Dynamical picture for the exotic XYZ states

M.A. Ivanov (JINR, Dubna)

QCD@Work

"International Workshop on QCD, theory and experiment"

25-28/June, 2018, Matera, Italy
Contents

Introduction

Dynamical picture for multiquark states

X(3872) as tetraquark

Z_c(3900)

Z_b(10610) and Z'_b(10610)

Summary
Exotics

- The elementary constituents in QCD are quarks $q$, antiquarks $\bar{q}$, and gluons $g$.

- They are confined into color-singlet hadrons.

- The most stable hadrons predicted by the quark model: conventional mesons $q\bar{q}$, baryons $qqq$ and antibaryons $\bar{q}\bar{q}\bar{q}$.

- This simple picture was changed since 2003 with the discovery of almost two dozen charmonium- and bottomonium-like $XYZ$ states that do not fit the naive quark-antiquark interpretation.
**J^PC = 1^{--}, neutral**

**Production** $e^+e^- \rightarrow Y$

**Y has $c\bar{c}$ pair**

**But Y is not simple charmonium**

**Examples:** $Y(4005), Y(4260), Y(4360), Y(4660)$
Z (Z_c and Z_b)

- Z_c has c\bar{c} pair and a charge
- Thus minimal quark content of Z_c^+ is c\bar{c}u\bar{d} (exotic state!)
- Usually the isospin of the Z is 1, neutral partner should exist.
- Z_b has b\bar{b} pair and a charge
- Examples: Z_b(10610), Z_b(10650), Z_c(3900), Z_c(4200), Z_c(4430), etc.
XYZ: short introduction

- $X'$s are the non-$q\bar{q}$ mesons other than $Y'$s and $Z'$s
- Most famous is $X(3872)$ observed by Belle in reaction

$$B^+ \rightarrow K^+ \underbrace{\pi^+ \pi^- J/\psi}_{X}$$
X(3872)

- X-mass is close to $D^0 - D^{*0}$ mass threshold:

$$M_X = 3872.0 \pm 0.6 \text{ (stat)} \pm 0.5 \text{ (syst)} \text{ MeV}$$

$$M_{D^0} + M_{D^{*0}} = 3871.81 \pm 0.25 \text{ MeV}$$

- Its width $\Gamma_X \leq 2.3 \text{ MeV}$ at 90% CL.

- Quantum numbers $J^{PC} = 1^{++}$.

- Strong isospin violation

$$\frac{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^- \pi^0)}{\mathcal{B}(X \rightarrow J/\psi \pi^+ \pi^-)} = 1.0 \pm 0.4 \text{ (stat)} \pm 0.3 \text{ (syst).}$$
An interpretation of the $X(3872)$ as a tetraquark was suggested in


$$X_q \rightarrow [cq]_{S=1} [\bar{c}\bar{q}]_{S=0} + [cq]_{S=0} [\bar{c}\bar{q}]_{S=1}, \quad (q = u, d)$$

The physical states are the mixing of $X_u$ and $X_d$

$$X_l \equiv X_{\text{low}} = X_u \cos \theta + X_d \sin \theta,$$

$$X_h \equiv X_{\text{high}} = -X_u \sin \theta + X_d \cos \theta.$$

The mixing angle $\theta$ is supposed to be found from the known ratio of the two-pion (via $\rho$) and three-pion (via $\omega$) decay widths.
Dynamical picture for multiquark states: covariant confined quark model

- Main assumption: hadrons interact via quark exchange only
- Interaction Lagrangian

\[ \mathcal{L}_{\text{int}} = g_H \cdot H(x) \cdot J_H(x) \]

- Quark currents

\[
\begin{align*}
J_M(x) &= \int dx_1 \int dx_2 \ F_M(x; x_1, x_2) \cdot \bar{q}_1^a(x_1) \Gamma_M q_2^a(x_2) \quad \text{Meson} \\
J_B(x) &= \int dx_1 \int dx_2 \int dx_3 \ F_B(x; x_1, x_2, x_3) \\
&\quad \times \Gamma_1 q_1^{a_1}(x_1) \left( q_2^{a_2}(x_2) C \Gamma_2 q_3^{a_3}(x_3) \right) \cdot \varepsilon^{a_1a_2a_3} \quad \text{Baryon} \\
J_T^\mu(x) &= \int dx_1 \ldots \int dx_4 \ F_T(x; x_1, \ldots, x_4) \\
&\quad \times \left( q_1^{a_1}(x_1) C \Gamma_1 q_2^{a_2}(x_2) \right) \left( \bar{q}_3^{a_3}(x_3) C \Gamma_2 q_4^{a_4}(x_4) \right) \cdot \varepsilon^{a_1a_2c} \varepsilon^{a_3a_4c} \quad \text{Tetraquark}
\end{align*}
\]
The vertex functions and quark propagators

- The vertex functions

\[ F_H(x, x_1, \ldots, x_n) = \delta^{(4)}(x - \sum_{i=1}^{n} w_i x_i) \Phi_H \left( \sum_{i < j} (x_i - x_j)^2 \right) \]

where \( w_i = \frac{m_i}{\sum_i m_i} \).

- We choose a Gaussian form for the function \( \Phi_H \) with the only dimensional parameter \( \Lambda_H \) characterizing the size of the hadron.

- The quark propagators

\[ S_q(x_1 - x_2) = \int \frac{d^4k}{(2\pi)^4i} \frac{e^{-ik(x_1-x_2)}}{m_q - k} \]

- The matrix elements of the physical processes are described by the Feynman diagrams which are the convolution of vertex functions and quark propagators.
Let us consider a general $\ell$-loop Feynman diagram with $n$ local propagators and $m$ Gaussian vertices.

Use the Schwinger representation of the propagator:

$$\frac{m + k}{m^2 - k^2} = (m + k) \int_0^\infty d\alpha \exp[-\alpha(m^2 - k^2)]$$

The general expression for the diagram

$$\Pi(p_1, \ldots, p_m) = \int_0^\infty d^n\alpha \int [d^4k]^\ell \text{Num} \exp[-\sum_{i=1}^n \alpha_i m_i^2 + \sum_j \tilde{\alpha}_j (K_j + P_j)^2]$$

where $K_i$ is the linear combination of the loop momenta and $P_i$ is the linear combination of the external momenta. $\text{Num}$ stands for the numerator product of propagators.
Go to integration over a simplex

- Generally speaking, the diagram contains the branch points and thresholds corresponding to quark production.

- After doing the loop integrations one obtains

\[
\Pi = \int_{0}^{\infty} d^{n} \alpha \ F(\alpha_{1}, \ldots, \alpha_{n}),
\]

where \( F \) stands for the whole structure of a given diagram.

- The set of Schwinger parameters \( \alpha_{i} \) can be turned into a simplex by introducing an additional \( t \)-integration via the identity

\[
1 = \int_{0}^{\infty} dt \ \delta(t - \sum_{i=1}^{n} \alpha_{i})
\]

leading to

\[
\Pi = \int_{0}^{\infty} dt \ t^{n-1} \int_{0}^{1} d^{n} \alpha \ \delta(1 - \sum_{i=1}^{n} \alpha_{i}) \ F(t\alpha_{1}, \ldots, t\alpha_{n}).
\]
Infrared confinement

- We cut the upper integration over “t” at $1/\lambda^2$ and obtain

$$\Pi^c = \frac{1}{\lambda^2} \int_0^1 dtt^{n-1} \int_0^1 d^n\alpha \delta \left( 1 - \sum_{i=1}^n \alpha_i \right) F(t\alpha_1, \ldots, t\alpha_n)$$

- By introducing the infrared cut-off one has removed all possible thresholds in the quark loop diagram.

- We take the cut-off parameter $\lambda$ to be the same in all physical processes.
Infrared confinement

- We consider the case of a scalar one-loop two-point function:

\[ \Pi_2(p^2) = \int \frac{d^4k_E}{\pi^2} \frac{e^{-s k_E^2}}{[m^2 + (k_E + \frac{1}{2} p_E)^2][m^2 + (k_E - \frac{1}{2} p_E)^2]} \]

where the numerator factor \( e^{-s k_E^2} \) comes from the product of nonlocal vertex form factors of Gaussian form. \( k_E, p_E \) are Euclidean momenta \( (p_E^2 = - p^2) \).

- Doing the loop integration one obtains

\[ \Pi_2(p^2) = \int_0^\infty dt \frac{t}{(s + t)^2} \int_0^1 d\alpha \exp \left\{ -t [m^2 - \alpha(1 - \alpha)p^2] + \frac{st}{s + t} \left(\alpha - \frac{1}{2}\right)^2 p^2 \right\} \]

A branch point at \( p^2 = 4m^2 \)
Infrared confinement

By introducing a cut-off in the $t$–integration one obtains

$$
\Pi_c^2(p^2) = \frac{1}{\lambda^2} \int_0^{1/\lambda^2} \int_0^1 dt \frac{t}{(s+t)^2} d\alpha \exp \left\{ -t \left[ m^2 - \alpha(1 - \alpha)p^2 \right] + \frac{st}{s+t} \left( \alpha - \frac{1}{2} \right)^2 p^2 \right\}
$$

where the one–loop two–point function $\Pi_c^2(p^2)$ no longer has a branch point at $p^2 = 4m^2$.

The confinement scenario also allows to include all possible both two-quark and multi-quark resonance states in our calculations.
The values of quark masses $m_{q_i}$, the infrared cutoff parameter $\lambda$ and the size parameters $\Lambda_{H_i}$ have been defined by the fit to the well-known physical observables.

<table>
<thead>
<tr>
<th>$m_u$</th>
<th>$m_s$</th>
<th>$m_c$</th>
<th>$m_b$</th>
<th>$\lambda$</th>
<th>GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.241</td>
<td>0.428</td>
<td>1.672</td>
<td>5.046</td>
<td>0.181</td>
<td></td>
</tr>
</tbody>
</table>
An effective interaction Lagrangian

$$\mathcal{L}_{\text{int}} = g_{X} \, X_{q \, \mu}(x) \cdot J_{X_{q}}^{\mu}(x), \quad (q = u, d).$$

The nonlocal version of the four-quark interpolating current

$$J_{X_{q}}^{\mu}(x) = \int dx_{1} \ldots \int dx_{4} \, \delta(x - \sum_{i=1}^{4} w_{i} x_{i}) \, \Phi_{X} \left( \sum_{i < j} (x_{i} - x_{j})^{2} \right) \, J_{4q}^{\mu}(x_{1}, \ldots, x_{4})$$

$$J_{4q}^{\mu} = \frac{1}{\sqrt{2}} \, \epsilon_{abc} \left[ q_{a}(x_{4}) C \gamma^{5} c_{b}(x_{1}) \right] \epsilon_{dec} \left[ \bar{q}_{d}(x_{3}) \gamma^{\mu} \bar{c}_{e}(x_{2}) \right] + (\gamma^{5} \leftrightarrow \gamma^{\mu}),$$

$$w_{1} = w_{2} = \frac{m_{c}}{2(m_{q} + m_{c})} \equiv \frac{w_{c}}{2}, \quad w_{3} = w_{4} = \frac{m_{q}}{2(m_{q} + m_{c})} \equiv \frac{w_{q}}{2}.$$
The coupling constant $g_X$ is determined from the compositeness condition

$$Z_X = 1 - \Pi'_X(M_X^2) = 0$$

where $\Pi_X(p^2)$ is the scalar part of the vector-meson mass operator.
Since the $X(3872)$ lies nearly the respective thresholds in both cases, 

$$m_X - (m_{J/\psi} + m_\rho) = -0.90 \pm 0.41 \text{ MeV},$$

$$m_X - (m_{D^0} + m_{D^{*0}}) = -0.30 \pm 0.34 \text{ MeV}$$

the intermediate $\rho(\omega)$ and $D^*$ mesons should be taken off-shell.
The narrow width approximation

\[
\frac{d\Gamma(X \to J/\psi + n\pi)}{dq^2} = \frac{1}{8 m_X^2 \pi} \cdot \frac{1}{3} |M(X \to J/\psi + v^0)|^2 \\
\times \Gamma_{v^0} \frac{m_{v^0}}{\pi} \frac{p^*(q^2)}{(m_{v^0}^2 - q^2)^2 + \Gamma_{v^0}^2 m_{v^0}^2} \text{Br}(v^0 \to n\pi),
\]

\[
\frac{d\Gamma(X_u \to \bar{D}^0 D^0 \pi^0)}{dq^2} = \frac{1}{2 m_X^2 \pi} \cdot \frac{1}{3} |M(X_u \to \bar{D}^0 D^{*0})|^2 \\
\times \Gamma_{D^{*0}} \frac{m_{D^{*0}}}{\pi} \frac{p^*(q^2) \mathcal{B}(D^{*0} \to D^0 \pi^0)}{(m_{D^{*0}}^2 - q^2)^2 + \Gamma_{D^{*0}}^2 m_{D^{*0}}^2},
\]
Strong decay widths

- Two new adjustable parameters: $\theta$ and $\Lambda_X$.

- The ratio
  \[
  \frac{\Gamma(X_u \to J/\psi + 3\pi)}{\Gamma(X_u \to J/\psi + 2\pi)} \approx 0.25
  \]
  is very stable under variation of $\Lambda_X$.

- Using this result and the central value of the experimental data
  \[
  \frac{\Gamma(X_{l,h} \to J/\psi + 3\pi)}{\Gamma(X_{l,h} \to J/\psi + 2\pi)} \approx 0.25 \cdot \left(\frac{1 \mp \tan \theta}{1 \mp \tan \theta}\right)^2 \approx 1
  \]
  gives $\theta \approx \pm 18.4^\circ$ for $X_l$ (" + ") and $X_h$ (" − "), respectively.

- This is in agreement with the results obtained by both Maiani: $\theta \approx \pm 20^\circ$ and Nielsen: $\theta \approx \pm 23.5^\circ$. 
Strong decay widths

\[ \Gamma(X \to D^0 \bar{D}^0 \pi^0), \text{ MeV} \]

\[ \Gamma(X \to J/\psi \pi^+ \pi^-), \text{ MeV} \]

\[ \frac{\Gamma(X \to D^0 \bar{D}^0 \pi^0)}{\Gamma(X \to J/\psi \pi^+ \pi^-)} = \begin{cases} 
4.5 \pm 0.2 & \text{theor} \\
10.5 \pm 4.7 & \text{expt} 
\end{cases} \]
Radiative $X$-decay

S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov, J. G. Koerner, P. Santorelli and G. G. Saidullaeva,

Phys. Rev. D 84, 014006 (2011)
If one takes \( \Lambda_X \in (3, 4) \) GeV with the central value \( \Lambda_X = 3.5 \) GeV then our prediction for the ratio of widths reads

\[
\frac{\Gamma(X \rightarrow \gamma + J/\psi)}{\Gamma(X \rightarrow J/\psi + 2\pi)} \bigg|_{\text{theor}} = 0.15 \pm 0.03
\]

which fits very well the experimental data from the Belle Collaboration

\[
\frac{\Gamma(X \rightarrow \gamma + J/\psi)}{\Gamma(X \rightarrow J/\psi 2\pi)} = \begin{cases} 
0.14 \pm 0.05 & \text{Belle} \\
0.22 \pm 0.06 & \text{BaBar}
\end{cases}
\]
$Z_c(3900)$: Data from BESIII and Belle

- **Discovery mode (mass and width measured)**
  
  $$e^+e^- \rightarrow \pi^+ \begin{array}{c} \pi^- J/\psi \\ Z_c^- \end{array} \quad \text{BESIII, Belle}$$

- **$D\bar{D}^*$ mode (mass and width measured)**
  
  $$e^+e^- \rightarrow \pi^\pm \begin{array}{c} (D\bar{D}^*)^\mp \\ Z_c^\mp \end{array} \quad \text{BESIII}$$

- **Angular distribution**
  
  $$\pi Z_c \Longrightarrow J^P = 1^+$$

- **Enhancement of $D\bar{D}^*$ mode compare with $\pi J/\psi$**
  
  \[
  \frac{\Gamma(Z_c(3885) \rightarrow D\bar{D}^*)}{\Gamma(Z_c(3900) \rightarrow \pi J/\psi)} = 6.2 \pm 1.1 \pm 2.7
  \]
$Z_c(3900)$: theoretical interpretation

F. Goerke, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli,

- Assume that $Z_c$ is a four-quark state with a tetraquark-type current:

\[ J^\mu = \frac{i}{\sqrt{2}} \epsilon_{abc} \epsilon_{dec} \left[ (u_a^T C \gamma_5 c_b)(\bar{d}_d \gamma^\mu C \bar{c}_e) - (u_a^T C \gamma^\mu c_b)(\bar{d}_d \gamma_5 C \bar{c}_e) \right] \]

- Matrix element of the decay $1^+(p, \mu) \rightarrow 1^-(q_1, \nu) + 0^-(q_2)$

\[ M = (A g^{\mu\nu} + B q_1^{\mu} q_2^{\nu}) \epsilon_\mu \epsilon_\nu^* \]

- We found that $A \equiv 0$ analytically in the case of the $D \bar{D}^*$ final state.

- This results in a significant suppression of the decay widths due to the D–wave suppression factor.

- Since this result contradict to the data, one has to conclude that the tetraquark-type current for $Z_c(3900)$ is in discord with experiment.
Assume that $Z_c$ is a four-quark state with a molecular-type current

$$J^\mu = \frac{1}{\sqrt{2}} \left[ (\bar{d}\gamma_5 c)(\bar{c}\gamma^\mu u) + (\bar{d}\gamma^\mu c)(\bar{c}\gamma_5 u) \right]$$

Now the form factor $A$ in the expansion of the amplitude is not equal to zero.

If the $\Lambda_{Z_c}$ is varied in the limits $\Lambda_{Z_c} = 3.3 \pm 1.1$ GeV then

$$\Gamma(Z_c^+ \to J/\psi + \pi^+) = (1.8 \pm 0.3) \text{ MeV},$$
$$\Gamma(Z_c^+ \to \eta_c + \rho^+) = (3.2^{+0.5}_{-0.4}) \text{ MeV},$$
$$\Gamma(Z_c^+ \to \bar{D}^0 + D^*+) = (10.0^{+1.7}_{-1.4}) \text{ MeV},$$
$$\Gamma(Z_c^+ \to \bar{D}^{*0} + D^+) = (9.0^{+1.6}_{-1.3}) \text{ MeV}.$$  

Thus a molecular-type current for the $Z_c$ is in accordance with the experimental observation.
Preliminary data from BESIII:

\[ R(Z) = \frac{\mathcal{B}(Z_c(3900) \to \rho \eta_c)}{\mathcal{B}(Z_c(3900) \to \pi J/\psi)} = 2.1 \pm 0.8. \]

Our result:

\[ R(Z) = 1.8 \pm 0.4 \]
$Z_b(10610)$ and $Z'_b(10610)$: experiment

- Observation of two charged bottomoniumlike resonances:


  $\Upsilon(5S) \rightarrow \pi^+ \pi^- \Upsilon(nS)$ and $\Upsilon(5S) \rightarrow \pi^+ \pi^- h_b(mP)$

  $(n=1,2,3)$ and $(m=1,2)$

- Masses and widths:

  $M_{Z_b} = (10607.2 \pm 2.0) \text{ MeV}$, \hspace{1cm} $\Gamma_{Z_b} = (18.4 \pm 2.4) \text{ MeV}$,

  $M_{Z'_b} = (10652.2 \pm 1.5) \text{ MeV}$, \hspace{1cm} $\Gamma_{Z'_b} = (11.5 \pm 2.2) \text{ MeV}$.

- Quantum numbers are $I^G(J^P) = 1^+(1^+)$.
Observation in $B\bar{B}\pi$ channels:

$e^+e^- \rightarrow \pi^+$ and $e^+e^- \rightarrow \pi^+$

It was found that the $B^{(*)}\bar{B}^*$-decays dominate among the corresponding final states.

Assuming that the $Z_b$-decays are saturated by $\Upsilon(nS)\pi$ ($n = 1, 2, 3$), $h_b(mP)\pi$ ($m = 1, 2$) and $B^{(*)}\bar{B}^*$ channels, the relative decay fractions were determined.
Since the masses of the $Z_b^+(10610)$ and $Z_b'(10650)$ are very close to the respective $B^*\bar{B}^*(10604\text{ MeV})$ and $B^*\bar{B}^*(10649\text{ MeV})$ thresholds, it was suggested that they have molecular-type binding structures.


\[
J_{Z_b^+}^{\mu} = \frac{1}{\sqrt{2}} \left[ (\bar{d}\gamma_5 b)(\bar{b}\gamma^\mu u) + (\bar{d}\gamma^\mu b)(\bar{b}\gamma_5 u) \right],
\]

\[
J_{Z_b'^+}^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}(\bar{d}\gamma_\alpha b)(\bar{b}\gamma_\beta u)
\]

Such a choice guarantees that the $Z_b$-state can only decay to the $[\bar{B}^* B + c.c.]$ pair whereas the $Z_b'$-state can decay only to a $\bar{B}^* B^*$ pair. Decays into the $BB$-channels are forbidden.

The nonlocal generalization of the above 4-quark currents is straightforward. Then we are able to calculate the matrix elements and the widths of all relevant two-body decays.
The bottomonium states $^{2S+1}L_J$.

<table>
<thead>
<tr>
<th>quantum number $I^G(J^{PC})$</th>
<th>name</th>
<th>quark current</th>
<th>mass (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+(0^{-+})$ (S = 0, L = 0)</td>
<td>$^1S_0 = \eta_b(1S)$</td>
<td>$\bar{b} i\gamma^5 b$</td>
<td>$9399.00 \pm 2.30$</td>
</tr>
<tr>
<td>$0^-(1^{--})$ (S = 1, L = 0)</td>
<td>$^3S_1 = \Upsilon$</td>
<td>$\bar{b} \gamma^\mu b$</td>
<td>$9460.30 \pm 0.26$</td>
</tr>
<tr>
<td>$0^+(1^{++})$ (S = 1, L = 1)</td>
<td>$^3P_0 = \chi_{b0}$</td>
<td>$\bar{b} b$</td>
<td>$9859.44 \pm 0.52$</td>
</tr>
<tr>
<td>$0^+(1^{++})$ (S = 1, L = 1)</td>
<td>$^3P_1 = \chi_{b1}$</td>
<td>$\bar{b} \gamma^\mu\gamma^5 b$</td>
<td>$9892.72 \pm 0.40$</td>
</tr>
<tr>
<td>$0^-(1^{+-})$ (S = 0, L = 1)</td>
<td>$^1P_1 = h_b(1P)$</td>
<td>$\bar{b} \gamma^\mu\gamma^5 b$</td>
<td>$9899.30 \pm 0.80$</td>
</tr>
</tbody>
</table>

Due to $G$-parity conservation the following decays are forbidden:

$$Z_b \rightarrow \Upsilon + \rho, \quad Z_b \rightarrow \eta_b + \pi, \quad Z_b \rightarrow \chi_{b1} + \pi, \quad Z_b \rightarrow h_b + \rho.$$ 

The decay $Z_b \rightarrow \chi_{b1} + \rho$ is not allowed kinematically.

There are therefore only the three allowed decays:

$$Z_b^+ \rightarrow \Upsilon + \pi^+, \quad Z_b^+ \rightarrow h_b + \pi^+, \quad Z_b^+ \rightarrow \eta_b + \rho^+.$$
All adjustable parameters of our model have been fixed in our previous studies by a global fit to a multitude of experimental data.

The only two new parameters are the size parameters of the two exotic $Z_b(Z'_b)$ states. As a guide to adjust them we take the experimental values of the largest branching fractions presented by Belle:

\[
\mathcal{B}(Z^+_b \to [B^+\bar{B}^*0 + \bar{B}^0B^{*+}]) = 85.6^{+1.5+1.5}_{-2.0-2.1}\ %, \\
\mathcal{B}(Z^{'+}_b \to \bar{B}^{*+}B^{*0}) = 73.7^{+3.4+2.7}_{-4.4-3.5}\ %.
\]

By using the central values of these branching rates and total decay widths we find the central values of our size parameters $\Lambda_{Z_b} = 3.45$ GeV and $\Lambda_{Z'_b} = 3.00$ GeV. Allowing them to vary in the interval

\[
\Lambda_{Z_b} = 3.45 \pm 0.05 \text{ GeV} \quad \Lambda_{Z'_b} = 3.00 \pm 0.05 \text{ GeV},
\]

we obtain the values of various decay widths.
### $Z_b(10610)$ and $Z'_b(10610)$: numerical results

<table>
<thead>
<tr>
<th>Channel</th>
<th>$Z_b(10610)$</th>
<th>$Z'_b(10650)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Upsilon(1S)\pi^+$</td>
<td>$5.9 \pm 0.4$</td>
<td>$9.5^{+0.7}_{-0.6}$</td>
</tr>
<tr>
<td>$h_b(1P)\pi^+$</td>
<td>$(0.14 \pm 0.01) \cdot 10^{-1}$</td>
<td>$0.74^{+0.05}_{-0.04} \cdot 10^{-3}$</td>
</tr>
<tr>
<td>$\eta_b\rho^+$</td>
<td>$4.4 \pm 0.3$</td>
<td>$7.5^{+0.6}_{-0.5}$</td>
</tr>
<tr>
<td>$B^+\bar{B}^*0 + \bar{B}^0B^{**}$</td>
<td>$20.7^{+1.6}_{-1.5}$</td>
<td>—</td>
</tr>
<tr>
<td>$B^{**}\bar{B}^{*0}$</td>
<td>—</td>
<td>$17.1^{+1.5}_{-1.4}$</td>
</tr>
</tbody>
</table>

**Total widths, MeV**

<table>
<thead>
<tr>
<th></th>
<th>Theory</th>
<th>Belle Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_b(10610)$</td>
<td>$30.9^{+2.3}_{-2.1}$</td>
<td>$25 \pm 7$</td>
</tr>
<tr>
<td>$Z'_b(10650)$</td>
<td>$34.1^{+2.8}_{-2.5}$</td>
<td>$23 \pm 8$</td>
</tr>
</tbody>
</table>
The Belle observations indicate that the decays involving bottomonium states are significantly suppressed compared with the $B$-meson modes.

In our calculation we find that the modes with $\Upsilon(1S)\pi^+$ and $\eta_b\rho^+$ are suppressed but not as much as in the data.

\[
\frac{\Gamma (Z_b \rightarrow \Upsilon(1S)\pi)}{\Gamma (Z_b \rightarrow B\bar{B}^* + \text{c.c.})} \approx 0.29, \quad \frac{\Gamma (Z_b \rightarrow \eta_b\rho)}{\Gamma (Z_b \rightarrow B\bar{B}^* + \text{c.c.})} \approx 0.21, \quad \frac{\Gamma (Z'_b \rightarrow \Upsilon(1S)\pi)}{\Gamma (Z'_b \rightarrow B^*\bar{B}^*)} \approx 0.56, \quad \frac{\Gamma (Z'_b \rightarrow \eta_b\rho)}{\Gamma (Z'_b \rightarrow B^*\bar{B}^*)} \approx 0.44.
\]

The decays into the $h_b(1P)\pi^+$ mode are suppressed by the $p$-wave suppression factor.
Summary

- We have studied the properties of the \(X(3872)\) as a tetraquark.

- We have calculated the strong decays \(X \rightarrow J/\psi + \rho(\rightarrow 2\pi)\), \(X \rightarrow J/\psi + \omega(\rightarrow 3\pi)\), \(X \rightarrow D + D^* (\rightarrow D\pi)\) and electromagnetic decay \(X \rightarrow \gamma + J/\psi\).

- The comparison with available experimental data allows one to conclude that the \(X(3872)\) can be a tetraquark state.
Summary

- We have critically checked two possible four-quark configurations for $Z_c(3900)$: tetraquark and molecular.

- We have calculated the partial widths of the decays $Z_c^+(3900) \rightarrow J/\psi\pi^+, \eta_c\rho^+$ and $\bar{D}^0D^*+, \bar{D}^*0D^+$.

- It turned out the decays $Z_c(3900) \rightarrow \bar{D}D^*$ are significantly suppressed on the case of a tetraquark configuration.

- Alternatively, in the case of a molecular configuration the partial widths of those decays are close to $\sim 15$ MeV and exceeded the partial widths for the decays $Z_c(3900) \rightarrow J/\psi\pi, \eta_c\rho$ by a factor of 6-7 in accordance with BESIII-experiment.
By using molecular-type four-quark currents for the recently observed resonances $Z_b(10610)$ and $Z_b(10650)$, we have calculated their two-body decay rates into a bottomonium state plus a light meson as well as into $B$-meson pairs.

We have fixed the model size parameters by adjusting the theoretical values of the largest branching fractions of the modes with the $B$-mesons in the final states to their experimental values.

We found that the modes with $\Upsilon(1S)\pi^+$ and $\eta_b\rho^+$ in the final states are suppressed but not as much as the Belle Collaboration reported.