



TMDs: A short Theory Overview

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QCD@Work

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Summary

- Historical motivations
- Phenomenological interest and puzzles
- Basic definition and properties of TMD PDFs and FFs
- Factorization, universality and process dependence
- TMD evolution, multiplicities
- Phenomenology in SIDIS and e^+e^- collisions
- TMDs in pp collisions, Drell-Yan processes (also in πp at COMPASS)
- Outlook and future prospects



Historical motivations

- Phenomenological implications of intrinsic (transverse) motion of partons in high energy hadronic collisions studied already at the start of parton model (Feynman and collab.), e.g. in the low q_T spectrum for Drell-Yan processes
- Formal and conceptual aspects studied in QCD, e.g. Ellis, Furmanski, Petronzio, 1983:
 - Lorentz invariance for on-shell partons implies a very narrow k_\perp distribution and a non Gaussian shape
- Recall the (very narrow) Lorentzian widths of spectral lines where pressure, Doppler broadening modify the shape and increase the width of the lines
 - (In our case: higher-twist effects, off-shellness etc, not well under control)
- Only unpolarized processes: no spin effects taken into account until the formulation of what is now called the Transverse Momentum Dependent (TMD) approach [for single hadron production in pp collisions also called Generalized Parton Model (GPM)]



Phenomenological problems and puzzles

- Huge transverse $\Lambda, \bar{\Lambda}$ polarization in unpolarized pp, pA collisions (1978-80)
 - Low c.m. energy, $p_T \leq 3$ GeV, no unpolarized cross section measurements
 - New measurements feasible at RHIC, LHC, SIDIS [HERMES PRD90 072007 (2014)], e^+e^- collisions [Belle 1611.06648] [talk by A. Martini on Belle II status?]
- Big transverse single spin asymmetries (SSAs) in $p^\uparrow p \rightarrow \pi X$ processes at large x_F and moderately large p_T (E704, 1990 \rightarrow today, RHIC). [LT collinear pQCD ~ 0 in this regime]
- Violation of QCD helicity selection rules in quarkonium exclusive decays (LC WFs)
- Dilepton c.m. angular distributions in Drell-Yan processes
- Proton spin puzzle: q, \bar{q} spin contributions apparently account for $\sim 30\%$ of the proton spin. The reminder due to gluon spin (Δg) and parton Orbital Angular momentum (OAM) contributions. [see next talks on GPDs by S. Niccolai and A. Sandacz]
 - Complications: spin decomposition: gauge invariance, uniqueness, measurability [see e.g. Leader, Lorcé, Phys. Rep. 541 (2014)]
 - Lattice QCD calculations seem to support huge OAM parton contributions

Proton spin: information on gluon Δg

Measurements of double longitudinal asymmetry

A_{LL} in gluon dominated processes at RHIC,

$pp + jet(s) + X$, $pp \rightarrow \pi^0 + X$

at mid- and forward rapidity and large p_T

SIDIS, $\ell^\pm p(d) \rightarrow \ell^\pm D + X$

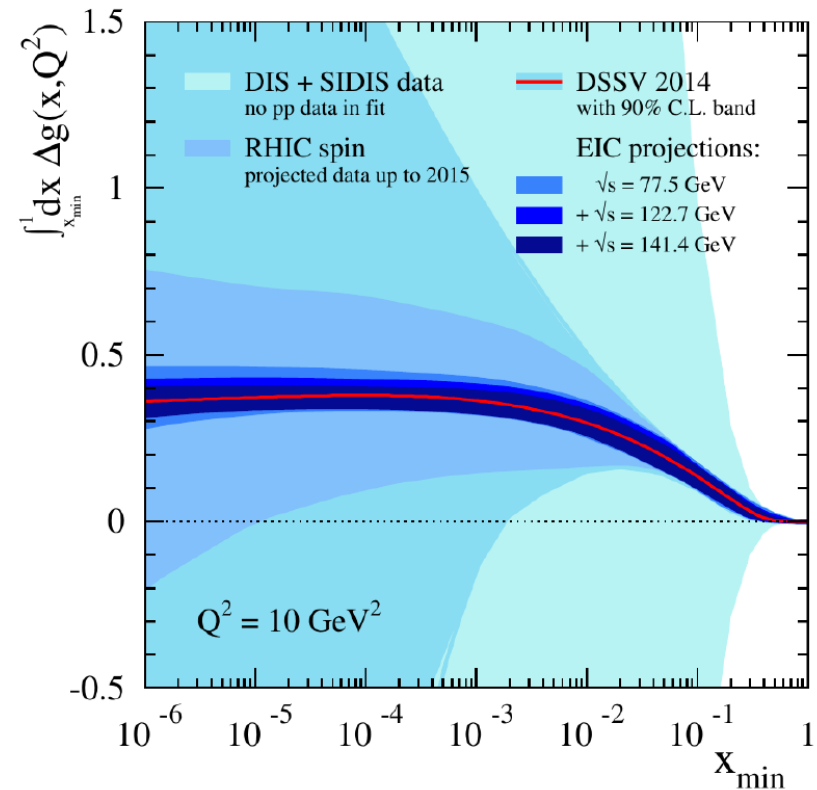
Using NLO QCD global fits to PDFs (CTEQ, DSSV, NNPDF, JAM, etc) one gets information on Δg

Main uncertainties due to low x contributions (EIC role crucial here)

NNPDF and DSSV estimates are well compatible

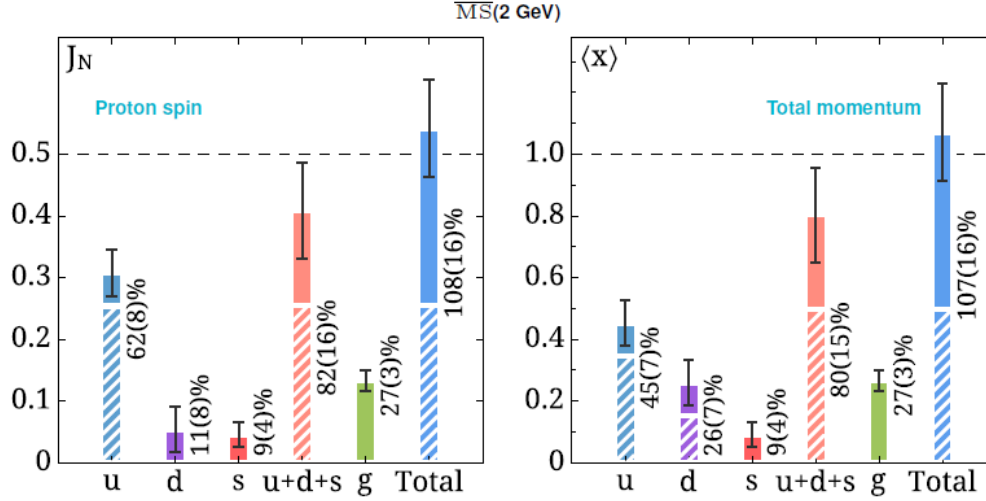
$$\int_{0.01}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = +0.23 \pm 0.15$$

$$\int_{0.01}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = +0.32 \pm 0.13$$



E. Nocera, NNPDF Collaboration

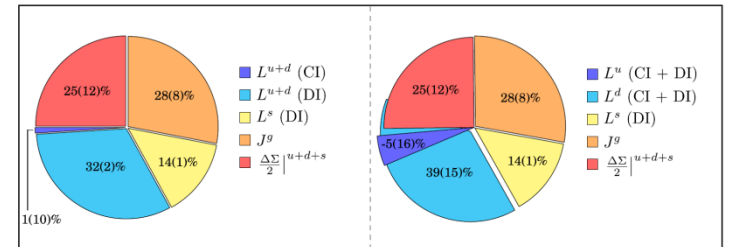
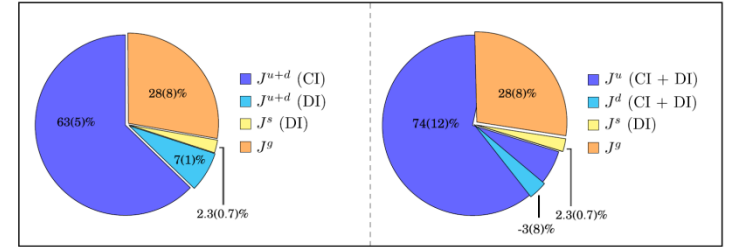
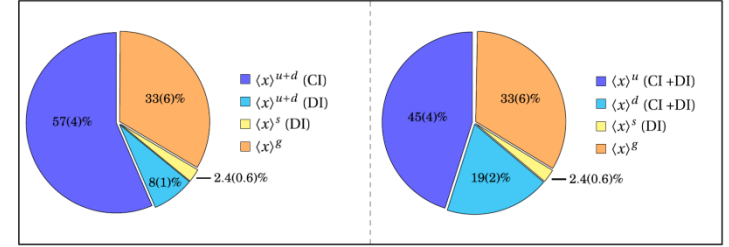
Proton spin and quark OAM on the lattice



Striped segments: valence quark contributions (connected)

Solid segments: sea quark & gluon contributions (disconnected)

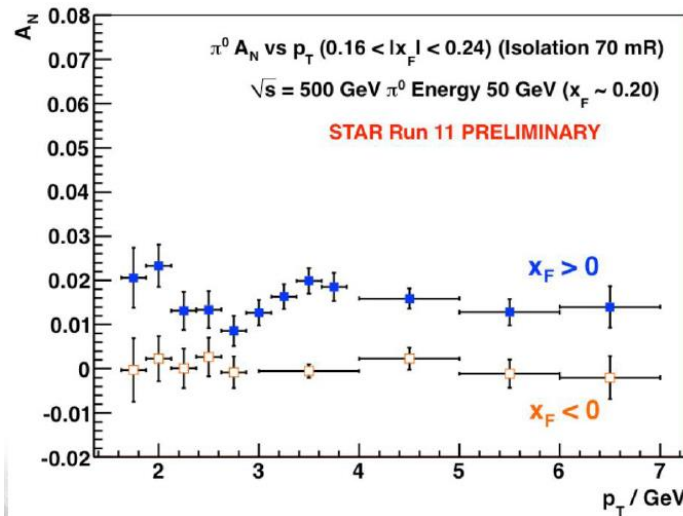
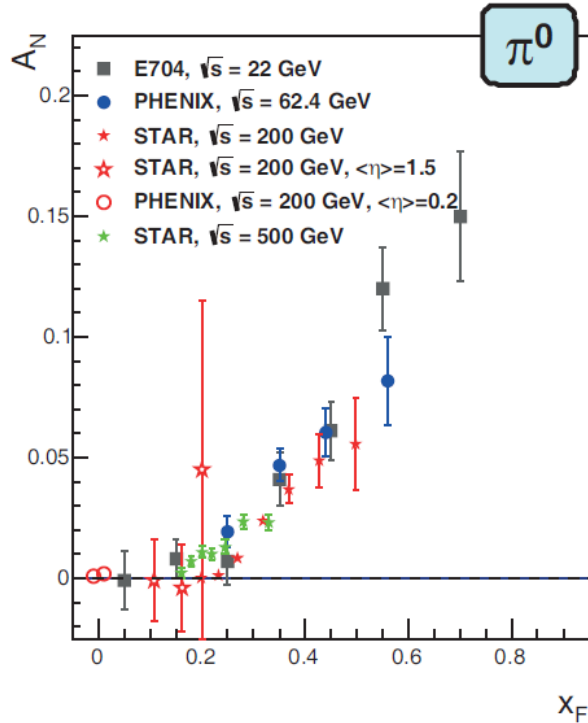
	$\frac{1}{2} \Delta \Sigma$	J	L	$\langle x \rangle$
u	0.415(13)(2)	0.308(30)(24)	-0.107(32)(24)	0.453(57)(48)
d	-0.193(8)(3)	0.054(29)(24)	0.247(30)(24)	0.259(57)(47)
s	-0.021(5)(1)	0.046(21)(0)	0.067(21)(1)	0.092(41)(0)
g	...	0.133(11)(14)	...	0.267(22)(27)
Tot.	0.201(17)(5)	0.541(62)(49)	0.207(64)(45)	1.07(12)(10)



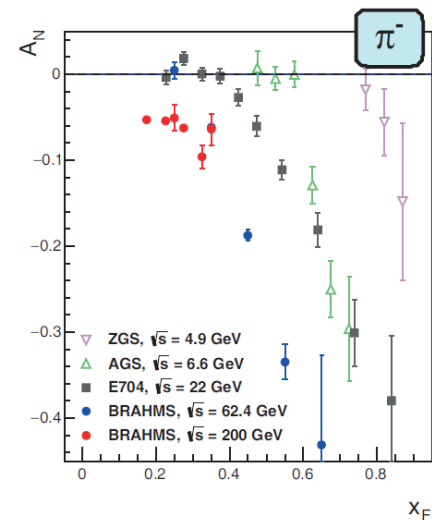
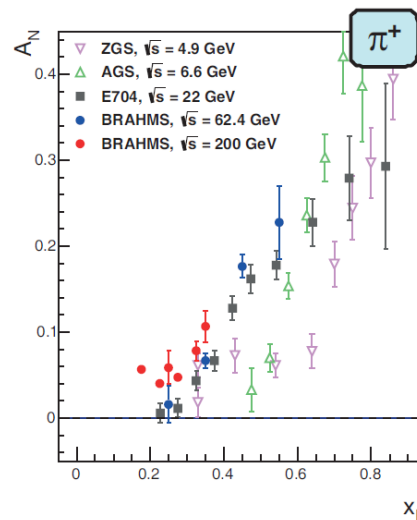
**χ QCD Collaboration
PRD 91, 014505 (2015)**

C. Alexandrou et al. PRL 119, 142002 (2017)

Transverse SSAs in $p^\uparrow p \rightarrow \pi + X$ processes



$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \frac{d\Delta\sigma}{2d\sigma}$$



Sivers effect in $p^\uparrow p \rightarrow \pi + X$ processes

$$d\sigma^{pp \rightarrow h X} \sim \sum_{a,b,c,d} f_{a/p}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab \rightarrow cd} \otimes D_{h/c}(z, p_{\perp c})$$

$$d\sigma^{p^\uparrow p \rightarrow h X} \sim \sum_{a,b,c,d} [f_{a/p}(x_a, k_{\perp a}) + \frac{1}{2} \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) \sin(\phi_a - \phi_S)] \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab \rightarrow cd} \otimes D_{h/c}(z, p_{\perp c})$$

$$[d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Sivers}} = d\Delta\sigma_{\text{Sivers}} = \sum \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) \sin(\phi_a - \phi_S) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab \rightarrow cd} \otimes D_{h/c}(z, p_{\perp c})$$



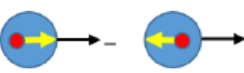
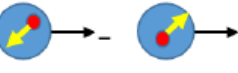




Another possible contribution to the SSA in $p^\uparrow p \rightarrow \pi + X$ is the Collins effect

$$[d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Collins}} = d\Delta\sigma_{\text{Collins}} = \sum \Delta_T f_{a/p}(x_a, k_{\perp a}) \sin(\dots) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \Delta \hat{\sigma}^{ab \rightarrow cd} \otimes \Delta^N D_{h/c^\uparrow}(z, p_{\perp c})$$

Physical parton-model inspired interpretation

Leading-Twist TMDs

8 TMDs with different polarization direction of nucleons and quarks

		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x, k_T^2)$ 		$h_1^\perp(x, k_T^2)$  <i>Boer-Mulders</i>
	L		$g_1(x, k_T^2)$  <i>Helicity</i>	$h_{1L}^\perp(x, k_T^2)$  <i>Long-Transversity</i>
	T	$f_1^\perp(x, k_T^2)$  <i>Sivers</i>	$g_{1T}(x, k_T^2)$  <i>Trans-Helicity</i>	$h_1(x, k_T^2)$  <i>Transversity</i> $h_{1T}^\perp(x, k_T^2)$  <i>Pretzelosity</i>

Leading twist gluon TMD PDFs

GLUONS	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

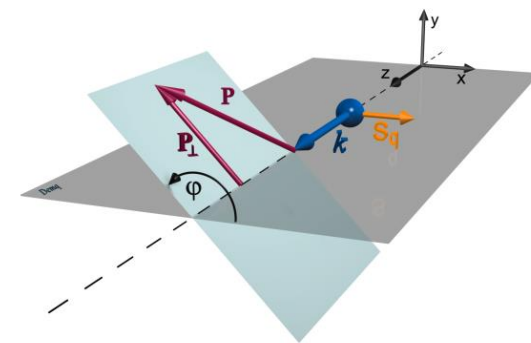
Beyond leading twist, a plethora of additional distributions
 Lorentz invariance, equation of motions give additional relations

Leading twist TMD FFs

Unpolarized or spin zero hadrons: unpolarized and Collins TMDs

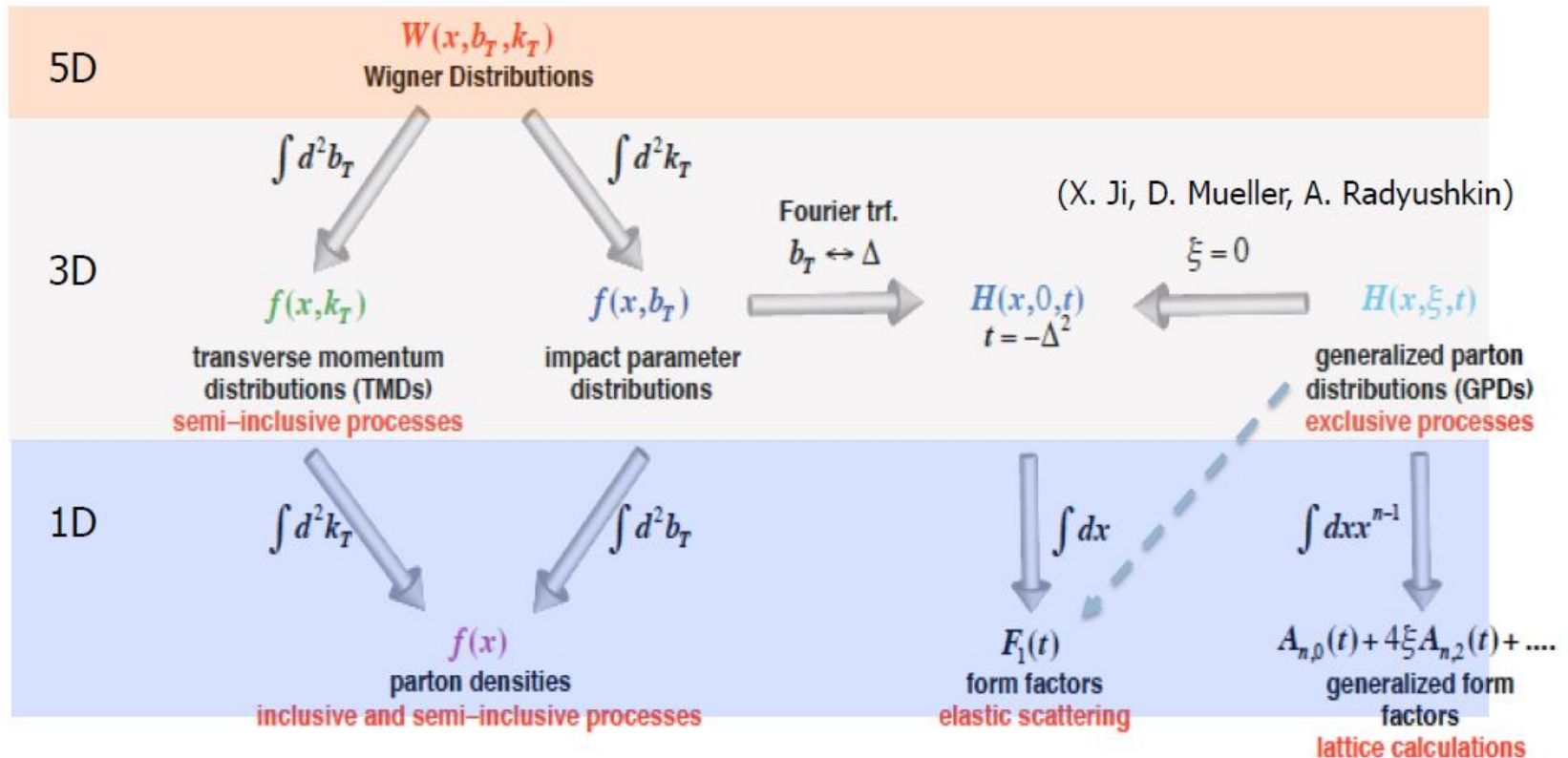
Spin ½ hadrons: similar to PDFs but reversing parton/hadron role

Polarizing FF: Analogous of Sivers PDF, relevant for Λ transv. pol.



A general scheme for TMDs, GPDs, FFs, 3D structure of the nucleon, hadron tomography

Wigner distributions (Belitsky, Ji, Yuan) (or GTMDs)



For GPDs, 3D structure and exclusive processes see next talks by S. Niccolai and A. Sandacz



A full TMD approach – (color) gauge invariance, Wilson lines, factorization

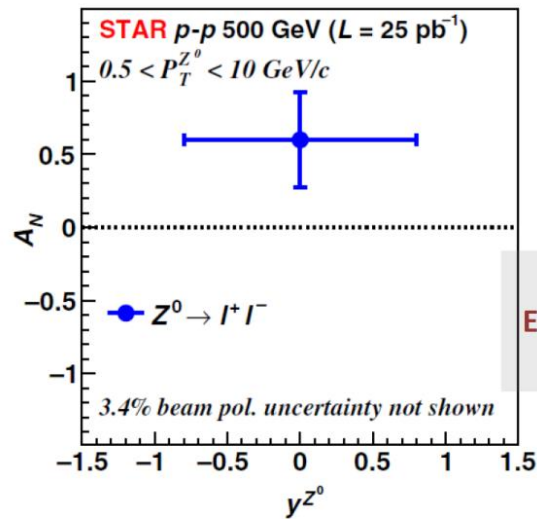
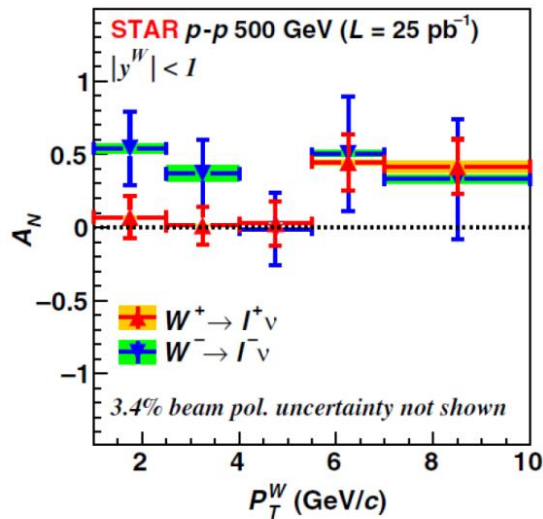
- Gauge invariance requires the introduction of proper gauge links in the hadronic correlators (both in the distribution and in the fragmentation sector) [See talk by G.A. Chirilli tomorrow]
- Correspond to initial (ISI) and final (FSI) state interactions between active partons and the hadron remnants [see e.g. Brodsky et al PLB 530 99 (2002)] in hard scattering processes
- Including transverse motion, these (staple-like) Gauge links cannot be reduced to the identity by a specific gauge choice, leading to process-dependent contributions
- In the proper TMD regime , $\lambda_{QCD} \sim q_T \ll Q$, these contributions are calculable (e.g. using eikonal approximation) and can be absorbed into modified hard scattering terms leading to process dependence - modified universality of TMDs
- Most important phenomenological predictions:
 - Change of sign of the Sivers distribution function when going from SIDIS to Drell-Yan processes (also mentioned as modified universality; same for Boer-Mulders function)
 - In the fragmentation sector, the Collins asymmetry is predicted to be universal
 - Proliferation of TMDs [e.g. Two independent gluon Sivers PDFs with different properties]
- TMD factorization theorems proved, in the $\lambda_{QCD} \sim q_T \ll Q$ regime, for SIDIS, Drell-Yan, e^+e^- collisions, two almost back-to-back jets/hadrons in pp collisions [??]
 - NOT for inclusive single hadron production in pp collisions



TMD evolution with energy scale

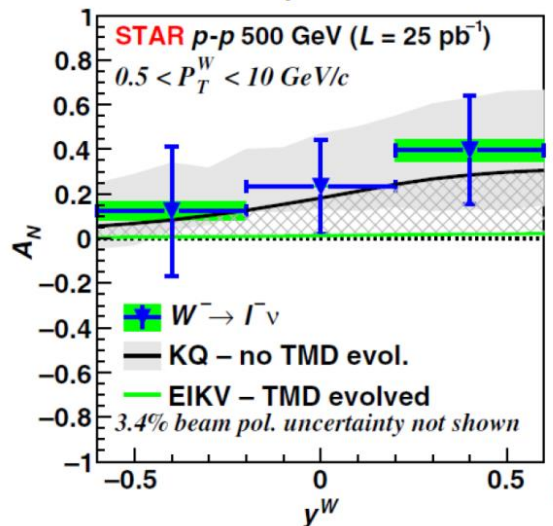
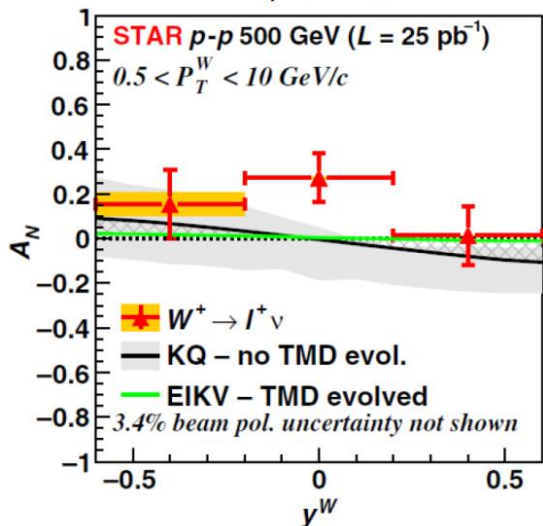
- Collins Soper Stermann (CSS) evolution scheme: b_T impact parameter space; Fourier transforming back to transverse momentum space needs integration in the full b_T range up to non-perturbative regime: needs prescription (b_* , freeze-out parameter)
- Different ways of implement non-perturbative input can lead to sizably different evolution behaviour in the large Q regime
- Several SSAs show at present little evolution effects (cancellations in the ratio)
- Study of TMD evolution on unpolarized cross sections for processes sensible to TMD effects: q_T spectrum in Drell-Yan processes, multiplicities in SIDIS and e^+e^- collisions (for FFs)
- Alternative approaches:
 - Soft Collinear Effective Theory (Scimemi, Idilbi et al)
 - Evolution within a NLO parton branching method with angular ordering for unpolarized TMD PDFs [F. Hautmann et al 1804.11152]

$pp^\uparrow \rightarrow W^\pm, Z + X$ and the change of sign of the Sivers function



STAR PRL 116 (2016)

Kang, Qiu PRL 103 (2009), PRD 81(2010)
 Echevarria, Idilbi, Kang, Vitev, PRD 89 (2014)
 Huang, Kang, Vitev, Xing, PRD 93 (2016)



Solid grey band:
 Uncertainty on sea quark Sivers

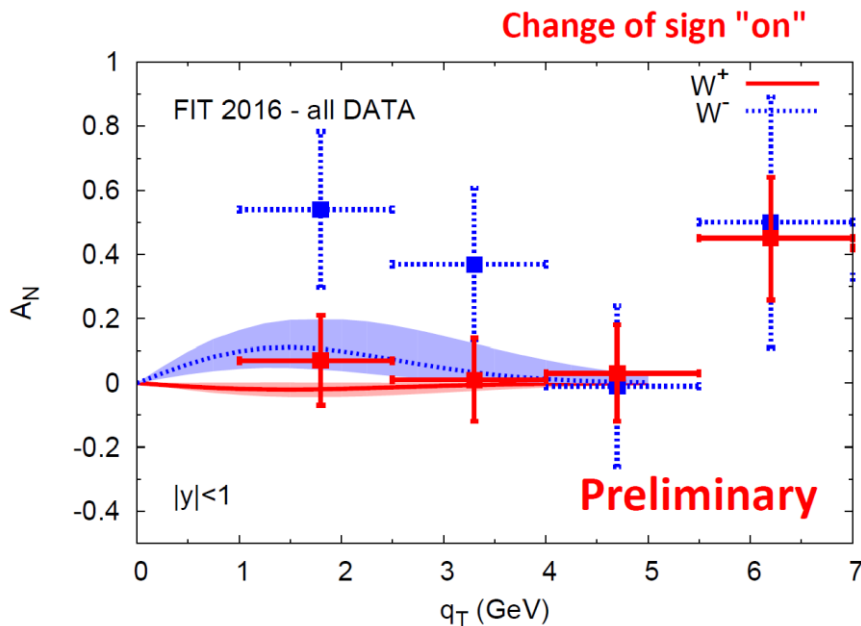
Crosshatched band:
 Theoretical uncertainty
 due to TMD evolution



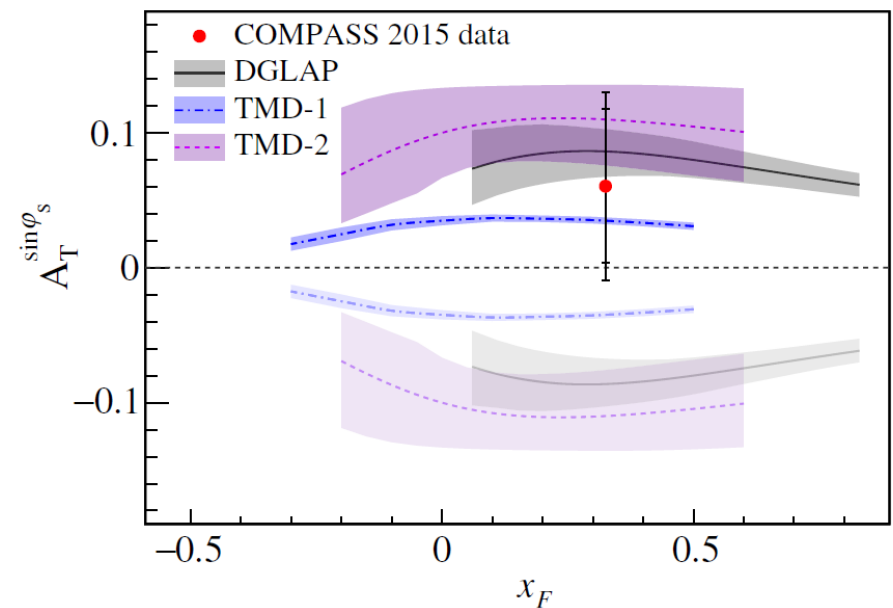
However:

Large exp. errors (low statistics), role of TMD evolution unclear, problems with q_T spectrum in the TMD regime

One data point also available from COMPASS for Siverts SSA in $\pi^- p^\uparrow \rightarrow \mu^+ \mu^- + X$ Drell-Yan process (hard scale comparable to that of recent SIDIS measurements)

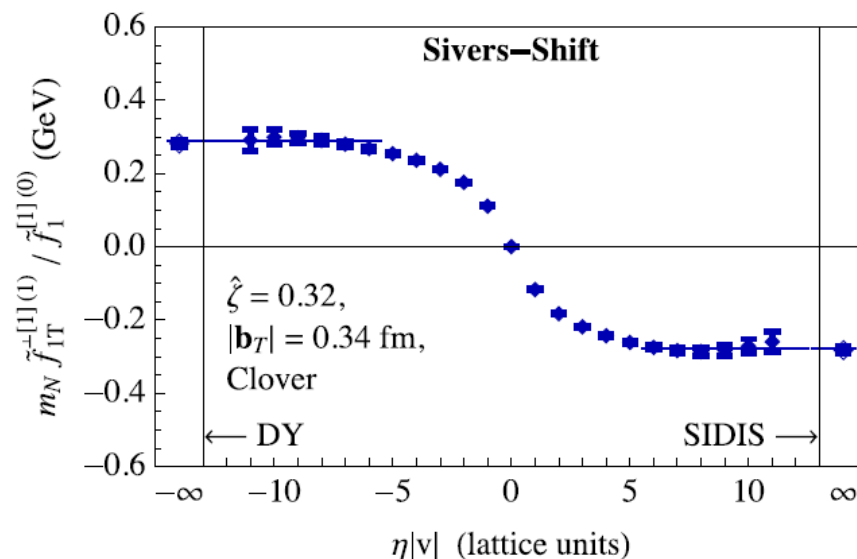


M. Anselmino et al, JHEP 04 (2017) 046

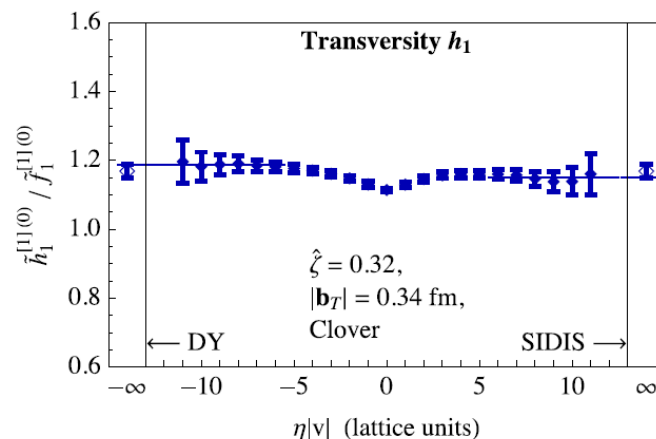
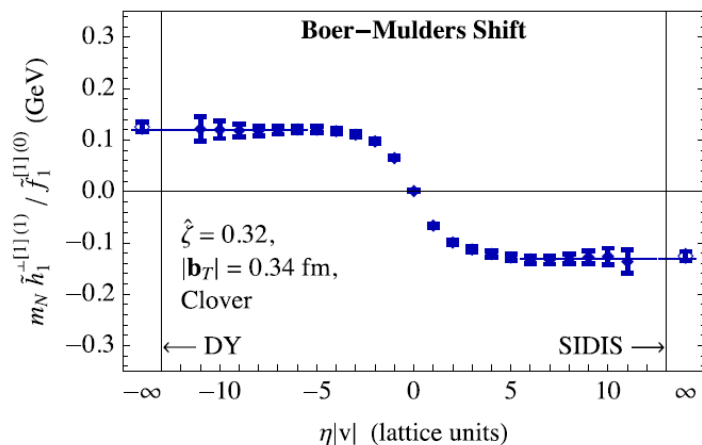


COMPASS, PRL 119, 112002 (2017)

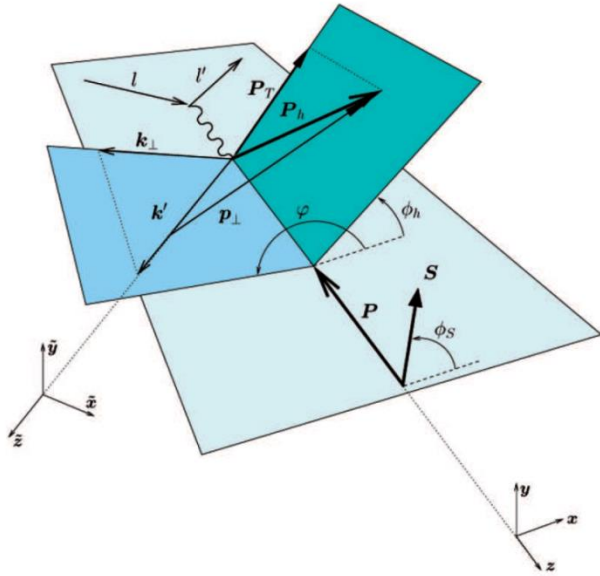
Sign change of Sivers function: a study from lattice QCD



B. Yoon et al
PRD96 094508 (2017)

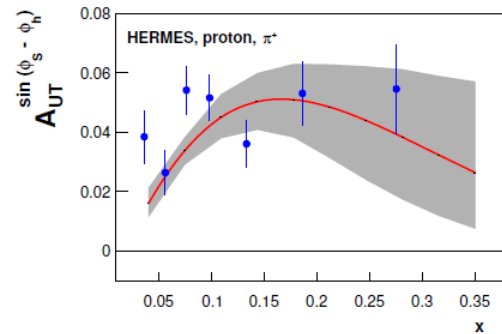


TMD Phenomenology: Semi-inclusive DIS (HERMES, Compass, Jlab)

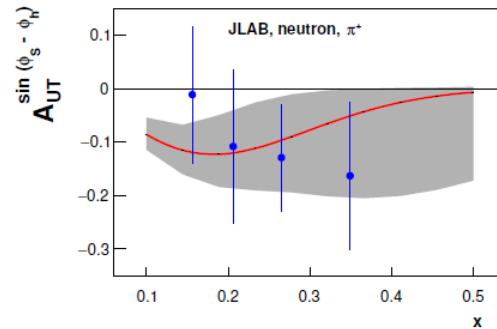


$$\begin{aligned}
 \frac{d\sigma^{\ell(S_e)+p(S)\rightarrow\ell'hX}}{dx_B dQ^2 dz_h d^2\mathbf{P}_T d\phi_S} = & \frac{2\alpha^2}{Q^4} \left\{ \frac{1 + (1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y} \cos\phi_h F_{UU}^{\cos\phi_h} + (1-y) \cos 2\phi_h F_{UU}^{\cos 2\phi_h} \right. \\
 & + S_L \left[(1-y) \sin 2\phi_h F_{UL}^{\sin 2\phi_h} + (2-y)\sqrt{1-y} \sin\phi_h F_{UL}^{\sin\phi_h} \right] \\
 & + S_L P_z^\ell \left[\frac{1 - (1-y)^2}{2} F_{LL} + y\sqrt{1-y} \cos\phi_h F_{LL}^{\cos\phi_h} \right] \\
 & + S_T \left[\frac{1 + (1-y)^2}{2} \sin(\phi_h - \phi_S) F_{UT}^{\sin(\phi_h - \phi_S)} + (1-y) (\sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} \right. \\
 & + \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)}) + (2-y)\sqrt{1-y} (\sin\phi_S F_{UT}^{\sin\phi_S} + \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)}) \left. \right] \\
 & + S_T P_z^\ell \left[\frac{1 - (1-y)^2}{2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + y\sqrt{1-y} (\cos\phi_S F_{LT}^{\cos\phi_S} \right. \\
 & + \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)}) \left. \right] \left. \right\}. \tag{79}
 \end{aligned}$$

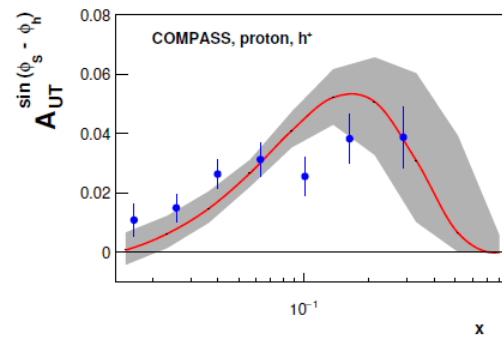
TMD Phenomenology in SIDIS: an example, the quark Sivers function



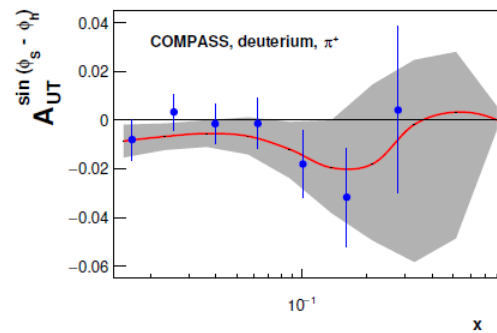
(a)



(b)

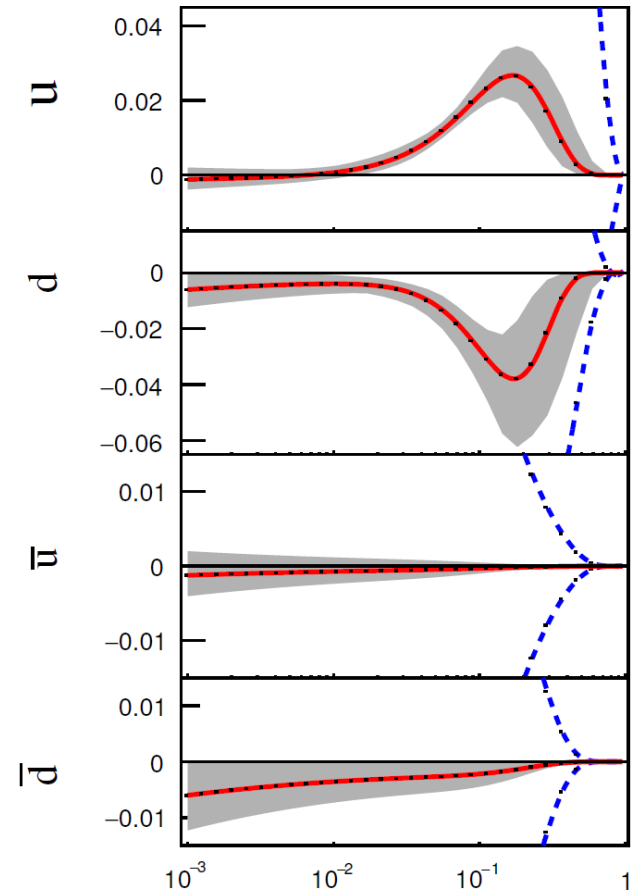


(c)



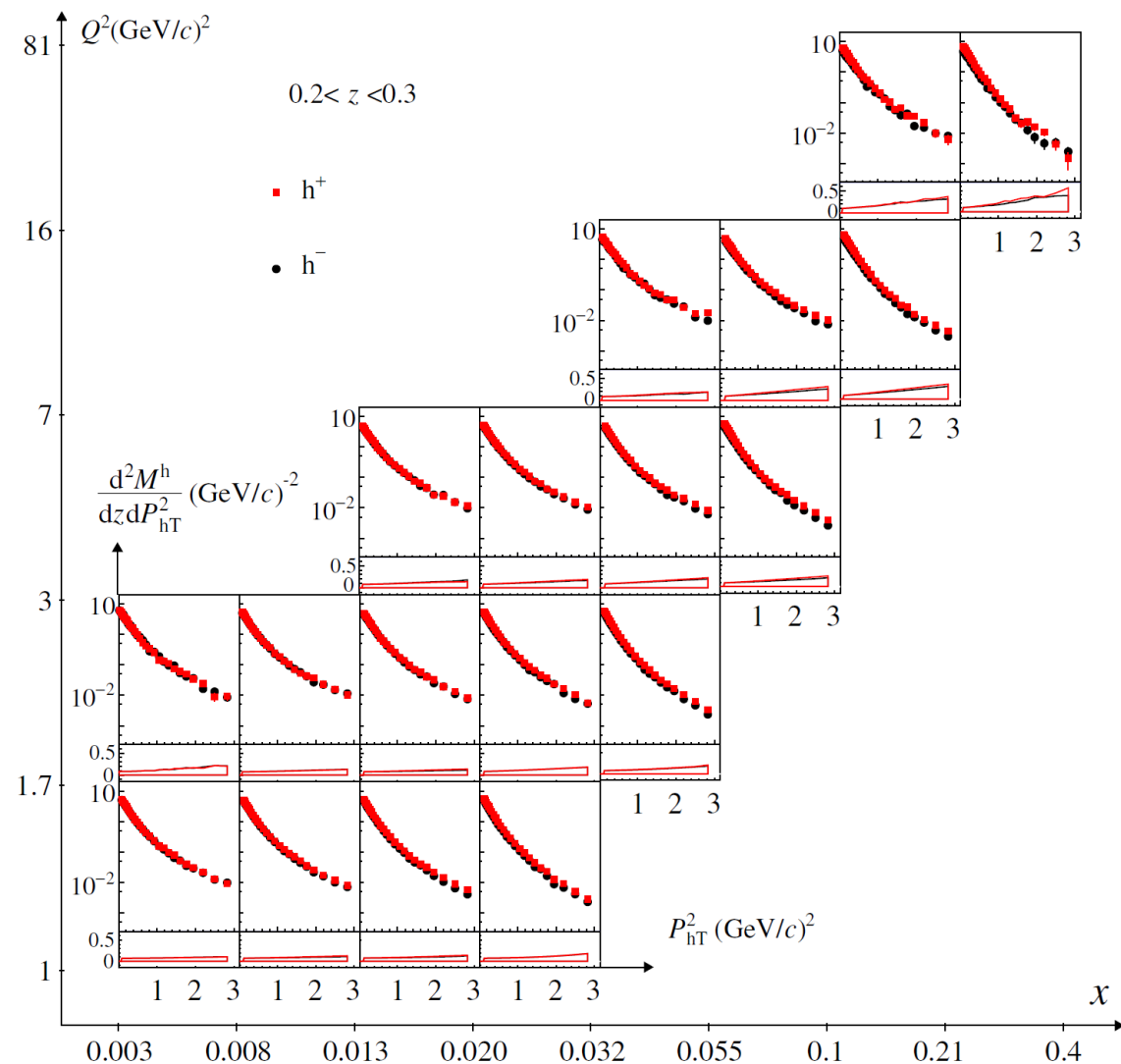
(d)

$$x\Delta^N f^{(1)}(x)$$

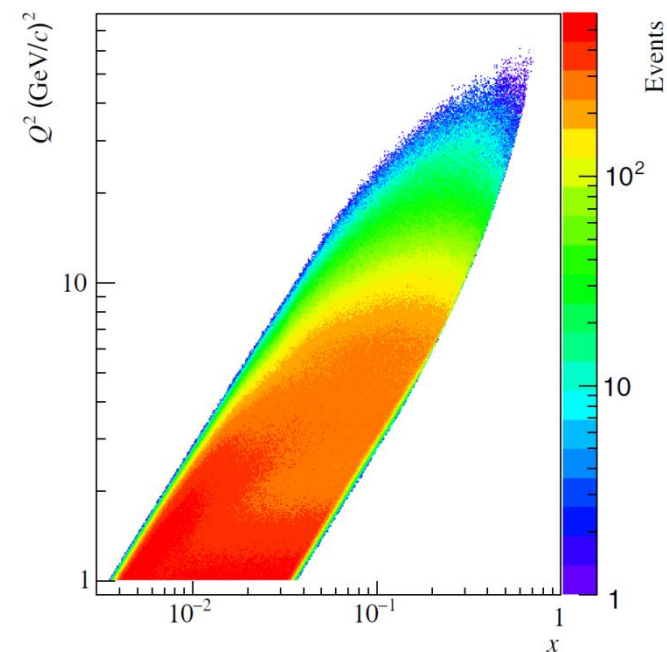


M. Anselmino et al, JHEP 04 (2017) 046

Phenomenology of unpolarized TMDs (and their evolution) from SIDIS multiplicities

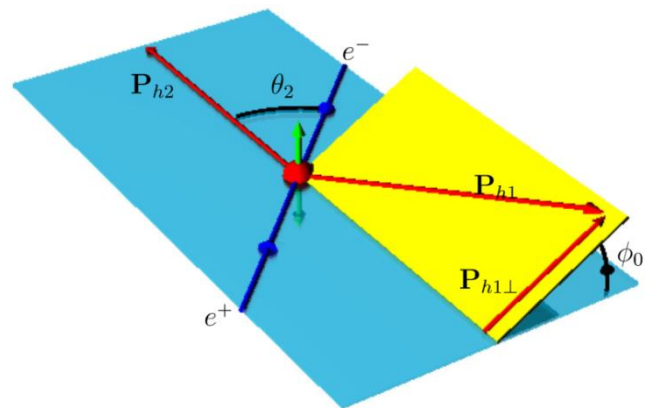
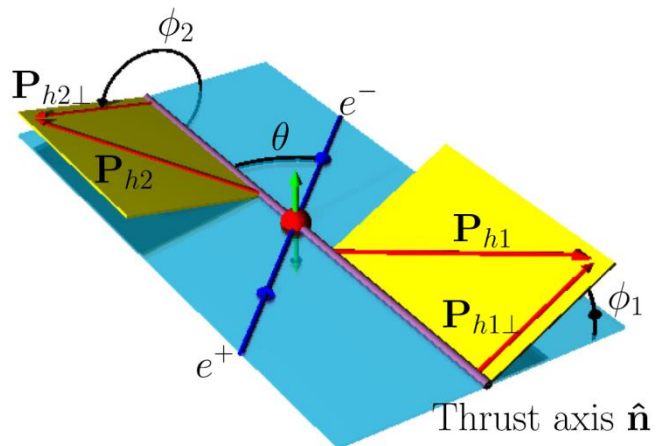


COMPASS PRD 97 032006 (2018)



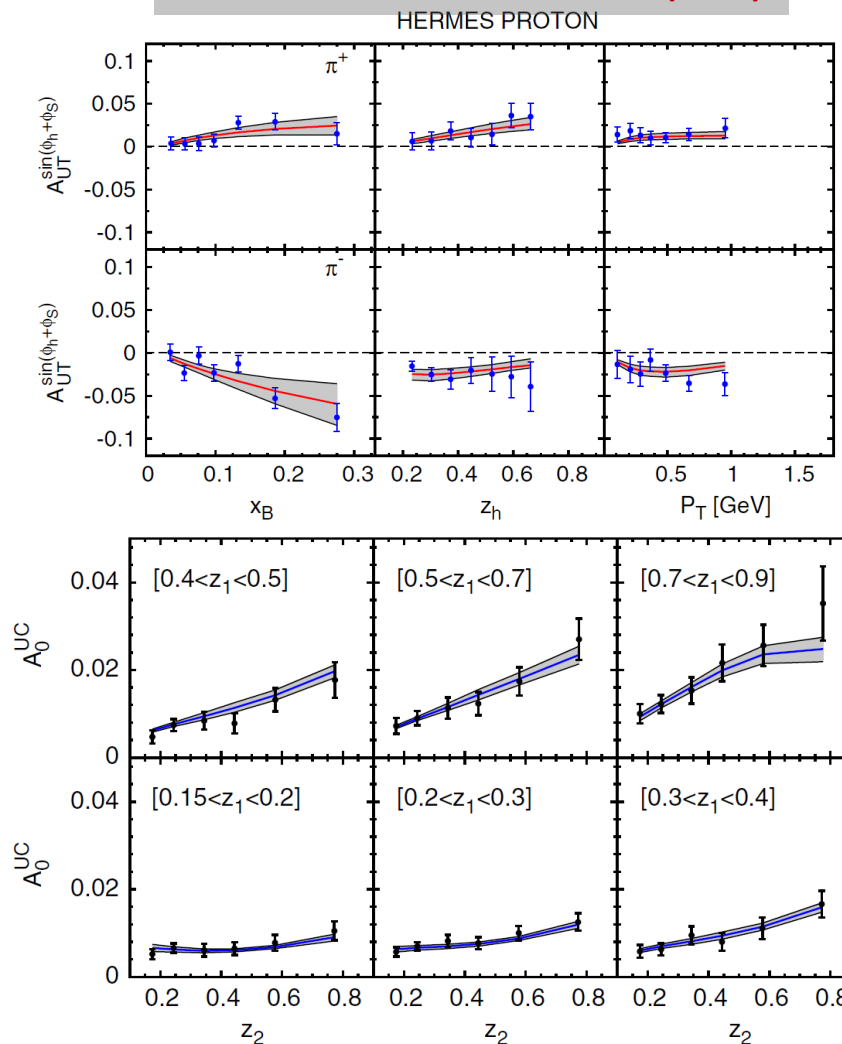
See also HERMES multiplicities
for charged pions and kaons
PRD 87 074029 (2013)

Phenomenology of TMD transversity and Collins FF: SIDIS and e^+e^- collisions

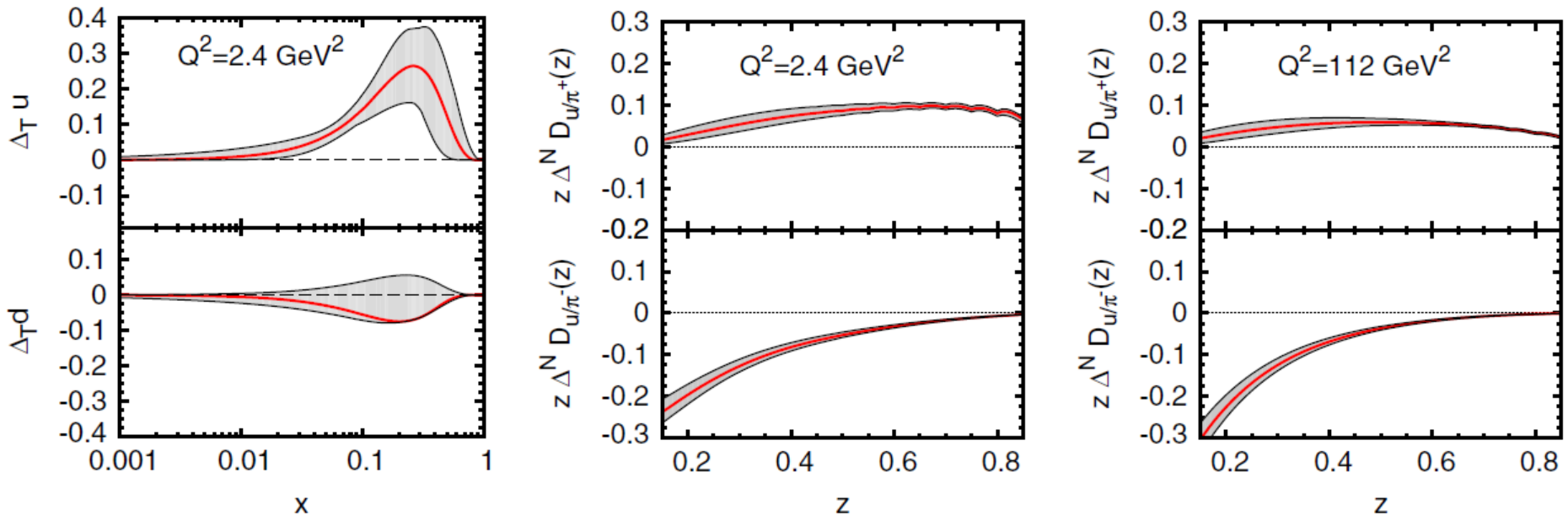


Belle, PRL 96 232002 (2006) – PRD 86 039905 (2012)
BaBar, PRD 90 052003 (2014) – PRD 92 111101 (2015)

Anselmino et al PRD 92 114023 (2015)



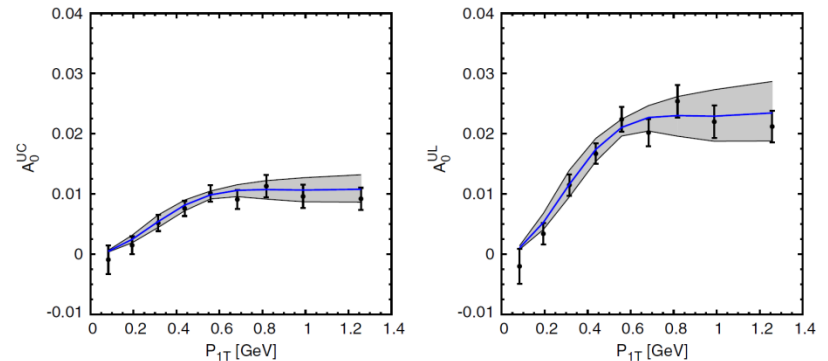
Phenomenology of TMD transversity and Collins FF: SIDIS and e^+e^- collisions



Anselmino et al PRD 92 114023 (2015)

The first ever phenomenological extraction of u, d quark transversity performed using this method in 2007

Anselmino et al PRD 75 054032 (2007)



Phenomenology of TMDs: Drell-Yan processes

$$\begin{aligned}
 \frac{d\sigma}{d^4q d\Omega} = & \frac{\alpha_{\text{em}}^2}{Fq^2} \{ ((1 + \cos^2\theta)F_{UU}^1 + (1 - \cos^2\theta)F_{UU}^2 + \sin 2\theta \cos\phi F_{UU}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{UU}^{\cos 2\phi}) \\
 & + S_{aL}(\sin 2\theta \sin\phi F_{LU}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{LU}^{\sin 2\phi}) + S_{bL}(\sin 2\theta \sin\phi F_{UL}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{UL}^{\sin 2\phi}) \\
 & + |\vec{S}_{aT}|[\sin\phi_a((1 + \cos^2\theta)F_{TU}^1 + (1 - \cos^2\theta)F_{TU}^2 + \sin 2\theta \cos\phi F_{TU}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TU}^{\cos 2\phi}) \\
 & + \cos\phi_a(\sin 2\theta \sin\phi F_{TU}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TU}^{\sin 2\phi})] + |\vec{S}_{bT}|[\sin\phi_b((1 + \cos^2\theta)F_{UT}^1 + (1 - \cos^2\theta)F_{UT}^2 \\
 & + \sin 2\theta \cos\phi F_{UT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{UT}^{\cos 2\phi}) + \cos\phi_b(\sin 2\theta \sin\phi F_{UT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{UT}^{\sin 2\phi})] \\
 & + S_{aL}S_{bL}((1 + \cos^2\theta)F_{LL}^1 + (1 - \cos^2\theta)F_{LL}^2 + \sin 2\theta \cos\phi F_{LL}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{LL}^{\cos 2\phi}) \\
 & + S_{aL}|\vec{S}_{bT}|[\cos\phi_b((1 + \cos^2\theta)F_{LT}^1 + (1 - \cos^2\theta)F_{LT}^2 + \sin 2\theta \cos\phi F_{LT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{LT}^{\cos 2\phi}) \\
 & + \sin\phi_b(\sin 2\theta \sin\phi F_{LT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{LT}^{\sin 2\phi})] + |\vec{S}_{aT}|S_{bL}[\cos\phi_a((1 + \cos^2\theta)F_{TL}^1 + (1 - \cos^2\theta)F_{TL}^2 \\
 & + \sin 2\theta \cos\phi F_{TL}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TL}^{\cos 2\phi}) + \sin\phi_a(\sin 2\theta \sin\phi F_{TL}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TL}^{\sin 2\phi})] \\
 & + |\vec{S}_{aT}||\vec{S}_{bT}|[\cos(\phi_a + \phi_b)((1 + \cos^2\theta)F_{TT}^1 + (1 - \cos^2\theta)F_{TT}^2 + \sin 2\theta \cos\phi F_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi F_{TT}^{\cos 2\phi}) \\
 & + \cos(\phi_a - \phi_b)((1 + \cos^2\theta)\bar{F}_{TT}^1 + (1 - \cos^2\theta)\bar{F}_{TT}^2 + \sin 2\theta \cos\phi \bar{F}_{TT}^{\cos\phi} + \sin^2\theta \cos 2\phi \bar{F}_{TT}^{\cos 2\phi}) \\
 & + \sin(\phi_a + \phi_b)(\sin 2\theta \sin\phi F_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi F_{TT}^{\sin 2\phi}) \\
 & + \sin(\phi_a - \phi_b)(\sin 2\theta \sin\phi \bar{F}_{TT}^{\sin\phi} + \sin^2\theta \sin 2\phi \bar{F}_{TT}^{\sin 2\phi}) \}.
 \end{aligned}$$

Angular distribution, unpolarized case: Lam-Tung relation: $\lambda + 2\nu = 1$

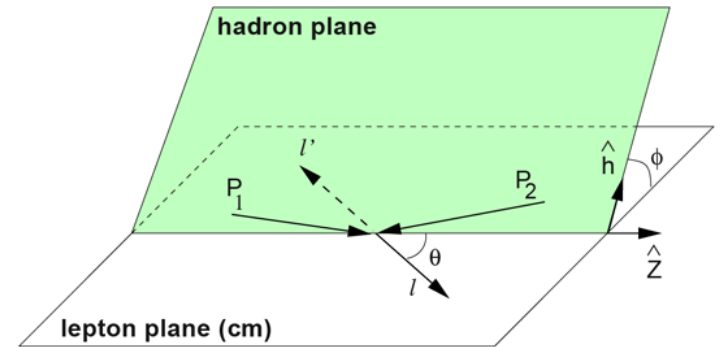
$$\frac{dN}{d\Omega} \equiv \frac{d\sigma}{d^4q d\Omega} \bigg/ \frac{d\sigma}{d^4q}$$

$$= \frac{3}{4\pi} \frac{1}{\lambda + 3} \left(1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right).$$

$$\lambda = \frac{F_{UU}^1 - F_{UU}^2}{F_{UU}^1 + F_{UU}^2},$$

$$\mu = \frac{F_{UU}^{\cos \phi}}{F_{UU}^1 + F_{UU}^2},$$

$$\nu = \frac{2F_{UU}^{\cos 2\phi}}{F_{UU}^1 + F_{UU}^2}.$$

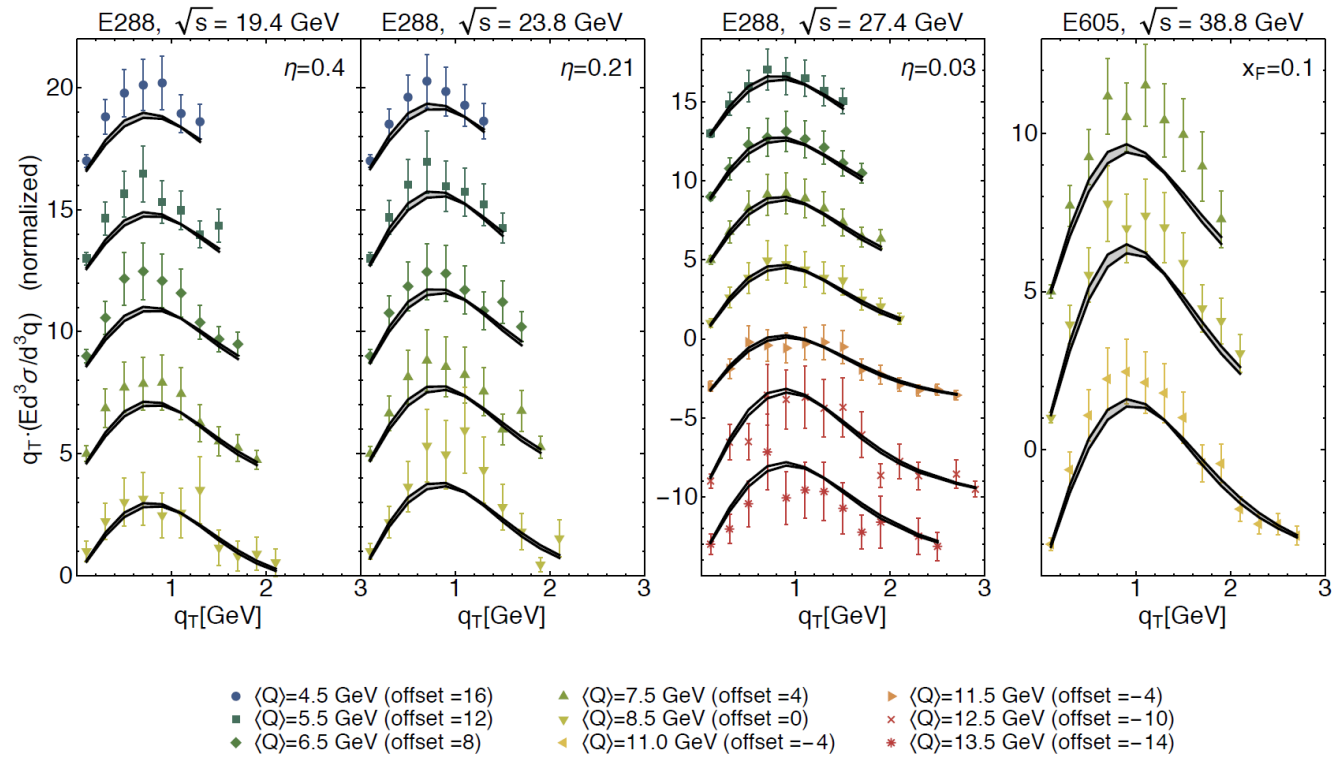


Boer-Mulders TMD

Phenomenological tests of Lam-Tung relation:

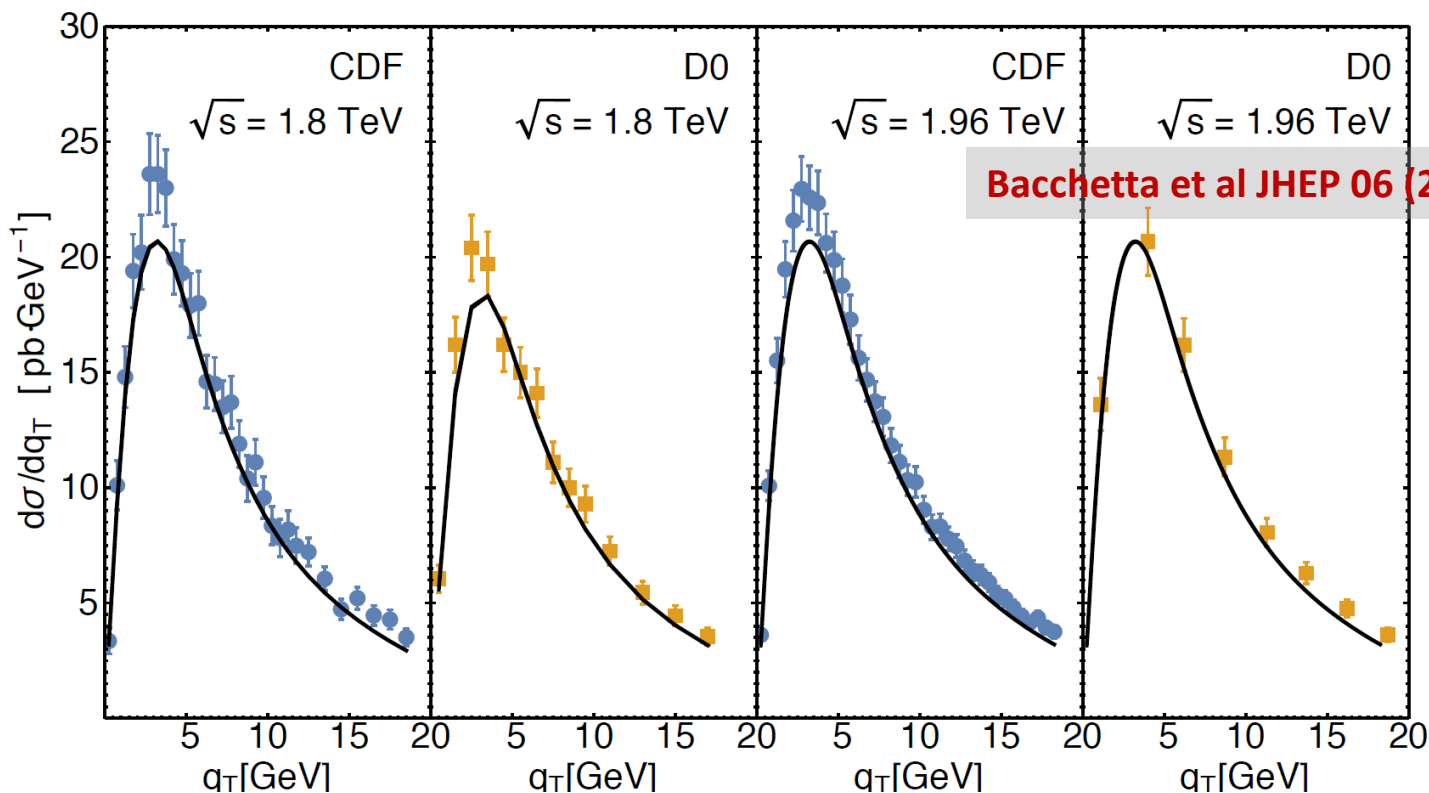
- Fixed target experiments, NA10 & E615, at low invariant dilepton mass and low q_T hint for sizable violations (TMD Boer-Mulders function may account for this)
- CDF results in the Z-boson mass region show agreement with LT relation
- CMS results at 8 TeV for Z boson production show violations of the LT relation
- Recent calculations by W. Vogelsang et al in NLO collinear pQCD in agreement with collider results and fixed target data at not too low q_T (TMD regime) - Vogelsang et al PRD93 114013 (2016)

Drell-Yan, unpolarized case: low q_T spectrum



Bacchetta et al JHEP 06 (2017) 081

Drell-Yan, unpolarized case: low q_T spectrum



Bacchetta et al JHEP 06 (2017) 081

See also Scimemi & Vladimirov
EPJC (2018) 78:89
Up to NNLL/NNLO



Outlook and perspectives

- After pioneering years, a new precision era for TMDs is starting
- Multiplicities in SIDIS and e^+e^- collisions will help in fixing unpolarized TMD PDFs and FFs and improving extraction of Sivers and Boer-Mulders PDFs, Collins FFs, etc.
- TMD evolution with energy scale needs further study and more experimental input
- Improved RHIC and Compass data on Sivers Drell-Yan asymmetries will allow to test the QCD sign-change prediction between SIDIS and DY for the Sivers function
- JLab12 crucial for exploring large Bjorken x regime ($\geq 0.3 - 0.4$) for TMD PDFs
- AFTER@LHC, (un)polarized fixed target at LHCb, (ALICE?)
- Future Electron Ion Collider: high luminosity, low- x regime, plenty of opportunities
- Gluon Sivers Function (almost unknown) probed in SSAs for quarkonium production in polarized pp collisions at RHIC and LHC
- Lambda hyperon transverse polarization in unpolarized pp collisions at RHIC and LHC: polarizing fragmentation function [also in SIDIS and e^+e^- collisions at BaBar and Belle]
- BaBar, Belle, BESIII: further information on multiplicities and unpolarized and Collins FFs



**Thanks a lot
for your attention!**