TMDs: A short Theory Overview
Francesco Murgia – INFN Sezione di Cagliari

QCD@Work
International Workshop on QCD Theory and Experiment
24-28 June 2018 – Matera, Italy
Summary

- Historical motivations
- Phenomenological interest and puzzles
- Basic definition and properties of TMD PDFs and FFs
- Factorization, universality and process dependence
- TMD evolution, multiplicities
- Phenomenology in SIDIS and $e^+e^-$ collisions
- TMDs in $pp$ collisions, Drell-Yan processes (also in $\pi p$ at COMPASS)
- Outlook and future prospects
Historical motivations

- Phenomenological implications of intrinsic (transverse) motion of partons in high energy hadronic collisions studied already at the start of parton model (Feynman and collab.), e.g. in the low $q_T$ spectrum for Drell-Yan processes

- Formal and conceptual aspects studied in QCD, e.g. Ellis, Furmanski, Petronzio, 1983:
  - Lorentz invariance for on-shell partons implies a very narrow $k_\perp$ distribution and a non Gaussian shape

- Recall the (very narrow) Lorentzian widths of spectral lines where pressure, Doppler broadening modify the shape and increase the width of the lines
  - (In our case: higher-twist effects, off-shellness etc, not well under control)

- Only unpolarized processes: no spin effects taken into account until the formulation of what is now called the Transverse Momentum Dependent (TMD) approach [for single hadron production in $pp$ collisions also called Generalized Parton Model (GPM)]
Phenomenological problems and puzzles

- Huge transverse $\Lambda, \bar{\Lambda}$ polarization in unpolarized $pp, pA$ collisions (1978-80)
  - Low c.m. energy, $p_T \leq 3$ GeV, no unpolarized cross section measurements
  - New measurements feasible at RHIC, LHC, SIDIS [HERMES PRD90 072007 (2014)], $e^+e^-$ collisions [Belle 1611.06648] [talk by A. Martini on Belle II status?]

- Big transverse single spin asymmetries (SSAs) in $p^\uparrow p \rightarrow \pi X$ processes at large $x_F$ and moderately large $p_T$ (E704, 1990 → today, RHIC). [LT collinear pQCD \(\sim 0\) in this regime]

- Violation of QCD helicity selection rules in quarkonium exclusive decays (LC WFs)

- Dilepton c.m. angular distributions in Drell-Yan processes

- Proton spin puzzle: $q, \bar{q}$ spin contributions apparently account for \(~30\%) of the proton spin. The reminder due to gluon spin ($\Delta g$) and parton Orbital Angular momentum (OAM) contributions. [see next talks on GPDs by S. Niccolai and A. Sandacz ]
  - Complications: spin decomposition: gauge invariance, uniqueness, measurability [see e.g. Leader, Lorcé, Phys. Rep.  541 (2014)]
  - Lattice QCD calculations seem to support huge OAM parton contributions


Proton spin: information on gluon $\Delta g$

Measurements of double longitudinal asymmetry $A_{LL}$ in gluon dominated processes at RHIC, $pp + jet(s) + X$, $pp \rightarrow \pi^0 + X$ at mid- and forward rapidity and large $p_T$

SIDIS, $\ell^\pm p(d) \rightarrow \ell^\pm D + X$

Using NLO QCD global fits to PDFs (CTEQ, DSSV, NNPDF, JAM, etc) one gets information on $\Delta g$

Main uncertainties due to low $x$ contributions (EIC role crucial here)

NNPDF and DSSV estimates are well compatible

$$\int_{0.01}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = +0.23 \pm 0.15$$

$$\int_{0.01}^{0.2} dx \Delta g(x, Q^2 = 10 \text{ GeV}^2) = +0.32 \pm 0.13$$

E. Nocera, NNPDF Collaboration
Proton spin and quark OAM on the lattice

C. Alexandrou et al. PRL 119, 142002 (2017)

\( J_N \)

<table>
<thead>
<tr>
<th>( J )</th>
<th>( L )</th>
<th>( \langle x \rangle )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>0.415(13)(2) 0.308(3) 0.046(2)</td>
<td>0.453(57)(4)</td>
</tr>
<tr>
<td>( d )</td>
<td>-0.193(8)(3) 0.247(3) 0.067(1)</td>
<td>0.259(57)(4)</td>
</tr>
<tr>
<td>( s )</td>
<td>-0.021(5)(1) 0.133(11)(1)</td>
<td>0.267(22)(27)</td>
</tr>
<tr>
<td>( g )</td>
<td>( \cdots ) 0.133(11)(1) ( \cdots )</td>
<td>0.267(22)(27)</td>
</tr>
<tr>
<td>Total</td>
<td>0.201(17)(5) 0.541(62)(49)</td>
<td>0.207(64)(45) 1.07(12)(10)</td>
</tr>
</tbody>
</table>

\( \chi \)QCD Collaboration
PRD 91, 014505 (2015)
Transverse SSAs in $p^+ p \to \pi + X$ processes

$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \frac{d\Delta\sigma}{2d\sigma}$$
Sivers effect in $p^\uparrow p \rightarrow \pi + X$ processes

\[ d\sigma_{pp\rightarrow h X} \sim \sum_{a,b,c,d} f_{a/p}(x_a, k_{\perp a}) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab\rightarrow cd} \otimes D_{h/c}(z, p_{\perp c}) \]

\[ d\sigma_{p^\uparrow p\rightarrow h X} \sim \sum_{a,b,c,d} [f_{a/p}(x_a, k_{\perp a}) + \frac{1}{2} \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) \sin(\phi_a - \phi_S)] \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab\rightarrow cd} \otimes D_{h/c}(z, p_{\perp c}) \]

\[ [d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Sivers}} = d\Delta\sigma_{\text{Sivers}} = \sum \Delta^N f_{a/p^\uparrow}(x_a, k_{\perp a}) \sin(\phi_a - \phi_S) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \hat{\sigma}^{ab\rightarrow cd} \otimes D_{h/c}(z, p_{\perp c}) \]

Another possible contribution to the SSA in $p^\uparrow p \rightarrow \pi + X$ is the Collins effect

\[ [d\sigma^\uparrow - d\sigma^\downarrow]_{\text{Collins}} = d\Delta\sigma_{\text{Collins}} = \sum \Delta_T f_{a/p}(x_a, k_{\perp a}) \sin(...) \otimes f_{b/p}(x_b, k_{\perp b}) \otimes \Delta\hat{\sigma}^{ab\rightarrow cd} \otimes \Delta^N D_{h/c^\uparrow}(z, p_{\perp c}) \]
## Physical parton-model inspired interpretation

### Leading-Twist TMDs

8 TMDs with different polarization direction of nucleons and quarks

<table>
<thead>
<tr>
<th>Nucleon Polarization</th>
<th>Quark Polarization</th>
<th>Unpolarized (U)</th>
<th>Longitudinally Polarized (L)</th>
<th>Transversely Polarized (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$f_1(x, k_T^2)$</td>
<td>$g_1(x, k_T^2)$</td>
<td>$h_1(x, k_T^2)$</td>
</tr>
<tr>
<td>U</td>
<td></td>
<td>$h_{1L}(x, k_T^2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>$h_{1T}(x, k_T^2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>$h_{1L}(x, k_T^2)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>$h_{1T}(x, k_T^2)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $f_1(x, k_T^2)$: Sivers
- $g_1(x, k_T^2)$: Helicity
- $h_{1T}(x, k_T^2)$: Transversity
- $h_{1L}(x, k_T^2)$: Long-Transversity
- $h_1(x, k_T^2)$: Boer-Mulders
Leading twist gluon TMD PDFs

<table>
<thead>
<tr>
<th>GLUONS</th>
<th>unpolarized</th>
<th>circular</th>
<th>linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>$f_1^g$</td>
<td></td>
<td>$h_1^{\perp g}$</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>$g_{1L}^g$</td>
<td>$h_{1L}^{\perp g}$</td>
</tr>
<tr>
<td>T</td>
<td>$f_{1T}^{\perp g}$</td>
<td>$g_{1T}^g$</td>
<td>$h_{1T}^{g}, h_{1T}^{\perp g}$</td>
</tr>
</tbody>
</table>

Beyond leading twist, a plethora of additional distributions
Lorentz invariance, equation of motions give additional relations

**Leading twist TMD FFs**
Unpolarized or spin zero hadrons: unpolarized and Collins TMDs
Spin ½ hadrons: similar to PDFs but reversing parton/hadron role
Polarizing FF: Analogous of Sivers PDF, relevant for $\Lambda$ transv. pol.
A general scheme for TMDs, GPDs, FFs, 3D structure of the nucleon, hadron tomography

Wigner distributions (Belitsky, Ji, Yuan) (or GTMDs)

For GPDs, 3D structure and exclusive processes see next talks by S. Niccolai and A. Sandacz
A full TMD approach – (color) gauge invariance, Wilson lines, factorization

- Gauge invariance requires the introduction of proper gauge links in the hadronic correlators (both in the distribution and in the fragmentation sector) [See talk by G.A. Chirilli tomorrow]

- Correspond to initial (ISI) and final (FSI) state interactions between active partons and the hadron remnants [see e.g. Brodsky et al PLB 530 99 (2002)] in hard scattering processes

- Including transverse motion, these (staple-like) Gauge links cannot be reduced to the identity by a specific gauge choice, leading to process-dependent contributions

- In the proper TMD regime, $\lambda_{QCD} \sim q_T \ll Q$, these contributions are calculable (e.g. using eikonal approximation) and can be absorbed into modified hard scattering terms leading to process dependence - modified universality of TMDs

- Most important phenomenological predictions:
  - Change of sign of the Sivers distribution function when going from SIDIS to Drell-Yan processes (also mentioned as modified universality; same for Boer-Mulders function)
  - In the fragmentation sector, the Collins asymmetry is predicted to be universal
  - Proliferation of TMDs [e.g. Two independent gluon Sivers PDFs with different properties]

- TMD factorization theorems proved, in the $\lambda_{QCD} \sim q_T \ll Q$ regime, for SIDIS, Drell-Yan, $e^+e^-$ collisions, two almost back-to-back jets/hadrons in $pp$ collisions [??]
  - NOT for inclusive single hadron production in $pp$ collisions
TMD evolution with energy scale

- Collins Soper Sterman (CSS) evolution scheme: \( b_T \) impact parameter space; Fourier transforming back to transverse momentum space needs integration in the full \( b_T \) range up to non-perturbative regime: needs prescription \( (b_*, \text{freeze-out parameter}) \)

- Different ways of implement non-perturbative input can lead to sizably different evolution behaviour in the large Q regime

- Several SSAs show at present little evolution effects (cancellations in the ratio)

- Study of TMD evolution on unpolarized cross sections for processes sensible to TMD effects: \( q_T \) spectrum in Drell-Yan processes, multiplicities in SIDIS and \( e^+e^- \) collisions (for FFs)

- Alternative approaches:
  - Soft Collinear Effective Theory (Scimemi, Idilbi et al)
  - Evolution within a NLO parton branching method with angular ordering for unpolarized TMD PDFs [ F. Hautmann et al 1804.11152 ]
pp$^+$ → $W^\pm$, $Z + X$ and the change of sign of the Sivers function

Star p-p 500 GeV (L = 25 pb$^{-1}$)

$|y^W| < 1$

$W^+ \rightarrow l^+\nu$

$W^- \rightarrow l^-\bar{\nu}$

3.4% beam pol. uncertainty not shown

$P^W_T$ (GeV/c)

$A_N$

3.4% beam pol. uncertainty not shown

Star p-p 500 GeV (L = 25 pb$^{-1}$)

$0.5 < P^Z_T < 10$ GeV/c

$Z^0 \rightarrow l^+ l^-$

Kang, Qiu PRL 103 (2009), PRD 81(2010)

Echevarria, Idilbi, Kang, Vitev, PRD 89 (2014)

Huang, Kang, Vitev, Xing, PRD 93 (2016)

Solid grey band:
Uncertainty on sea quark Sivers

Crosshatched band:
Theoretical uncertainty
due to TMD evolution
However:
Large exp. errors (low statistics), role of TMD evolution unclear, problems with $q_T$ spectrum in the TMD regime

One data point also available from COMPASS for Sivers SSA in $\pi^- p^+ \rightarrow \mu^+ \mu^- + X$ Drell-Yan process (hard scale comparable to that of recent SIDIS measurements)

M. Anselmino et al, JHEP 04 (2017) 046

COMPASS, PRL 119, 112002 (2017)
Sign change of Sivers function: a study from lattice QCD

B. Yoon et al
PRD96 094508 (2017)
TMD Phenomenology: Semi-inclusive DIS (HERMES, Compass, Jlab)
TMD Phenomenology in SIDIS: an example, the quark Sivers function

M. Anselmino et al, JHEP 04 (2017) 046
Phenomenology of unpolarized TMDs (and their evolution) from SIDIS multiplicities

See also HERMES multiplicities for charged pions and kaons
PRD 87 074029 (2013)
Phenomenology of TMD transversity and Collins FF: SIDIS and $e^+e^-$ collisions


Anselmino et al PRD 92 114023 (2015)
Phenomenology of TMD transversity and Collins FF: SIDIS and $e^+e^-$ collisions

The first ever phenomenological extraction of $u,d$ quark transversity performed using this method in 2007

Anselmino et al PRD 75 054032 (2007)
Phenomenology of TMDs: Drell-Yan processes

\[
\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha_e^2}{F q^2} \left\{ (1 + \cos^2\theta) F_{UU}^1 + (1 - \cos^2\theta) F_{UU}^2 + \sin 2\theta \cos \phi F_{UU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{UU}^{\cos 2\phi} \right. \\
+ S_{aL}(\sin 2\theta \sin \phi F_{LU}^{\sin \phi} + \sin^2 \theta \sin 2\phi F_{LU}^{\sin 2\phi}) + S_{bL}(\sin 2\theta \sin \phi F_{UL}^{\sin \phi} + \sin^2 \theta \sin 2\phi F_{UL}^{\sin 2\phi}) \\
+ |\tilde{S}_{aT}|^2 [\sin \phi_a ((1 + \cos^2\theta) F_{TU}^1 + (1 - \cos^2\theta) F_{TU}^2 + \sin 2\theta \cos \phi F_{TU}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{TU}^{\cos 2\phi})] + |\tilde{S}_{bT}|^2 [\sin \phi_b ((1 + \cos^2\theta) F_{UT}^1 + (1 - \cos^2\theta) F_{UT}^2 + \sin^2 \theta \cos 2\phi F_{UT}^{\cos 2\phi})] \\
+ S_{aL} S_{bL} ((1 + \cos^2\theta) F_{LL}^1 + (1 - \cos^2\theta) F_{LL}^2 + \sin 2\theta \cos \phi F_{LL}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{LL}^{\cos 2\phi}) \\
+ S_{aL} |\tilde{S}_{bT}| [\sin \phi_b ((1 + \cos^2\theta) F_{LT}^1 + (1 - \cos^2\theta) F_{LT}^2 + \sin 2\theta \cos \phi F_{LT}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{LT}^{\cos 2\phi})] + |\tilde{S}_{aT}| S_{bL} [\sin \phi_a ((1 + \cos^2\theta) F_{TL}^1 + (1 - \cos^2\theta) F_{TL}^2 + \sin^2 \theta \cos 2\phi F_{TL}^{\cos 2\phi})] \\
+ |\tilde{S}_{aT}| |\tilde{S}_{bT}| [\cos (\phi_a + \phi_b) ((1 + \cos^2\theta) F_{TT}^1 + (1 - \cos^2\theta) F_{TT}^2 + \sin 2\theta \cos \phi F_{TT}^{\cos \phi} + \sin^2 \theta \cos 2\phi F_{TT}^{\cos 2\phi})] + \cos (\phi_a - \phi_b) ((1 + \cos^2\theta) \tilde{F}_{TT}^1 + (1 - \cos^2\theta) \tilde{F}_{TT}^2 + \sin 2\theta \cos \phi \tilde{F}_{TT}^{\cos \phi} + \sin^2 \theta \cos 2\phi \tilde{F}_{TT}^{\cos 2\phi}) \\
+ \sin (\phi_a + \phi_b) (\sin 2\theta \sin \phi \tilde{F}_{TT}^{\sin \phi} + \sin^2 \theta \sin 2\phi \tilde{F}_{TT}^{\sin 2\phi}) \\
+ \sin (\phi_a - \phi_b) (\sin 2\theta \sin \phi \tilde{F}_{TT}^{\sin \phi} + \sin^2 \theta \sin 2\phi \tilde{F}_{TT}^{\sin 2\phi})].
\]
Angular distribution, unpolarized case: Lam-Tung relation: \( \lambda + 2\nu = 1 \)

\[
\frac{dN}{d\Omega} = \frac{d\sigma}{d^4q d\Omega} \frac{1}{d^4q} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos \phi \right) + \frac{\nu}{2} \sin^2\theta \cos 2\phi.
\]

\[
\lambda = \frac{F_{UU}^1 - F_{UU}^2}{F_{UU}^1 + F_{UU}^2},
\]

\[
\mu = \frac{F_{UU}^1 \cos \phi}{F_{UU}^1 + F_{UU}^2},
\]

\[
\nu = \frac{2F_{UU}^1 \cos 2\phi}{F_{UU}^1 + F_{UU}^2}.
\]

**Phenomenological tests of Lam-Tung relation:**

- **Fixed target experiments**, NA10 & E615, at low invariant dilepton mass and low \( q_T \) hint for sizable violations (TMD Boer-Mulders function may account for this)

- **CDF results** in the Z-boson mass region show agreement with LT relation

- **CMS results at 8 TeV** for Z boson production show violations of the LT relation

- **Recent calculations** by W. Vogelsang et al in NLO collinear pQCD in agreement with collider results and fixed target data at not too low \( q_T \) (TMD regime) - Vogelsang et al PRD93 114013 (2016)
Drell-Yan, unpolarized case: low $q_T$ spectrum

Bacchetta et al JHEP 06 (2017) 081
Drell-Yan, unpolarized case: low $q_T$ spectrum

See also Scimemi & Vladimirov
EPJC (2018) 78:89
Up to NNLL/NNLO
Outlook and perspectives

- After pioneering years, a new precision era for TMDs is starting

- Multiplicities in SIDIS and $e^+ e^-$ collisions will help in fixing unpolarized TMD PDFs and FFs and improving extraction of Sivers and Boer-Mulders PDFs, Collins FFs, etc.

- TMD evolution with energy scale needs further study and more experimental input

- Improved RHIC and Compass data on Sivers Drell-Yan asymmetries will allow to test the QCD sign-change prediction between SIDIS and DY for the Sivers function

- JLab12 crucial for exploring large Bjorken $x$ regime ($\geq 0.3 - 0.4$) for TMD PDFs

- AFTER@LHC, (un)polarized fixed target at LHCb, (ALICE?)

- Future Electron Ion Collider: high luminosity, low-$x$ regime, plenty of opportunities

- Gluon Sivers Function (almost unknown) probed in SSAs for quarkonium production in polarized $pp$ collisions at RHIC and LHC

- Lambda hyperon transverse polarization in unpolarized $pp$ collisions at RHIC and LHC: polarizing fragmentation function [ also in SIDIS and $e^+ e^-$ collisions at BaBar and Belle ]

- BaBar, Belle, BESIII: further information on multiplicities and unpolarized and Collins FFs
Thanks a lot for your attention!