $1^-(0,1,2)^+$, it is predicted also for light quarks by LQCD.

Physical gauge QCD (Hamiltonian)

$1^{+-} \times 0^{-+} = 1^{--}$

$1^{+-} \times 1^{--} = 0^{-+}, 1^{--}, 2^{--}$

Guo, Szczepaniak, Galatà, Vassallo, E.S., PRD2008

20XX experimental confirmation - discovery?
If, on the other hand, we place a constituent gluon as a P-wave gluonic field excitation, so that $J_g^P = 1^-+$, we appear to be able to successfully describe both the lightest hybrid supermultiplet of $(0, 1, 2)^-^+$, $1^-^+$ (by having $qar{q}$ in an S-wave) and the heavier exotic states, $0^+^-$, $(2^+^-)^2$ (with $qar{q}$ in a P-wave).

Y(4260) discovered by BaBar in $J/\psi \pi^+\pi^-$ (2005) confirmed by CLEO, Belle other modes from BaBar $J^{PC} = 1^-^-$ (from $e^+e^-$) width $\mathcal{O}(100\text{MeV})$

$M = 4252 \pm 6^{+2}_{-3}\text{MeV}$
$\Gamma = 105 \pm 18^{+4}_{-6}\text{MeV}$

*Belle (2007)*

*Theory: Hybrid candidate*
Pentaquark states
The LHCb observation [1] was further supported by another two articles by the same group [2,3]:

\[ \Lambda_b^0 \rightarrow J/\psi + \Lambda^*, \Lambda^* \rightarrow K^- + p \]
\[ \Lambda_b^0 \rightarrow P^{0+} + K^-, P^{0+} \rightarrow J/\Psi + p \]

\[ M_{P_c'}(4450) = (4449.8 \pm 8 \pm 29) \text{MeV} \]
\[ \Gamma = (39 \pm 5 \pm 19) \text{MeV} \]
\[ M_{P_c'}(4380) = (4380 \pm 1.7 \pm 2.5) \text{MeV} \]
\[ \Gamma = (205 \pm 18 \pm 86) \text{MeV} \]

statistic significance greater than 9 sigma!
Pentaquarks as compact five quark states,
E. S., A. Giachino, Phys. Rev. D 96 (2017), 014014

- Using group theory techniques we found that the compact pentaquark states belong to an SU(3) flavour octet.
- The masses of the octet pentaquark states were calculated by means of a Gürsey-Radicati mass formula extension.
- The partial decay widths were calculated by means of an effective Lagrangian:

\[
\Gamma^- = \left( \begin{array}{c} \gamma_5 \\ \gamma_\mu \end{array} \right), \quad \Gamma^- = \left( \begin{array}{c} \gamma_5 \\ 1 \end{array} \right).
\]

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Channel</th>
<th>Partial width [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p^{10} )</td>
<td>( \Lambda J/\Psi )</td>
<td>7.94</td>
</tr>
<tr>
<td>( p^{1-}, p^{10}, p^{1+} )</td>
<td>( \Sigma J/\Psi )</td>
<td>7.21</td>
</tr>
<tr>
<td>( p^{2-}, p^{20} )</td>
<td>( \Xi J/\Psi )</td>
<td>6.35</td>
</tr>
</tbody>
</table>
Near the thresholds, resonances are expected to have an exotic structure, like the hadronic molecules.

The observed pentaquarks are found to be just below the $\bar{D}^* \Sigma_c^+ (P_0^+(4380))$ and the $\bar{D}^* \Sigma^*_c (P_0^+(4450))$ thresholds. Moreover, the $\bar{D}^* \Lambda_c$ threshold is only 25 MeV below the $\bar{D} \Sigma_c$ threshold. For this reason, the $\bar{D} \Lambda_c, \bar{D}^* \Lambda_c$ channels are not irrelevant in the hidden-charm meson-baryon molecules.

In Phys. Rev. D96 (2017) no. 1, 014018 E. Santopinto e Y. Yamaguchi considered the coupled channel systems of $\bar{D} \Lambda_c, \bar{D}^* \Lambda_c, \bar{D} \Sigma_c, \bar{D} \Sigma^*_c, \bar{D}^* \Sigma_c$ and $\bar{D}^* \Sigma^*_c$ to predict the bound and the resonant states in the hidden-charm sector. The binding interaction between the meson and the baryon is given by the One Meson Exchange Potential (OMEP).
In particular the bound and resonant states with $J^P = \frac{3^+}{2}, \frac{3^-}{2}, \frac{5^+}{2}$ and $\frac{5^-}{2}$ with isospin $I = \frac{1}{2}$ are studied by solving the coupled channel Schrödinger equations.

Free parameter of the model: the cut-off parameter $\Lambda$;

$\Lambda$ is fixed to reproduce the heaviest resonant...
Coupled channel between the meson-baryon states

Good agreement for the mass and quantum numbers of the lightest pentaquark $P_c^+(4380)$

The masses and widths of the two observed pentaquark states; BE AWARE: the mass of the lightest one is a prediction, while the mass of the heaviest is fitted to fix the cut-off parameter $\Lambda$

<table>
<thead>
<tr>
<th>$\Lambda$ [MeV]</th>
<th>1300</th>
<th>1400</th>
<th>1500</th>
<th>1600</th>
<th>1700</th>
<th>1800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^P = 3/2^-$</td>
<td>4236.9 − i0.8</td>
<td>4136.0</td>
<td>4006.3</td>
<td>3848.2</td>
<td>3660.0</td>
<td>3438.26</td>
</tr>
<tr>
<td></td>
<td>4381.3 − i11.4</td>
<td>4307.9 − i18.8</td>
<td>4242.6 − i1.4</td>
<td>4150.1</td>
<td>4035.2</td>
<td>3897.3</td>
</tr>
<tr>
<td></td>
<td>4368.5 − i64.9</td>
<td>4348.7 − i21.1</td>
<td>4312.7 − i16.0</td>
<td>4261.0 − i7.0</td>
<td>4187.7 − i0.9</td>
<td>4092.5</td>
</tr>
<tr>
<td>$J^P = 3/2^+$</td>
<td>4223.0 − i97.9</td>
<td>4206.7 − i41.2</td>
<td>4169.3 − i5.3</td>
<td>4104.2</td>
<td>3996.7</td>
<td>3855.8</td>
</tr>
<tr>
<td></td>
<td>4363.3 − i57.0</td>
<td>4339.7 − i26.8</td>
<td>4311.8 − i6.6</td>
<td>4268.5 − i1.3</td>
<td>4193.2 − i0.1</td>
<td>4091.6</td>
</tr>
<tr>
<td>$J^P = 5/2^-$</td>
<td>—</td>
<td>4428.6 − i89.1</td>
<td>4391.7 − i88.8</td>
<td>4338.2 − i56.2</td>
<td>4286.8 − i27.3</td>
<td>4228.3 − i7.4</td>
</tr>
<tr>
<td>$J^P = 5/2^+$</td>
<td>—</td>
<td>—</td>
<td>4368.0 − i9.2</td>
<td>4305.8 − i1.9</td>
<td>4222.7 − i1.4</td>
<td>4111.1</td>
</tr>
</tbody>
</table>

The masses and widths of the two observed pentaquark states; BE AWARE: the mass of the lightest one is a prediction, while the mass of the heaviest is fitted to fix the cut-off parameter $\Lambda$.
Upgrade of the model: Coupled channel between the meson-baryon states and the five quark states

In the current problem of pentaquark $P_c$, there are two competing sets of channels: the meson-baryon (MB) channels and the five-quark channels.

Can a couple channel between the MB channels and the core contribution describe in a more realistic way the pentaquark states?
Coupled channel between the meson-baryon states and the five quark states


- Hidden-charm pentaquarks as $\bar{D} \Lambda_c, \bar{D}^* \Lambda_c, \bar{D} \Sigma_c$, $\bar{D}^* \Sigma_c$, $\bar{D} \Sigma^*_c$, and $\bar{D}^* \Sigma^*_c$, and molecules coupled to the five-quark states

**ADDITION OF THE CORE CONTRIBUTION**

- For the first time some predictions for the hidden-bottom pentaquarks as $\bar{D} \Lambda_c, \bar{D}^* \Lambda_c, \bar{D} \Sigma_c, \bar{D}^* \Sigma_c, \bar{D} \Sigma^*_c$ and $\bar{D}^* \Sigma^*_c$ molecules coupled to the five-quark states are provided.

- In particular, by solving the coupled channel Schrödinger equation, we study the the bound and resonant hidden-charm
The meson-baryon channels describe the dynamics at long distances, while the five-quark part describes the dynamics at short distances (of the order of 1 fm or less).
Results for the hidden-charm sector

The lowest threshold $\bar{D} \Lambda_c$ is at 4153.46 MeV and the state whose energy is lower than the threshold is a bound state.

No resonant states and no bound states for $\frac{f}{f_0} = 0$

In the hidden-charm sector the OPEP is not enough strong to produce bound and resonant $P_c$ states.
We found that, unlike the charm-sector, in which the five quark potential is needed to produce bound states, in the bottom sector the OPEP provides sufficiently strong attraction to generate several bound and resonant states.

Dot-dashed lines are the $\bar{B} \Lambda_B$ and $\bar{B}^* \Lambda_B$ thresholds. Dashed lines are the $\bar{B} \Sigma_B, \bar{B} \Sigma^*_B, \bar{B}^* \Sigma_B$ and $\bar{B}^* \Sigma^*_B$ thresholds.
Moreover, many states appear, when the 5q potential is switched on.

The hidden-bottom pentaquarks are more likely to form than the hidden-charm pentaquarks.

The hidden-bottom sector is an interesting environment to search for pentaquark states.
In the hidden bottom sector, the kinetic energy of the meson-baryon system is suppressed with respect to the charm sector due to the higher mass of the system.

Why bound and resonant states are more likely to be found in the bottom sector?

- In the hidden-bottom sector, the OPEP is strong enough to produce states due to the mixing effect enhanced by the small mass splitting between $B, B^*$ and $\Sigma_B, \Sigma^*_B$.
• What does it happen if one consider a coupled channel MB-core with a OMEP?

• So far in our analysis we have studied only the negative parity states dominated by the s-wave configurations. For positive parity states, we need p-wave excitations for both meson-baryon and for 5q states.

These tasks require further technical developments which will be a future work.
Unquenching the quark model for the MESONS & Why Unquenching?

Santopinto, Galatà, Ferretti, Vassallo
UQM: Meson Self Energies & couple channels

- Hamiltonian:

\[ H = H_0 + V \]

- \( H_0 \) act only in the bare meson space and it is chosen the Godfray and Isgur model

- \( V \) couples \(|A>\) to the continuum \(|BC>\)

- Dispersive equation

\[ \Sigma(E_a) = \sum_{BC} \int_0^\infty q^2 dq \frac{|V_{a, bc}(q)|^2}{E_a - E_{bc}} \]

- from non-relativistic Schrödinger equation

- Bare energy \( E_a \) (\( H_0 \) eigenvalue) satisfies:

\[ M_a = E_a + \Sigma(E_a) \]

- \( M_a \) = physical mass of meson A

- \( \Sigma(E_a) \) = self energy of meson A
• Coupling $V_{a, bc}(q)$ in $\Sigma(E_a)$ calculated as:

  Sum over a complete set of accessible ground state (1S) mesons

  Coupling calculated in the $^3P_0$ model

• Two possible diagrams contribute:

• Self energy in the UQM:

  $$\Sigma(E_a) = \sum_{BC\ell J} \int_0^\infty q^2 dq \frac{|\langle BCq\ell J | T^i | A \rangle|^2}{E_a - E_b - E_c}$$
Godrey and Isgur model as bare mass

- Bare energies $E_a$ calculated in the relativized G.I.Model for mesons
- Hamiltonian:
  \[ H = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + V_{\text{conf}} + V_{\text{hyp}} + V_{\text{so}} \]
- Confining potential:
  \[ V_{\text{conf}} = -\left( \frac{3}{4} c + \frac{3}{4} br - \frac{\alpha_s(r)}{r} \right) \vec{F}_1 \cdot \vec{F}_2 \]
- Hyperfine interaction:
  \[ V_{\text{hyp}} = -\frac{\alpha_s(r)}{m_1 m_2} \left[ \frac{8\pi}{3} \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r}) \right. \]
  \[ + \left. \frac{1}{r^3} \left( \frac{3}{r^2} \vec{S}_1 \cdot \vec{r} \vec{S}_2 \cdot \vec{r} - \vec{S}_1 \cdot \vec{S}_2 \right) \right] \vec{F}_i \cdot \vec{F}_j \]
- Spin-orb. :
  \[ V_{\text{so,cm}} = -\frac{\alpha_s(r)}{r^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j \]
  \[ V_{\text{so,tp}} = -\frac{1}{2r} \frac{\partial H_{ij}^{\text{conf}}}{\partial r} \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \]
UQM or couple channel Quark Model

- Parameters of the relativized QM fitted to

\[ M_a = E_a + \Sigma(E_a) \]

- Recursive fitting procedure

- \( M_a \) = calculated physical masses of q bar-q mesons \( \rightarrow \) reproduce experimental spectrum [PDG]

- Intrinsic error of QM/UQM calculations: 30-50 MeV
UQM: charmonium with self-energy corr.

- Parameters of the UQM ($^3P_0$ vertices)

- fitted to:
M [X(3872); UQM] = 3908 MeV
**UQM**: charmonium with self-energy corr.

- Experimental mass: $3871.68 \pm 0.17$ MeV [PDG]
- Several predictions for $X(3872)$'s mass. Here: $c\overline{c} +$ continuum effects

<table>
<thead>
<tr>
<th>$\chi_{c1}(2^3P_1)$'s mass (MeV)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>3908</td>
<td>[1]</td>
</tr>
<tr>
<td>4007.5</td>
<td>[2]</td>
</tr>
<tr>
<td>3990</td>
<td>[3]</td>
</tr>
<tr>
<td>3920.5</td>
<td>[4]</td>
</tr>
<tr>
<td>3896</td>
<td>[5]</td>
</tr>
</tbody>
</table>

Interpretation of the X(3872) as a charmonium state plus an extra component due to the coupling to the meson-meson continuum


- UCQM results used to study the problem of the X(3872) mass, meson with $J^{PC} = 1^{++}, 2^{3}P_{1}$ quantum numbers
- Experimental mass: 3871.68 ± 0.17 MeV [PDG]
- X(3872) very close to D bar-D* decay threshold
- Possible importance of continuum coupling effects?
- Several interpretations: pure $c$ bar-$c$
  - D bar-D* molecule
  - tetraquark
  - $c$ bar-$c$ + continuum effects

necessary to study strong and radiative decays to understand the situation
Radiative decays

Ferretti, Galatà, Santopinto, Phys. Rev. D90 (2014) 5, 054010

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_\gamma$ [MeV]</th>
<th>$\Gamma_{cc}$ [KeV]</th>
<th>$\Gamma_{DD^*}$ [KeV]</th>
<th>$\Gamma_{DD^*}$ [KeV]</th>
<th>$\Gamma_{cc+DD^*}$ [KeV]</th>
<th>$\Gamma_{exp.}$ [KeV]</th>
<th>PDG [43]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872) \rightarrow J/\Psi\gamma$</td>
<td>697</td>
<td>11</td>
<td>8</td>
<td>64 – 190</td>
<td>125 – 251</td>
<td>2 – 17</td>
<td>$\approx$ 7</td>
</tr>
<tr>
<td>$X(3872) \rightarrow \Psi(2S)\gamma$</td>
<td>181</td>
<td>70</td>
<td>0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X(3872) \rightarrow \Psi(3770)\gamma$</td>
<td>101</td>
<td>4.0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X(3872) \rightarrow \Psi_2(1^3D_2)\gamma$</td>
<td>34</td>
<td>0.35</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[7] Swanson: molecular interpretation
[59]-[60] Faessler: molecular; ccbar +molecular

The Molecular model does not predict radiative decays into $\Psi(3770)$ and $\Psi_2(1^3D_2)\gamma$. Possible way to distinguish between the two interpretations
Bottomonium spectrum
(in couple channel calculations)
Ferretti, Santopintio, Phys.Rev. D90, 094022 (2014)

- Parameters of the UQM ($^3P_0$ vertices)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_0$</td>
<td>0.732</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.500 GeV</td>
</tr>
<tr>
<td>$r_Q$</td>
<td>0.335 fm</td>
</tr>
<tr>
<td>$m_n$</td>
<td>0.330 GeV</td>
</tr>
<tr>
<td>$m_s$</td>
<td>0.550 GeV</td>
</tr>
<tr>
<td>$m_c$</td>
<td>1.50 GeV</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4.70 GeV</td>
</tr>
</tbody>
</table>

- Pair-creation strength $\gamma_0$ fitted to:

$$\Gamma_{\Upsilon(4S) \rightarrow BB} = 2\Phi_{A \rightarrow BC} \left| \langle BC\bar{q}_0 \ell J | T^\dagger | A \rangle \right|^2$$
$$= 2\Phi_{\Upsilon(4S) \rightarrow B\bar{B}} \left| \langle B\bar{B}\bar{q}_0 11 | T^\dagger | \Upsilon(4S) \rangle \right|^2$$
$$= 21 \text{ MeV} ,$$
### Bottomonium Strong Decays

Ferretti, Santopinto, Phys.Rev. D90 094022 (2014)

<table>
<thead>
<tr>
<th>State</th>
<th>Mass [MeV]</th>
<th>$J^{PC}$</th>
<th>$B B^*$</th>
<th>$B^* B$</th>
<th>$B_s B_{s^*}$</th>
<th>$B_s B^*$</th>
<th>$B_{s^<em>} B^</em>$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(4^3,S_1)$</td>
<td>10.595</td>
<td>1$^{--}$</td>
<td>21</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\chi_b(2^3,D_2)$</td>
<td>10585</td>
<td>2$^{++}$</td>
<td>34</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Upsilon(3^3,D_1)$</td>
<td>10661</td>
<td>1$^{--}$</td>
<td>23</td>
<td>4</td>
<td>15</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Upsilon_2(3^3,D_2)$</td>
<td>10667</td>
<td>2$^{--}$</td>
<td>-</td>
<td>37</td>
<td>30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\Upsilon(3^3,D_2)$</td>
<td>10668</td>
<td>2$^{++}$</td>
<td>-</td>
<td>55</td>
<td>57</td>
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Bottomonium spectrum (in couple channel calculations)

Ferretti, Santopinto, Phys.Rev. D90, 094022 (2014)

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Results:
Couple Channels corrections to Bottomonium, the $\chi_b(3P)$ system

Ferretti, Santopintio, Phys.Rev. D90 (2014) 9, 094022  : 1306.2874

- Results used to study some properties of the $\chi_b(3P)$ system, meson multiplet with N=3, L=1 quantum numbers
- $\chi_b(3P)$ states close to first open bottom decay thresholds
- Possible importance of continuum coupling effects?
- **Pure c bar-c and c bar-c + continuum effects** interpretations
- Necessary to study decays (strong, e.m., hadronic, ...) to confirm one interpretation
Couple Channels corrections to Bottomonium, the $\chi_b(3P)$ system
Ferretti, Santopintio, Phys.Rev. D90 (2014) 9, 094022

- Some experimental results for the mass barycenter of the system:
  - $M[\chi_b(3P)] = 10.530 \pm 0.005$ (stat.) $\pm 0.009$ (syst.) GeV
  - $M[\chi_b(3P)] = 10.551 \pm 0.014$ (stat.) $\pm 0.017$ (syst.) GeV
  - Abazov et al. [D0 Coll.], Phys. Rev. D 86, 031103 (2012)

- Mass barycenter in the UQM:
Back-up Slides