The improved Ginzburg-Landau technique

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S.Carignano, F.Anzuini, O. Benhar, MM Phys.Rev. D97 (2018), 036009

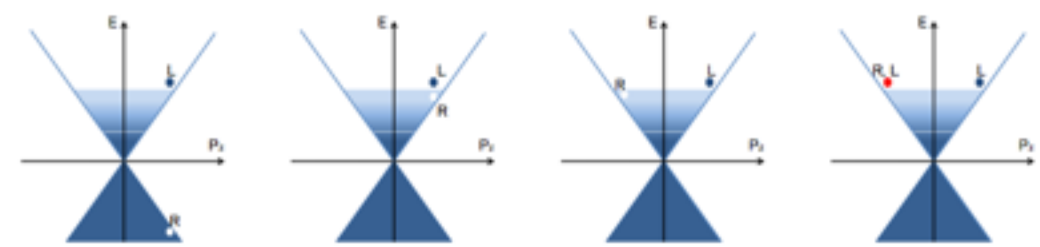
QCD@work
Bari, June 28, 2018

Outline

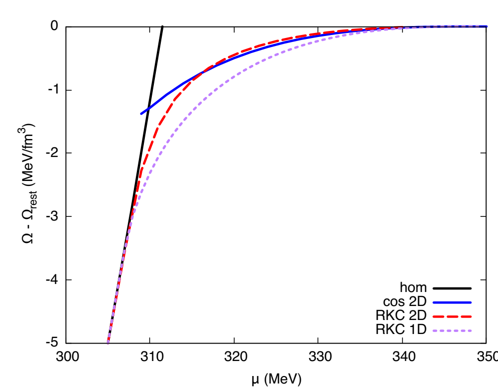
- **Background**

$$\underbrace{SU(3)_\mathrm{c} \times SU(3)_\mathrm{L} \times SU(3)_\mathrm{R}}_{\supset\,[U(1)_\mathrm{e.m.}]} \times U(1)_\mathrm{B}$$

- **Competing condensates**



- **Improved Ginzburg-Landau expansion**



BACKGROUND

Symmetries of QCD

Symmetries of the three flavor massless QCD Lagrangian

color gauge group **global chiral symmetry** **global baryonic number**

$$SU(3)_{\mathrm{c}} \times \underbrace{SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}}}_{\supset\, [U(1)_{\mathrm{e.m.}}]} \times U(1)_{\mathrm{B}}$$

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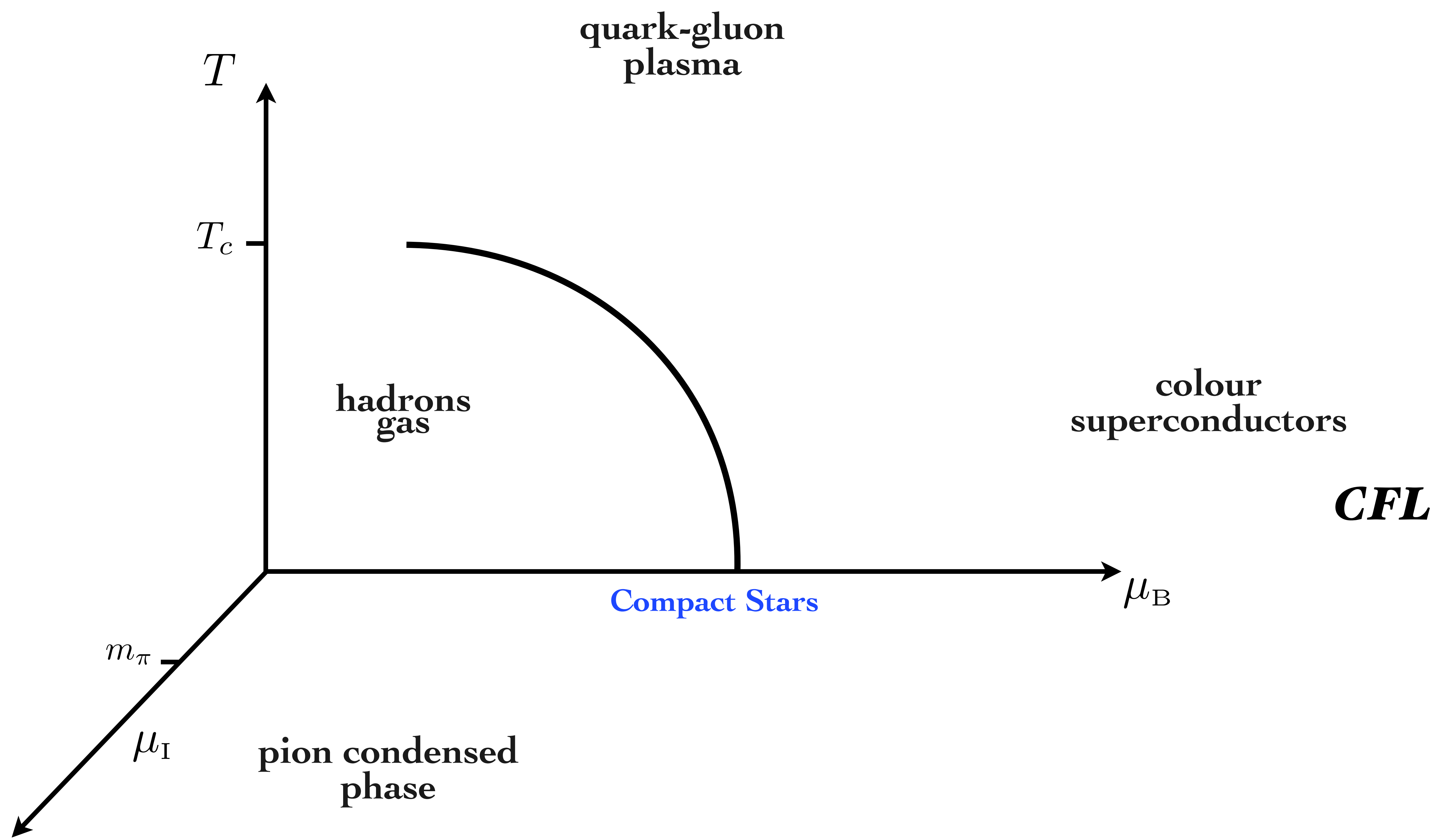
The ground state may have a lower symmetry because of quark condensates

$\langle\bar\psi\psi\rangle$ **Chiral condensate: Locks chiral rotations**

$\langle\bar\psi\sigma_2\gamma_5\psi\rangle$ **Pion condensate: Locks chiral rotations and breaks** $U(1)_{\mathrm{e.m.}}$

$\langle\psi C\gamma_5\psi\rangle$ **Diquark condensate: Breaks the gauge group and may lock chiral rotations**

Quark matter phase diagram



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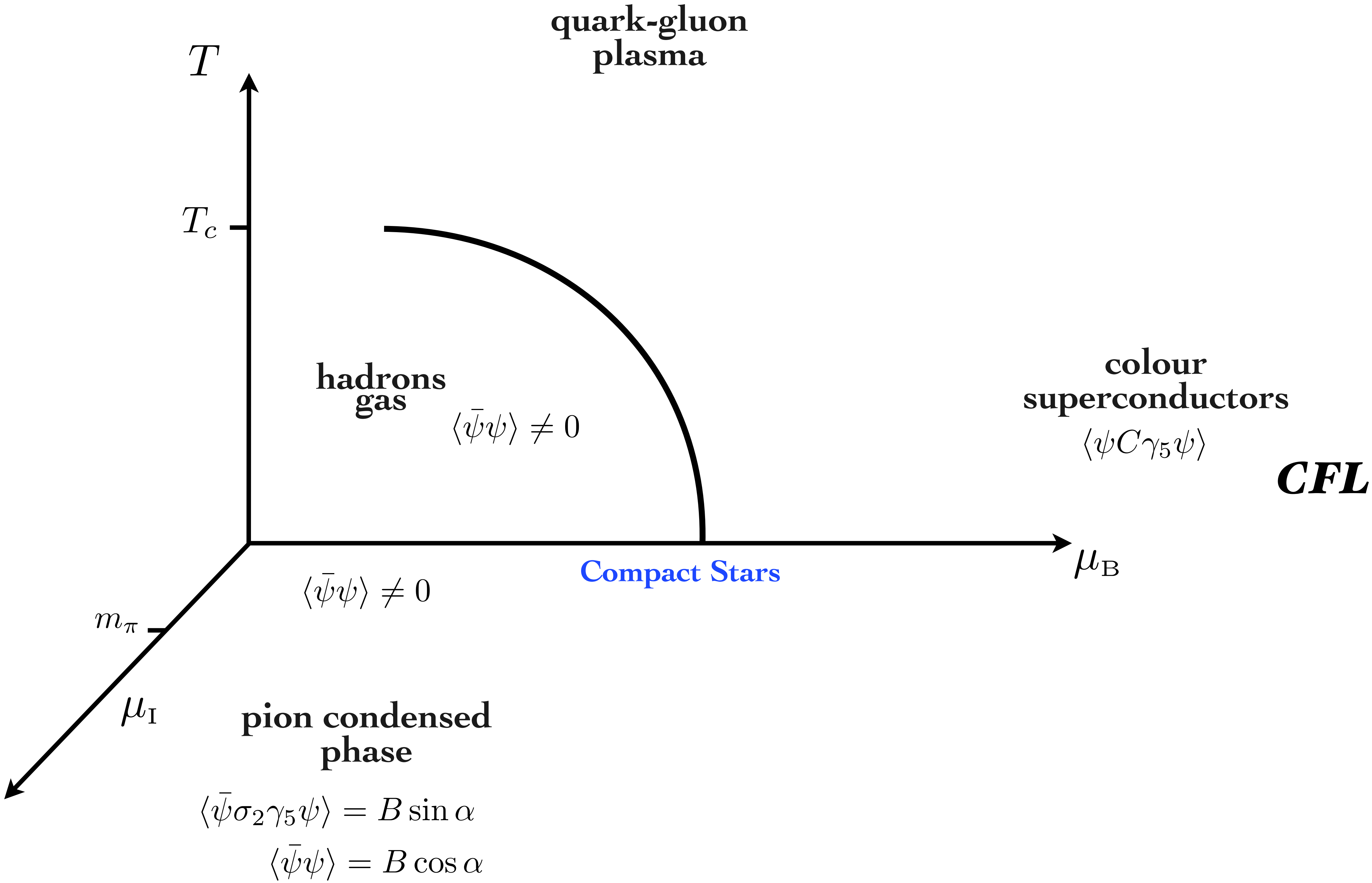
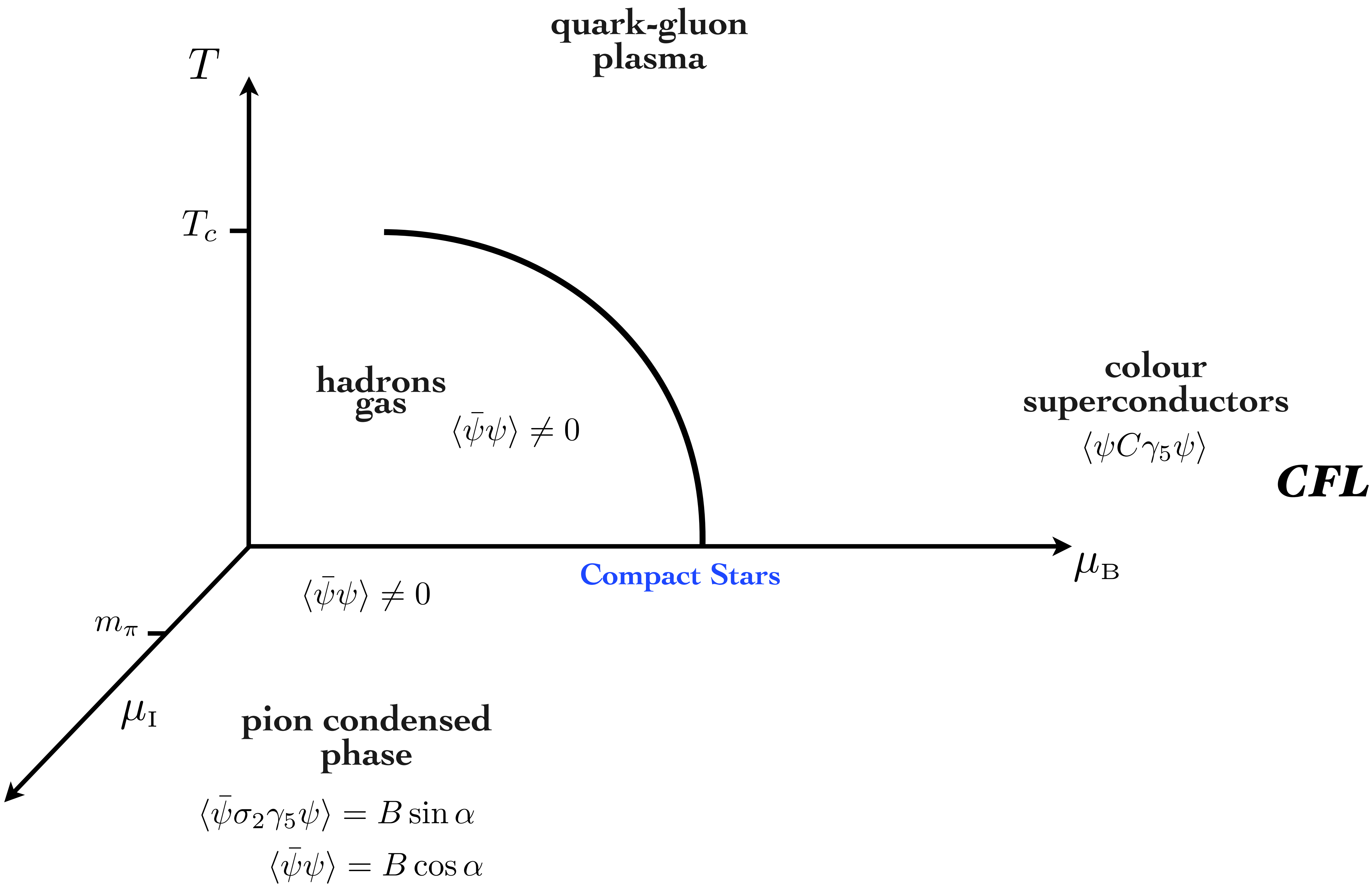


Quark matter phase diagram

SOME METHODS



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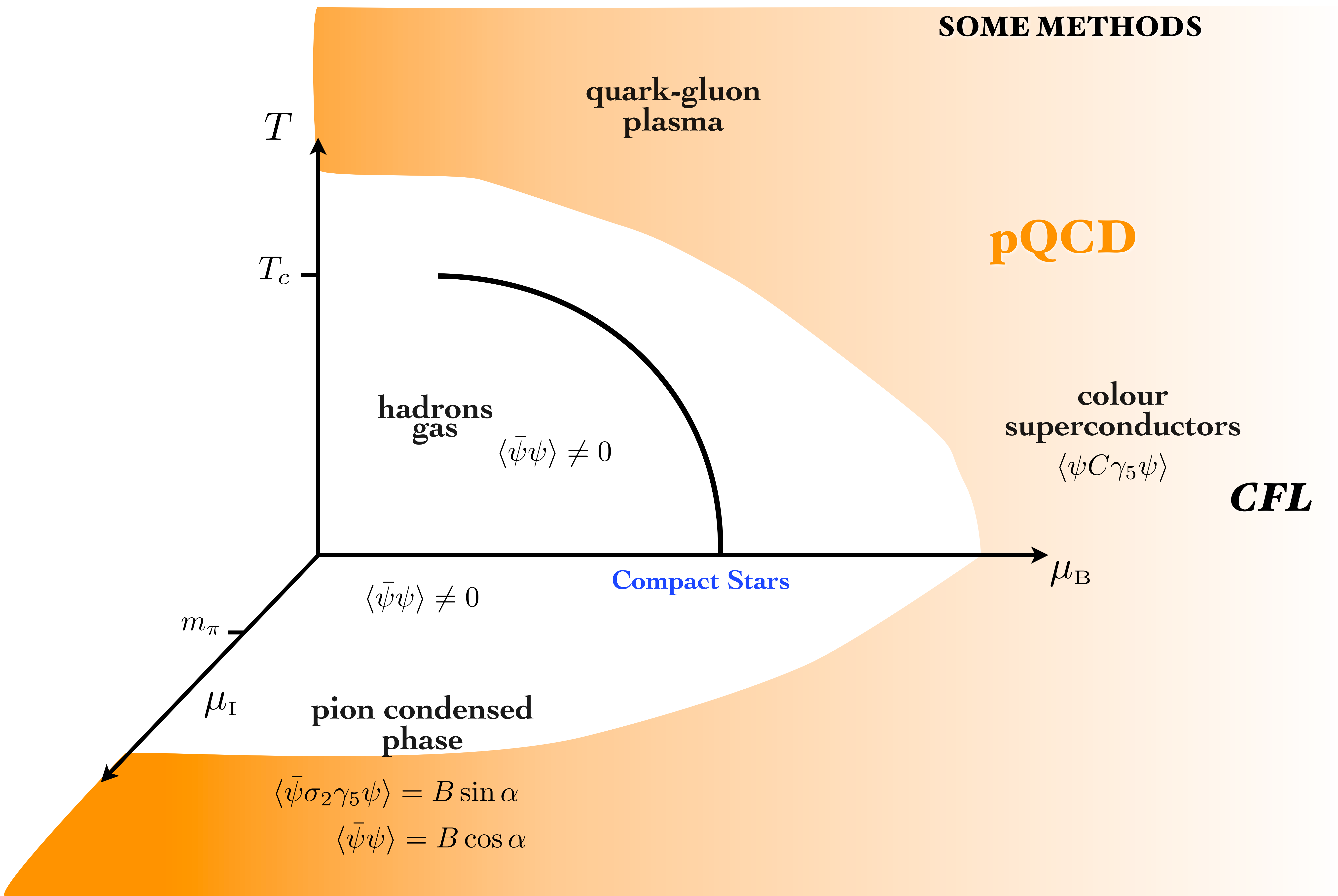

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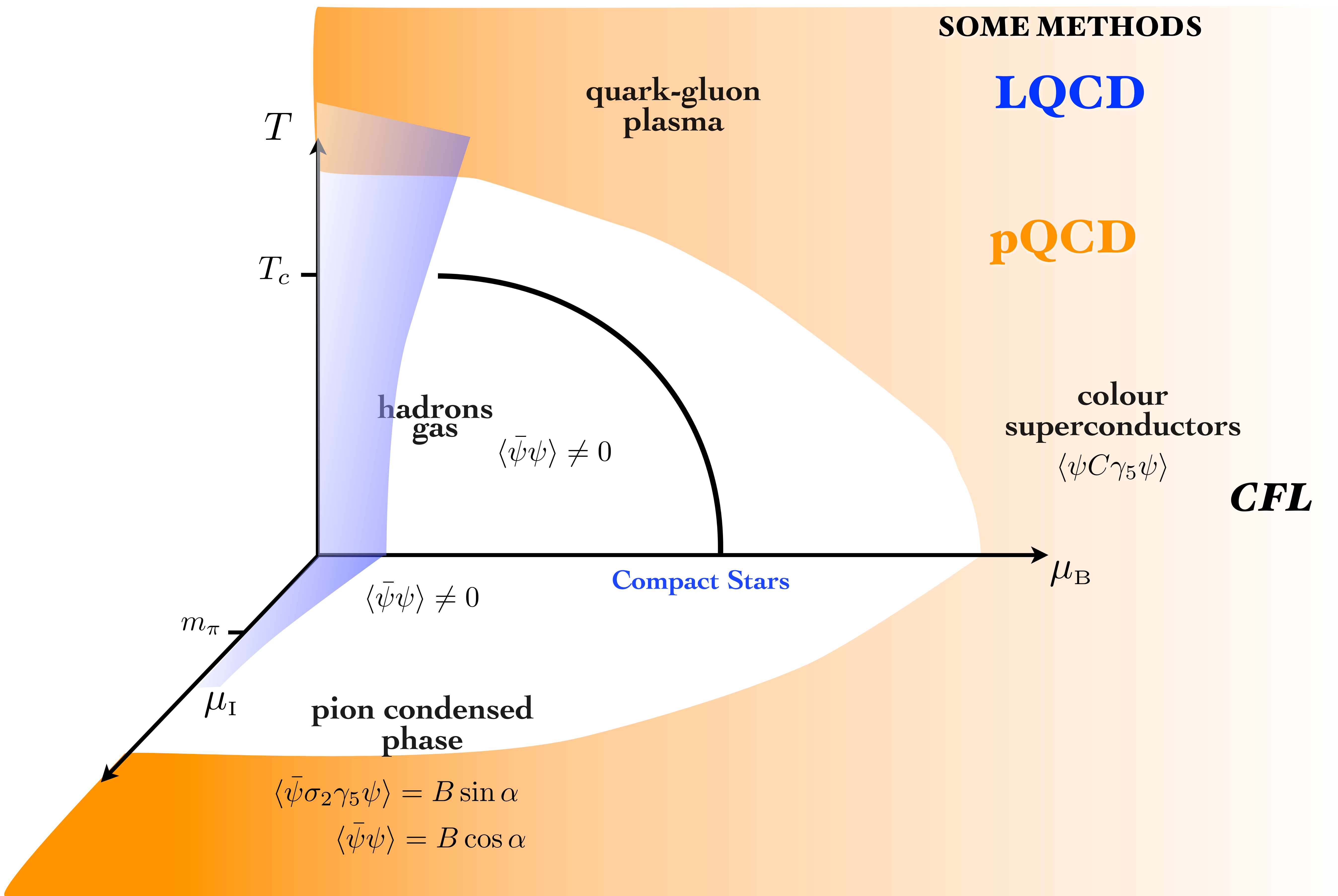

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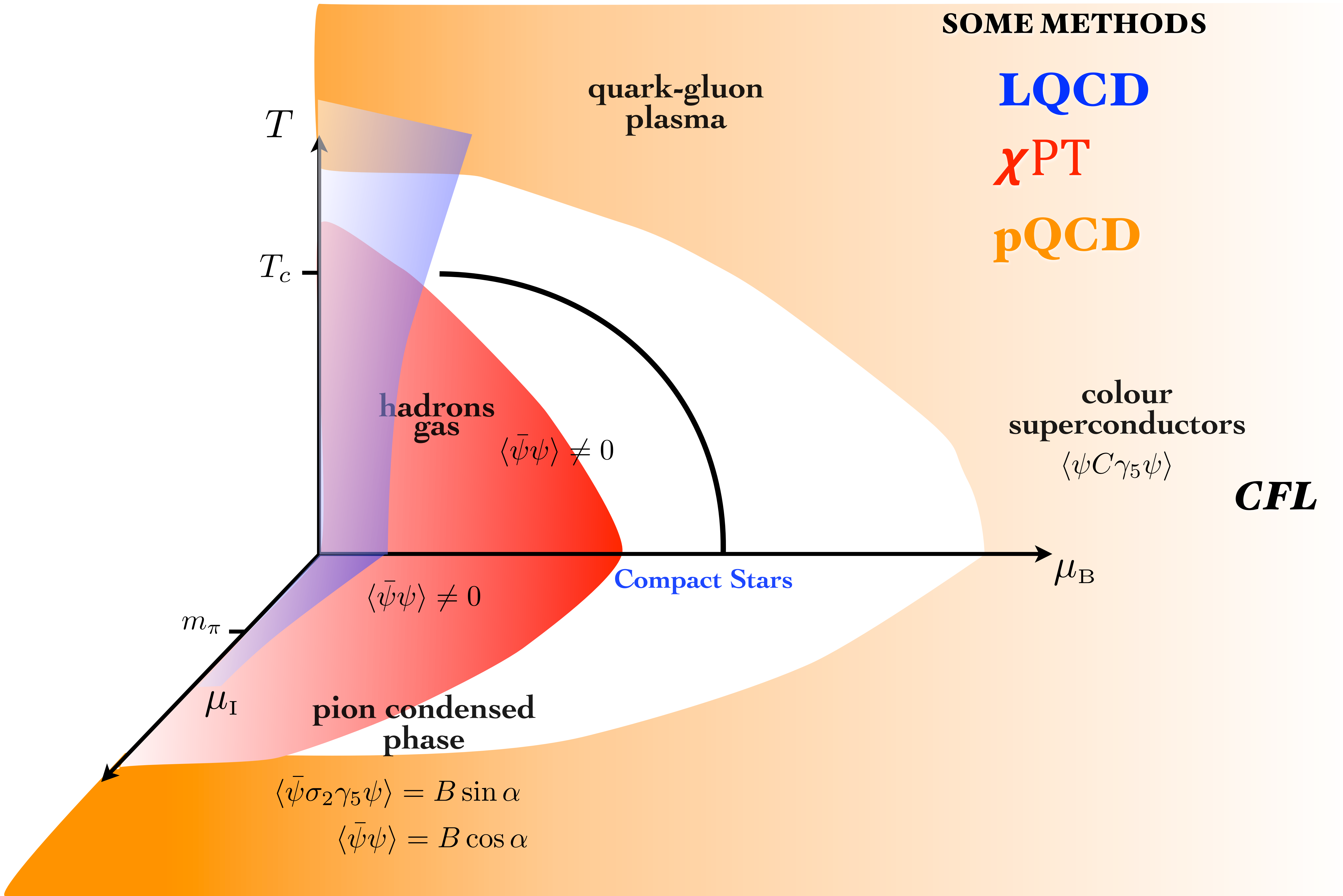

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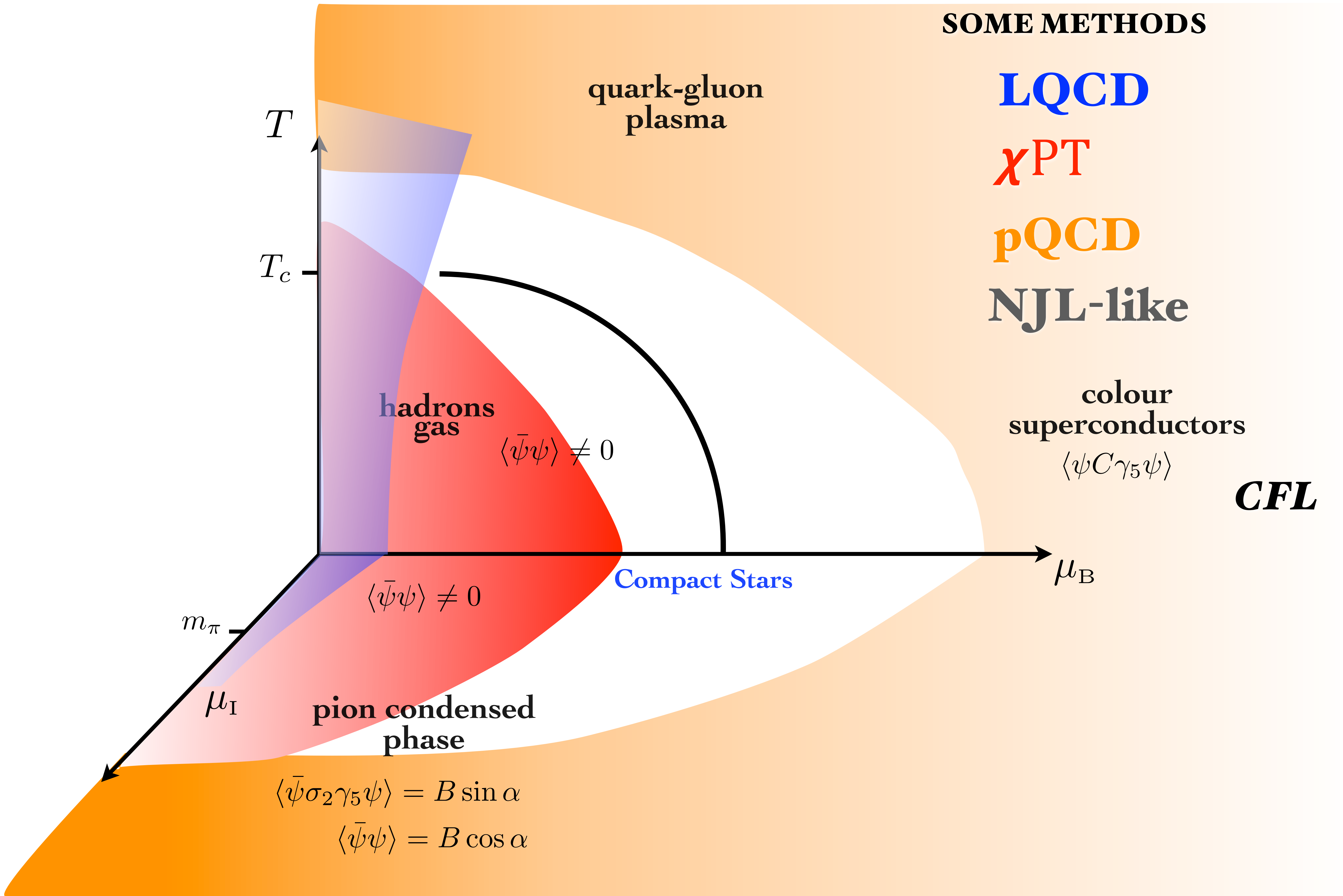

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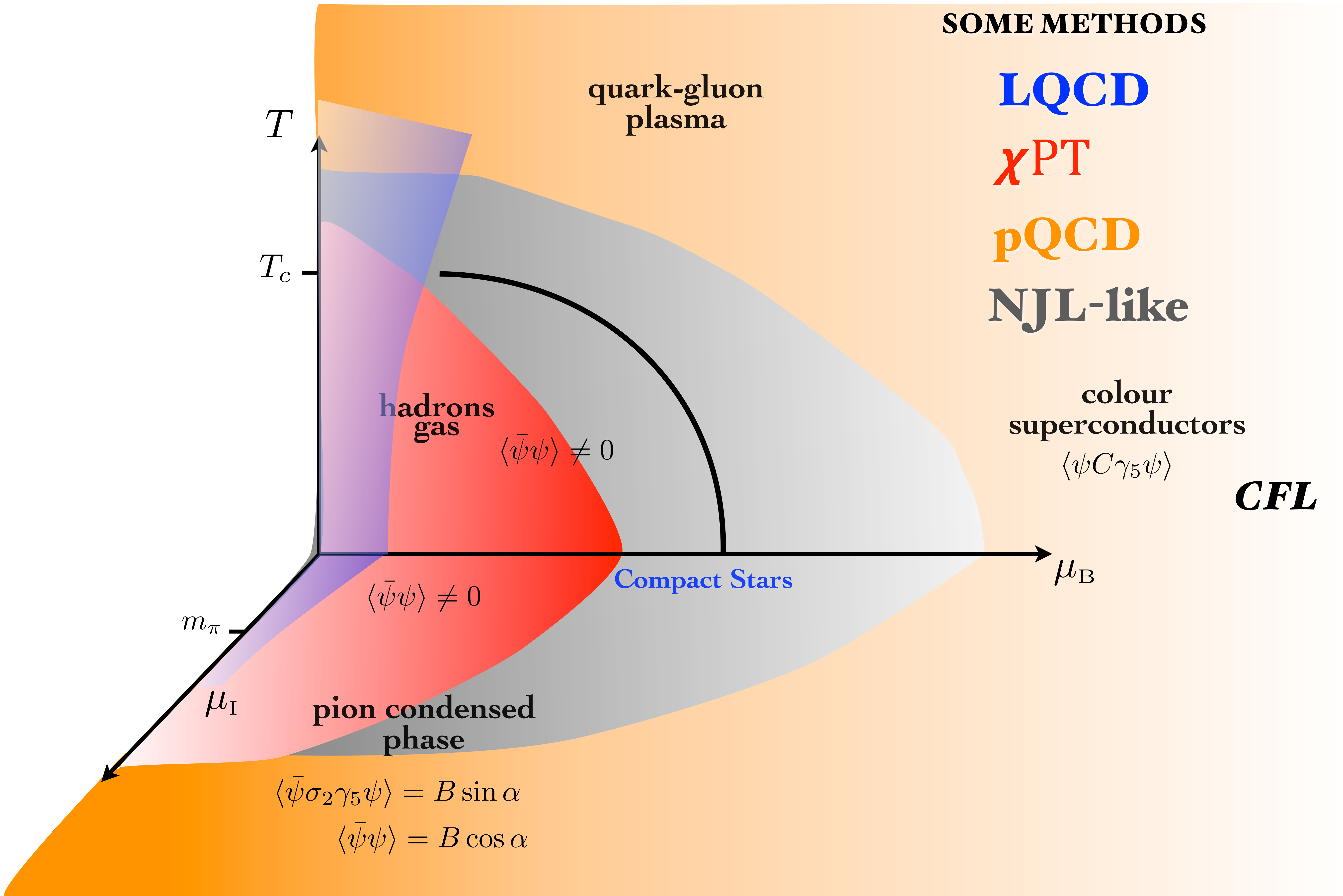
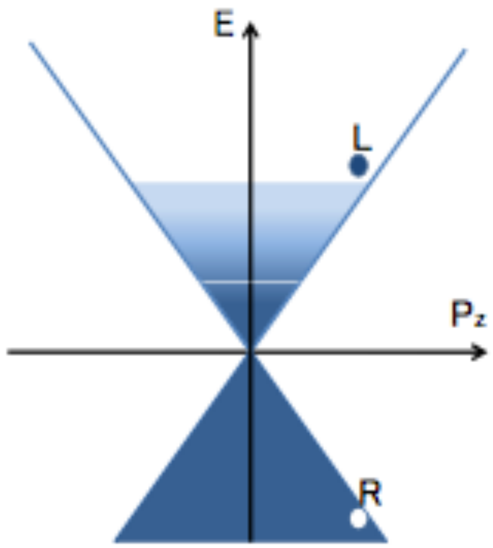


COMPETING CONDENSATES

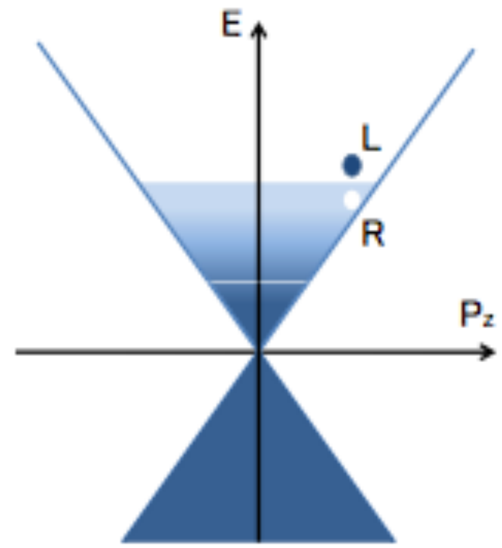
Fight of condensates

Different kind of pairings

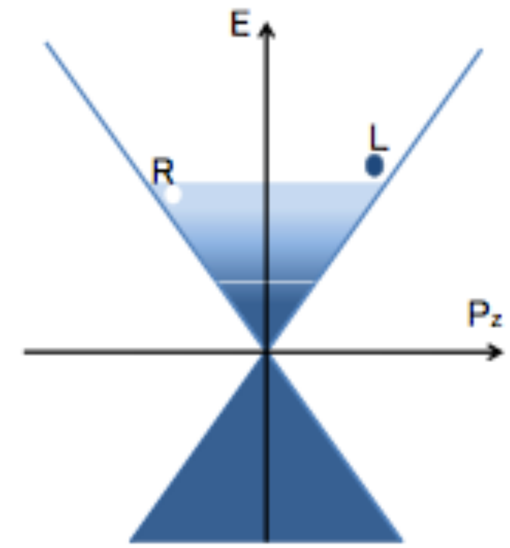
T. Kojo, Y. Hidaka, L. McLerran, R. D. Pisarski, Nucl. Phys. A 843 (2010) 37



quark-antiquark



quark-hole
(exciton-like)



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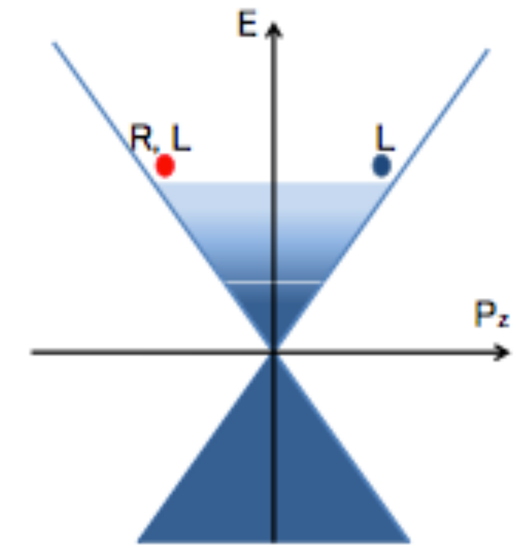

quark-quark
color superconductor

Not obvious which of these is energetically favored

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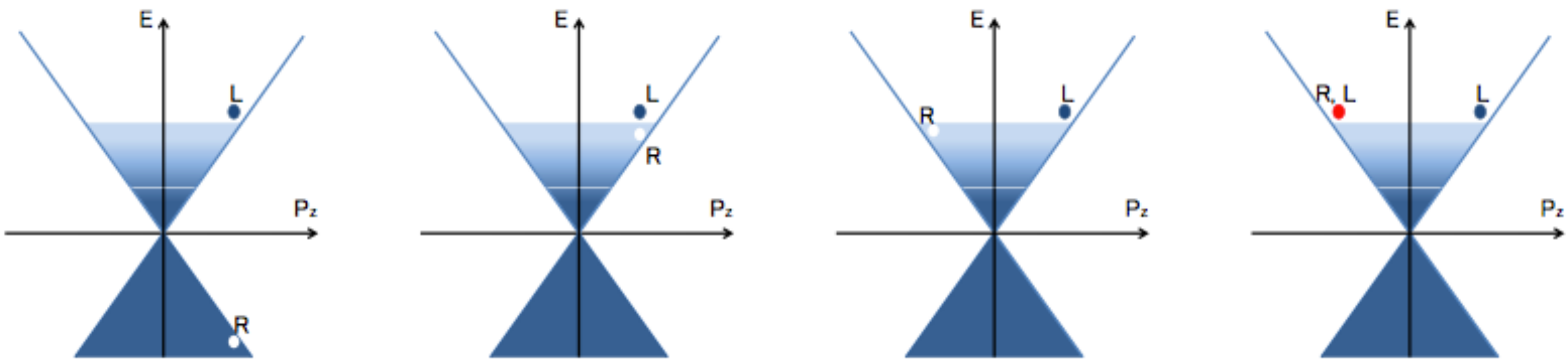

quark-antiquark

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(exciton-like)

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Not obvious which of these is energetically favored

Unfortunately it seems that the favored condensate is somehow model dependent.
The appearance of inhomogeneous phases makes the picture even more complicated

Melting the chiral condensate

The chiral condensate becomes disfavored with increasing density

It can melt in different ways:
1) By a second order phase transition
2) By a first order phase transition
3) Passing through an inhomogeneous phase

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NJL-model analysis

Pauli-Villars regulator Λ = 757.048 MeV
Scalar coupling constant G = 6/Λ^2

variational parameters

CDW ansatz

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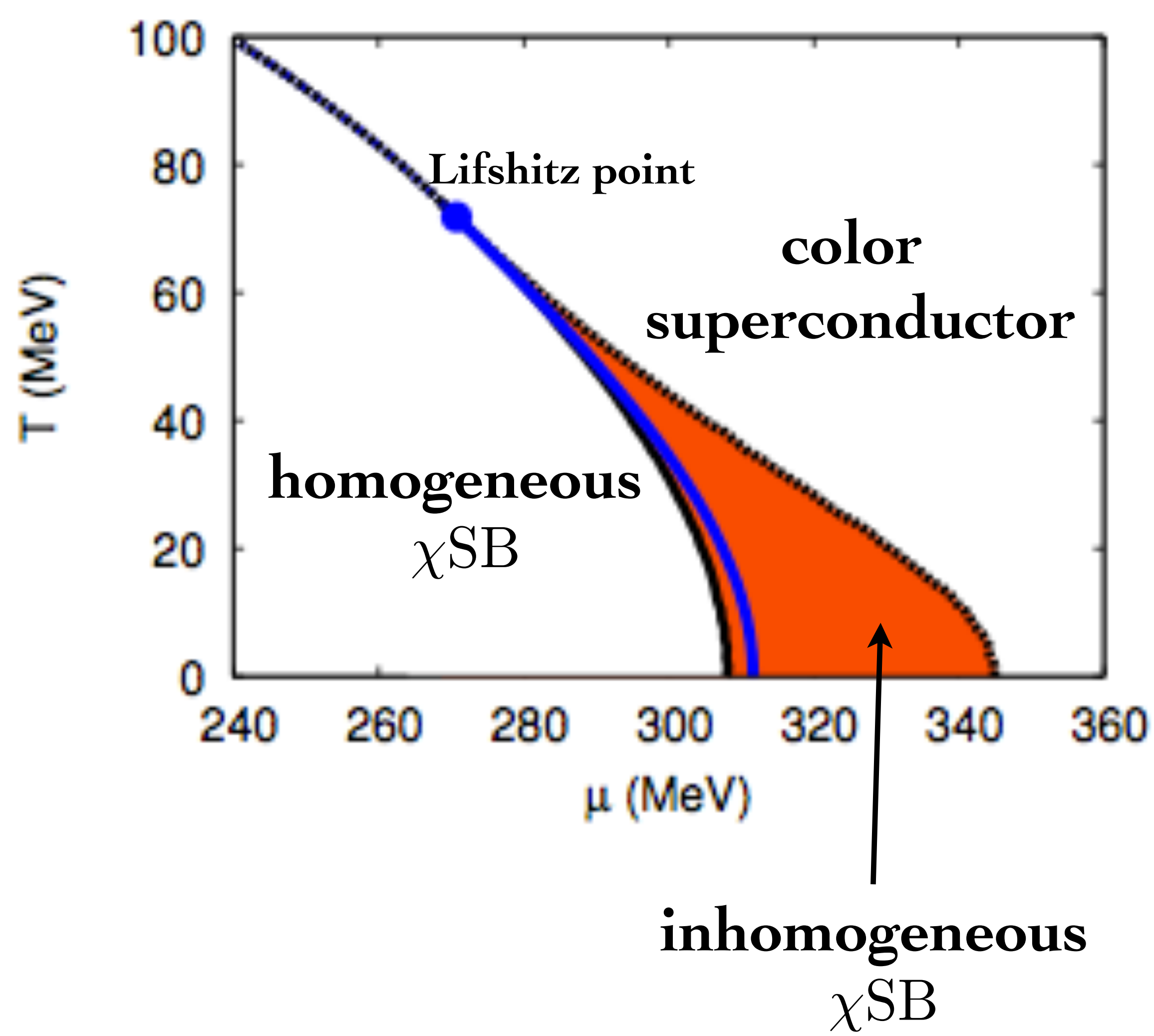

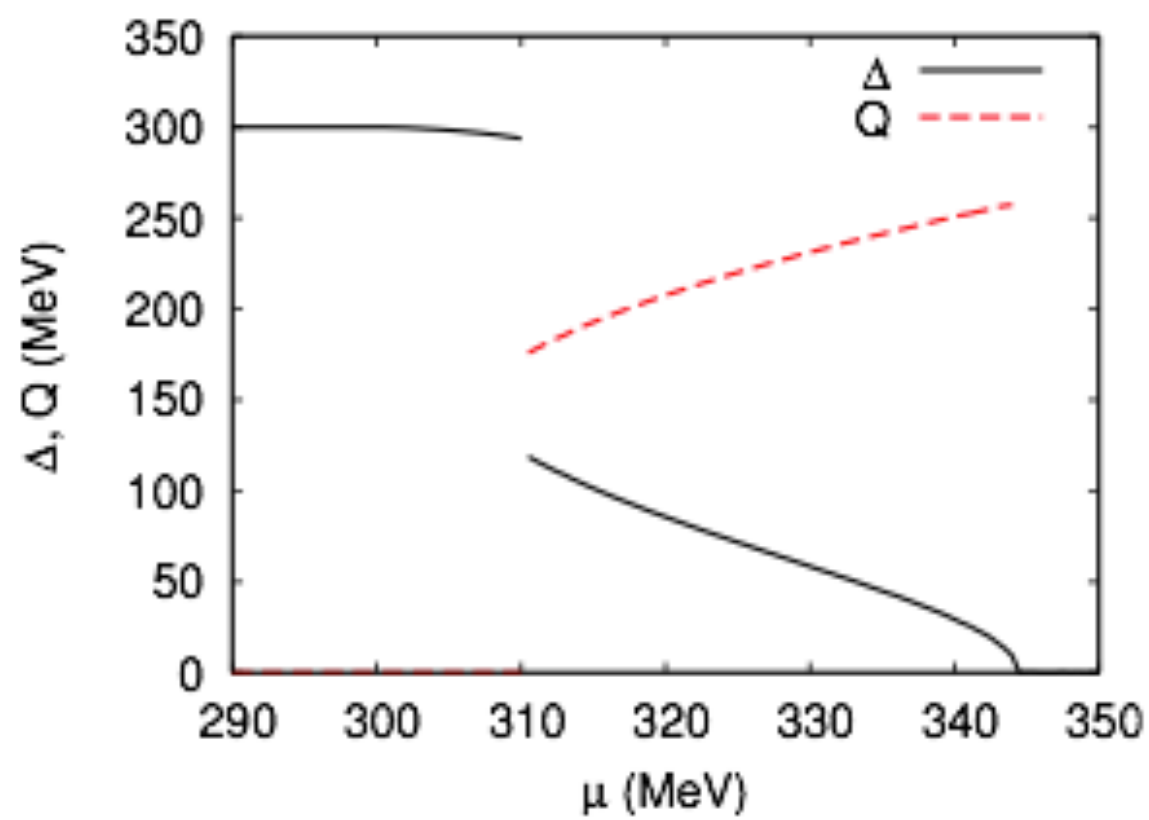

Buballa and Carignano
Prog.Part.Nucl.Phys. 81 (2015) 39-96

IMPROVED GINZBURG-LANDAU EXPANSION

Standard GL expansion

We focus on inhomogeneous chiral symmetry breaking

$$\Omega_{\rm GL} =\Omega[0] + \frac{1}{V}\int d\boldsymbol{x} \Big[\alpha_2 M^2 + \alpha_4 \left(M^4 + (\nabla M)^2\right) + \alpha_6 \left(M^6 + 3(\nabla M)^2 M^2 + \frac{1}{2}(\nabla M^2)^2 + \frac{1}{2}(\nabla^2 M)^2\right)$$
$$+\alpha_8 \left(14M^4(\nabla M)^2 - \frac{1}{5}(\nabla M)^4 + \frac{18}{5}M(\nabla^2 M)(\nabla M)^2 + \frac{14}{5}M^2(\nabla^2 M)^2 + \frac{1}{5}(\nabla^3 M)^2\right) + \dots\Big]$$

D. Nickel, Phys. Rev. Lett. 103, 072301 (2009)
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This is correct **close to the Lifshitz point** where both M and ∇M are small.
But is not in general true.

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But is not in general true.

What is the "correct expansion" **away from the Lifshitz point**?
How to compute the relevant terms?
Which are the characteristic scales of fluctuations?

“Universality”

The α_n coefficients are “universal”, for the considered system.

They shoul be derived from the microscopic theory. Since in this regime QCD is non perturbative, they depend on the effective model we use.

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Using a NJL model, they do not only depend on μ , but also on the regularization scale Λ

$$
\begin{aligned}
\alpha_2 &= \frac{1}{4G} - \frac{N_f N_c}{8\pi^2}\left(3\Lambda^2 \log\left(\frac{4}{3}\right) - 2\mu^2\right) \\
\alpha_4 &= -\frac{N_f N_c}{16\pi^2} \log\left(\frac{32\mu^2}{3\Lambda^2}\right) \\
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Even within the NJL model they are not easy to compute. **Brute force is not very rewarding**.

Improved GL expansion (for chiral symmetry breaking)

Idea: scale separation between slow and fast fluctuations + use a simple model to compute some of the coefficients.

Long wavelengths: dominant at the onset of the inhomogeneous phase
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Captures short-wavelength oscillations by larger number of gradients.

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Computing the additional terms

We compute the $\tilde{\alpha}_{2n+2}$ using a simple model

We must have an analytical or semi-analytical expression of the free energy in one simple case:

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Double expansion

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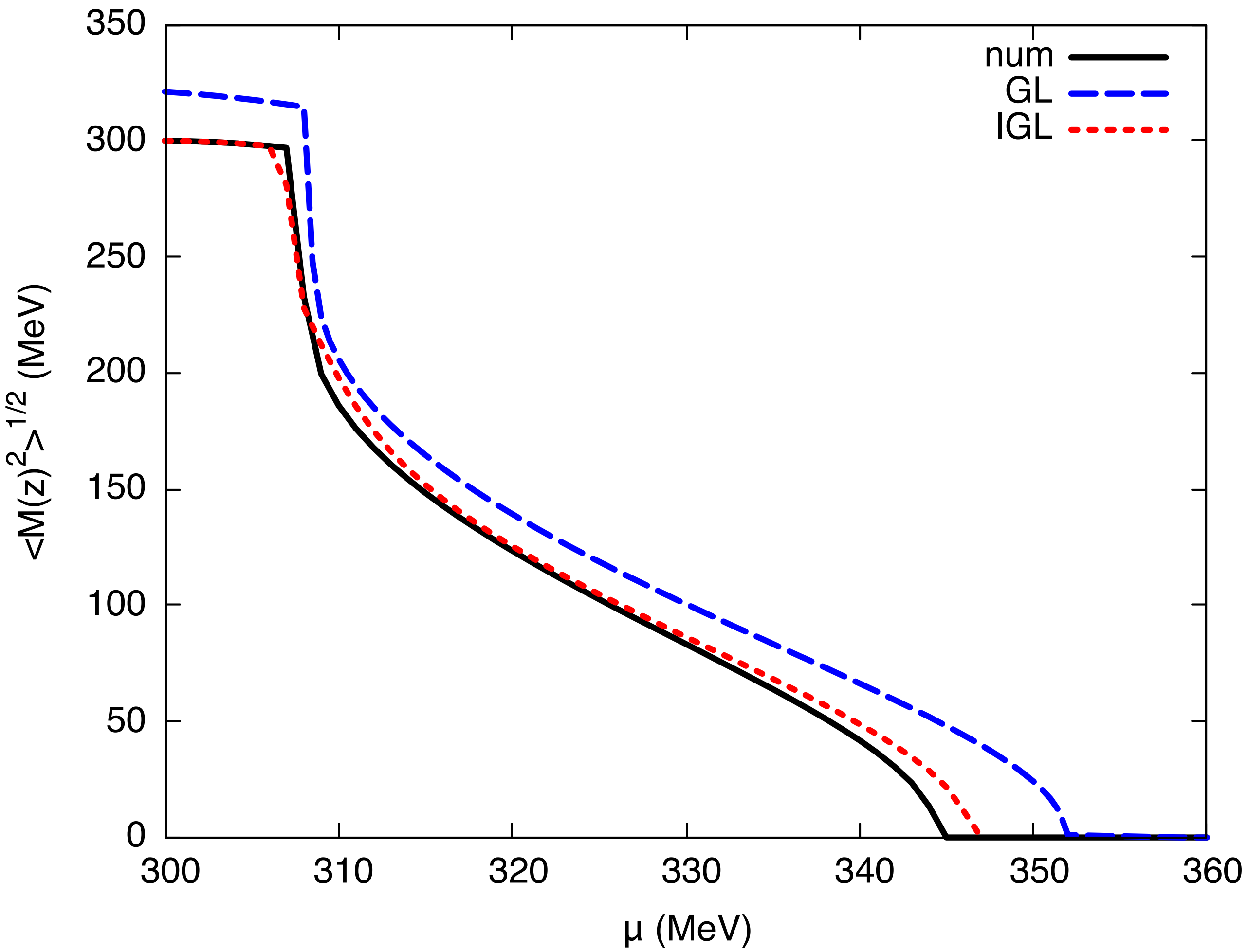
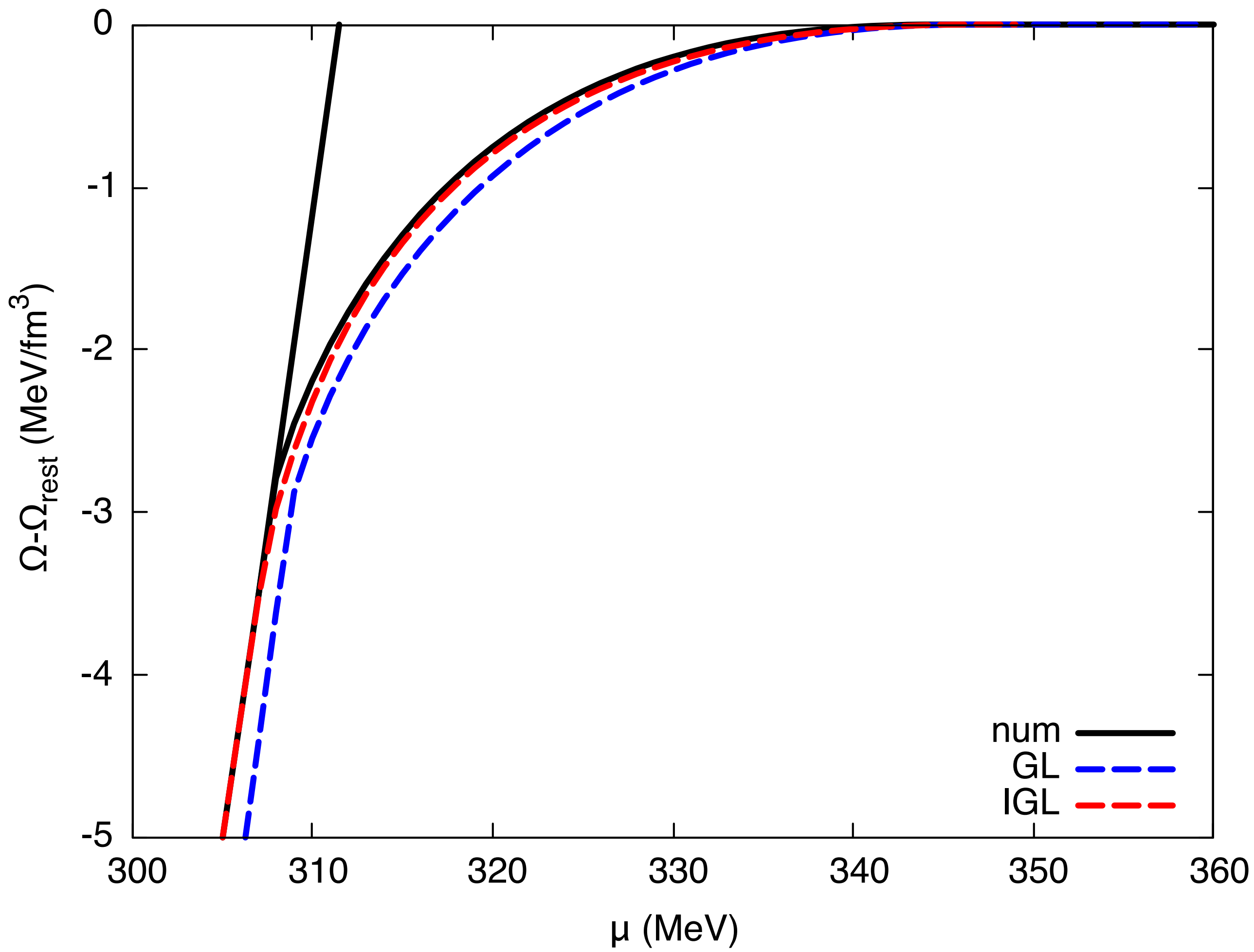
Therefore $\quad \tilde{\alpha}_{10} = \dfrac{N_f N_c}{1024\pi^2}\left(\dfrac{230}{567\Lambda^6}+\dfrac{1}{21\mu^6}\right)$ $\quad$ and we can in principle extract more terms

Comparison: kink case

Real kink

$$M(z) = \Delta\sqrt{\nu}\,\mathrm{sn}(\Delta z|\nu)$$





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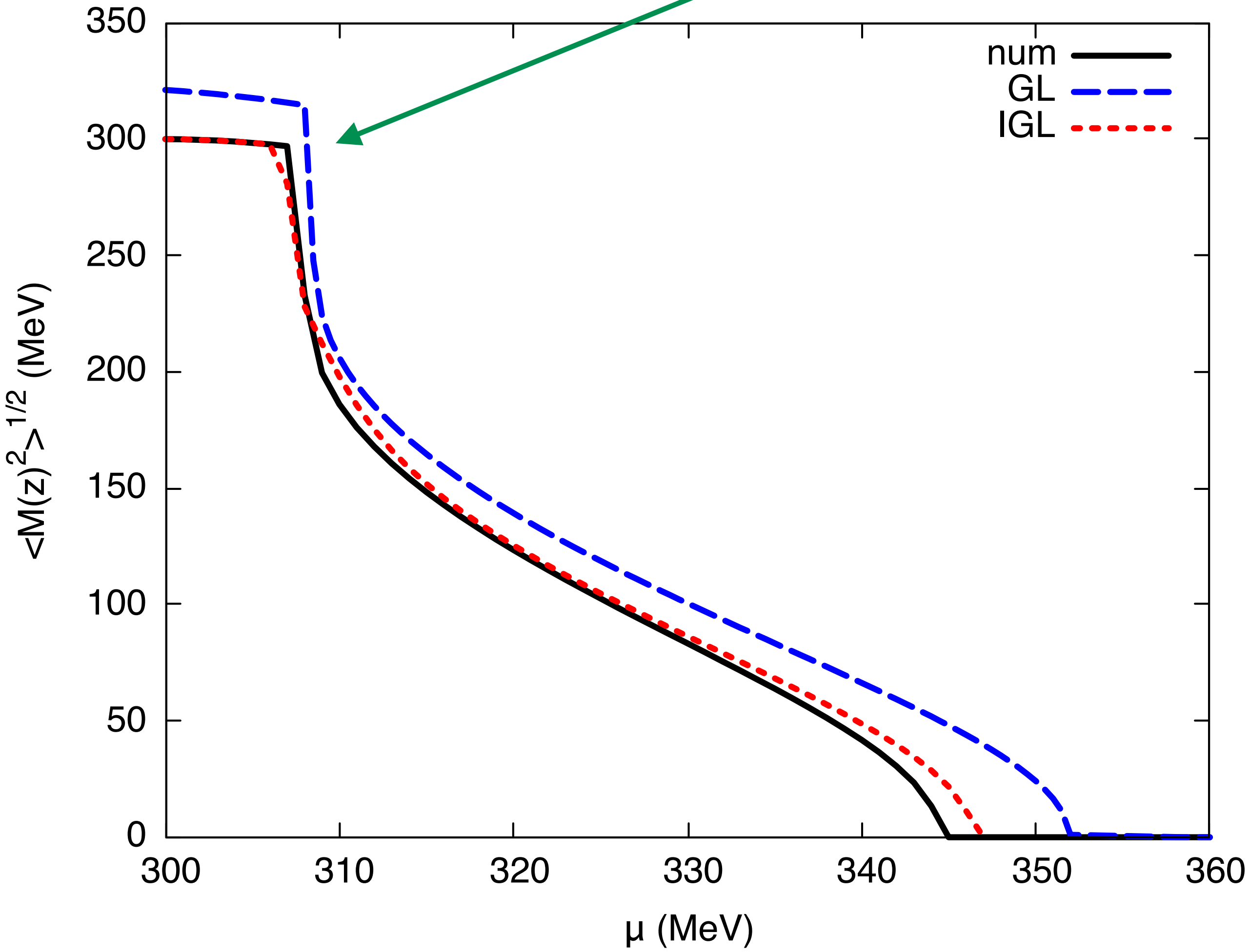
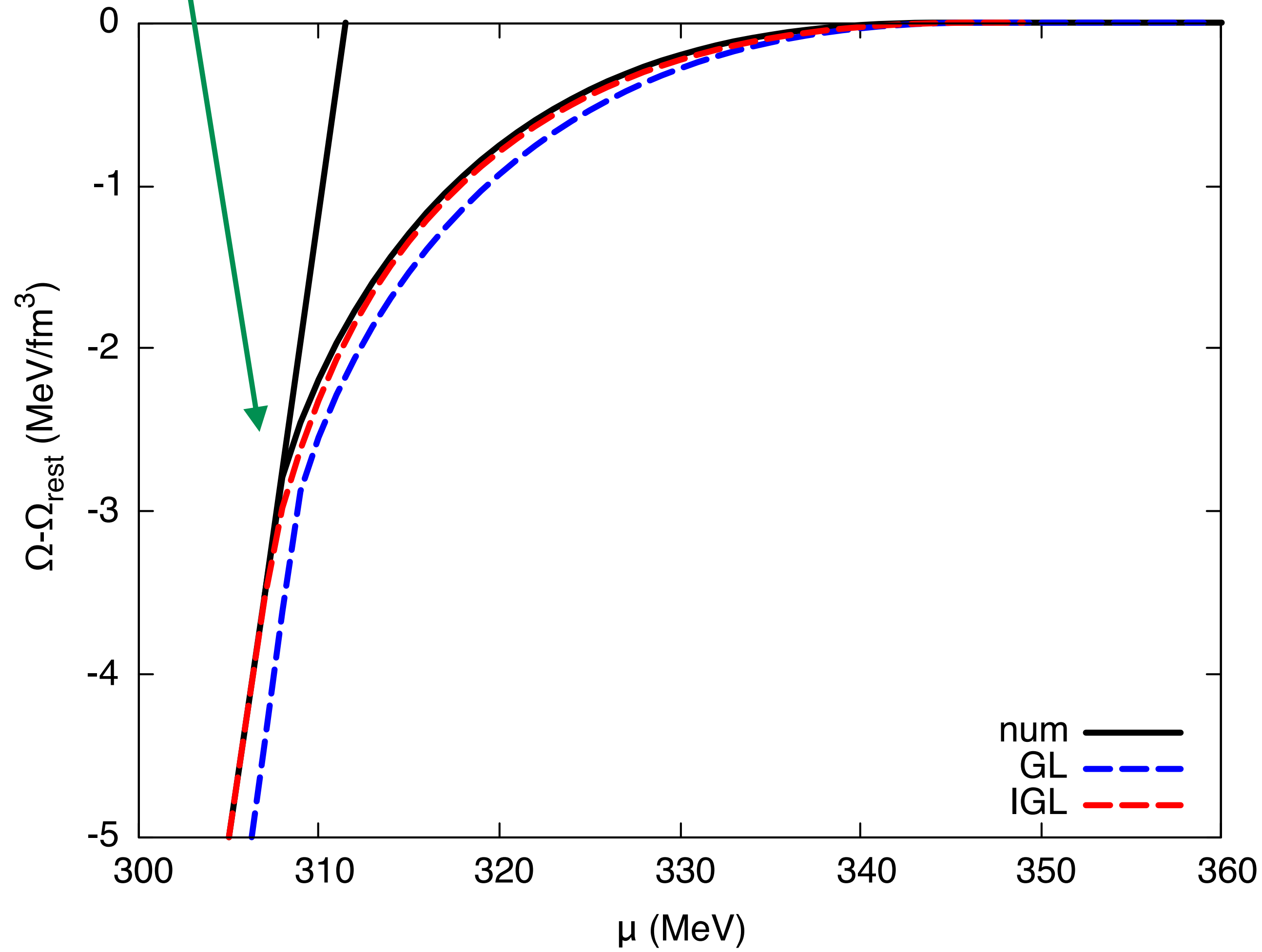
The IGL smoothly leads to the homogeneous phase





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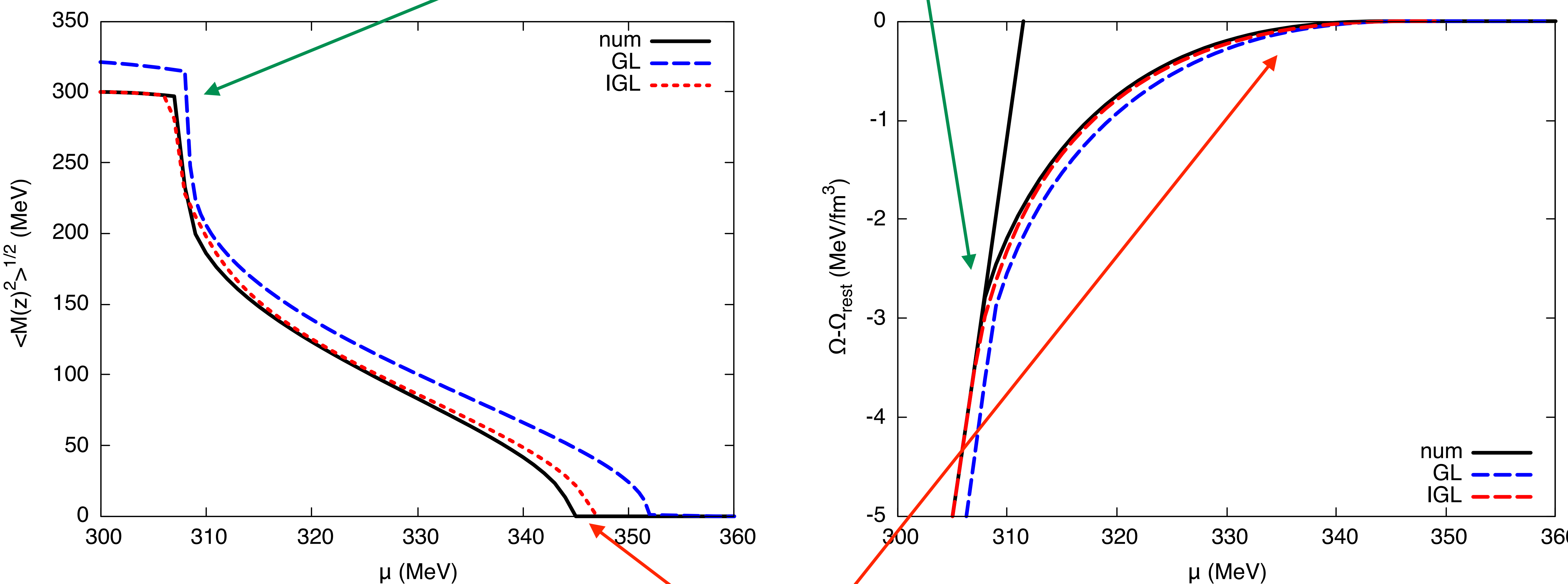
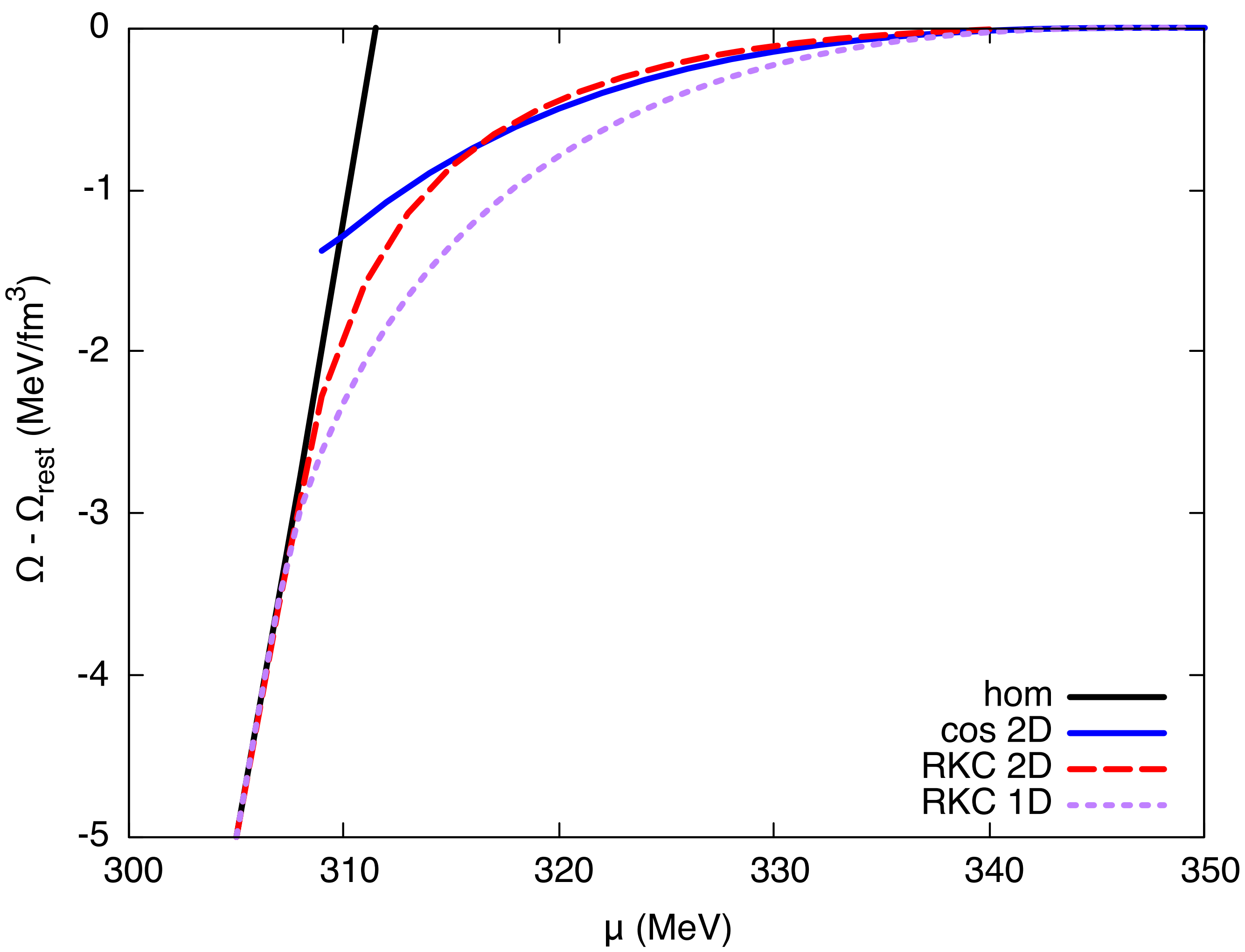


Improvement due to the inclusion of higher order terms

Comparison of some 1D and 2D modulations

Free energy of various phases in the IGL approximation



Why 1D modulations always win? Where does pairing occur?

Conclusions

1)We have proposed a novel GL expansion, which improves the description of the phase transitions to the inhomogeneous phases

2)It requires the knowledge of one (semi-)analytical expression of the free energy for a smple case

3)We have applied it to the inhomogeneous chiral symmetry breaking

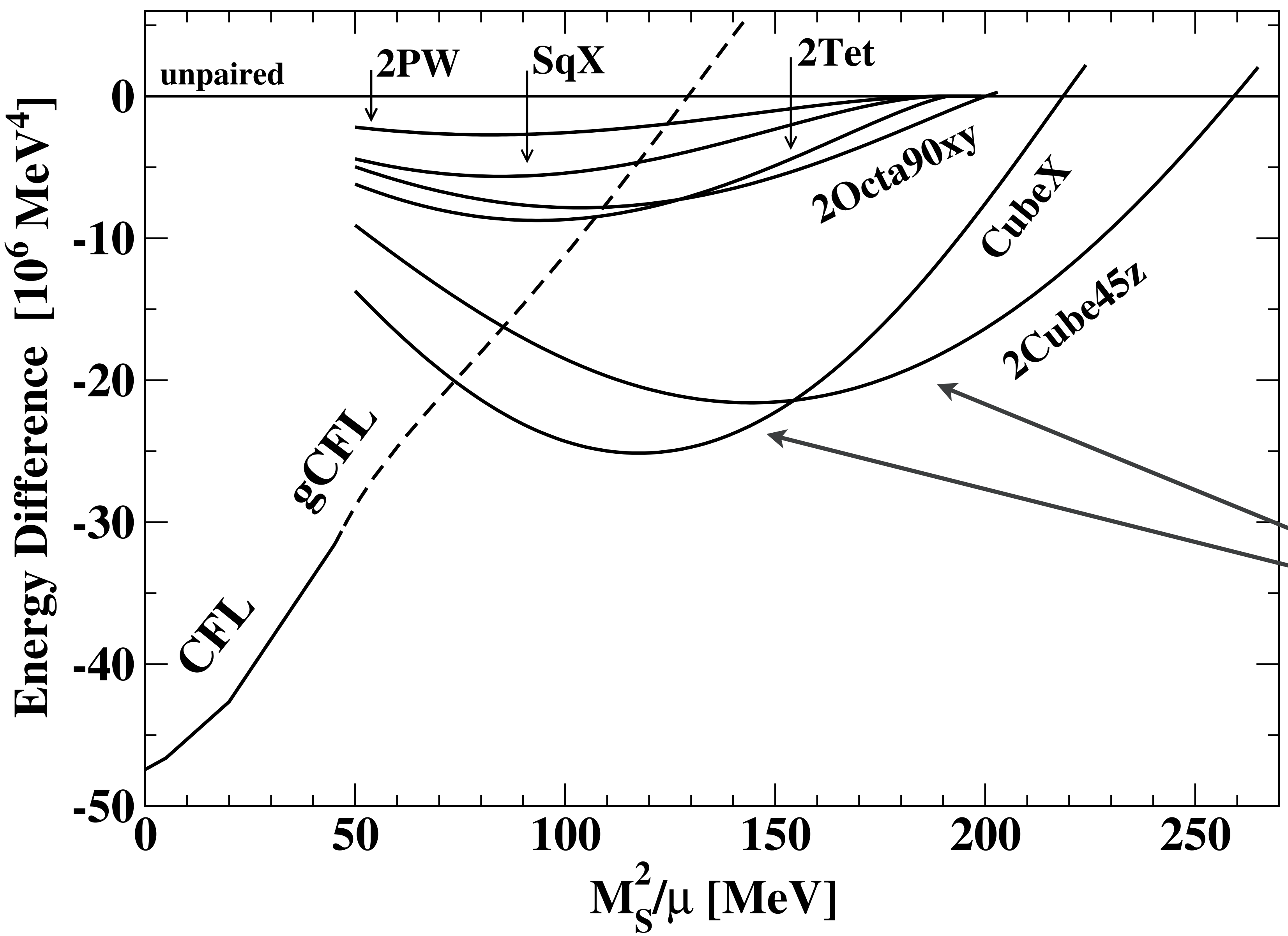
4)It can be applied to any inhomogeneous phase transition that satisfies the condition 2), as for example the crystalline color superconducting phase

BACKUP

Outlook

The IGL technique can be applied to the Crystalline Color Superconductors

NJL + GL expansion

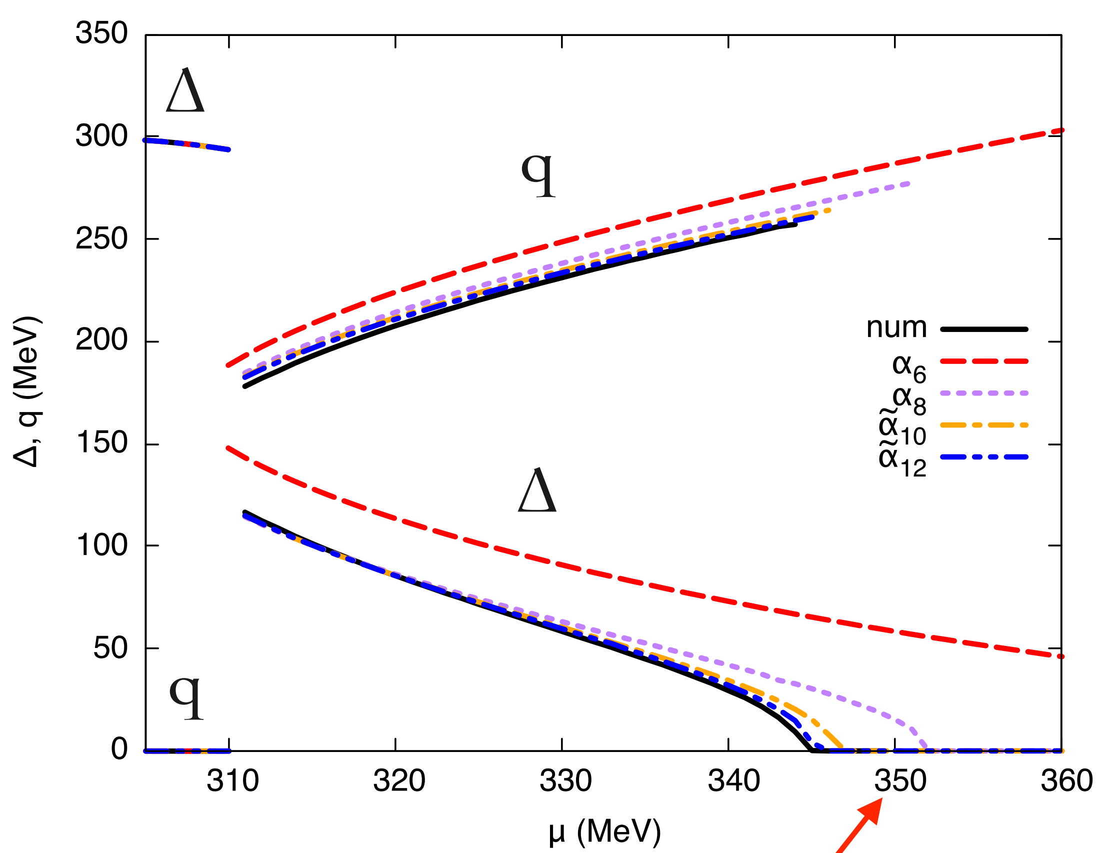


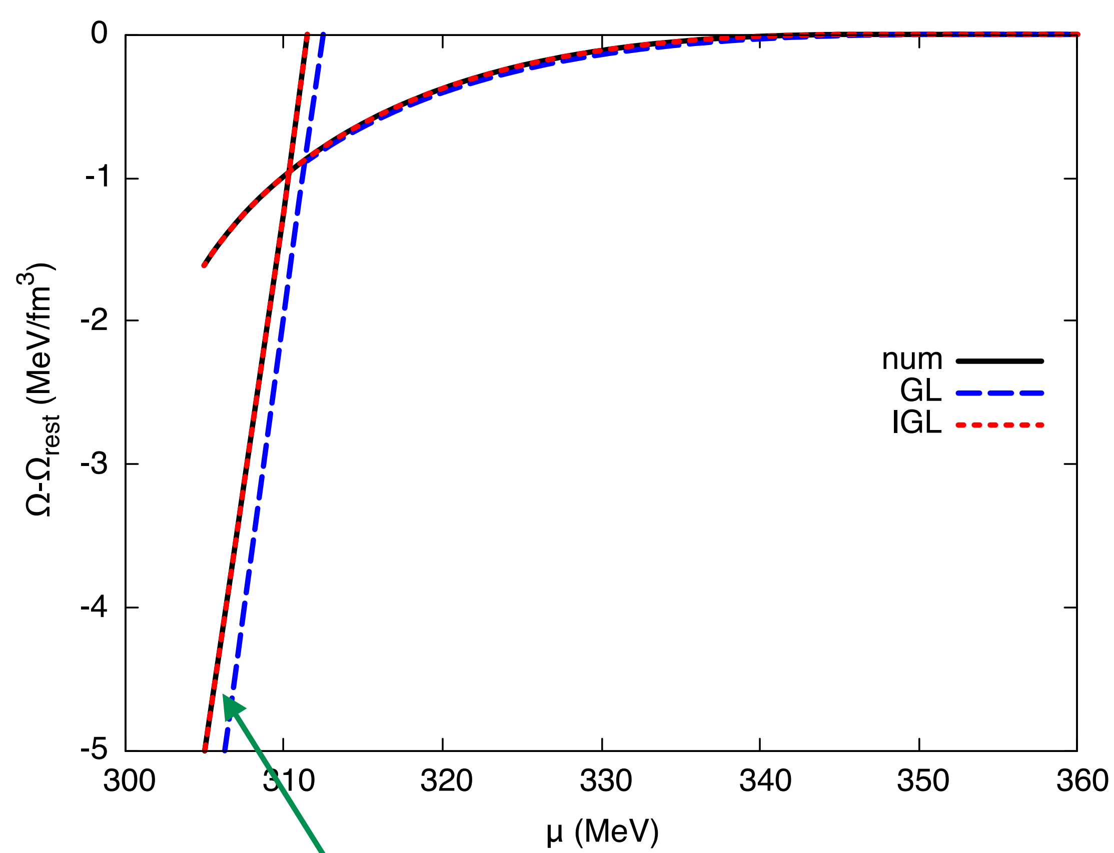
Weird non-monotonic behavior

Rajagopal and Sharma Phys.Rev. D74 (2006) 094019

Comparison: CDW case

Let us see what happens for the CDW anstaz $M(z) = \Delta e^{2iqz}$
In this case we have the numerical solution.





Improvement due to the inclusion of higher order terms

By construction IGL reproduces the homogeneous phase

Qualitative analysis of pairing

We closely inspect the integrand of the CDW ansatz

$$\Omega_{\mathrm{CDW}} = -\frac{N_f N_c}{4\pi^2}\int_0^{\infty} dp_\perp p_\perp \int_{-\infty}^{\infty} dp_z \sum_{\epsilon=\pm}\Big[E^{\epsilon}_{PV} + (\mu - E^{\epsilon})\theta(\mu - E^{\epsilon})\Big] + \frac{\Delta^2}{4G}$$

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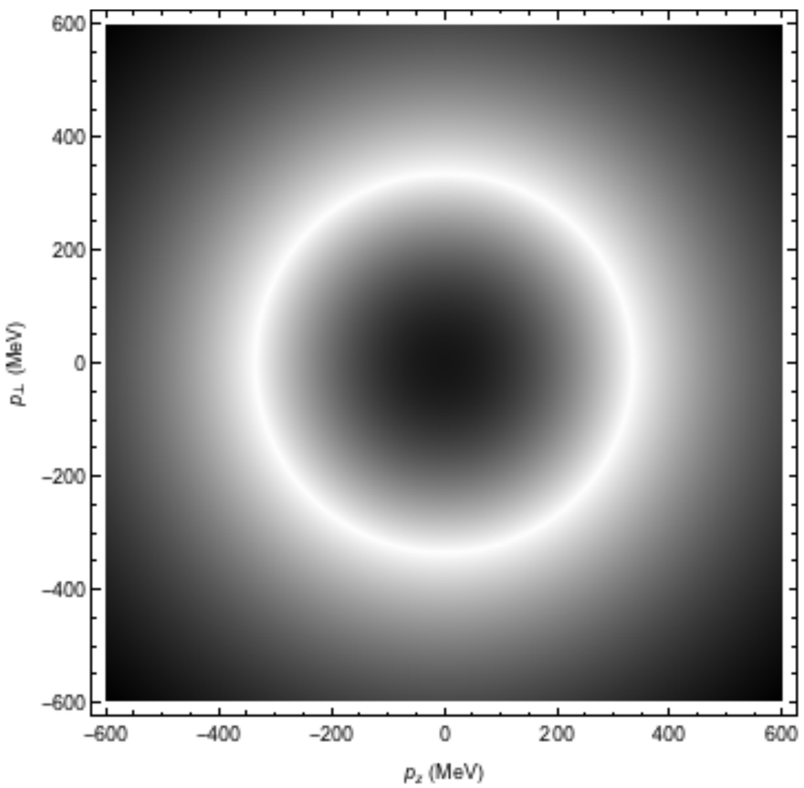
2D projection of the Fermi spheres for $\mu = 335$ MeV.
Light region: the energy cost for exciting quasiparticle is small

$\Delta = 0, Q = 0$



2 coincident Fermi spheres

Qualitative analysis of pairing

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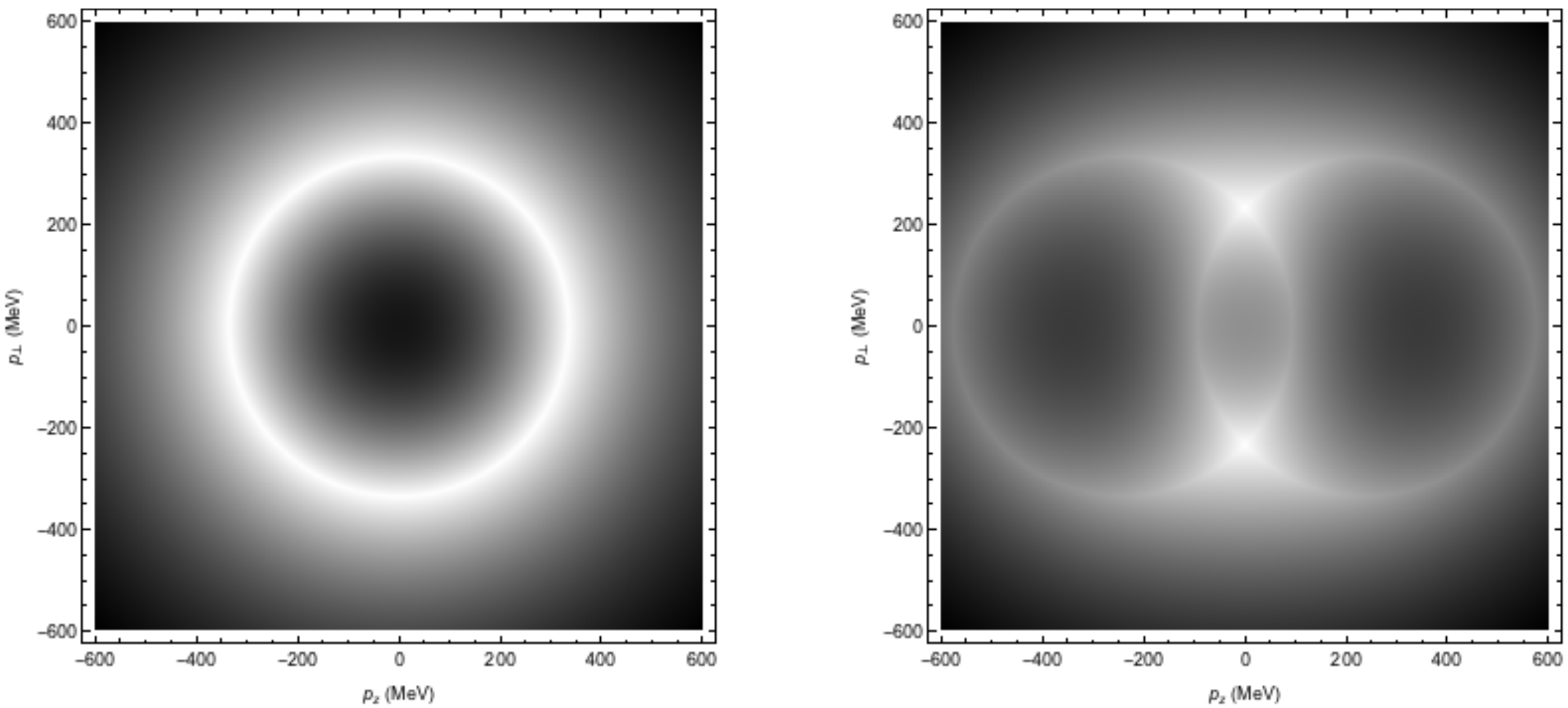
$$\Omega_{\mathrm{CDW}} = -\frac{N_f N_c}{4\pi^2}\int_0^\infty dp_\perp p_\perp \int_{-\infty}^{\infty} dp_z \sum_{\epsilon=\pm}\left[E^\epsilon_{PV} + (\mu - E^\epsilon)\theta(\mu - E^\epsilon)\right] + \frac{\Delta^2}{4G}$$

2D projection of the Fermi spheres for $\mu = 335$ MeV.
Light region: the energy cost for exciting quasiparticle is small

$\Delta = 0, Q = 0$ $\qquad$ $\Delta = 0, Q = 241$ MeV



2 coincident Fermi spheres $\qquad$ 2 displaced Fermi spheres

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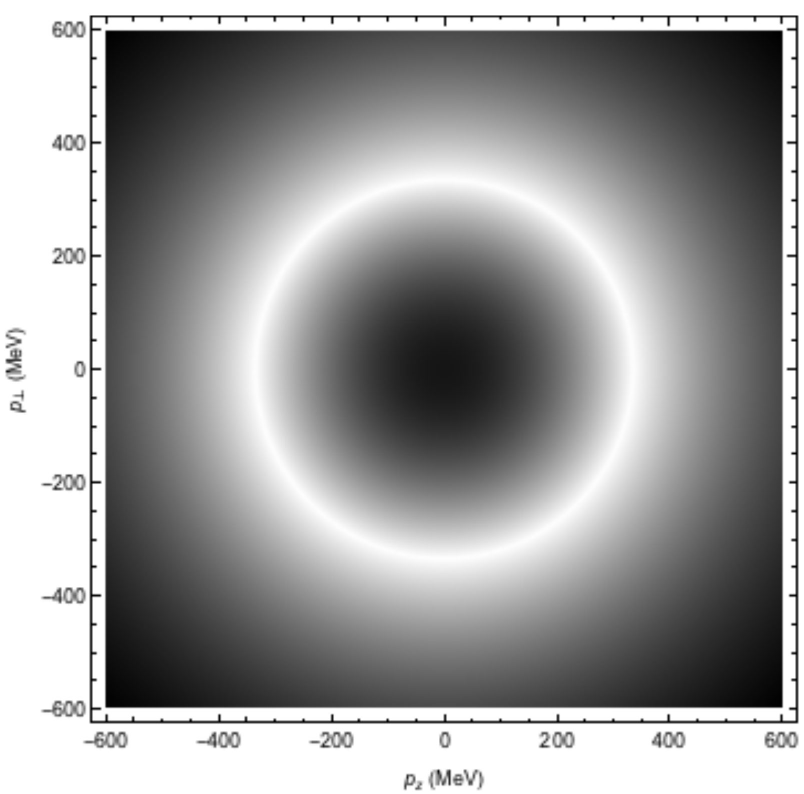

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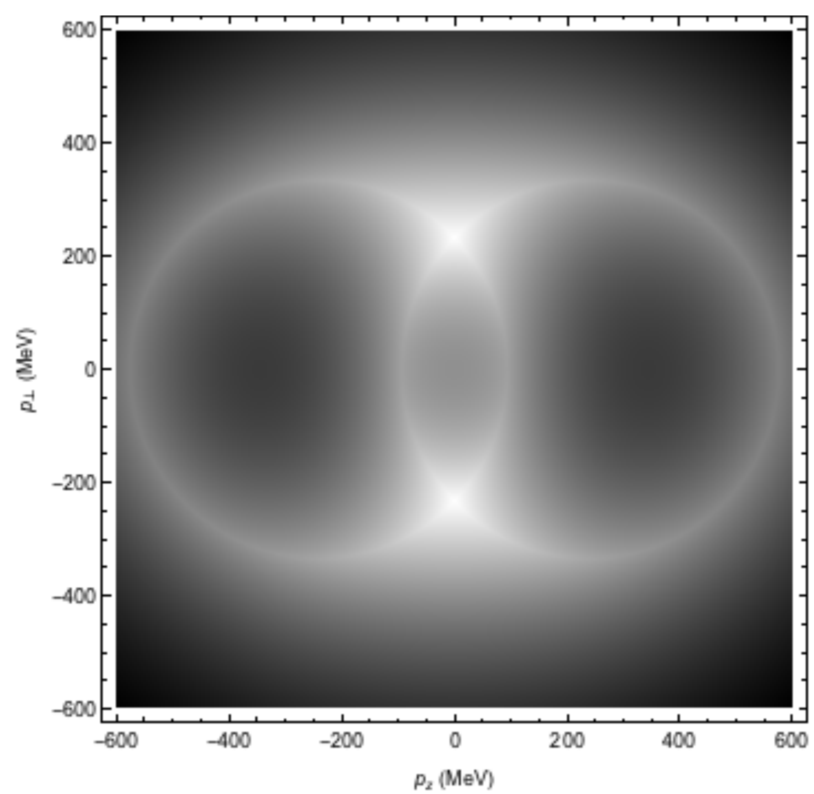
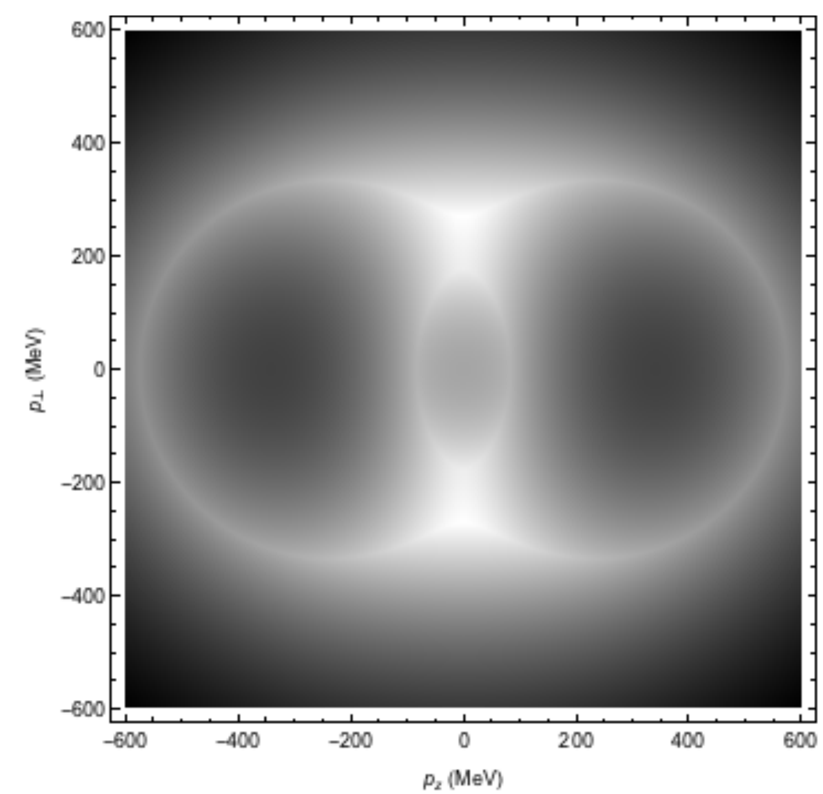


2 displaced Fermi spheres

$\Delta = 44$ MeV$, Q = 241$ MeV

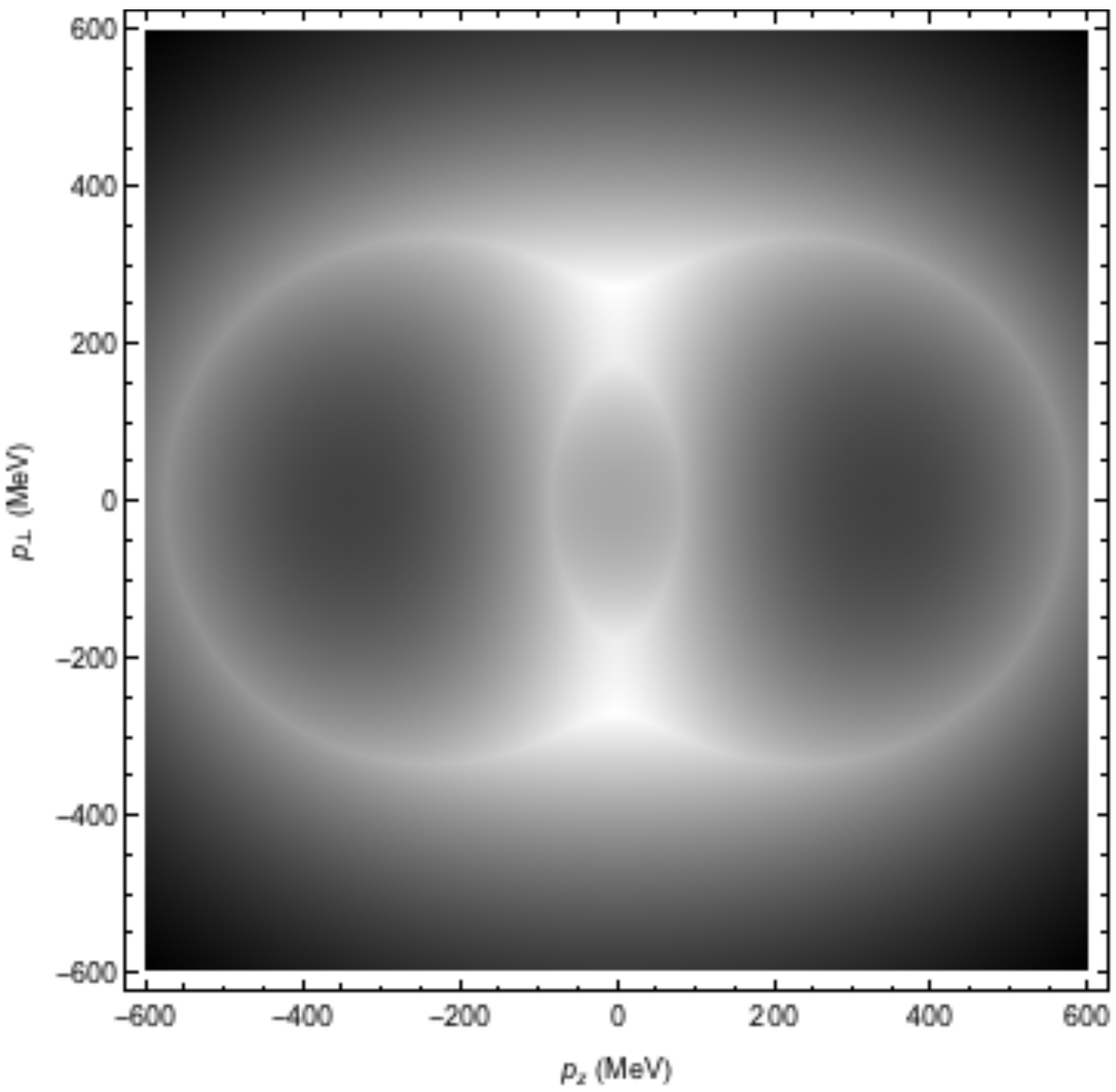


Turning on pairing

Analysis of pairing

The Fermi spheres are strongly modified. As far as Q is large, only 1D modulation can be favored

$$\Delta = 44 \text{ MeV}, Q = 241 \text{ MeV}$$



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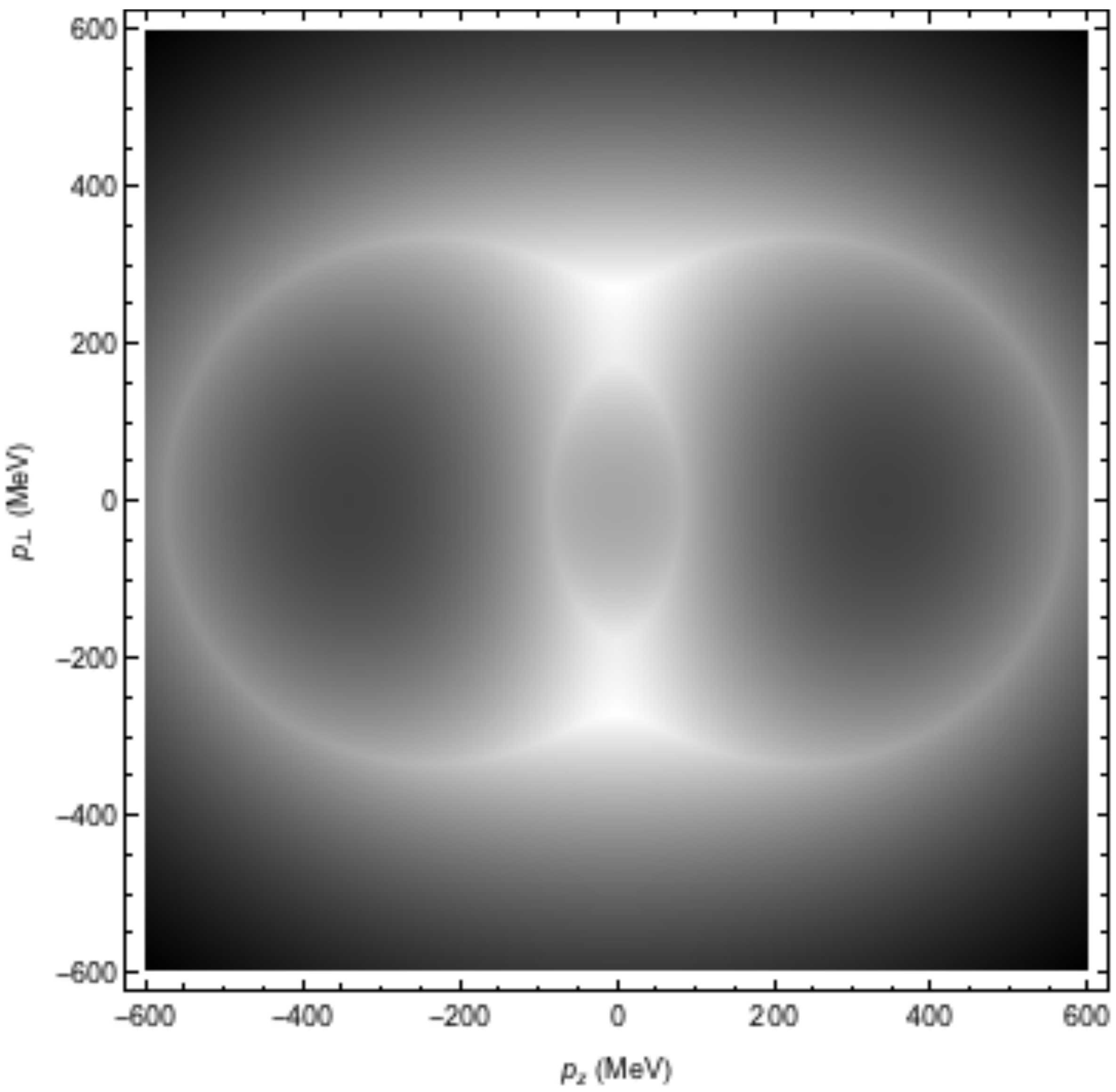

For example: pairing

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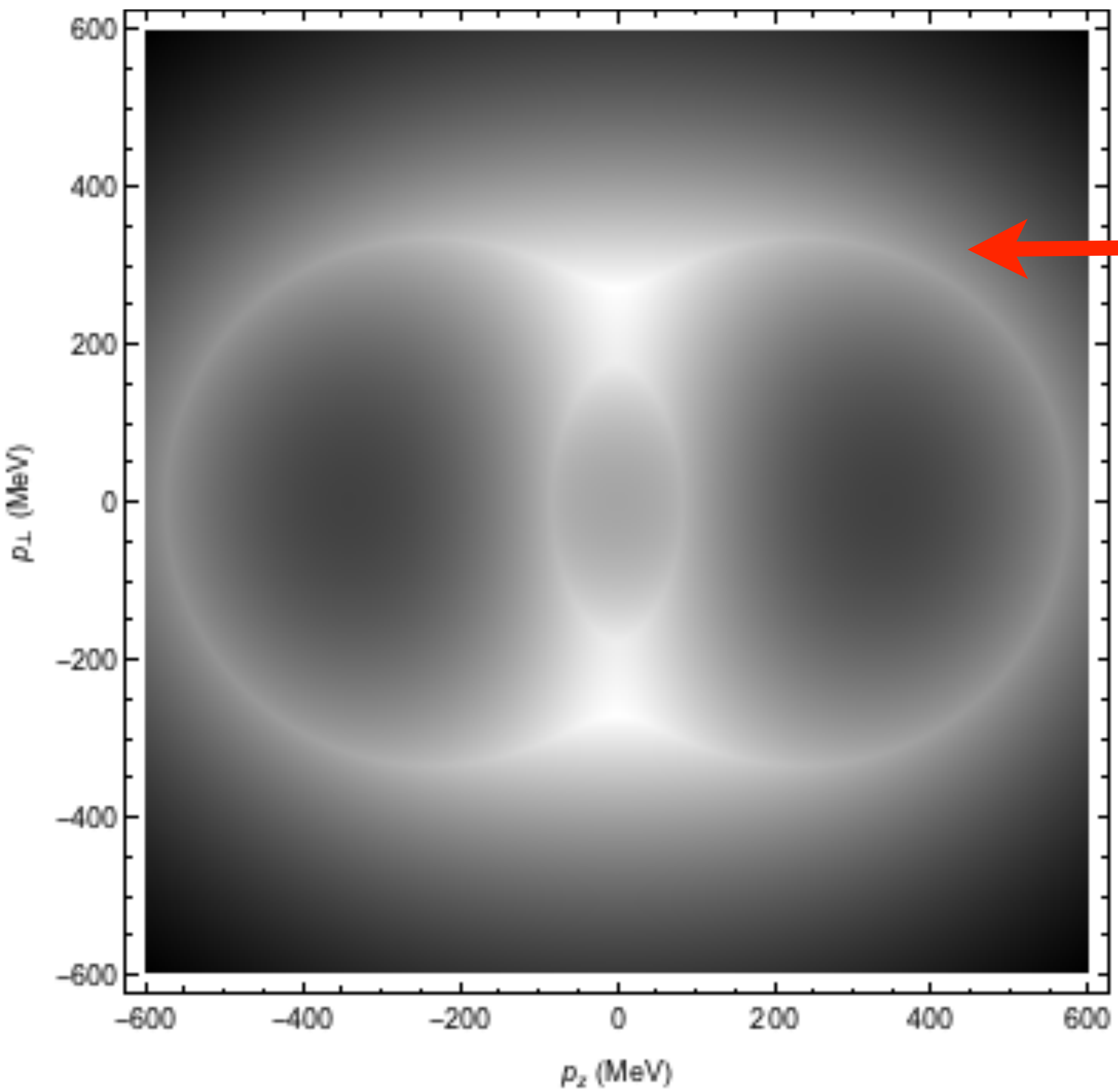

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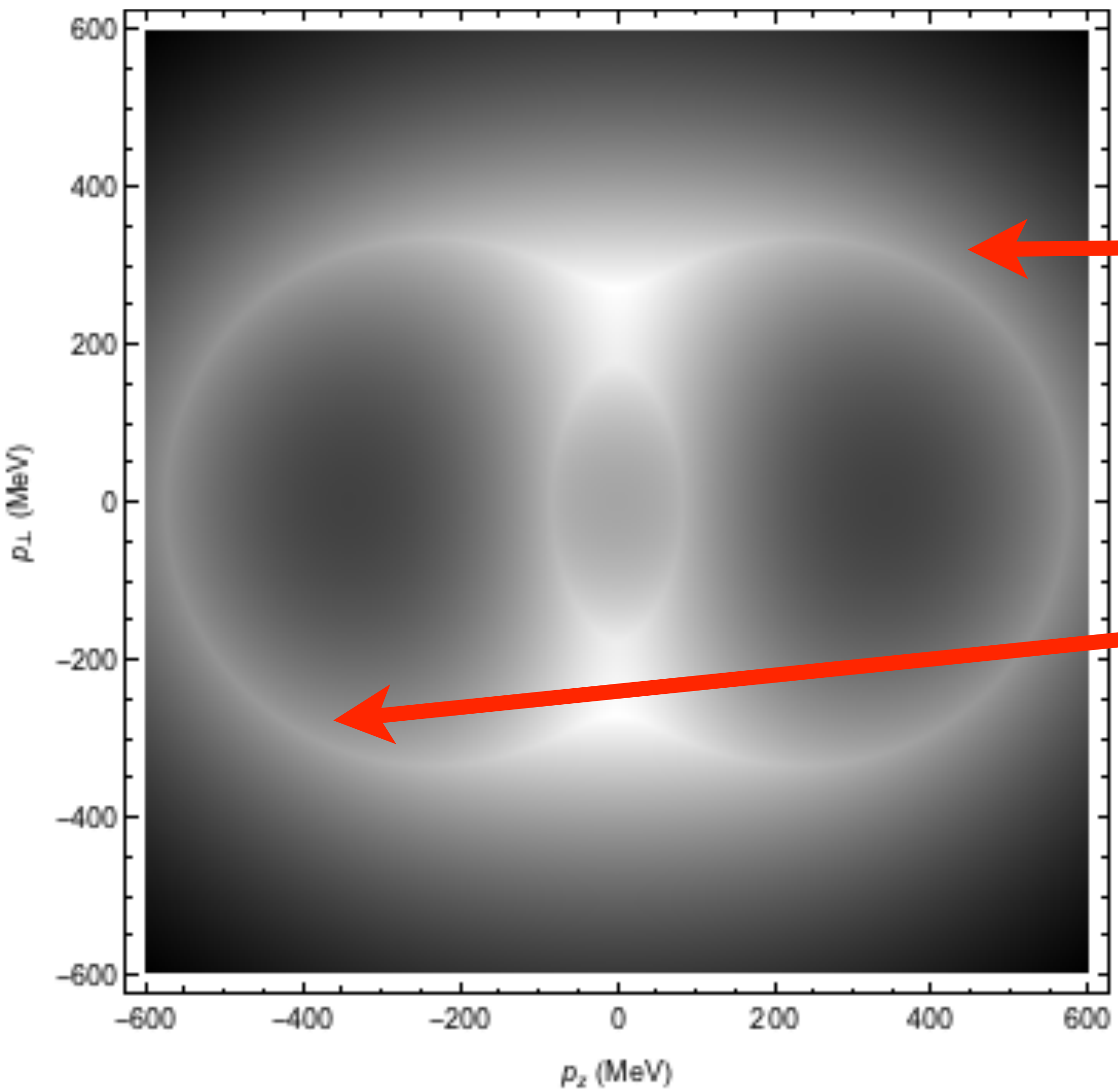

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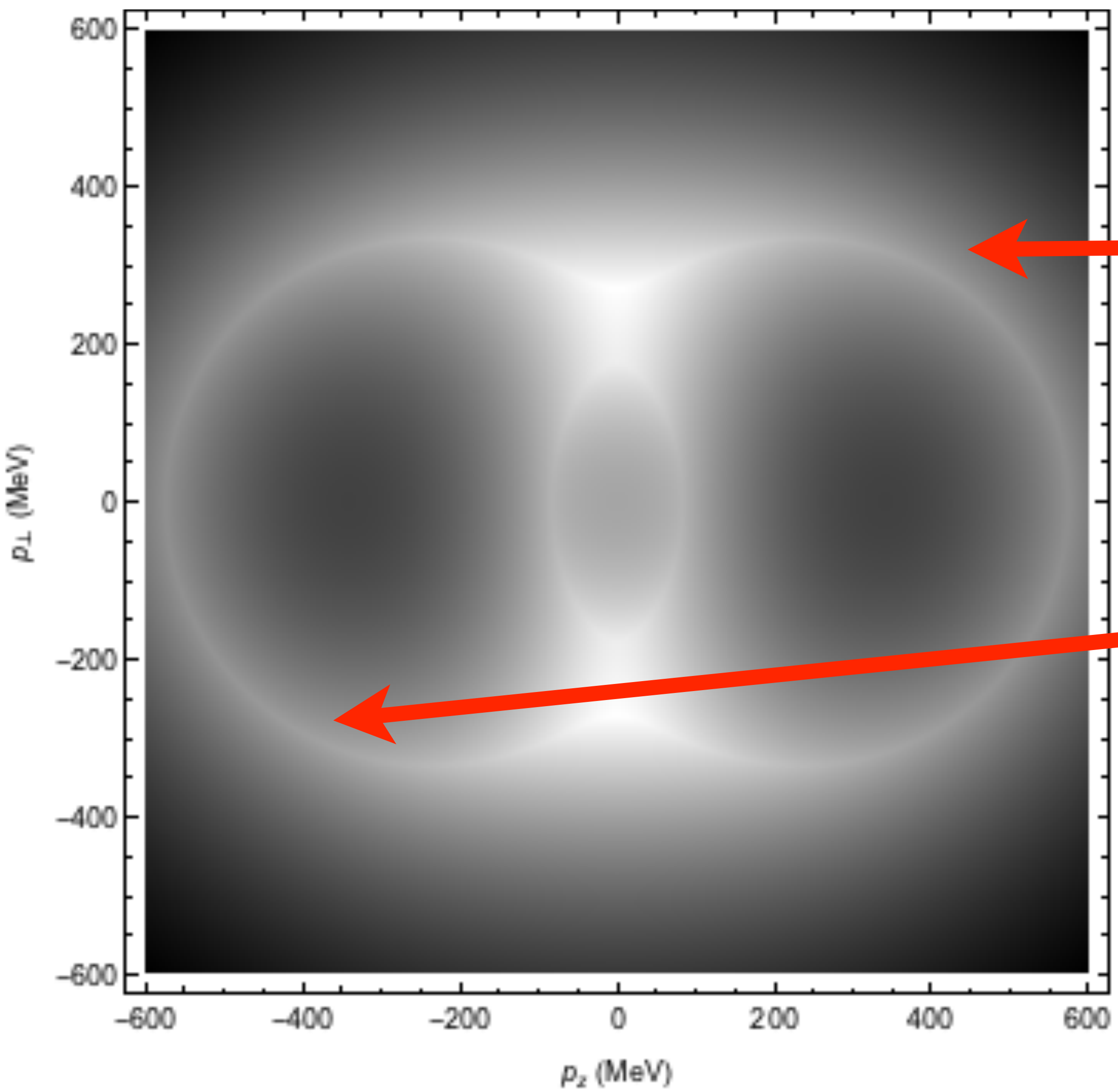

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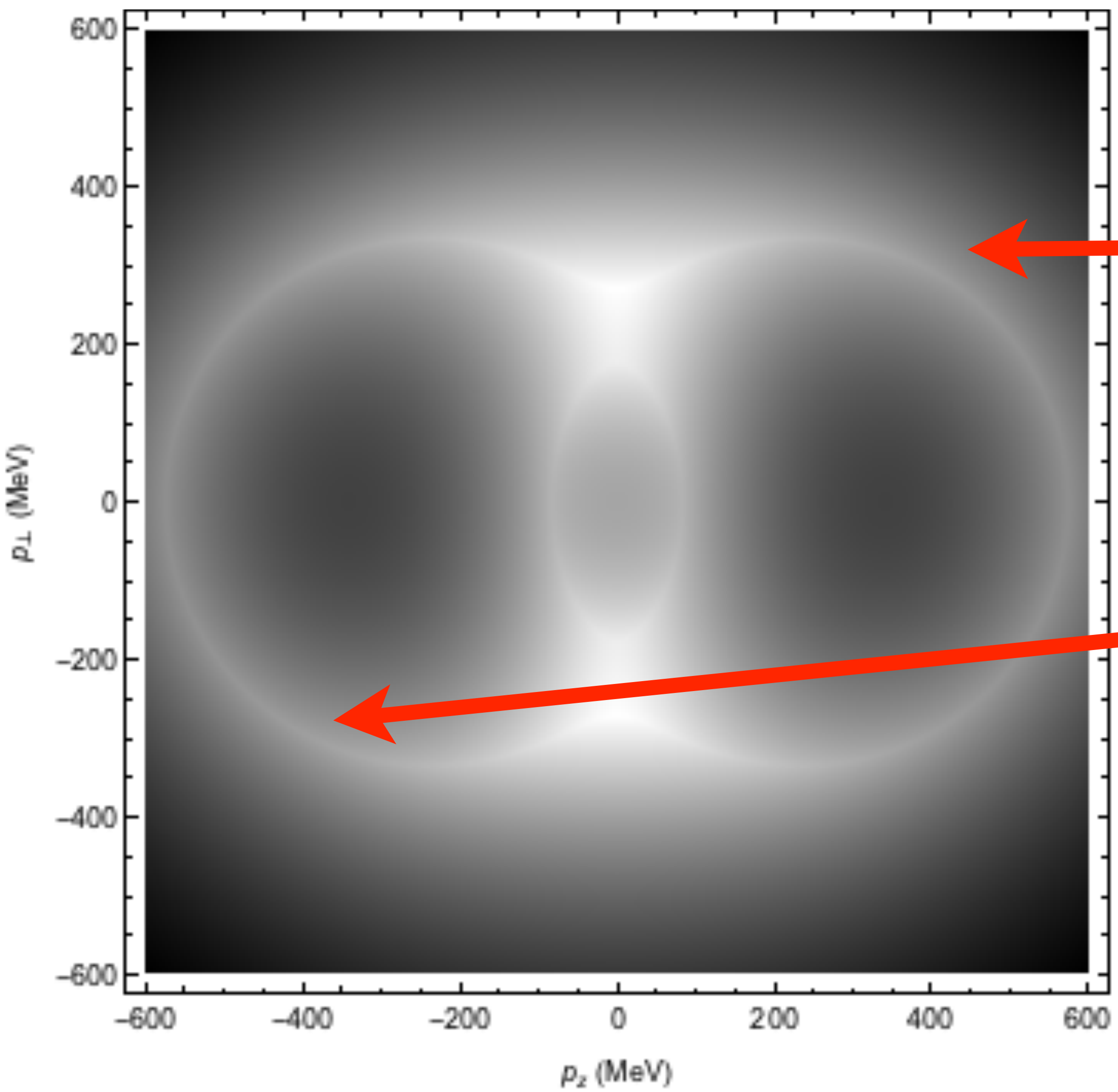

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There seems no way to have a crystalline phase.

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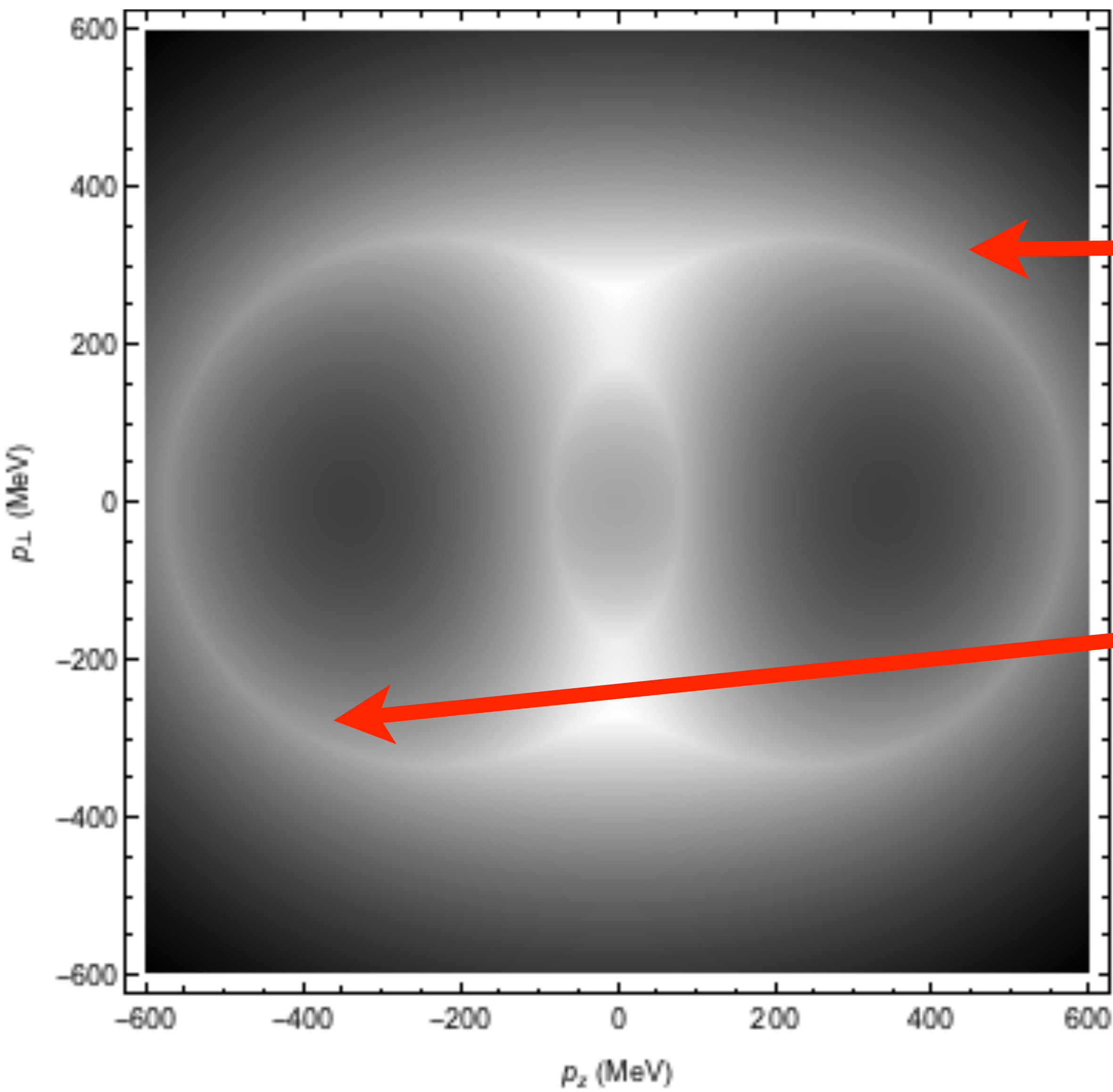

For example: pairing

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quarks here

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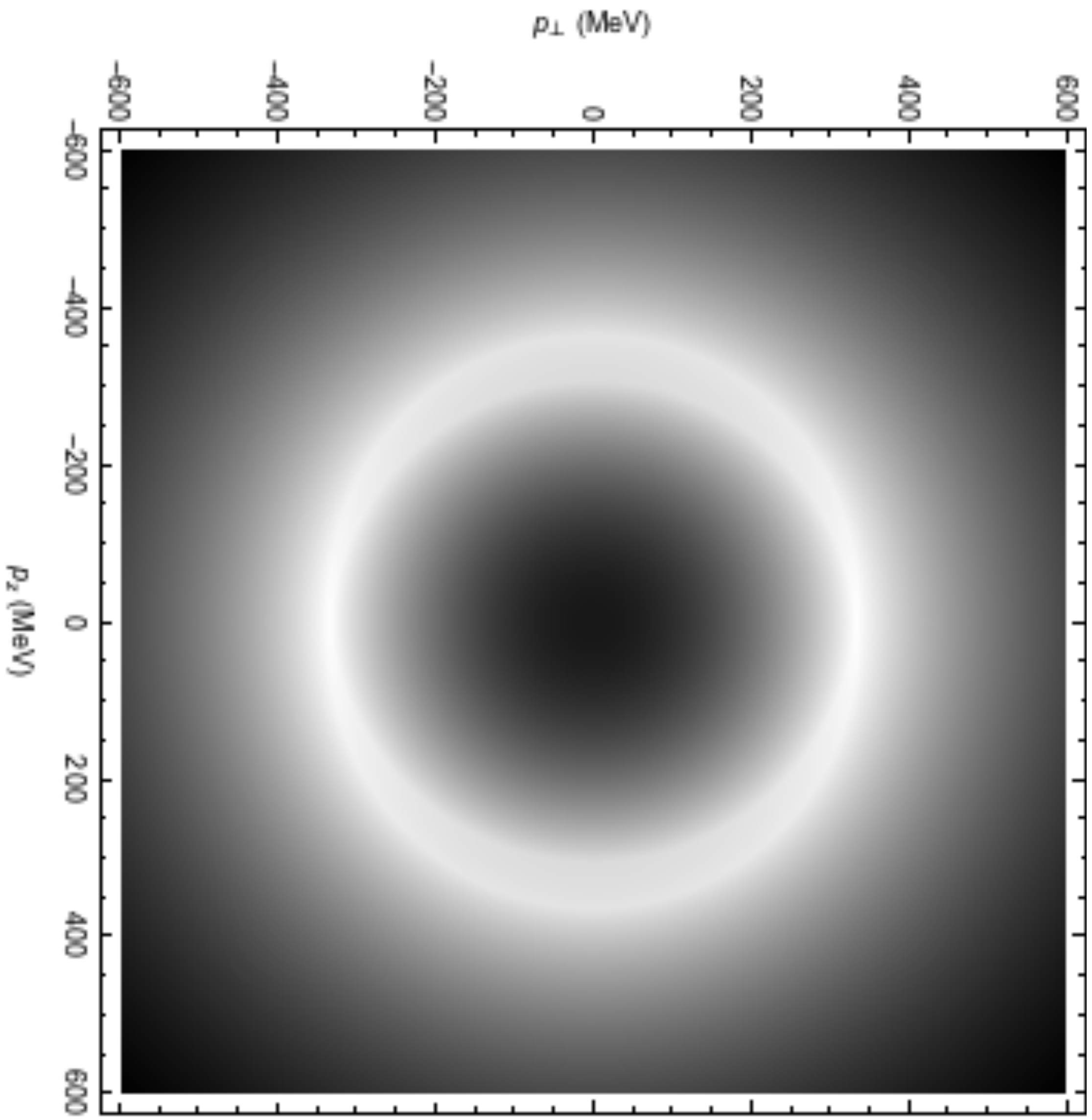
There seems no way to have a crystalline phase.

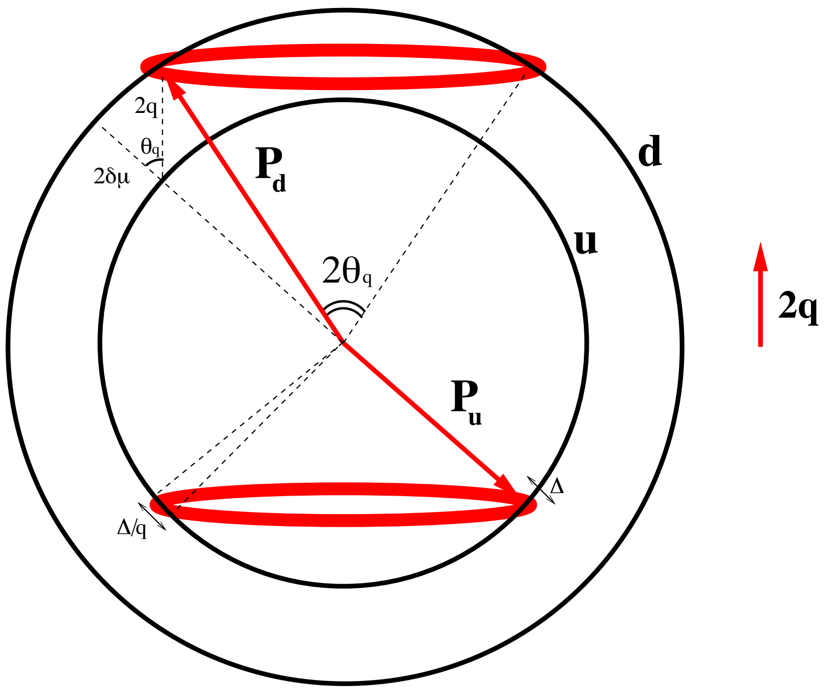
But then why a crystalline phase is realized in color superconductors?

FFLO-phase (quark-quark pairing)

Mismatched two flavor quark matter

For $\delta\mu_1 < \delta\mu < \delta\mu_2$ the superconducting phase with Cooper pairs of non-zero total momentum is favored





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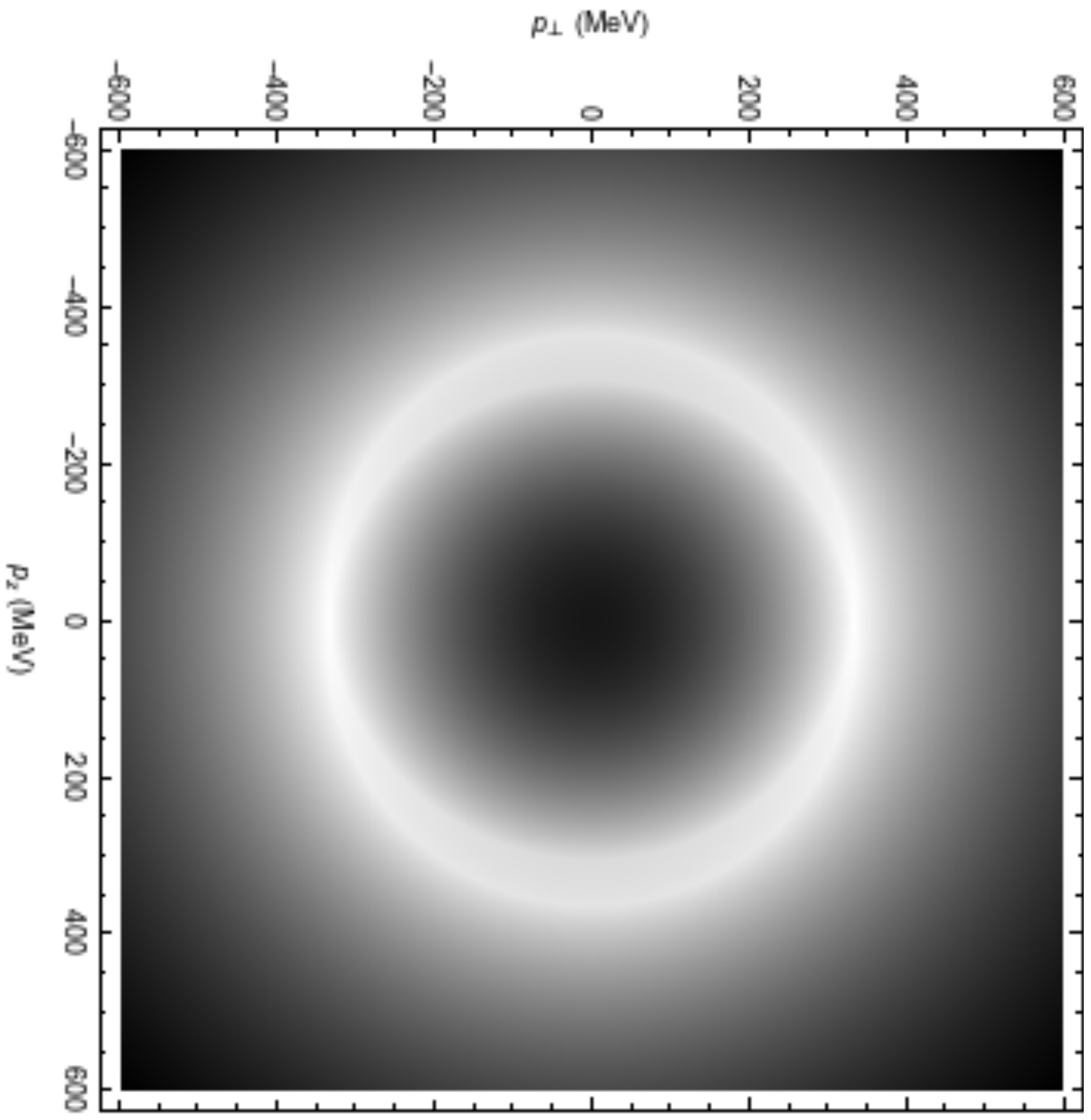

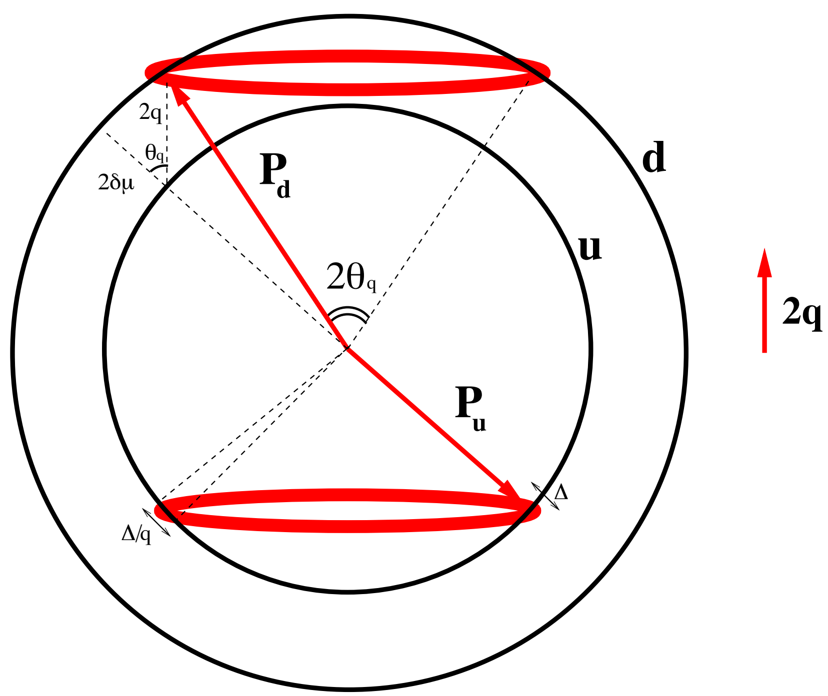

In weak coupling

$$\delta\mu_1 \simeq \frac{\Delta_0}{\sqrt{2}} \qquad \delta\mu_2 \simeq 0.75\,\Delta_0$$

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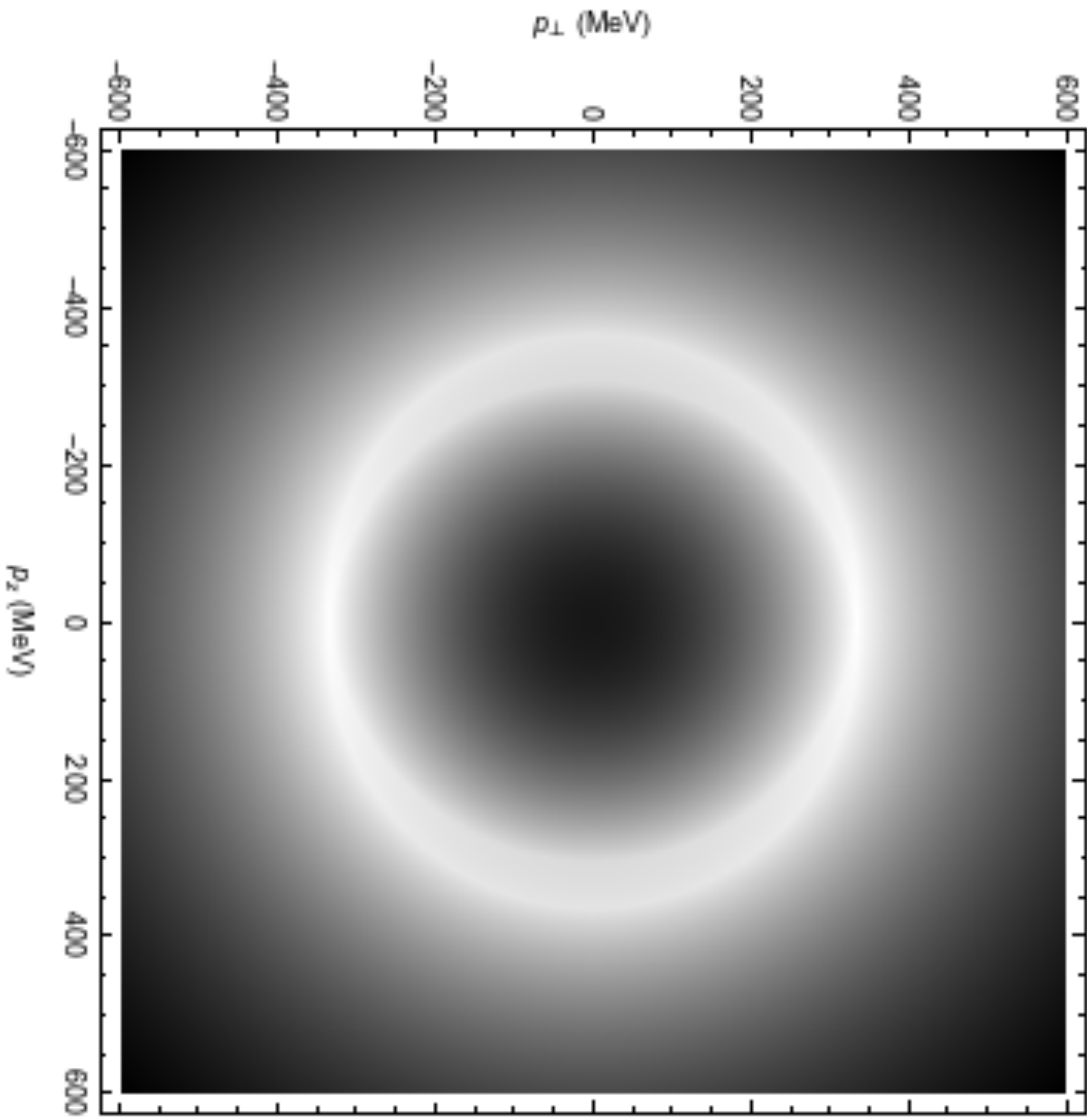

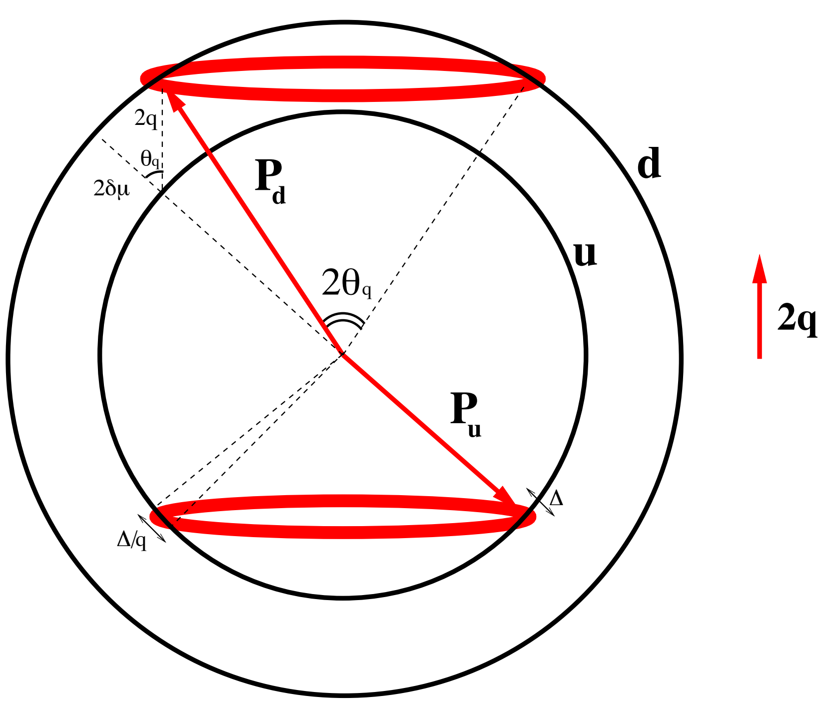

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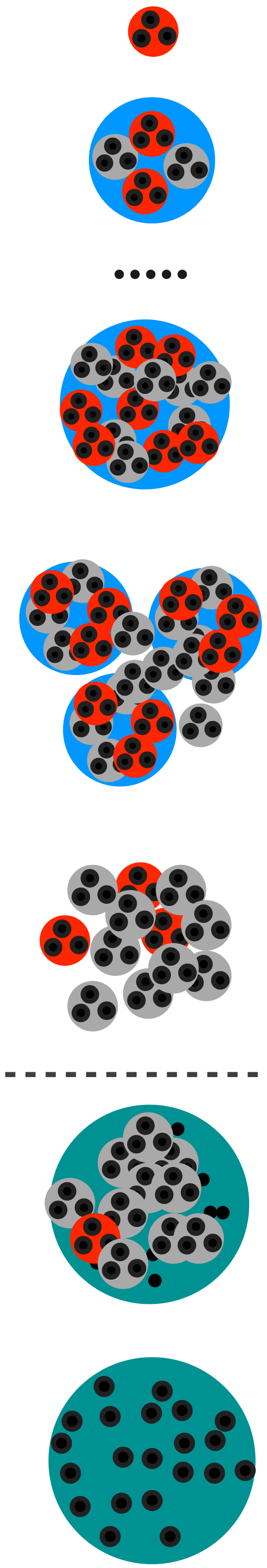
Deforming the Fermi sphere does not cost too much!
The free energy gain due to pairing overcompensates this co

Increasing baryonic density



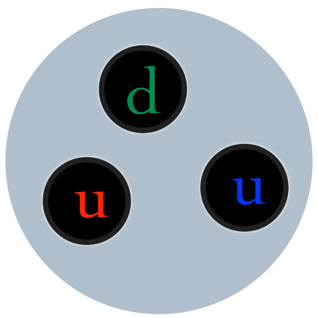
	Density	CSO part	$\alpha_s \equiv \alpha_s(\mu)$	Degrees of freedom
H				
He	......	atmosphere		light nuclei
......				.....
Fe	10 g cm^{-3}	outer crust	confining	heavy nuclei
neutron drip	10^{11} g cm^{-3}			
		inner crust		
neutron proton soup	10^{14} g cm^{-3}			neutrons and protons
		core		
quark drip	very large		strong coupling	quarks and gluons Cooper pairs of quarks?
quark soup	extreme		weak coupling	Cooper pairs of quarks NGBs

ρ

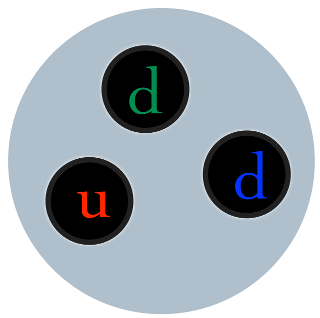
Quark model

BARYONS

proton



neutron

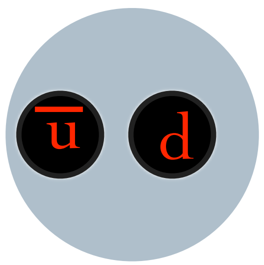


....

$$M_n \sim 1\text{GeV} \gg m_{u,d}$$

MESONS

pions



....

$$M_\pi \sim 135 \text{ MeV} \gg m_{u,d}$$

Quarks and gluons are the building blocks of hadrons

Q	quark flavor (mass in MeV)		
$+2/3$	u (3)	c (1300)	t (170000)
$-1/3$	d (5)	s (130)	b (4000)

The theory describing quarks and gluons is **Quantum Chromodynamics (QCD)**: a nonabelian SU(3) gauge theory.
Quarks form a triplet in the fundamental representation
Gluons are the vector gauge bosons associated to the octet adjoint representation